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Abstract

We give fairly easy conditions under which a multidimensional diffusion with jumps of compound-Poisson type possess several global-stability properties: (exponential) ergodicity, (exponential) β-mixing property, and also boundedness of moments. These are important to statistical inference under long-time asymptotics. The underlying technique used in this article is based on Masuda (2007), but we here utilize an explicit "T-chain", which enables us to include almost arbitrary finite-jump parts under nondegeneracy of the diffusion part without reference to topological continuity of the original transition semigroup.

AMS mathematics subject classification (2000) : 37A25, 60J25, 65C30.

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1 Introduction and statement of results

When attempting statistical inference for a continuous-time stochastic process $X = (X_t)_{t\in[0,T]}$ based on "long-time asymptotics", that is, $T \to \infty$, most often (but not always!) required are a law of large numbers, typically referred to as *ergodicity*. Moreover, in case of higher-order inference a *fast decay of a mixing coefficient* (see Section 1.3) is indispensable; of course, this is also the case for discrete-time time series, see Liebscher (2005) and the references therein. Previously, for general multidimensional diffusions with possibly infinitely many jumps, Masuda (2007), henceforth referred to as [M] (with the corrections to as [M-Corrections]), derived sufficient conditions for such global stabilities. Although the results in [M] and [M-Corrections] are sufficiently general to cover a wide range of diffusions with jumps, the conditions include a kind of topological continuity of the transition semigroup (see [M, Assumption 2] and [M-Corrections, Assumption 2(a)′]), for which one may be forced to consult some advanced results on existence and smoothness of a transition density: this may cause some inconvenience to readers unfamiliar with such results.

The purpose of this article is to provide fairly easy conditions for the above-mentioned global stabilities of X when the diffusion coefficient is non-degenerate and the jump intensity is finite. Our emphasize here is put on ease of verification of the conditions rather than pursuing the greatest generality. The scenario of the proofs we will take here is in parallel with that in [M], except that we will utilize the "T-chain property" of a skeleton chain of X in an explicit way: actually, this enables us to pick out a nice property of the diffusion coefficient with leaving the finite-jump part almost arbitrary.

In the rest of this section we describe our objective and results, part of which are applicable to much more general diffusions with jumps than our main objective (1) below. The proofs are given in Section 2.

1.1 Objective

Let $X = (X_t)_{t\in\mathbb{R}_+}$ be a d-dimensional càdlàg [1] Markov process given by the time-homogeneous Itô's stochastic differential equation

$$dX_t = b(X_t)dt + \sigma(X_t)dw_t + \int_0^t \int \zeta(X_{t-}, z)\mu(dt, dz), \tag{1}$$

where X is defined on some filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{R}_+}, \mathbb{P})$ and:

- the coefficients $b : \mathbb{R}^d \to \mathbb{R}^d$, $\sigma : \mathbb{R}^d \to \mathbb{R}^d \otimes \mathbb{R}^d$, and $\zeta : \mathbb{R}^d \times \mathbb{R}^r \to \mathbb{R}^d$ are measurable;
- w is a d-dimensional standard Wiener process;
- μ is a time-homogeneous Poisson random measure on $\mathbb{R} \times \mathbb{R}^r \backslash \{0\}$ with Lévy measure $\nu(dz)$; and,
- the initial variable X_0 is $\mathcal{F}_0$-measurable and independent of (w, μ).

We will suppose that $\nu(\mathbb{R}^r) < \infty$, which implies that the number of jumps of X is a.s. locally finite, and that the stochastic integral with respect to μ in the right-hand side of (1) is well defined. The process X is called diffusion with compound-Poisson jumps, which extends the diffusion process (where $\zeta \equiv 0$) and constitutes a broad class of stochastic processes accommodating accidental large variation in addition to diffusive small fluctuation; consult the references in [M] for a comprehensive account for theory of general diffusion with jumps.

We will write $\mathbb{E}$ for the expectation operator and η for the law of X_0, and denote by $(P_t)_{t\in\mathbb{R}_+}$ the transition semigroup of X, namely, $P_t(x, dy) = \mathbb{P}[X_t \in dy | X_0 = x]$. Also, we will write $\mathbb{P}_\eta$ (resp. $\mathbb{E}_\eta$) instead of $\mathbb{P}$ (resp. $\mathbb{E}$) when emphasizing the dependence on η. The symbol $\mathbb{P}_x$ corresponds to the case of $\eta = \delta_x$ for some $x \in \mathbb{R}^d$, where δ_x stands for the Dirac delta measure at x.

1.2 Assumptions

For a matrix $M = (M^{ij})$, let $|M| := \{\sum_{i,j}(M^{ij})^2\}^{1/2}$ and $M^{\otimes 2} := MM^\top$. Let ∂_{x_i} (resp. $\partial^2_{x_i x_j}$) stand for the partial derivations with respect to x_i (resp. x_j and then x_i). Write $a \lesssim a'$ if $a \le ca'$ for some generic constant $c > 0$.

We introduce the following conditions on X given by (1).

