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On Galileo's Conception of Mathematical Physics

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Abstract

Galileo's theory of motion was the first successful attempt in the history of science towards the establishment of mathematical physics. This paper discusses the overall structure and genesis of his mathematical physics. The structure is presented and analyzed from three aspects: (1) Galilean abstraction or idealization, with its philosophical and methodological difficulties, (2) his mathematical method and conceptual problems that he actually confronted, with a tentative proposal of a five-stage development of his study of motion, and (3) experimental method and its five types he employed in the course of his research, as well as a concrete examination of fol.116v in Galilean MS 72. As for the origin of Galileo's mathematico-physical approach, the author stresses the significant influence of Archimedes on Galileo, not only in theoretical contents but also in a successful presentation of an "exemplar" in the Kuhnian sense, thus rejecting a medieval scholastic influence on Galileo as well as his stereotyped image as a "Platonist" or an "experimentalist."

1. Introduction

At the very beginning of the Third Day of his *Discorsi* (1638), Galileo states brilliantly that "We bring forward a brand new science concerning a very old subject,"[(1)] that is, concerning motion. He was undoubtedly right because his two chief achievements in this field, namely the discoveries of the law of naturally falling bodies and the parabolic path of a projectile, were brand new and ever-lasting. The most significant achievement, however, was not concerned with these particular results, but rather with his realization and clarification of the possibility potential for future research by combining mathematics and physics. He entered this new field by applying mathematics (geometry) successfully to his analysis of local motions. He was well aware of the originality of this endeavor, as the following remark shows: "(what is in my opinion more worthwhile) there will be opened a gateway and a road to a large and excellent of science of which these labors of ours shall be the elements, ..."[(2)] The overall structure of Galileo's mathematical physics will be developed further in section two.

The most important question is why and how it was possible for Galileo to propose the idea of mathematical physics. This question is relevant to investigating a still bigger problem which is the modern scientific view of nature. As is well known, Aristotle divided the theoretical sciences into three kinds: metaphysics (or theology), mathematics, and physics. This tripartition demanded restrictions on the applicability of mathematics to physics, and in some cases, he did not hesitate to make an even stronger claim: "mathematical accuracy is not to be demanded in everything, but only in things which do not contain matter. Hence this [mathematical] method is not that of natural science, because presumably all nature is concerned with matter."[(3)] We should remember that Galileo started and continued his academic career in a time when the Aristotelian understanding of the sciences was so dominant that it was supported by many scholars. It was a time when the application of mathematics to physics was supposed to be methodologically invalid, certainly not the simple exercise that it is today. With this general background in mind, there seemed to be many problems Galileo had to overcome in order to gain

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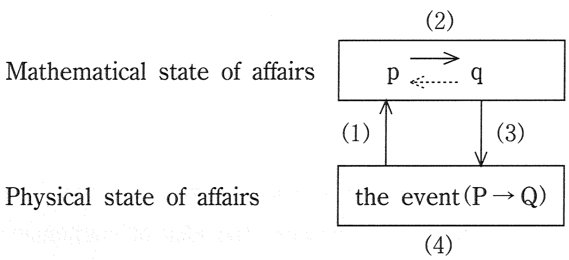
acceptance for his mathematical approach to physics and thus bear successful fruits. In this paper, I would like to clearly sketch out some of the most important aspects of his approach. In the following section I will present the general framework of his new approach with its detailed explanations, and in the last, and concluding section, I would like to call attention to some salient features of his approach, including an interpretative proposal of Galileo as an Archimedean.

2. The Overall Structure of Galileo's Mathematical Physics

First, in order to analyze the relationship between physics and mathematics in Galileo's thought, let us examine the diagram below. While it is an oversimplification, it will help one to grasp the fundamentals.



The following are some remarks that help explain the above diagram.

- step (1): abstraction or idealization
- step (2): mathematical analysis and deduction
- step (3): experimental (or observational) verification or falsification, and heuristics
- step (4): explanation (or establishment) of relationships between natural phenomena

The basic structure of Galileo's mathematical physics is summarized as follows: In order to explain phenomenal relationships found in nature, he first transposes his discussion domain to mathematics in step (1). Then in step (2) he makes, on the one hand, an analysis to find fundamental principles or rules, and, on the other, deduces some conclusions mathematically from his principles or previously established rules. Finally, he comes back in step (3) to the natural material world and tests whether his mathematical conclusions are physically true or not. It is important to note that Galileo did not intend to investigate the phenomena exclusively within the domain of physics, as it had traditionally been done by Aristotelians. In other words, step (4) consists of steps (1)−(3), and is always a result of those three steps. Physical conclusions are obtained not by direct consideration of the phenomena themselves in terms of qualitative language, but by bypassing them through a mathematical language circuit.

It is then clear that the most crucial and problematic point is how to correlate mathematical and physical states, a correlation that requires two distinct approaches: historical and philosophical. For simplification, let us consider, in order, the important points of each step.

