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Introducing Viewpoints of Mechanics into Basic Growth Analysis – (IV) Hypothetic Aspects of Growth Mechanics compared with Momentum, Impulse and Kinetic Energy in Motion –

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The present study was designed to introduce momentum, impulse and kinetic energy of motion into growth mechanics in order to construct corresponding items hypothetically. The results obtained were as follows. Growth mechanics versions of momentum, impulse and kinetic energy were constructed according to definitions in motion. Growth mechanics versions of three items might look like those of motion in the form of equations. This was due probably to the resemblance between equation of growth force and Newton's equation of motion. However, coefficients of equations were different between growth mechanics and Newtonian mechanics. This difference was probably caused by symmetric properties of exponential function with base *e* in both differential and integral that relate coefficients to exponents. The way of construction and interpretation of growth mechanics versions of momentum, impulse and kinetic energy should be investigated further.

INTRODUCTION

It was suggested in a report (Shimojo *et al.*, 2006) that there were three aspects of growth mechanics compared with three laws of motion, using relationships among weight, absolute growth rate and growth acceleration in basic growth analysis. It is known that there are further items of mechanics that explain various aspects of motion (Kawabe, 2006).

The present study was designed to introduce concepts of momentum, impulse and kinetic energy of motion into growth mechanics in order to construct corresponding items hypothetically.

THREE ASPECTS OF GROWTH MECHANICS COMPARED WITH THREE LAWS OF MOTION

A series of calculations leading to growth mechanics (Shimojo *et al.*, 2006) is as follows.

$$\frac{1}{W}\cdot\frac{dW}{dt}=\mathrm{RGR},\ (1)\ \xrightarrow{\text{Integral}}\ W=W_0\cdot\exp(\mathrm{RGR}\cdot t),\ (2)$$

$$\xrightarrow{\text{Differential}}\quad \frac{dW}{dt}=\mathrm{AGR}=W_0\cdot\mathrm{RGR}\cdot\exp(\mathrm{RGR}\cdot t),\ (3)$$

$$\xrightarrow{\text{Differential}}\quad \frac{d(\mathrm{AGR})}{dt}=\mathrm{GA}=W_0\cdot(\mathrm{RGR})^2\cdot\exp(\mathrm{RGR}\cdot t),\ (4)$$

where W=weight, t=time, W_0=the weight at $t=0$, RGR=relative growth rate, AGR=absolute growth rate, GA=growth acceleration.

Then, relating equations (2)～(4) gives

$$\frac{\mathrm{AGR}}{W}=\frac{\mathrm{GA}}{\mathrm{AGR}}=\mathrm{RGR}. \qquad (5)$$

Relating the first and second terms in (5) gives three aspects of growth mechanics. Thus,

$$(\mathrm{AGR})^2=W\cdot\mathrm{GA},\quad \text{or}\quad \mathrm{AGR}=\pm\sqrt{W\cdot\mathrm{GA}}\ . \qquad (6)$$

$$W=W_0\cdot\exp(\mathrm{RGR}\cdot t)\ \xrightarrow{\mathbf{GA=0\ \ namely\ \ RGR=0}}\ W=W_0\ . \quad (7)$$

$$\mathrm{AGR}_1=\sqrt{W\cdot\mathrm{GA}}\ \text{and}\ \ \mathrm{AGR}_2=-\sqrt{W\cdot\mathrm{GA}}$$

$$\rightarrow\ \ \mathrm{AGR}_1+\mathrm{AGR}_2=0\ (\mathrm{AGR}_1=-\mathrm{AGR}_2). \qquad (8)$$

Equation (6) might look like the second law of motion (Newton's equation of motion), namely something like growth force. Equation (7) might look like the first law of motion (law of inertia). Equation (8) might look like the third law of motion (law of action and reaction). These are superficial resemblances, allowing me to give a rough mechanical interpretation of growth phenomena.

In the next section I would like to take up other three items of motion in order to search for corresponding aspects in growth mechanics.