[C1] *For every $x_1, x_2 \in \mathbb{R}^d$ and $z_1, z_2 \in \mathbb{R}^r$, we have $\zeta(x_1, 0) = 0$ and*

$$\begin{aligned}
&|b(x_1) - b(x_2)| + |\sigma(x_1) - \sigma(x_2)| \lesssim |x_1 - x_2|, \\
&|\zeta(x_1, z_1) - \zeta(x_2, z_1)| \lesssim |z_1||x_1 - x_2|, \\
&|\zeta(x_1, z_1) - \zeta(x_1, z_2)| \lesssim \rho(x_1)|z_1 - z_2|,
\end{aligned}$$

where $\rho : \mathbb{R}^d \to \mathbb{R}_+$ is a locally bounded function such that $|\zeta(x, z)| \le \rho(x)|z|$ for every (x, z) and that $\lim_{|x|\to\infty} \rho(x)/|x| = 0$.

[C2] *ν is nonnull and satisfies that $\nu(\mathbb{R}^r) < \infty$.*

[C3] *The functions $x \mapsto b(x)$ and $x \mapsto \sigma(x)$ are of class $\mathcal{C}^2$, and satisfy:*

- $\sup_{x\in\mathbb{R}^d} \max_{1\le i\le d} \{|\partial_{x_i} b(x)| + |\partial_{x_i} \sigma(x)|\} < \infty$;
- *$^\exists r > 0$ s.t. $^\forall x \in \mathbb{R}^d$ $\max_{1\le i,j\le d} \{|\partial^2_{x_i x_j} b(x)| + |\partial^2_{x_i x_j} \sigma(x)|\} \lesssim (1 + |x|^r)$; and*
- *$^\exists R \ge 1$ s.t. $^\forall x \in \mathbb{R}^d$ $R^{-1} I_d \le \sigma^{\otimes 2}(x) \le R I_d$, with I_d denoting the d-dimensional identity matrix.*

[1] A function $t \mapsto x_t$ on $\mathbb{R}_+$ is called càdlg̀ if it is right-continuous and if $\lim_{s\uparrow t, s<t} x_s$ exists for each $t > 0$.

Recall that under **[C1]** the stochastic differential equation (1) admits a unique solution, which is $(\mathcal{F}_t)$-adapted, non-explosive, càdlàg, strong-Markov, and weak-Feller.

We need two more conditions. For $q > 0$ and $x = (x_i)_{i=1}^d \in \mathbb{R}^d \backslash \{0\}$, we write

$$\begin{aligned} B_q(x) &= q|x|^{q-2} x^\top b(x), \\ D_q(x) &= \frac{1}{2} q|x|^{q-2} \mathrm{trace}\left\{ \left((q-2)[x_i x_j]_{i,j=1}^d |x|^{-2} + I_d \right) \sigma(x)\sigma(x)^\top \right\}, \\ G_q(x) &= B_q(x) + D_q(x), \\ J_q(x) &= \{\rho(x)\}^2 |x|^{q-2} + \{\rho(x)\}^q + |x|^{q-1}\rho(x) \mathbf{1}_{(1,\infty)}(q), \end{aligned}$$

where $\mathbf{1}_{(1,\infty)}(q)$ is defined to be 0 or 1 according as $q \in (0,1]$ or $q \in (1,\infty)$: note that $x \mapsto G_q(x)$ is formally the diffusion part of the generator of X applied to the function $x \mapsto |x|^q$.

[E] *At least one of the following two holds true.*

- *There exists a constant* $q > 0$ *such that* $\int_{|z|>1} |z|^q \nu(dz) < \infty$ *and that:*
  - (i) $^\exists c > 0$ *s.t.* $G_q(x) \le -c$ *for every* $|x|$ *large enough; and*
  - (ii) $\lim_{|x|\to\infty} J_q(x) = 0$.
- *There exists a constant* $q > 0$ *such that* $\int_{|z|>1} |z|^q \nu(dz) < \infty$ *and that:*
  - (i') $B_q(x) \to -\infty$ *as* $|x| \to \infty$*; and*
  - (ii') $\lim_{|x|\to\infty} \{D_q(x) \vee J_q(x)\}/B_q(x) = 0$.

[EE] *There exist constants* $q > 0$ *and* $c' > 0$ *such that* $\int_{|z|>1} |z|^q \nu(dz) < \infty$ *and that*

$$G_p(x) \le -c'|x|^q$$

for every $|x|$ *large enough.*

Of course, we can simplify the conditions if (σ, ρ) does not become so large for $|x| \to \infty$. Clearly, we can replace "$G_q(x)$" with "$B_q(x)$" in **[E]**(i) if $\sigma(x) = o(|x|^{1-q/2})$. Also, as

$$\left| \frac{D_q(x) \vee J_q(x)}{B_q(x)} \right| \lesssim \frac{\{|\sigma(x)| \vee \rho(x)\}^2}{|x^\top b(x)|} + \left\{ \left(\frac{\rho(x)}{|x|} \right)^q + \mathbf{1}_{(1,\infty)}(q) \frac{\rho(x)}{|x|} \right\} \frac{|x|^2}{|x^\top b(x)|},$$

the condition **[E]**(ii') is fulfilled as soon as $\{|\sigma(x)| \vee \rho(x)\}^2/|x^\top b(x)| \to 0$ and $|x|^2/|x^\top b(x)| \to 0$. Moreover, we can replace "$G_q(x) \le -c'|x|^q$" with "$x^\top b(x)/|x|^2 \le -c'$" in **[EE]** if $\sigma(x) = o(|x|)$.

