Introducing Viewpoints of Mechanics into Basic Growth Analysis - (VI) Some Solutions to a Simple Differential Equation Associated with Growth Mechanics -

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Introducing Viewpoints of Mechanics into Basic Growth Analysis – (VI) Some Solutions to a Simple Differential Equation Associated with Growth Mechanics –

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The present study was designed to investigate some solutions to a simple differential equation associated with growth mechanics and whether they were related under some concept. The results obtained were as follows. Some solutions to the simple differential equation for ruminant agriculture were: (1) a function for the growth of a forage plant or a ruminant animal, (2) a function for the light attenuation in forage plant canopy, (3) a function for the degradable residue of forage protein in the rumen, (4) a function suggesting field–forage–ruminant relationships through matter circulation, (5) a function for spirals topologically similar to micro– and macro–structures of forages or ruminants. Other solutions in the field of physics that might be related to ruminant agriculture through the energy issue were: (6) a function for the mass–energy relation applying to a forage plant or a ruminant animal, (7) a function for the wave including energy that was a solution to partial differential equations replacing but corresponding to the ordinary differential equation, (8) a function for the exponential expansion of the space with almost constant density of energy. It was suggested that some solutions to a simple differential equation associated with growth mechanics ranged from some aspects of ruminant agriculture to energy issues of physics, provided that they were described using exponential functions with base *e*.

INTRODUCTION

It was suggested in reports (Shimojo *et al.*, 2006a; Shimojo, 2007a) that introducing mechanical viewpoints into basic growth analysis of forages and ruminants showed some resemblances to laws of motion (Kawabe, 2006). This hypothetic resemblance might be due to the concept of acceleration involved in both growth and motion, though they are different phenomena. It was also shown by Shimojo (2006b, 2007b) that wave functions including imaginary unit were derived from introducing growth jerk, the derivative of growth acceleration. These issues, however, should be investigated from the viewpoint of solutions to differential equations.

The present study was designed to investigate some solutions to a simple differential equation associated with growth mechanics and whether they were related under some concept.

A DIFFERENTIAL EQUATION FOR GROWTH MECHANICS AND ITS SIMPLE FORM

A differential equation for growth mechanics

As shown in reports (Shimojo *et al.*, 2006a; Shimojo, 2007a), a differential equation for growth mechanics is given by applying a series of differential procedures to the following basic growth function,

$$W = W_0 \cdot \exp(\mathrm{RGR} \cdot t), \tag{1}$$

where W=weight, t=time, RGR=relative growth rate, W_0=the weight at t=0.
The derivative of W with respect to t gives absolute growth rate (AGR),

$$\mathrm{AGR} = \frac{dW}{dt} = W_0 \cdot \mathrm{RGR} \cdot \exp(\mathrm{RGR} \cdot t). \tag{2}$$

The second derivative of W gives growth acceleration (GA),

$$\mathrm{GA} = \frac{d^2W}{dt^2} = W_0 \cdot (\mathrm{RGR})^2 \cdot \exp(\mathrm{RGR} \cdot t). \tag{3}$$

Relating functions (1), (2) and (3) gives

$$\frac{\frac{dW}{dt}}{W} = \frac{\frac{d^2W}{dt^2}}{\frac{dW}{dt}} = \mathrm{RGR}. \tag{4}$$

Combining the first and second terms in (4) gives

$$W \cdot \frac{d^2W}{dt^2} - \left(\frac{dW}{dt}\right)^2 = 0. \tag{5}$$

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A siimple form of differential equation (5)

A siimple form of equation (5) is given by its generalization,

$$y \cdot \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0. \tag{6}$$

This is the differential equation that we take up in the present study to investigate its some solutions for not only ruminant agriculture but also other related fields of natural sciences. There is, of course, a possibility that functions that take a form of exponential function with base e are solutions to the differential equation (6).

SOME SOLUTIONS TO DIFFERENTIAL EQUATION (6) FOR RUMINANT AGRICULTURE

In this chapter we replace (x, y) in the differential equation (6) with various variables that are related to ruminant agriculture.

Growth of forages and ruminants

As shown above, the growth of a forage plant (Watson, 1952) or a ruminant animal (Brody, 1945b) is given by

$$W(t) = W_0 \cdot \exp(\mathrm{RGR} \cdot t). \tag{1}$$

This issue requires the replacement in the differential equation (6): $(x, y) \to (t, W)$.