HYPOTHETIC ASPECTS OF GRWOTH MECHANICS COMPARED WITH MOMENTUM, IMPULSE AND KINETIC ENERGY OF MOTION

Growth mechanics version of momentum

Momentum (p) in motion is defined as the product of mass of an object and its velocity (Kawabe, 2006),

$$p=m\cdot v, \qquad (9)$$

where m=mass of an object, v=velocity.

The differentiation of equation (9) with respect to time gives the second law of motion, Newton's equation of motion ($F=m\cdot a$). Thus,

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$$\frac{dP}{dt}=\frac{d(m\cdot v)}{dt}$$
$$=\frac{dm}{dt}\cdot v+m\cdot\frac{dv}{dt}$$
$$=m\cdot\frac{dv}{dt}\qquad\left(\because\frac{dm}{dt}=0\right)$$
$$=m\cdot a\,,\tag{10}$$

where a=acceleration.

Introducing this concept into growth mechanics might be expected to give growth mechanics version of momentum (GMV of momentum), though it is a hypothetic one. Thus,

$$\text{GMV of momentum}=W\cdot\text{AGR},\tag{11}$$

where W=weight, AGR=absolute growth rate.

The differentiation of equation (11) with respect to t gives

$$\frac{d(\text{GMV of momentum})}{dt}=\frac{d(W\cdot\text{AGR})}{dt}$$
$$=\frac{dW}{dt}\cdot\text{AGR}+W\cdot\frac{d(\text{AGR})}{dt}$$
$$=(\text{AGR})^2+W\cdot\text{GA}\qquad[\because\text{ equations (3), (4)}]$$
$$=W\cdot\text{GA}+W\cdot\text{GA}\qquad[\because\text{ equation (6)}]$$
$$=2\cdot W\cdot\text{GA}\,.\tag{12}$$

Relating equation (11) with equation (12) shows that the rate of change in GMV of momentum is equal to twice the growth force, a feature that comes from the symmetric property of exponential function with base e. This is different from what occurs in motion, where the rate of change in momentum is equal to force.

Growth mechanics version of impulse

Impulse (Δp) in motion is given by the integral of force with respect to time (Kawabe, 2006). Thus,

$$\Delta p=p(t_2)-p(t_1)$$
$$=\int_{t_1}^{t_2}Fdt.\tag{13}$$

Introducing this concept into growth mechanics might be expected to give growth mechanics version of impulse (GMV of impulse), though it is a hypothetic one. Thus,

$$\text{GMV of impulse}=\int_{t_1}^{t_2}(W\cdot\text{GA})\,dt.\tag{14}$$

Then,

$$\text{GMV of impulse}=\frac{1}{2}\cdot[W\cdot\text{AGR}]_{t_1}^{t_2}\qquad[\because\text{ equation (12)}]$$
$$=\frac{1}{2}\cdot[\text{GMV of momentum}]_{t_1}^{t_2}\qquad[\because\text{ equation (11)}]$$
$$=\frac{1}{2}\cdot\{\text{GMV of momentum}(t_2)-\text{GMV of momentum}(t_1)\}\tag{15}$$

Relating equation (14) with equation (15) shows that GMV of impulse is equal to half the change in GMV of momentum, a feature that comes from the symmetric property of exponential function with base e. This is different from what occurs in motion, where impulse is equal to the change in momentum.

Growth mechanics version of kinetic energy

Kinetic energy in motion comes from the integral of work (W) (Kawabe, 2006),

$$W_{ab}=\int_{t_1}^{t_2}\left(m\cdot\frac{dv}{dt}\right)\cdot(v\,dt)$$
$$=m\cdot\int_{v_a}^{v_b}v\,dv$$
$$=\frac{m\cdot v_b^2}{2}-\frac{m\cdot v_a^2}{2}\,.\tag{16}$$