2.1 Step(1): Galilean Idealization

Galileo tried to correlate mathematics with physics by introducing the concept of the "external or material impediments," such as the resistance of air, the friction between bodies, the shapes of bodies, and so on. According to him, if you can abstract such impediments, you are entitled to enter the mathematical domain. Let us introduce sign ε (or ε') to denote such impediments and express the correlation of the two domains by an equation, $P-\varepsilon=p$ (or $Q-\varepsilon'=q$). The problem is then expressed as: How could he think of and handle the epistemological status of ε (or ε')? This is the crucial point that would determine whether or not his attempt at establishing mathematical physics would be accepted by scholars and would require him to pursue. To anticipate my conclusion, in Galileo's historical context, there seemed to be at least two possible alternatives which are symbolized as follows:

$$P=p+\varepsilon \begin{cases} \to (\mathrm{A1}) \quad P \neq p \quad (\varepsilon \neq 0) \\ \to (\mathrm{A2}) \quad P \fallingdotseq p \quad (\varepsilon \to 0) \end{cases}$$

To use the interlocutors in his two masterpieces, *Dialogo* (1632) and *Discorsi* (1638), alternatives (A1) and (A2) are Simplicio's and Salviati's respectively. Naturally, the former is the position of Aristotelians while the latter is that of Galileo. According to (A1), the idea of mathematical physics must be denied in principle, because, as Simplicio says, "these mathematical subtleties do very well in the abstract, but they do not work out when applied to sensible and physical matters."(4) But

(A2) enables Sagredo, another interlocutor who is an uncommitted citizen, to deduce his opinion that "it must be admitted that trying to deal with physical problems without geometry is attempting the impossible"[5], so that the idea of mathematical physics is affirmed. Therefore, whether Galileo's idea would be accepted among his contemporary scholars or readers depends on his presentation of the argument, an argument that could be presented so that it denied (A1) or affirmed (A2). Did he succeed in his presentation? Let us look at two relevant passages in his works and examine them.

The first passage taken from the *Dialogo* is concerned with the problem of whether or not a sphere can touch a plane surface at a single point. Although this is trivially true in mathematics, the point is to ask if this is also true in physics. Galileo gets Simplicio to say that this geometrical theorem does not apply to physics because the weights of material spherical objects cause some deviation from their original shapes and have plenty of gaps. In short, because of "the imperfection of matter" (*l'imperfezion della materia*): "doubtless it is the imperfection of matter which prevents things taken concretely from corresponding to those considered in the abstract."[6] Salviati's response to this problem is quite simple and straightforward: since those material things do not satisfy the mathematical definitions of sphere and plane surfaces, it is quite natural that a non-spherical thing does not touch a non-smooth thing at a single point, and this is not contradictory to mathematical truth. "The geometrical philosopher" (*il filosofo geometra*) has to think of things after removing "the impediments of matter" (*gli impedimenta della materia*), and then by doing so "things in the abstract have precisely the same requirements as in the concrete."[7] In this way Galileo could defend the universal validity of mathematics in the domain of physics too. After a long discussion between interlocutors, Galileo puts an end to this discussion by letting Salviati summarize: "it seems to me that we have gone off woolgathering. Since our arguments should continue to be about serious and important things, let us waste no more time on frivolous and trivial altercations."[8]

In my opinion, however, Simplicio was not engaged in a trivial dispute. This could have been a real challenge to Galileo's research program. To use our symbols, the most important point is whether ε is a removable material impediment or a permanent property on account of "the imperfection of matter." If Galileo had made Simplicio smarter than he really was in the *Dialogo*, Simplicio could have offered a sufficient argument, emphasizing the fact that ε is not zero. For example, Simplicio could have argued in the following manner: If material bodies inevitably have complicated shapes, how can they be legitimately subjected to mathematical analysis in terms of such simple figures as spheres and planes? This was essentially the same question that Benedetti and Guidobaldo raised in their discussions of the proof-procedure for the law of the lever.[9] In order to gain greater insight in this matter, let us go to the second passage where we can find an example that shows how Simplicio, as well as Sagredo, could have been smarter.

The second passage, which is found in his *Discorsi*, contains the argument developed after one of Galileo's greatest achievements, namely the mathematical proof of the parabolic path of a projectile.[10] But this important conclusion has become a source of objections against his procedure. If a projectile really follows a parabolic path, as proved just before, it departs further and further from the axis of the parabola as it falls downwards. This inevitable conclusion seemed to be contradictory both to a "physical" understanding, and to everyone's intuition, that a heavy body would tend forward and finally reach the center of the earth, whatever path it took during its intermediate course. We should bear in mind this paradoxical nature of his famous achievement. To cite what Sagredo says, a citizen of common sense in the *Discorsi*:

> ... the axis of our parabola is vertical, just as we assume the natural motion to be, and it goes to end at the center of the earth. Yet the parabolic line goes ever widening from its axis, so that no projectile would ever end at the center [of the earth], or if it did, as it seems it must, then the path of the projectile would become transformed into some other line, quite different from the parabolic.[11]

In this connection, it is noteworthy that Galileo expressed this common sense view more than once, as revealed in his discussion of the trajectory of naturally falling bodies: one of the premises of his discussion was that "the descending weight tends to end at the center of the earth."[12] In any event, since there is no mathematical error in deducing that conclusion, some premises should be responsible for this physical impossibility. Sagredo raised his voice in criticism. After characterizing Galileo's proof procedure as an *ex suppositione* argument, he enumerated the following premises that seemed to him impossible:

[1] The uniform motion along the horizontal and the accelerated motion along the vertical maintain, respectively, their own states. (*Cf.* the conservation of motion)

[2] The composition of these two motions does not affect any change in either of them.