Inserting the function (1) into the differential equation (6) gives

$$W(t) \cdot \frac{d^2W(t)}{dt^2} - \left(\frac{dW(t)}{dt}\right)^2 = W(t) \cdot \left((\mathrm{RGR})^2 \cdot W(t)\right) - \left(\mathrm{RGR} \cdot W(t)\right)^2 = 0. \tag{7}$$

Thus, the function (1) for the growth of a forage plant or a ruminant animal is one of the solutions to the differential equation (6).

Light attenuation in the canopy of forage plants

The light interception by canopy leaves is described using the following function for light attenuation (Monsi und Saeki, 1953),

$$I(F) = 100 \cdot \exp(-K \cdot F), \tag{8}$$

where F=cumulative leaf area index from the top to a certain layer of leaves, 100= relative light intensity above the canopy, $I(F)$=relative light intensity below a certain layer of leaves, K=light extinction coefficient of the canopy.
This issue requires the replacement in the differential equation (6): $(x, y) \to (F, I)$.

Inserting the function (8) into the differential equation (6) gives

$$I(F) \cdot \frac{d^2I(F)}{dF^2} - \left(\frac{dI(F)}{dF}\right)^2 = I(F) \cdot \left((-K)^2 \cdot I(F)\right) - \left((-K) \cdot I(F)\right)^2 = 0. \tag{9}$$

Thus, the function (8) for the light attenuation in forage plant canopy is one of the solutions to the differential equation (6).

Degradation of forage protein in the rumen

The function for the degradation of forage protein in the rumen of ruminant animals is given by Ørskov (1982). Thus,

$$p = a + b \cdot \left(1 - \exp(-c \cdot t)\right), \tag{10}$$

where p=the amount of degraded forage protein at time t, a=the rapidly–soluble fraction, b=the amount which in time will degrade, c=the fractional–rate constant at which b will be degraded.
Then, the decrease in the degradable residue of forage protein is given by

$$D(t) = b \cdot \exp(-c \cdot t). \tag{11}$$

This issue requires the replacement in the differential equation (6): $(x, y) \to (t, D)$.

Inserting the function (11) into the differential equation (6) gives

$$D(t) \cdot \frac{d^2D(t)}{dt^2} - \left(\frac{dD(t)}{dt}\right)^2 = D(t) \cdot \left((-c)^2 \cdot D(t)\right) - \left((-c) \cdot D(t)\right)^2 = 0, \tag{12}$$

Thus, the function (11) for the degradable residue of forage protein in the rumen is one of the solutions to the differential equation (6).

Cyclic phenomena in ruminant agriculture

There are cases where cyclic phenomena are described using polar form on the complex plane, for example, Euler's formula that is given by

$$\exp(\boldsymbol{i} \cdot \theta) = \cos\theta + \boldsymbol{i}\sin\theta, \tag{13}$$

where $\boldsymbol{i}$=imaginary unit.
In this section we take up two things (A) and (B) that may be related to cyclic phenomena.
(A) Field–forage–ruminant relationships through matter circulation

As shown by Shimojo *et al.* (2003a, 2003b), the following function is taken up to suggest field–forage–ruminant relationships through matter circulation. Thus,

$$\begin{aligned} M(\theta) &= \sqrt{D^2+I^2} \cdot \exp(\boldsymbol{i} \cdot \theta) \\ &= \sqrt{D^2+I^2} \cdot \cos\theta + \boldsymbol{i}\sqrt{D^2+I^2} \cdot \sin\theta \\ &= D + \boldsymbol{i}I. \end{aligned} \tag{14}$$

where D= forage digestible weight, I= forage indigestible weight, $D+I=W$ (forage weight).

Firstly, the function (14) suggests the field with standing forage plants, because both D and I, namely W takes positive values.

Secondly, the derivative of (14) gives

$$\frac{dM(\theta)}{d\theta} = -I + \boldsymbol{i}D, \tag{15}$$

where the positive D suggests ruminant production from forage digestible matter, but the negative I suggests its absence.

Thirdly, the second derivative of (14) gives

$$\frac{d^2M(\theta)}{d\theta^2}=-D-\boldsymbol{i}I, \tag{16}$$

where the negative D and negative I suggest their absence, namely the field without standing forage plants because of having been harvested to be fed to ruminant animals.

Fourthly, the third derivative of (14) gives

$$\frac{d^3M(\theta)}{d\theta^3}=I-\boldsymbol{i}D, \tag{17}$$

where the positive I suggests feces excreted from ruminant animals, but the negative D suggests its absence.