Thus, kinetic energy in motion is given by

$$\text{Kinetic energy}=\frac{m\cdot v^2}{2}\,.\tag{17}$$

Applying the same process to growth mechanics might be expected to give growth mechanics version of work (GMV of work). Thus,

$$\text{GMV of work}=\int_{t_1}^{t_2}\left(W\cdot\frac{d(\text{AGR})}{dt}\right)\cdot(\text{AGR})dt$$
$$=\int_{t_1}^{t_2}(W\cdot\text{GA})\cdot(\text{AGR})dt\qquad[\because\text{ equation (4)}]$$
$$=\int_{t_1}^{t_2}(\text{AGR})^3dt\qquad[\because\text{ equation (6)}]$$
$$=\int_{t_1}^{t_2}(W_0^3\cdot(\text{RGR})^3\cdot(\exp(\text{RGR}\cdot t))^3)dt\qquad[\because\text{equation (3)}]$$
$$=\frac{1}{3}\cdot[W_0^3\cdot(\text{RGR})^2\cdot(\exp(\text{RGR}\cdot t))^3]_{t_1}^{t_2}$$
$$=\frac{1}{3}\cdot[(W_0\cdot\exp(\text{RGR}\cdot t))\cdot(W_0\cdot\text{RGR}\cdot\exp(\text{RGR}\cdot t))^2]_{t_1}^{t_2}\qquad[\because\text{ equations (2), (3)}]$$
$$=\frac{1}{3}[W\cdot(\text{AGR})^2]_{t_1}^{t_2}.\tag{18}$$

There is a correspondence between time passage ($t_1\rightarrow t_2$) and AGR changes ($\text{AGR}_a\rightarrow\text{AGR}_b$). Thus,

$$\text{GMV of work}=\frac{1}{3}\cdot[W\cdot(\text{AGR})^2]_{\text{AGR}_a}^{\text{AGR}_b}$$
$$=\frac{1}{3}\cdot\{W\cdot(\text{AGR}_b)^2-W\cdot(\text{AGR}_a)^2\}.\tag{19}$$

Equation (19) suggests that growth mechanics version of kinetic energy (GMV of kinetic energy) is given by

$$\text{GMV of kinetic energy} = \frac{W \cdot (\text{AGR})^2}{3} \quad . \tag{20}$$

The numerator of GMV of kinetic energy [equation (20)] might look like that of kinetic energy in motion [equation (17)]. However, the denominator of GMV of kinetic energy, which comes from the symmetric property of exponential function with base e, is different from that of kinetic energy in motion.

Discussion

Growth mechanics superficially look like Newtonian mechanics in the form of equations for momentum, impulse and kinetic energy, when growth mechanics versions of them are constructed hypothetically. This may be caused by the superficial resemblance between equation of growth force and Newton's equation of motion (Shimojo *et al.*, 2006). However, coefficients show differences between growth mechanics and Newtonian mechanics. This may be due to symmetric properties of exponential function with base e in both differential and integral that relate coefficients to exponents. One of the cases for this difference is also given by the following equations (21) and (22). The relationship between momentum (p) and kinetic energy in motion is given by

$$\text{kinetic energy in motion} = \frac{1}{2} \cdot m \cdot v^2$$
$$= \frac{m^2 \cdot v^2}{2 \cdot m} = \frac{p^2}{2 \cdot m} = \frac{(\text{momentum in motion})^2}{2 \cdot m} \quad . \tag{21}$$

The corresponding relationship in growth mechanics is given by

$$\text{GMV of kinetic energy} = \frac{1}{3} \cdot W \cdot (\text{AGR})^2$$
$$= \frac{W^2 \cdot (\text{AGR})^2}{3 \cdot W} = \frac{(\text{GMV of momentum})^2}{3 \cdot W} \quad . \tag{22}$$

Comparing equations (21) and (22) shows a difference in coefficient despite the resemblance in the form of equation.

In the present study, concepts of momentum, impulse and kinetic energy in motion were introduced directly into growth mechanics. This is based on the hypothesis that growth mechanics versions of these three concepts are considered of interest as the next step toward understanding growth phenomena more deeply. There are, however, problems that should be investigated further: (1) whether this way of constructing them is appropriate or not, (2) clarifying the significance of them from the viewpoint of growth mechanics.

Conclusions

On the basis of concepts of momentum, impulse and kinetic energy in motion, growth mechanics versions of them were constructed hypothetically. The way of construction and interpretation of them should be investigated in further studies.

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