1.3 Main results

The β-mixing (absolute-regular) coefficient of X say $\beta_X(t)$ is given by

$$\beta_X(t) = \sup_{s \in \mathbb{R}_+} \int \|P_t(x, \cdot) - \eta P_{s+t}(\cdot)\| \eta P_s(dx),$$

where $\eta P_t = \mathcal{L}(X_t)$ and $\|m\|$ stands for the total variation norm of a signed measure m. Then X is called:

- *β-mixing* if $\beta_X(t) = o(1)$ for $t \to \infty$;

- *exponentially β-mixing* if there exists a constant $\gamma > 0$ such that $\beta_X(t) = O(e^{-\gamma t})$ for $t \to \infty$.

Now we can state our main result of this article.

Theorem 1.1. *Suppose* **[C1]**, **[C2]**, *and* **[C3]**.

(a) *Suppose additionally* **[E]**. *Then (P_t) admits a unique invariant law π for which*

$$\|P_t(x,\cdot) - \pi(\cdot)\| \to 0$$

as $t \to \infty$ for every $x \in \mathbb{R}^d$, and X is β-mixing for any η.

(b) *Suppose additionally* **[EE]**. *Then (P_t) admits a unique invariant law π fulfilling*

$$\int |x|^q \pi(dx) < \infty, \tag{2}$$

for which there exist constants $a, c > 0$ such that

$$\|P_t(x,\cdot) - \pi(\cdot)\| \leq c(1 + |x|^q)e^{-at}$$

for every $x \in \mathbb{R}^d$ and $t \in \mathbb{R}_+$. If moreover $\int |x|^q \eta(dx) < \infty$, then there exist constants $a' > 0$ and $c' > 0$ such that $\beta_X(t) \leq c' e^{-a't}$ for each $t \in \mathbb{R}_+$, hence exponentially β-mixing.

In both cases we have the ergodic theorem: for every π-integrable F

$$\frac{1}{T}\int_0^T F(X_t)dt \to \int F(x)\pi(dx) \tag{3}$$

as $T \to \infty$ in $\mathbb{P}_\eta$-probability whatever η is.

Remark 1.2. *We can also consult the preceding Kulik (2007) for an exponential β-mixing result for $\sigma \equiv 0$; in this case, we inevitably need some nondegeneracy conditions on the jump part.*

Remark 1.3. *Even in the first-order inference (such as M-estimation) concerning X, the boundedness of moments in the sense that, e.g.,*

$$\sup_{t \in \mathbb{R}_+} \mathbb{E}_\eta[|X_t|^k] < \infty \tag{4}$$

for sufficiently large $k > 0$, may be also crucial in order to deduce suitable limit theorems for estimating functions. We note that (4) can be readily verified by using [M, Theorem 2.2 (i)] without any topological continuity condition of (P_t). The tail behavior of ν determines possible range of k in (4); specifically, one can take $k \leq q$ with q appearing in **[E]** *or* **[EE]**, *so that we have (4) for every $k > 0$ as soon as $\int_{|z|>1} |z|^q \nu(dz) < \infty$ for every $q > 0$.*

Finally, let us mention that it is possible to deduce the (exponential) β-mixing property and variants of the uniform boundedness (4) under different sets of conditions. Among others, we here focus on the case where the drift function b is bounded and ν admits an exponential moments outside a neighborhood of the origin. In this instance we can derive the same conclusion as in Theorem 1.1(b), and moreover an exponential-moment version of (4) as was done by a part of Gobet (2002) for diffusions. Before stating the result, we introduce new sets of (stronger) conditions.

[C1b] *In addition to* **[C1]**, *b and ρ are bounded.*

[C3b] $^\exists R \geq 1$ *s.t.* $^\forall x \in \mathbb{R}^d$ $R^{-1}I_d \leq \sigma^{\otimes 2}(x) \leq RI_d$.

[EEb] *There exist constants $r > 0$ and $c_0 > 0$ such that* $\int_{|z|>1} \exp(r|z|)\nu(dz) < \infty$ *and that*

$$x^\top b(x) \leq -c_0|x|$$

for every $|x|$ large enough.

Then we have:

Theorem 1.4. *Suppose* **[C1b]**, **[C2]**, **[C3b]**, *and* **[EEb]**. *Then the same statement as in Theorem 1.1*(b) *with "***[EE]***" replaced by "***[EEb]***" holds true. Moreover, there exists a constant $r_0 > 0$ such that:*

- *for any $r_1 \in [0, r_0)$ we have $\int \exp(r_1|x|)\pi(dx) < \infty$; and that*
- *for any $r_2 \in [0, r_0)$ meeting $\int \exp(r_2|x|)\eta(dx) < \infty$ we have*

$$\sup_{t\in\mathbb{R}_+} \mathbb{E}_\eta[\exp(r_2|X_t|)] < \infty. \tag{5}$$

We proceed to some useful lemmas.

1.4 Some lemmas applicable to more general setup

Here we prepare some lemmas, part of which will be used in the proof of Theorem 1.1. The following Lemma 1.5, 1.6, and 1.7 are slight refinements of Lemma 2.4, 2.5(i), and 3.9 of [M], respectively, and they can actually work on much more general diffusions with jumps than (1).