Moreover, Simplicio adds two uncertain assumptions:

[3] The uniform motion along the lateral direction is impossible, since the horizontal, extended in a straight line, would never be equidistant from the center of the earth. (*Cf.* Galilean circular inertia.)

[4] The resistance caused by the surrounding medium cannot be removable.

To summarize in Simplicio's words, "All these difficulties make it highly improbable that anything demonstrated from such fickle assumptions can ever be verified in actual experiences [*nella praticate esperienze*]."(13)

Facing these objections, Salviati, who is supposed to represent Galileo's point of view, gives the following answer, which is interesting and significant: "All the difficulties and objections you advance are so well founded that I deem it impossible to remove them. For my part, *I grant them all, as I believe our Author* [Galileo himself] *would also concede them.* I admit that the conclusions demonstrated in the abstract are altered in the concrete, are so falsified that horizontal [motion] is not equable; nor is the line of the projectile parabolic, and so on."(14) Using our symbols, the gist of his answer is that $P \neq p$ because $\varepsilon \neq 0$. Considering exactly the concrete physical state of affairs in question, no other answer would be admissible. It is worth emphasizing that Galileo could not completely deny (A1) as we so indicated before.

However, Galileo intended to take (A2). He tried to defend this position both by bringing forward "the authority of Archimedes" in his mathematical establishment of the law of the lever and by alluding to the practical operations "of architects." Surprisingly, his reply could be summarized as follows: Although, to be precise, the premises are false ($P \neq p$), it is unnecessary to consider these minute things in our practical activities ($P \fallingdotseq p$). In other words, he insists on the utility of such mathematico-physical conclusions within a certain range of their applicability to the real world. It was sufficient for Galileo to get an *approximate* (and *useful*) understanding rather than an exact (but often sterile) understanding. We find here in Galileo, who failed to offer convincing reasons against (A1) in favor of (A2), a still more important and innovative notion implied which can adequately be characterized as *a semantical transformation of the concept of science*: Science is a field of study searching not for exactitude (that is, an *epistēmē* or *scientia*), but rather for an approximation. This transformation was needed to move from (A1) to (A2), which Galileo did achieve in an unnoticed way. In order to buttress the interpretation presented so far— one which, I am afraid, may seem highly speculative — I would like to call attention to Galileo's letter to Baliani dated 7 January 1639 which was written after his publication of the *Discorsi*. In this letter, Galileo frankly admitted that he was "lucky" in his theory of motion, and I think that his words should be taken at face value:

> But getting back to my treatise on motion, I argue *ex suppositione* concerning the motion defined in the above way [$v \propto T$], so that even if the consequences [deduced] did not correspond to the event of natural motion of descending heavy things, it would matter little to me, just as it in no way derogates from the demonstration of Archimedes that there is found in nature no moveable that is moved through spiral lines. But in this I have been, as I shall put it, *lucky* [*avventurato*], since the motion of heavy things and its events correspond punctually [*punctualmente*] to the events demonstrated by me of the motion defined by me.(15)

It is worth emphasizing here that although Galileo's application of mathematical analysis to naturally falling bodies was restricted only to mechanics (to be strict, to kinematics), it would lead to demands for the transformation of the concept of matter: that is, to say, since motion was the key concept in Aristotelian physics in the sense of *physis* (the intrinsic nature of things to move), then the mathematical treatment of motion could lead to the mathematization of the whole field of physics. As we have seen thus far, the subtle distinction in phraseology Galileo introduced, namely "the *imperfection* of matter" and "the *impediments* of matter," can be identified as a watershed between Aristotelian and Galilean physics. Galileo's famous distinction of primary and secondary qualities, which was presented in his polemical *Il Saggiatore* (1623), seems to have been closely connected with his idea of mathematical physics.

2.2 Step(2): Mathematical Method

The following is Galileo's famous statement in his polemical *Il Saggiatore*: "Nature is written in the language of mathematics,"[16] which is made up of triangles, circles, and other geometric figures. It seems to me, however, that he failed to give us a precise description of the nature of this language, as often happens in polemical books like this. The core of this mathematical language does not consist of geometrical figures, but of proportional calculations based on them. It was the theory of proportion that was Galileo's principal mathematical weapon used to attack the problems of nature, especially those of naturally falling bodies. This was not incidental since the theory of proportion was the only theory of general magnitudes (both discrete and continuous) available at that time. To understand the development of his theory of motion, examining this language is critical. Because I have already written in an article on this topic, I will not discuss it at this time.[17]

In order to gain a clearer understanding of Galileo's theoretical investigation in step (2), a historical, not a philosophical, approach is required. In other words, we have to reconstruct the steps he followed in his investigations instead of following the theoretical results contained in his final, written version. Since 1972, when S. Drake published his study on one part of Galilean manuscripts (these have been preserved as Galilean MS 72 in the Central National Library, Florence), many historians of science have proposed historical reconstructions. Instead of examining these reconstructions, I would like to propose my own version which differs from these in some important ways. (*See* the Appendix A which gives an outline of the development of his theory of motion according to my reconstruction.)