Fifthly, the fourth derivative of (14) gives

$$\frac{d^4M(\theta)}{d\theta^4}=D+\boldsymbol{i}I. \tag{18}$$

where the positive D and positive I suggest returning to the field with standing forage plants, namely the onset of the next cycle for ruminant production from forage plants.

Functions (14)～(18) give a series of events: the field with standing forage plants → ruminant production from digestible part of forages → the field without standing forage plants → feces excreted from ruminants → the field with the next standing forage plants. This suggests a matter circulation in field–forage–ruminant relationships in ruminant agriculture, except for the urine excreted from ruminants. This issue requires the replacement in the differential equation (6): $(x, y) \rightarrow (\theta, M)$.

Inserting the function (14) into the differential equation (6) gives

$$M(\theta)\cdot\frac{d^2M(\theta)}{d\theta^2}-\left(\frac{dM(\theta)}{d\theta}\right)^2=M(\theta)\cdot\left(\boldsymbol{i}^2\cdot M(\theta)\right)-\left(\boldsymbol{i}\cdot M(\theta)\right)^2=0. \tag{19}$$

Thus, the function (14) for field–forage–ruminant relationships through matter circulation is one of the solutions to the differential equation (6).

(B) Spirals topologically similar to micro– and macro–structures of forages or ruminants

Let us take up the following two functions as shown by Shimojo *et al.* (2003c).

$$R(\theta)=\exp(\boldsymbol{i}\cdot\theta)=\cos(\theta)+\boldsymbol{i}\sin(\theta), \tag{20}$$
$$L(\theta)=\exp(-\boldsymbol{i}\cdot\theta)=\cos(\theta)-\boldsymbol{i}\sin(\theta). \tag{21}$$

The stereographic description of functions (20) and (21) gives a right–handed spiral by $(\theta, \cos\theta, \boldsymbol{i}\sin\theta)$ and a left–handed spiral by $(\theta, \cos\theta, -\boldsymbol{i}\sin\theta)$. These spirals might show topological similarities to vines of forage plants and micro–structures of both ruminants and forages (Shimojo *et al.*, 2003c). These issues require the replacement in the differential equation (6): $(x, y) \rightarrow (\theta, R)$ and $(x, y) \rightarrow (\theta, L)$.

Inserting the function (20) or (21) into the differential equation (6) gives

$$R(\theta)\cdot\frac{d^2R(\theta)}{d\theta^2}-\left(\frac{dR(\theta)}{d\theta}\right)^2=R(\theta)\cdot\left(\boldsymbol{i}^2\cdot R(\theta)\right)-\left(\boldsymbol{i}\cdot R(\theta)\right)^2=0, \tag{22}$$

$$L(\theta)\cdot\frac{d^2L(\theta)}{d\theta^2}-\left(\frac{dL(\theta)}{d\theta}\right)^2=L(\theta)\cdot\left((-\boldsymbol{i})^2\cdot L(\theta)\right)-\left((-\boldsymbol{i})\cdot L(\theta)\right)^2=0. \tag{23}$$

Thus, functions (20) and (21) for spirals associated with micro– and macro–structures of forages or ruminants are two of the solutions to the differential equations (6).

SOME SOLUTIONS TO DIFFERENTIAL EQUATION (6) FOR OTHER FIELDS RELATED TO RUMINANT AGRICULTURE

One of other fields related to ruminant agriculture might be physics, because the differential equation (5) for growth mechanics might look like the second law of motion (Shimojo *et al.*, 2006a; Shimojo, 2007a). In addition, the issue of energy is common to ruminant agriculture and physics, because energy is indispensable to both growth and motion.

Correspondences between growth and motion

The differential equation (5) for growth mechanics might look like the second law of motion, and the latter law also applies to an object when its mass shows changes (Kawabe, 2006). Thus,

$$\text{Growth mechanics: } \left(\frac{dW}{dt}\right)^2=W\cdot\frac{d^2W}{dt^2}\ . \tag{5}$$

$$\text{Second law of motion: } \frac{dp}{dt}=m\cdot\frac{d^2x}{dt^2}\ . \tag{24}$$

Between equations (5) and (24), the following correspondences are suggested:

$$\text{Growth force: } \left(\frac{dW}{dt}\right)^2 \longleftrightarrow \text{Motion force: } \left(\frac{dp}{dt}\right), \tag{25}$$

$$\text{Weight: } (W) \longleftrightarrow \text{Mass: } (m), \tag{26}$$

$$\text{Growth acceleration: } \left(\frac{d^2W}{dt^2}\right)\longleftrightarrow\text{Motion acceleration: } \left(\frac{d^2x}{dt^2}\right). \tag{27}$$