In Lemmas 1.5 and 1.6 below, we forget the objective (1), and instead deal with the general diffusion with jumps given by

$$\begin{aligned} dX'_t &= b(X'_t)dt + \sigma(X'_t)dw_t \\ &\quad + \int_0^t \int_{|z|\leq 1} \zeta(X'_{t-}, z)\tilde{\mu}(dt, dz) + \int_0^t \int_{|z|>1} \zeta(X'_{t-}, z)\mu(dt, dz), \end{aligned} \tag{6}$$

where $\tilde{\mu}(dt, dz) = \mu(dt, dz) - \nu(dz)dt$ denotes the compensated Poisson random measure; note that X' is much more general than X. We used the same notation as in (1) for the coefficient of the stochastic differential equation (6) just for the convenience; for X', we will consistently put the descriptions of **[C1]**, **[E]**, and **[EE]** to use.

To state the lemmas we need some more notation. As in [M], let $\mathcal{Q}$ denote the set of all $\mathcal{C}^2$ functions $f : \mathbb{R}^d \to \mathbb{R}_+$ such that there exists a locally bounded measurable function $\bar{f}$ for which

$$\int_{|z|>1} f(x + \zeta(x, z))\nu(dz) \leq \bar{f}(x)$$

for every $x \in \mathbb{R}^d$, and put $\mathcal{Q}^* = \mathcal{Q} \cap \{f : \mathbb{R}^d \to \mathbb{R}_+ |\ f(x) \to \infty \text{ as } |x| \to \infty\}$. Define the extended generator $\mathcal{A}$ of X' by

$$\mathcal{A}f = \mathcal{G}f + \mathcal{J}_* f + \mathcal{J}^* f \tag{7}$$

for $f \in \mathcal{Q}$, where

$$\begin{aligned} \mathcal{G}f(x) &= \nabla f(x)b(x) + \frac{1}{2}\text{trace}\{\nabla^2 f(x)\sigma(x)\sigma(x)^\top\}, \\ \mathcal{J}_* f(x) &= \int_{|z|\leq 1} \Big(f(x + \zeta(x, z)) - f(x) - \nabla f(x)\zeta(x, z)\Big)\nu(dz), \\ \mathcal{J}^* f(x) &= \int_{|z|>1} \Big(f(x + \zeta(x, z)) - f(x)\Big)\nu(dz). \end{aligned}$$

The function $x \mapsto \mathcal{A}f(x)$ is actually well defined and locally bounded as soon as $f \in \mathcal{Q}$ (see [M, Section 3.1.2] for details). Now let us recall the drift conditions used in [M] (the conditions **[D]** and **[D*]** below are termed Assumption 3 and Assumption 3* in [M], respectively):

[D] *There exist $f \in \mathcal{Q}$ and a constant $c > 0$ such that $\mathcal{A}f(x) \leq -c$ for every $|x|$ large enough.*

[D*] *There exist $f \in \mathcal{Q}^*$ and a constant $c' > 0$ such that $\mathcal{A}f(x) \leq -c'f(x)$ for every $|x|$ large enough.*

In [M], we have seen that the β-mixing property (resp. the exponential β-mixing property) can be derived under the following three kinds of conditions (see Section 2.1 for more detail): **[C1]**, a kind of irreducibility and continuity of the transition semigroup (cf. Assumption 2 of [M]), and drift conditions **[D]** (resp. **[D*]**). The next lemma serves as a tool for verification of the last one.

Lemma 1.5. *Under* **[C1]** *there exists an $f \in \mathcal{Q}^*$ for which* **[D]** *(resp.* **[D*]***) holds true, if* **[E]** *(resp.* **[EE]***) is additionally fulfilled;*

The scenario of the proof is equal to Kulik (2007, Proposition 4.1), which previously obtained **[D*]** in case of $\sigma \equiv 0$. However, in Section 2.3 we will give a full proof in order to clarify how to derive **[D]**.

The next one is a refinement of [M, Lemma 2.5(i)] dealing with a very heavy-tailed ν, but we do not use it in this article.

Lemma 1.6. *Suppose* **[C1]** *and*

$$\int_{|z|>1} \log(1+|z|)\nu(dz) < \infty, \tag{8}$$

and that $|\sigma(x)| = o(|x|)$ for $|x| \to \infty$. Furthermore, suppose that

$$\limsup_{|x|\to\infty} \frac{x^\top b(x)}{|x|(1+|x|)} < 0. \tag{9}$$

Then there exists an $f \in \mathcal{Q}^*$ for which **[D]** *holds true.*

We end with the following lemma, which can apply to general continuous-time Markov processes.