For Galileo to develop the mathematical theory of motion of falling bodies, it was absolutely necessary for him to know at least one mathematical relationship concerning the motion of natural fall. The starting point for his development of the theory was probably supplied by his discovery of the times-squared rule ($S \propto T^2$). He had discovered this rule by 1604 at the latest, because in his letter to Sarpi, dated 16 October 1604, Galileo reported the possible proof of this rule. Moreover, almost all Galilean students concur that the very proof hinted in the letter is preserved in fol.128rv. Thus, this folio is deemed, beyond a doubt, to have been composed around 1604. As for this folio and the letter, we should keep in mind two important facts which give us important clues to our interpretative attempt for the historical reconstruction of his theory of motion: (1) The new principle, from which he deduced the times-squared rule, was in fact false, because it stated the direct proportionality of the instantaneous velocity (v) and the space traversed (S) [that is, ($v \propto S$)]; and (2) he had at this time two concepts of velocity, that is, instantaneous velocity (v) and overall, or total (to use the medieval

Table I

Abbreviations	Rules or Techniques	Corresponding Propositions in *Discorsi*
GP	Galileo's Postulate	Before Prop.1
Ar-tech	the Area Technique	Cf. Prop.1
TS-rule	the Times-Squared Rule	Prop.2 ($S \propto T^2$)
ON-rule	the Odd Numbers Rule	Prop.2, Corollary 1
LT-rule	the Lengths-Times Rule	Prop.3 ($T \propto L$)
Ch-rule	the Chords Rule	Prop.6
RA-rule	the Right-Angle Rule	Cf. Prop.9
TS-appl	the Application of the TS	Prop.11
2V-rule	the Double Velocity Rule	the Scholium of Prop.23
2D-rule	the Double Distance Rule	the Scholium of Prop.23

terminology), velocity (V), as is evident from his proof.

Therefore, from 1604 on, Galileo's study of motion was mainly devoted to manipulating these two facts. In his struggle to find their solutions, Galileo devoted his energy to the mathematical analysis and synthesis of the theoretical assertions implied in the problems, and we find him engaged in *conceptual* problems, rather than experimental or empirical ones, as will be shown in the next section. His efforts brought him several theoretical results, including both erroneous and accurate conclusions. To show the plenitude of his results, and to simplify the explanation, let us introduce some abbreviations of these results in the tabular form (Table I).

In table I, I arranged the rules, techniques and such in the order of propositions found in his final version, the *Discorsi*. But we should bear in mind that the order did not reflect the actual chronology of their discovery. My investigation of both Galilean MS 72 and other published books and letters in his *Le Opere* led me to a proposal of the following five stages of his study of motion, as shown in Table II.

Let us quickly look at the essentials of each stage in order to see clearly how he struggled in the mathematical domain.

At stage (I), he knew the old form of the LT-rule[18] with the exclusive use of the V-concept and the mechanical consideration of an inclined plane. Even in the first stage it is conspicuously evident that he was critical of Aristotle's theory of motion and that he proposed his own theory of motion, which had been strongly influenced by Archimedes's study of statics. Moreover, he formulated the Ch-rule to prove the isochronism of a pendulum by using mechanical concepts, as his letter to Guidobaldo del Monte on 29 November 1602 indicates.[19] Finally, we can surmise from his letter to Sarpi that Galileo discovered experimentally the ON- and TS-rules before 16 October 1604.[20]

While stage (I) was only preliminary, stage (II) was actually the starting point for his study of naturally falling bodies. The introduction of the concept of instantaneous velocity (v) was thought to be revolutionary by Galileo because in the preceding stage he had considered the acceleration of natural fall to be a temporary phenomenon.[21] However, as I mentioned before, he erroneously deduced the TS-rule from the false principle ($v \propto S$) via the Ar-tech (which was supposed to offer a correspondence between v and V). After recognizing his computational error in compounded ratios, he first questioned the Ar-tech and then the V-concept. In his mathematical analyses of several rules which were known to him, especially of the LT- and RA-rules which were closely related to the V-concept, he seems to have obtained his GP as a principle to deduce the LT-rule in 1609. This is clear from Galileo's letter, which was lost, to Valerio of 5 June. In this letter, Galileo asked Valerio, among other things, the validity of GP, which can be surmised from Valerio's answer to Galileo dated on 18 July, 1609.[22]