Growth mechanics version (GMV) of kinetic energy was also hypothetically suggested according to the concept of motion (Shimojo, 2007a). Thus,

$$\text{GMV of kinetic energy: } \left(\frac{1}{3}\cdot W\cdot\left(\frac{dW}{dt}\right)^2\right) \longleftrightarrow \text{kinetic energy of motion: } \left(\frac{1}{2}\cdot m\cdot\left(\frac{dx}{dt}\right)^2\right). \tag{28}$$

The correspondence (28) suggests that GMV of kinetic energy might look like the kinetic energy of motion, though what GMV of kinetic energy suggests is obscure.

In this chapter, the following three sections will take up the issue of energy.

Mass–energy relation

The combustion energy is used for evaluating forages in ruminant nutrition (Brody, 1945a; Kleiber, 1987). It was also shown by Brody (1945a) that the combustion energy is included in the energy of mass that is given by the well–known equality,

$$E=m\cdot c^2, \tag{29}$$

where E=energy, m=mass of an object, c=the speed of light.
The energy of sunlight absorbed by forage plants comes from the nuclear fusion through the mass–energy equivalence. Therefore, the equality (29) is related indirectly to ruminant agriculture. If the energy of mass of a forage plant or a ruminant animal is estimated by the equality (29) on condition that mass is replaced by weight, then the following function will be taken up,

$$E(t)=\left(W_0\cdot\exp(\mathrm{RGR}\cdot t)\right)\cdot c^2. \tag{30}$$

This issue requires the replacement in the differential equation (6): $(x, y) \to (t, E)$.

Inserting the function (30) into the differential equation (6) gives

$$E(t)\cdot\frac{d^2E(t)}{dt^2}-\left(\frac{dE(t)}{dt}\right)^2=E(t)\cdot\left((\mathrm{RGR})^2\cdot E(t)\right)-\left((\mathrm{RGR})\cdot E(t)\right)^2=0. \tag{31}$$

Thus, the function (30) for mass–energy relation for a forage plant or a ruminant animal is one of the solutions to the differential equation (6).

Wave function including energy

Shimojo (2006b, 2007b) reported that wave functions were derived from introducing growth jerk, the third derivative, into the basic growth function. Many chemical reactions, which occur in forages and ruminants to support the life and production of them, are based ultimately on quantum mechanics. In this section, a wave function of the simplest form (French and Taylor, 1978) will be taken up, though this is far behind describing actual quantum phenomena. Thus,

$$\Psi(x, t)=A\cdot\exp\left(\boldsymbol{i}\cdot\frac{p}{\hbar}\cdot x-\boldsymbol{i}\cdot\frac{E}{\hbar}\cdot t\right), \tag{32}$$

where Ψ=wave function, $\boldsymbol{i}$=imaginary unit, $\hbar=h/2\pi$ (h: Planck's constant, π: circular constant), p=momentum, E=energy.
This issue requires the replacement in the differential equation (6): $x \to (x, t)$ and $y \to \Psi$, namely the ordinary differential equation (6) is replaced by the following four kinds of partial differential equation. Thus, inserting the function (32) into the four partial differential equations gives

$$\Psi(x, t)\cdot\frac{\partial^2\Psi(x, t)}{\partial x^2}-\left(\frac{\partial\Psi(x, t)}{\partial x}\right)^2$$
$$=\Psi(x, t)\cdot\left(\left(\boldsymbol{i}\cdot\frac{p}{\hbar}\right)^2\cdot\Psi(x, t)\right)-\left(\left(\boldsymbol{i}\cdot\frac{p}{\hbar}\right)\cdot\Psi(x, t)\right)^2$$
$$=0, \tag{33}$$

$$\Psi(x, t)\cdot\frac{\partial^2\Psi(x, t)}{\partial t^2}-\left(\frac{\partial\Psi(x, t)}{\partial t}\right)^2$$
$$=\Psi(x, t)\cdot\left(\left(-\boldsymbol{i}\cdot\frac{E}{\hbar}\right)^2\cdot\Psi(x, t)\right)-\left(\left(-\boldsymbol{i}\cdot\frac{E}{\hbar}\right)\cdot\Psi(x, t)\right)^2$$
$$=0, \tag{34}$$