Lemma 1.7. *Let $Y = (Y_t)_{t\in\mathbb{R}_+}$ be a Markov process taking its values in a locally compact separable metric space $(\mathbb{Y}, \mathcal{B}(\mathbb{Y}))$, $\mathcal{B}(\mathbb{Y})$ denoting the Borel field on $\mathbb{Y}$. Let η, $(P_t)_{t\in\mathbb{R}_+}$, and $\beta_Y(t)$ respectively denote initial distribution, transition semigroup, and β-mixing coefficient of Y. Suppose that there exists a probability measure π on $(\mathbb{Y}, \mathcal{B}(\mathbb{Y}))$ for which*

$$V_t(y) := \|P_t(y, \cdot) - \pi(\cdot)\| \to 0$$

as $t \to \infty$ for every $y \in \mathbb{Y}$. Then, for each $t \in \mathbb{R}_+$ and $u \in (0, t)$ we have

$$\beta_Y(t) \leq \eta(V_t) + 2\eta(V_u) + \pi(V_{t-u}). \tag{10}$$

Especially:

(a) *Y is β-mixing for any η;*

(b) *$\beta_Y(t) \lesssim \delta(t/2)$ for each $t \in \mathbb{R}_+$ if $V_t(y) \leq h(y)\delta(t)$ for a finite measurable function $h : \mathbb{Y} \to \mathbb{R}_+$ and a nonincreasing function $\delta : \mathbb{R}_+ \to \mathbb{R}_+$ and if $\pi(h) \vee \eta(h) < \infty$.*

2 Proofs

2.1 Proof of Theorem 1.1

The scenario of the proof is essentially based on [M, Theorems 2.1 and 2.2] for the most part, that is, we will achieve the proof through "weak-Feller property", "open-set irreducibility and T-chain property of some skeleton chain", and "Foster-Lyapunov drift conditions". However, as mentioned in Introduction, differently from [M] we will here utilize an explicit T-chain kernel given by (12) below.

As in [M], we will apply Meyn and Tweedie (1993b, Theorems 5.1 and 6.1) for the ergodic properties, i.e., $\|P_t(x,\cdot)-\pi(\cdot)\| \to 0$ or $\|P_t(x,\cdot)-\pi(\cdot)\| \le c(1+|x|^q)e^{-at}$ mentioned in the statement; see also [M, Corrections]. There one of the crucial steps is to prove that

$$\text{every compact sets are petite for some skeleton chain } (X_{\Delta m})_{m\in\mathbb{Z}_+}, \tag{$\star$}$$

where $\mathbb{Z}_+ := \mathbb{N}\cup\{0\}$, and $\Delta>0$ is some constant: see Meyn and Tweedie (1993a) for a detailed account for the notion of *petite sets*. X is a non-explosive right process under **[C1]**, so that, in order to prove Theorem 1.1 it is sufficient to show:

- "($\star$) and **[D]**" for (a);
- "($\star$) and **[D*]**" for (b).

(see Section 1.4 for the definitions of **[D]** and **[D*]**.) This sufficiency follows from the argument in the fist paragraph of [M, Section 3.1.1]. The drift conditions **[D]** and **[D*]** can be verified by means of Lemma 1.5. Also, under **[EE]** we immediately get $\int |x|^q\pi(dx)<\infty$; see [M, the last paragraph in p.50]. Furthermore, the ergodic theorem (3) is a direct consequence of **[E]**; see [M, Theorem 2.1].

On the other hand, the property ($\star$) is not straightforward to verify as such. Let us first describe the outline of the proof. We will make use of the fact that ($\star$) is implied by the following two conditions (at least for one $\Delta>0$):

[T1] (Open-set irreducibility) *For every open set $O\subset\mathbb{R}^d$ and every $x\in\mathbb{R}^d$, there exists a constant $m=m(x,O)\in\mathbb{N}$ for which $\mathbb{P}_x[X_{m\Delta}\in O]>0$;*

[T2] (T-chain property with sampling distribution being δ_Δ) *there exists a kernel $T_\Delta:\mathbb{R}^d\times\mathcal{B}(\mathbb{R}^d)\to[0,1]$ such that*

(i) $x\mapsto T_\Delta(x,A)$ *is lower semicontinuous for every $A\in\mathcal{B}(\mathbb{R}^d)$, that*

(ii) $P_\Delta(x,A)\ge T_\Delta(x,A)$ *for every $x\in\mathbb{R}^d$ and $A\in\mathcal{B}(\mathbb{R}^d)$, and that*

(iii) $T_\Delta(x,\mathbb{R}^d)>0$ *for every $x\in\mathbb{R}^d$.*

Actually, if **[T1]** and **[T2]** are fulfilled for some $\Delta>0$, then ($\star$) follows from Meyn and Tweedie (1993a, the last half of Theorem 6.2.5(ii)). This together with direct applications of Lemmas 1.5 and 1.7 yields the claims of Theorem 1.1; we need to verify (2) when applying Lemma 1.7(b), but this readily follows from Meyn and Tweedie (1993b) under the conditions, see [M, Theorem 2.2(ii)] for details.

Building on the observations above, we see that it suffices to prove **[T1]** and **[T2]**.