Contrary to the current interpretations regarding the development of Galileo's theory of motion, which assume the completion of his theory of motion at this stage (II), we should bear in mind that although the GP succeeded in giving a new meaning to instantaneous velocity, it did not succeed in solving the problem of whether ($v \propto S$) was correct or not. The facts are diametrically opposite to what current interpretations assume, which becomes manifest in an examination of stage (III). The most important event at this stage was the fact that Galileo asked his friends, Arrighetti and Guiducci, to make fair copies of his Paduan notes on motion. According to S. Drake,[23] the copyists worked for him around 1618 and produced 35 copies. I have examined the theoretical contents of these copies[24] and have come to the following conclusions: (1) Galileo still held to the old assumption ($v \propto S$), because there is no evidence in the manuscripts to support the hypothesis that he had come to the correct principle at this time, namely ($v \propto T$); (2) he was still ambivalent as to which was the most theoretically significant and fundamental of the three rules, TS-, LT-, and Ch-rules; (3) Prop.1 which has often

Table II

(I) c.1590-1604	from *De motu* until Galileo's letter to Sarpi
(II) 1604-1610	from Galileo's letter to Sarpi until Galileo's return to Florence
(III) 1610-c.1625	from Galileo's return to Florence until his composition of the *Dialogo*
(IV) c.1625-1638	from his composition of the *Dialogo* until the publication of the *Discorsi*
(V) 1638-1642	from the publication of the *Discorsi* until his death

been regarded as a reproduction of the medieval "Merton-rule (or the Mean-Speed theorem)" does not appear at all in the fair copies, although we find almost all propositions of Part II of Galileo's treatise being copied at that time; and finally, (4) we do not find any copies of Parts I (uniform motion) and III (projectile motion) of his treatise.

I propose that stage (IV) is a distinct epoch, because the correct principle, ($v \propto T$), was discovered in this stage. I believe that the principle first appeared when he tried to prove the 2D-rule in the *Dialogo* (1632), the drafts of which were written between 1625 and 1629. Galileo mentioned the 2D-rule twice in the First and Second Day sections of the book. The complete proof of the 2D-rule was given in the Second Day, whereas it was only mentioned in the First Day. It is very important to note that the second appearance seems to have been inserted as an interlude, interrupting the main stream of argument on the diurnal motion of the earth.[25] While the exact date of the discovery of the proof of the 2D-rule cannot be determined, Galileo probably hit upon this proof in either 1625 or 1626. This is because the 2D-rule was inserted into the Second Day with a proof. This seemingly unusual appearance was meant to supplement Galileo's "proof-less" mention of the 2D-rule in the First Day.

At stage (V) the most significant problem was how to validate his GP, as shown by the fact that his disciple, Viviani, inserted its proof before proposition 3 as a posthumous work in the 1656 edition of the *Discorsi*.

We have thus far seen Galileo's study of motion in its main outline. What strikes us most during this long and hard period of his search for the fundamentals is the extremely long duration of the false principle ($v \propto S$), as well as the tenacity with which he held it. Even to Galileo the mathematical language of nature was so difficult to read that even if he adopted the mathematical reading of nature, he could not immediately obtain the full-fledged theoretical fruits (say, in a couple of years). It is true that he knew some rules at the earliest stage of his study, and many at the next stage, but his principal task was to find fundamental principles or a definition of natural fall that would enable him to deduce and establish the correlations of these rules. And this sort of work was purely mathematical.

In concluding this section, I would like to pay attention to some benefits of the application of mathematical method to physics. We can recognize at least three benefits. First, contrary to Aristotelian physics, Galileo could bring a clear structure of inference into his theory of motion, which did not permit any kind of equivocation. To use Popperian terms, it increased the falsifiability of the theory. Second, in his thought he could attain certitude in his theory by way of a mathematical circuit, since he had a unique notion of mathematics, insisting that only in mathematics could human beings obtain certain knowledge, equal to God's knowledge, even though in a restricted sense. Thus Galileo wrote:

> ...... it is best to have recourse to a philosophical distinction and to say that the human understanding can be taken in two modes, the *intensive* or the *extensive*. *Extensively*, that is, with regard to the multitude of intelligibles, ... But taking man's understanding *intensively*, in so far as this term denotes understanding some propositions perfectly, I say that the human intellect does understand some of them perfectly, and thus in these *it has as much absolute certainty as Nature itself has. Of such are the mathematical sciences alone; that is, geometry and arithmetic,* in which the Divine intellect indeed knows infinitely more propositions, since it knows all. But with regard to those few which the human intellect does understand, *I believe that its knowledge equals the Divine in objective certainty,* for here it succeeds in understanding necessity, beyond which there can be no greater sureness.[26]

Third, his theory of motion served a heuristic function. We can find a good example of this, which is concerned with propositions 7 and 8 of part III of the *Discorsi*. Let us hear what Salviati says:

> The knowledge of one single effect acquired through its causes opens the mind to the understanding and certainty of other effects without need of recourse to experiments. That is exactly what happens in the present instance [prop.7]; for having gained by demonstrative reasoning the certainty that the maximum of all ranges of shots is that of elevation at half a right angle [prop.7], the Author

demonstrates to us something that *has perhaps not been observed through experiment;* and this is that of the other shots, those are equal [in range] to one another whose elevations exceed or fall short of half a right angle by equal angles [prop.8].[27]