$$\Psi(x, t)\cdot\left(\frac{\partial^2\Psi(x, t)}{\partial x\,\partial t}\right)-\left(\frac{\partial\Psi(x, t)}{\partial x}\right)\cdot\left(\frac{\partial\Psi(x, t)}{\partial t}\right)$$
$$=\Psi(x, t)\cdot\left(\left(\boldsymbol{i}\cdot\frac{p}{\hbar}\right)\cdot\left(-\boldsymbol{i}\cdot\frac{E}{\hbar}\right)\cdot\Psi(x, t)\right)$$
$$-\left(\left(\boldsymbol{i}\cdot\frac{p}{\hbar}\right)\cdot\Psi(x, t)\right)\cdot\left(\left(-\boldsymbol{i}\cdot\frac{E}{\hbar}\right)\cdot\Psi(x, t)\right)$$
$$=0, \tag{35}$$

$$\Psi(x, t)\cdot\left(\frac{\partial^2\Psi(x, t)}{\partial t\,\partial x}\right)-\left(\frac{\partial\Psi(x, t)}{\partial t}\right)\cdot\left(\frac{\partial\Psi(x, t)}{\partial x}\right)$$
$$=\Psi(x, t)\cdot\left(\left(-\boldsymbol{i}\cdot\frac{E}{\hbar}\right)\cdot\left(\boldsymbol{i}\cdot\frac{p}{\hbar}\right)\cdot\Psi(x, t)\right)$$
$$-\left(\left(-\boldsymbol{i}\cdot\frac{E}{\hbar}\right)\cdot\Psi(x, t)\right)\cdot\left(\left(\boldsymbol{i}\cdot\frac{p}{\hbar}\right)\cdot\Psi(x, t)\right)$$
$$=0. \tag{36}$$

Thus, the function (32) for the simplest wave is one of the solutions to the partial differential equations that replace but correspond to the ordinary differential equation (6).

Space expansion with constant density of energy

Enlightening books on the universe (Guth, 1997; Sato, 2005) show a hypothesis that huge amounts of energy in the present universe, which include the energy of forages and ruminants, came from the exponential expansion of the space where the energy density was kept almost constant, and this expansion lasted for extremely short period in the very early stage after the birth of the universe. A function for the exponential expansion of the universe (Sakai, 2000) is estimated by

$$a(t) \propto \exp(H\cdot t), \tag{37}$$

where $H^2\equiv(8\pi G/3)\cdot\rho_v$, π=circular constant, G=gravitational constant, ρ_v=energy density, H=expansion coefficient.
This issue requires the replacement in the differential equation (6): $(x, y) \to (t, a)$.

Inserting the function (37) into the differential equation (6) gives

$$a(t)\cdot\left(\frac{d^2a(t)}{dt^2}\right)-\left(\frac{da(t)}{dt}\right)^2=a(t)\cdot\left(H^2\cdot a(t)\right)-\left(H\cdot a(t)\right)^2=0. \tag{38}$$

Thus, the function (38) for the exponential expansion of the space with almost constant density of energy is one of the solutions to the differential equation (6).

DISCUSSION

Exponential functions with base e are often used to describe natural phenomena, because those functions are easy to treat in both differential and integral that are essential tools to analyze dynamics of events occurring in the world of nature. This applies to analyzing the growth of forages and ruminants that support ruminant agriculture, where relating weight, absolute growth rate and growth acceleration gives the differential equation (5). Its simple form is the differential equation (6) that might be expected to have some solutions covering things of different fields, if they are described using exponential functions with base e. What are considered to be of interest are non–linearity of the differential equation (6) and the following two things (i) and (ii). (i) In ruminant agriculture, not only the issue of growth phenomena [equation (1)] but also that of cyclic phenomena [equations (14), (20) and (21)] is one of the solutions to the differential equation (6). (ii) In the field of physics that might be related to ruminant agriculture through the energy issue, the issue of matter [equation (30)] and that of wave [equation (32)] are two of the solutions to the equation (6) in the ordinary differential and partial differential, respectively. These two things (i) and (ii) suggest that matter and wave, which have different characteristics, are two aspects of one thing from the viewpoint of mechanics. One of the possible reasons for this coexistence of matter and wave might be due to the description of them using exponential function with base e. It is also known that there are many natural phenomena that have nonlinear characteristics. Roughly speaking at the risk of making mistakes, does the non–linearity of the differential equation (6) have something to do with covering different things in the world of nature?

CONCLUSIONS

It was suggested that some solutions to a simple differential equation associated with growth mechanics ranged from some aspects of ruminant agriculture to energy issues of physics, provided that they were described using exponential functions with base e.

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