Proof of **[T1]**. Take any $x\in\mathbb{R}^d$, and define a diffusion $Y=(Y_t)_{t\in\mathbb{R}_+}$ by

$$Y_t = x+\int^t b(Y_s)ds+\int_0^t\sigma(Y_s)dw_s. \tag{11}$$

In view of [M, the proof of Claim 1 under Assumption 2(a) in Proposition 3.1] (see also [M-Corrections]), we see that it suffices to show that $\mathbb{P}_x[Y_\Delta\in O']>0$ for any open $O'\subset\mathbb{R}^d$; for any $\Delta>0$, the event $E_\Delta := \{\omega\in\Omega|\mu((0,\Delta],\mathbb{R}^r\backslash\{0\})=0\}$ on which X and Y coincide over $[0,\Delta]$

has positive probability as $\mathbb{P}[E_\Delta] = e^{-\lambda\Delta} > 0$ under **[C2]**, where $\lambda := \nu(\mathbb{R}^r) > 0$. Now, on the event E_Δ we can apply Gobet (2002, Proposition 1.2), see also Azencott (1984), under **[C1]** and **[C3]** to conclude the existence of an everywhere positive transition density $(t, x, y) \mapsto p_t(x, y)$ fulfilling $\sup_{y\in\mathbb{R}^d} \sup_{x\in K} p_t(x, y)$ for each $t > 0$ and compact $K \subset \mathbb{R}^d$. Thus **[T1]** holds true for *any* $\Delta > 0$ under **[C1]**, **[C2]**, and **[C3]**.

Proof of **[T2]**. Again fix any $x \in \mathbb{R}^d$. In [M], the existence of a bounded transition density of the "original X" was supposed. We will weaken this point by reducing the situation to Y given by (11). Specifically, we set $T_\Delta(x, A) = \mathbb{P}_x[\{X_\Delta \in A\} \cap E_\Delta]$. Since on the event E_Δ the original X agrees with Y over the time interval $[0, \Delta]$ ($\mathbb{P}_x$-a.s.), by means of the independence between w and μ we have

$$T_\Delta(x, A) = \mathbb{P}_x[Y_\Delta \in A]\mathbb{P}[E_\Delta] \tag{12}$$

for every $x \in \mathbb{R}^d$ and $A \in \mathcal{B}(\mathbb{R}^d)$. Obviously we have **[T2]**(ii). Reminding that $\mathbb{P}[E_\Delta] > 0$ under **[C2]**, we also get **[T2]**(iii). So it remains to prove **[T2]**(i), and to this end we will utilize Cline and Pu (1998, Lemma 3.1) as in [M].

Fix any $\epsilon > 0$ and compact $K_1, K_2 \subset \mathbb{R}^d$. Now take any $\delta > 0$ such that

$$\delta < \epsilon\left(\mathbb{P}[N_\Delta] \sup_{y\in\mathbb{R}^d} \sup_{x\in K_1} p_\Delta(x, y)\right)^{-1}, \tag{13}$$

and then fix any $A \subset K_2$ such that $\ell(A) < \delta$, where ℓ stands for the Lebesgue measure on $\mathbb{R}^d$. According to (12) and (13) we have

$$\begin{aligned}
\sup_{x\in K_1} T_\Delta(x, A) &= \mathbb{P}[N_\Delta] \sup_{x\in K_1} \mathbb{P}_x[Y_\Delta \in A] \\
&= \mathbb{P}[N_\Delta] \sup_{x\in K_1} \int_A p_\Delta(x, y)dy \\
&\le \ell(A)\left(\mathbb{P}[N_\Delta] \sup_{y\in\mathbb{R}^d} \sup_{x\in K_1} p_\Delta(x, y)\right) \\
&< \delta\left(\mathbb{P}[N_\Delta] \sup_{y\in\mathbb{R}^d} \sup_{x\in K_1} p_\Delta(x, y)\right) \\
&< \epsilon,
\end{aligned}$$

verifying the condition (i) of Cline and Pu (1998, Lemma 3.1). On the other hand, since the diffusion Y is weak-Feller under **[C1]**, the lower semicontinuity of $x \mapsto T_\Delta(x, O')$ for every open $O' \subset \mathbb{R}^d$ follows on account of (12), cf. Meyn and Tweedie (1993a, Proposition 6.1.1(i)):

$$\liminf_{y\to x} T_\Delta(y, O') = \left(\liminf_{y\to x} \mathbb{P}_y[Y_\Delta \in O']\right)\mathbb{P}[E_\Delta] \ge \mathbb{P}_x[Y_\Delta \in O']\mathbb{P}[E_\Delta] = T_\Delta(x, O').$$

This verifies the condition (ii) of Cline and Pu (1998, Lemma 3.1), thereby yielding the lower semicontinuity of $x \mapsto T_\Delta(x, A)$ for any $A \in \mathcal{B}(\mathbb{R}^d)$. Thus the proof of **[T2]** is complete.

2.2 Proof of Theorem 1.4

This can be achieved in much the same way as in the proof of Theorem 1.1, so we will only mention the points.

We note that in the proof of Theorem 1.1 the condition **[C3]** is used in order to the existence of an everywhere positive bounded transition density of the diffusion Y given by (11), which leads to $(\star)$. Under the present situation the existence can be verified by invoking Stroock and Varadhan (1979, Theorem 3.2.1). Therefore, it remains to look at the drift condition **[D*]** and (5); as in Theorem 1.1, that "for any $r_1 \in [0, r_0)$ we have $\int \exp(r_1|x|)\pi(dx) < \infty$" in the statement follows from **[D*]**.