2.3 Step(3): Experimental Method

Let us begin with the general discussion of the concept of experiment according to the framework given in section 2. I proposed to express the relationship between two domains (physics and mathematics) as $P-\varepsilon=p$ or $Q-\varepsilon'=q$. For Galileo who wanted to advocate (A2) instead of (A1), it was requisite and indispensable to make a setting that would reduce ε or ε' as smaller and smaller as possible. If otherwise, the mathematical deduction of q from p would have nothing to do with the natural course of the event $(P \rightarrow Q)$. He was obliged to have an artificial setting for minimizing the impediments of matter. This approach is exactly the employment of modern scientific experiment which is differentiated from that of observation in the strict sense of the word. It deserves special emphasis that the notion of mathematical physics requires experimentation, not the other way around. Moreover, an artificial setting of experiment requires in turn the human involvement with natural events. Galileo's experiment of inclined planes[28] offers a good illustration of this point: before making a very hard bronze ball descend, he prepared a wooden beam and rabbeted a very straight channel on it, and finally glued a piece of vellum within that channel to make it as smooth and clean as possible. The active human involvement is manifest, however primitive it may seem to us, and this involvement can be seen as one of the features of technology. That Galileo's mathematical physics demanded experiment means in depth the beginning of the unification of science and technology. This aspect of experiment is in excellent harmony with his semantical change of the meaning of science that we alluded to in section 2.1.

In regard to his alternative (A2), I would like to emphasize the notion of "experimental errors." Galileo did not develop a theory of experimental errors, but his practice of various experiments did show that he took full advantage of that notion. In my opinion this practice should also be counted as one of his greatest contributions. In his working notes, however, we find on his part some uneasiness and reluctance against the positive use of that notion, as will be shown in the examination of fol. 116v. But we can understand well the reason why he not only had the uneasiness but also achieved that contribution *implicitly* in his practice: because in Galileo's time "experimental error" meant nothing but the "error of experiment" which invited one immediately to assume the *in*applicability of mathematics to physics and consequently to repudiate Galileo's conception of mathematical physics.

Now let us take up Galileo's experiments and examine their methodological implications. But before doing so, let us see in advance why and where one feels the needs for experiments, in order to make our analysis systematic. In general, mathematical deduction takes the form of the conditional statement: if p, then q, that is $(p \rightarrow q)$. If one succeeds in establishing the truth of p, there is no need at all to test the truth of q as long as the latter is correctly deduced. This means that Galileo did not need to test all the conclusions he got mathematically. In Galileo's final version of the theory of motion, there remain only two such fundamental propositions that *can* serve as p for the subsequent propositions: one is the definition of naturally accelerated motion, $(v \propto T)$, while the other is the GP (Galileo's Postulate) which demands the equality of instantaneous velocities acquired at any points on an inclined plane and on the vertical whose vertical distances are the same. Although both propositions contain the concept of *instantaneous* velocity, so that its direct measurement is logically impossible on account of no lapse of time, nevertheless in Galileo's judgment the latter can be subject to indirect measurement, whereas the former cannot be.

Moreover, if we turn our eyes from his final version to the context of his discovery of such fundamentals, we find two other possibilities for the need of experiment. One is concerned with the discovery of the basic proposition q which could serve as the foundation for subsequent research. In Galileo's case this was nothing but the TS-rule discovered before 1604. The other is concerned, to use the Baconian terms, with the "crucial experiment." As mentioned before in section 2.2, Galileo was fooled by the false principle $(v \propto S)$, and found the true principle $(v \propto T)$ after a long interval of time. In such a case or its equivalent case where he held two incompatible propositions at the same time and moreover had no other theoretical reason for deciding which was correct, then the need for an experiment was in order. Therefore, it is reasonable to assume *five* possible cases in regard to whether or not Galileo needed an experiment. In what follows, the first two cases are related to the context of discovery while the last three are concerned with the context of justification.

(1) The case in which the basic proposition q is discovered.

As said earlier, the TS-rule was the proposition of this type. Its discovery document is probably fol.107v, the interpretation of which is in opposition to Drake's recent interpretation but in agreement with his initial one.[29] In addition, the ON-rule, which is equivalent to the TS-rule and received also an explicit mention in Galileo's letter to Sarpi, may be counted as of this type. It is important to note that Galileo never doubted and could not doubt the validity of these rules once he had discovered them, for to cast doubt on them would have meant the impossibility for him to develop the mathematical theory of natural fall.

(2) The case in which two incompatible propositions are at his hand.

As mentioned before, this case is concerned with the so-called "crucial experiment." As far as I know, there is only one Galilean note, fol.116v, that belongs to this category. This folio has received much attention of Galilean scholars from their different points of view ever since the first analysis done by Drake.[30] The characterization above is, of course, my interpretation. I will take up this folio again in greater detail in order to support my interpretation of it as well as to get a deeper insight into the actual (not the purported) state of the experiment that Galileo performed.

(3) The case in which q is subject to experiment while p is not.

The case is illustrated by Galileo's famous experiment of an inclined plane where p is $(v \propto T)$ and q is $(S \propto T^2)$. Although the direct verification of p, if possible, was preferably Galileo's ideal, this sort of work was well beyond his experimental capability and the technology of his time. Therefore he was obliged to be content with presenting a plausible argument for p being true by a mere allusion to "the close affinity ... between time and motion."[31] As for the experimental verification of q, Galileo stressed the truth of q by letting Salviati say that

> ... we made the same ball descend only one-quarter the length of this channel, and the time of its descent being measured, this was found *always to be precisely to the point* [*sempre puntualissimamente*] one-half the other. Next making the experiment for other lengths, examining now the time ..., by experiments repeated a full hundred times, the spaces were *always* found to be to one another as the squares of times. And this [held] for all inclinations of the plane; ... we observed also that the times of descent for diverse inclinations maintained among themselves *accurately* [*esquisitamente*] that ratio that we shall find later assigned and demonstrated by our Author.[32]

The empirical establishment of q is usually thought to be closely related to that of p. In a sense this procedure in case (3) resembles "the Hypothetico-Deductive and Experimental Method" in modern science. But we should not jump to a conclusion that Galileo was satisfied with this method, as will be discussed later. Moreover, what surprises us most is the fact that Galileo did not give any real experimental data in his text that we may expect him to provide, although he tried to persuade readers into accepting the validity and accuracy of his experimental results, as is clear by the italicized parts of the quotation above. We will consider later the implications of this fact.

(4) The case in which p is subject to experiment.

A good example of this case is provided by the experiment of his GP which Galileo himself called as "one single principle [*un solo principio*]."[33] This principle as well as his definition of the naturally accelerated motion was one of the cornerstones of his theory of motion. The ingenious experiment for establishing the truth of this principle was the device of the pendulum with some nails placed at different vertical positions. It is unnecessary to go into the details of the experiment. Rather what deserves our attention here is the significance Galileo attached to this experiment. Before introducing this experiment, Galileo first let Sagredo say that "this assumption [the GP] truly seems to me to be *so probable* [*tanto* *del probabile*] as to be granted without argument, ..."[34] and let him admit the validity of GP. But second, immediately before the explanation of the experiment, Galileo has Salviati say very cautiously: "You [Sagredo] reason from good probability [*probabilmente discorrete*]. But apart from mere probability [*oltre al verisimile*], I wish to increase the probability so much by an experiment that *it will fall little short of equality with necessary demonstration* [*volio con una esperienza accrescer tanto la probabilità, che poco gli manchi all' agguagliarsi ad una ben necessaria dimostrazione*]."[35] And as to the effectiveness of the experiment, we find several such laudable expressions scattered in the dialogue: "this experiment leaves no room for doubt as to the truth of our assumption, ..." (Salviati) and "The argument appears to me conclusive, and the experiment is so well adapted to verify the postulate that it may very well be worthy of being conceded as if it had been proved." (Sagredo).[36] To our astonishment, however, Galileo's conclusion was that the experiment was insufficient for establishing the truth of GP on account of two reasons: (1) because an accelerated motion along a straight surface (the case necessary for the GP) is not the same as that along a curved one (the case in the experiment); and (2) a ball descending along an inclined straight plane would inevitably encounter obstruction from the ascending plane at the junction point of both planes. Let us see Salviati's concluding remarks:

> Hence let us take this [GP] for the present as a *postulate* [*postulato*], of which *the absolute truth* [*la verità assoluta*] will be later established for us by our seeing that other conclusions, built on this hypothesis [*sopra tale ipotesi*], do indeed correspond with and *exactly* [*puntualmente*] conform to experience.[37]

In short, this case (4) was reduced by him to case (3) in his last analysis. At any event, it is an impressive (and, one might say, obsessive) fact that Galileo tried to establish the GP as exactly as possible. His attitude of seeking for an exactitude is diametrically opposite to that which we saw in section 1 for his deduction of parabolic trajectory, a fact which shows that he was still, though ambivalently, captured by Aristotelian ideal of demonstrative knowledge.

(5) The case in which there is no need of experiment.

Typical examples of this case are both his assertion that "the stone falling from the ship's mast strikes in the same place whether the ship moves or stands still."[38] and propositions 7 and 8 concerning the angles of shots we quoted before. His rejection of the need of experiment seems to derive from his conviction of the truth of p, because from p and $(p \rightarrow q)$ the truth of q is deductively established by the *modus ponens*.[39]

An Examination of fol.116v

In concluding this section on Galileo's experimental method, I would like to take a concrete example, an experiment on fol.116v, that shows his actual activities in this field, and to give a capsule summary of the preceding discussions.

To begin with, let us see the folio itself and the partial transcription by Drake which are respectively reproduced in Appendices B1 and B2. Note that Drake omitted two portions of the folio, which caused a grave misunderstanding of the intent and the significance of this experiment. A schematic representation of the experiment is given in Appendix B3, as well as the theoretical and experimental values in the tabulated form, the contents of two omitted portions, and the three formulae behind his calculations.