Let $f : \mathbb{R}^d \to \mathbb{R}_+$ be a $\mathcal{C}^2$ function such that $f(x) = \exp(\alpha|x|)$ for $|x| \geq 1$, where $\alpha > 0$ is a constant, and that $f(x) \leq \exp(\alpha|x|)$ for every $x \in \mathbb{R}^d$. Suppose $\alpha \in (0, r/m)$, then, since $m := \sup_x |b(x)| \vee \sup_x \rho(x)$ is finite under **[C1b]**, we have

$$\int_{|z|>1} f(x+\zeta(x,z))\nu(dz) \leq \exp(\alpha|x|) \int_{|z|>1} \exp(\alpha m|z|)\nu(dz) \lesssim \exp(\alpha|x|),$$

so that $f \in \mathcal{Q}^*$. Below we will control $\alpha \in (0, r/m)$ in order to verify **[D*]** and (5).

Let us recall (7), which here reads (X is given by (1))

$$\mathcal{A}f(x) = \mathcal{G}f(x) + \int \Big(f(x+\zeta(x,z)) - f(x)\Big)\nu(dz). \tag{14}$$

Let $|x| \geq 1$ in the sequel. Simple algebra leads to

$$\nabla f(x) = \frac{\alpha}{|x|}x, \tag{15}$$

$$\nabla^2 f(x) = \alpha f(x)\bigg(\frac{\alpha}{|x|^2}[x_i x_j]_{i,j=1}^d + \frac{1}{|x|}I_d - \frac{1}{|x|^3}[x_i x_j]_{i,j=1}^d\bigg). \tag{16}$$

First, for the jump part Taylor's formula gives

$$\begin{aligned}\bigg|\int \Big(f(x+\zeta(x,z)) - f(x)\Big)\nu(dz)\bigg| &\leq \alpha\rho(x)\int \bigg\{\sup_{0\leq u\leq 1} |\nabla f(x+u\zeta(x,z))|\bigg\} |z|\nu(dz) \\ &\leq \alpha m f(x) \int |z| \exp(\alpha m|z|)\nu(dz), \\ &\lesssim \alpha f(x). \end{aligned} \tag{17}$$

Also, it follows from **[C3b]**, (15), and (16) that $\mathcal{G}f(x)$ takes the form of

$$\mathcal{G}f(x) = \alpha f(x)\bigg\{\frac{x^\top b(x)}{|x|} + D_\alpha(x)\bigg\}, \tag{18}$$

where $|D_\alpha(x)| \lesssim \alpha + o(1)$ for $|x| \to \infty$. Substituting (17) and (18) together with **[EEb]** into (14), we arrive at the relation

$$\mathcal{A}f(x) \leq \alpha f(x)\{-c_0 + c_1\alpha + o(1)\} \tag{19}$$

for some constant $c_1 > 0$ and for $|x| \to \infty$. Hence **[D*]** follows on letting α be sufficiently small. Once **[D*]** is verified, we can readily derive the moment bound (5) as in [M, pp.50–51] and [M-Corrections, Remark 3]. The proof of Theorem 1.4 is thus complete.

2.3 Proof of Lemma 1.5

Let $q > 0$ be as given in **[E]** or **[EE]**. In analogy with [M] and [M-Corrections], we will target at a $\mathcal{C}^2$ function $f : \mathbb{R}^d \to \mathbb{R}_+$ such that $f(x) = |x|^q$ for $|x| \geq 1$, and that $f(x) \leq |x|^q$ for every $x \in \mathbb{R}^d$: we know that $f \in \mathcal{Q}^*$, see [M, Lemma 2.3]. We are going to show that this kind of f serves as the required function.

First we look at **[D]**. For every $|x| \geq 1$ we have

$$\mathcal{G}f(x) = G_q(x). \tag{20}$$

By means of **[C1]** we can find a constant $K' \geq 1$ such that

$$\frac{1}{2}|x| \leq \inf_{|z|\leq 1, u\in[0,1]} |x + u\zeta(x,z)| \leq \sup_{|z|\leq 1, u\in[0,1]} |x + u\zeta(x,z)| \leq \frac{3}{2}|x|$$

as soon as $|x| \geq K'$, since we have

$$1 - \frac{\rho(x)}{|x|} \leq \frac{|x + u\zeta(x,z)|}{|x|} \leq 1 + \frac{\rho(x)}{|x|}$$

for every $x \in \mathbb{R}^d$, $z \in \mathbb{R}^r$ such that $|z| \leq 1$, and $u \in [0,1]$. Therefore Taylor's formula yields that

$$\mathcal{J}_* f(x) \lesssim |x|^{q-2}\{\rho(x)\}^2 \int_{|z|\leq 1} |z|^2 \nu(dz) \lesssim |x|^{q-2}\{\rho(x)\}^2 \tag{21}$$

for every $|x| \geq 2K'$. As for $\mathcal{J}^* f$, first we consider $q \in (0,1]$. we can apply the inequality $|A+B|^q \leq |A|^q + |B|^q$ valid for $q \in (0,1]$ to get

$$\mathcal{J}^* f(x) \leq \int_{|z|>1} (|x + \zeta(x,z)|^q - |x|^q)\nu(dz) \leq \{\rho(x)\}^q \int_{|z|>1} |z|^q \nu(dz) \lesssim \{\rho(x)\}^q, \tag{22}$$

using the presupposed property $f(x) \leq |x|^q$ for every $x \in \mathbb{R}^d$. In case of $q > 1$ we similarly obtain

$$\begin{aligned} \mathcal{J}^* f(x) &\lesssim |x|^{q-1}\rho(x) \int_{|z|>1} |z|\nu(dz) + \{\rho(x)\}^q \int_{|z|>1} |z|^q \nu(dz) \\ &\lesssim |x|^{q-1}\rho(x) + \{\rho(x)\}^q \end{aligned} \tag{23}$$

Putting (21), (22) and (23) together, we have

$$\mathcal{J}_* f(x) + \mathcal{J}^* f(x) \lesssim J_q(x) \tag{24}$$

for every $|x| \geq 2K'$. It follows from (20) and (24) that there exists a constant $c_0 > 0$ such that

$$\mathcal{A} f(x) \leq G_q(x) + c_0 J_q(x) = B_q(x) + D_q(x) + c_0 J_q(x) \tag{25}$$

for every $|x|$ large enough, from which **[D]** follows on **[E]**.

As for **[D*]**, in view of (25), **[C1]**, and **[EE]** we see that

$$\begin{aligned} \mathcal{A} f(x) &\leq |x|^q \left[\frac{G_q(x)}{|x|^q} + c_0 \left\{ \left(\frac{\rho(x)}{|x|} \right)^2 + \left(\frac{\rho(x)}{|x|} \right)^q + \mathbf{1}_{(1,\infty)}(q) \frac{\rho(x)}{|x|} \right\} \right] \\ &\lesssim |x|^q \{-c' + o(1)\} \end{aligned}$$

for $|x| \to \infty$, yielding [D*]. The proof of Lemma 2.3 is thus complete.

2.4 Proof of Lemma 1.6

The proof is analogous to [M-Corrections], so we only give a sketch.

Let $f : \mathbb{R}^d \to \mathbb{R}_+$ fulfil that $f(x) = \log(1 + |x|)$ for $|x| \geq 1$, and that $f(x) \leq \log(1+|x|)$ for every $x \in \mathbb{R}^d$. In this case we have $f \in \mathcal{Q}^*$ (cf. [M, Lemma 2.3]), and

$$\begin{aligned} \nabla f(x) &= \frac{1}{|x|(1+|x|)} x^\top, \quad |x| \geq 1, \\ |\nabla^2 f(x)| &= O(|x|^{-2}), \quad |x| \to \infty. \end{aligned}$$

Therefore Taylor's formula and **[C1]** give $\mathcal{J}_* f(x) \lesssim (\rho(x)/|x|)^2 = o(1)$. Further, in view of the choice of f made above we get

$$\mathcal{J}^* f(x) \leq \int_{|z|>1} \log\left(1 + \frac{\rho(x)}{1+|x|}|z|\right)\nu(dz)$$

for $|x|$ large enough, the upper bound tending to 0 as $|x| \to \infty$ by means of the condition (8), the dominated convergence theorem, and **[C1]**. Thus, taking the condition on σ into account we have

$$\mathcal{A} f(x) \leq \frac{x^\top b(x)}{|x|(1+|x|)} + o(1),$$

and accordingly Lemma 1.6 follows on (9).

2.5 Proof of Lemma 1.7

The proof consists of a modification of Liebscher (2005, Proposition 3). By triangular inequality we see that $\beta_Y(t) \leq \beta_{Y,1}(t) + \beta_{Y,2}(t)$, where

$$\beta_{Y,1}(t) := \sup_{s\in\mathbb{R}_+} \|\eta P_{t+s}(y,\cdot) - \pi(\cdot)\|,$$
$$\beta_{Y,2}(t) := \sup_{s\in\mathbb{R}_+} \int \|P_t(y,\cdot) - \pi(\cdot)\| \eta P_s(dy).$$

Since $t \mapsto V_t(y)$ is nonincreasing for each $y \in \mathbb{Y}$, we have

$$\beta_{Y,1}(t) \leq \int \sup_{s\in\mathbb{R}_+} \|P_{t+s}(y,\cdot) - \pi(\cdot)\|\eta(dy) \leq \int \|P_t(y,\cdot) - \pi(\cdot)\|\eta(dy) = \eta(V_t). \tag{26}$$

Next, fix any $u \in (0,t)$. Using the Chapman-Kolmogorov relation $P_{t_1+t_2} = P_{t_2}P_{t_1}$ for any $t_1, t_2 \in \mathbb{R}_+$, and using the fact $\sup_{t\in\mathbb{R}_+, y\in\mathbb{Y}} |V_t(y)| \leq 2$, we get

$$\begin{aligned}
\beta_{y,2}(t) &\leq \sup_{s\in\mathbb{R}_+} \iint V_{t-u}(x)P_u(y,dx)\eta P_s(dy) \\
&= \sup_{s\in\mathbb{R}_+} \iint V_{t-u}(x)P_{s+u}(z,dx)\eta(dz) \\
&\leq 2 \sup_{s\in\mathbb{R}_+} \int V_{s+u}(z)\eta(dz) + \pi(V_{t-u}) \\
&\leq 2\eta(V_u) + \pi(V_{t-u}). \qquad (27)
\end{aligned}$$

Combining (26) and (27) thus yields (10). Now (a) is obvious by taking $u = t/2$ in (10) and then applying the dominated convergence theorem. Also, under the assumptions it directly follows from (10) that $\beta_Y(t) \lesssim \delta(u \wedge (t-u))$, leading to (b) again by taking $u = t/2$. The proof of Lemma 1.7 is complete.

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