Generally speaking, an experiment is artificially devised. In other words, it is a "theory-laden" activity. In this instance, Galileo used many theoretical assumptions for the construction of this experiment except for material hindrances. These were as follows:

[1] the GP (his implicit assumption of the equality of instantaneous velocities at B and F: $v(B)=v(F)$)[40]
[2] the conservation of uniform horizontal motion after the ball's departure from the table
[3] the constancy of the time of fall from B to C for every fall (this is, of course, based on the TS-rule.)
[4] the composition of velocities or the independence of motions

The involvement of many assumptions in an experiment causes a variety of interpretations of its true intent. However, if one does not ignore any calculations written on the folio as ignored by Drake, I think it an inevitable conclusion that Galileo here tried a crucial experiment as to whether ($v \propto S$) or ($v \propto \sqrt{S}$). The experimental results would seem to lend support to the second hypothesis, not to the first. There are some historians who, wishing to see in Galileo "an experimental scientist," have applauded the accuracy of this experiment.[41] However, if Galileo had truly given the same whole-hearted support to the modern Hypothetico-Deductive and Experimental method (*abbr.* H-D-E method), he could have established the correct law, ($v \propto \sqrt{S}$), at this moment. But I think that to make Galileo a modern experimental scientist as well as an advocate of the H-D-E method is wrong for two reasons.

One is concerned with the inconsistency with other facts. If ($v \propto \sqrt{S}$) were established at this moment, then the correct principle ($v \propto T$) would have been obtained, since this was easily deduced when combined with the TS-rule ($S \propto T^2$). But this conclusion is quite contrary to the facts that we mentioned in regard to the stage (III) in section 2.2.

The other reason is supplied from an examination of the omitted portion at the lower right-hand corner. The calculation done there clearly shows that Galileo changed the standard value D_0 from 800 to 820. This fact indicates that the discrepancies between theoretical and experimental values were far from negligible to Galileo himself, contrary to the expectation that the advocates of H-D-E method might deem them to be permissible and satisfactory. But even the increase of the standard by 20 punti (=19 mm) did not succeed in giving the experimental value, 1172. Moreover, Galileo crossed out number 2 in his new standard 820 by a slash mark in the same way as found in other calculations (*See* the folio in Appendix B1). This fact lends full support to the interpretation that this experiment was unsuccessful in his thought.

We can learn an important fact from this experiment – namely, that Galileo had a strong quest for the agreement between theoretical and experimental results. This quest goes well beyond the limit that we, who are already accustomed to the notion of experimental errors, might allow. It is very instructive that Galileo never mentioned this experiment in his published books, though this received a fine sophistication and was one of the well-designed experiments done by Galileo. If we take into account this sort of strong quest revealed in his private notes, we can well understand one of the main reasons why in his published books he sometimes put strong emphasis on the accuracy of his experiments to the extent of overstatement. This was exactly one aspect of his rhetorical strategy.

Moreover, other aspects of the same are easily discernible for us who have seen various types of his experiments up to this point: for example (1) the lack of any real experimental data; (2) the deliberate silence as to some of his basic assumptions, such as assumptions [2] and [4] in fol.116v and premises [1]-[3] for the proof of the parabolic trajectory mentioned in section 2.1; and (3) the excessive scrutiny of a particular principle [GP]. (Cf. our foregoing discussion of case (4).) The employment of a dialogue style in his masterpieces was perfectly suitable for his control of these rhetorical skills. In order to have the conception of mathematical physics accepted among his contemporaries, Galileo absolutely needed these kinds of rhetorical strategy, since he knew well more than anyone else that there were always some discrepancies between theoretical and experimental values.

3. Concluding Remarks

Let us summarize in capsule some salient features of Galileo's approach. Galileo employed a mathematical approach to the problem of natural fall. The mathematical idealization of physical states of affairs [Step (1)] required a new look at matter as an *impediment*, something preferably removable for the purpose of developing his theory of motion. After having entered the mathematical domain, once he found the TS-rule and its equivalent ON-rule as his theoretical footings [Step (2)], his whole energy was devoted both to the investigation of some further fundamental principles and to the establishment of theoretical relationships between a variety of rules. As far as this aspect of his research was concerned, his experimental activities did not figure as heavily among his concerns as would be expected from Galileo the experimentalist. Therefore the stereotyped image of Galileo is far from the reality. The intervention of experiment into his concerns was almost restricted to a few crucial instances [Step (3)]: the TS-rule (and the ON-rule), the selection of incompatible assertions when there were no other means, and the candidate principles for the whole edifice of his theory (the definition of natural accelerated motion and the GP). Steps (1) and (3), which are bridges between two domains (the mathematical and the physical), are asymmetric in a sense: in step (1) Galileo could abstract some factors as being irrelevant and neglect them, whereas in step (3) he was inevitably involved in these neglected and often uncontrolled factors. As we have seen in section 2.1, he confessed that "the conclusions demonstrated in the abstract were altered in the concrete." Nevertheless, he was convinced of the practical utility of his conclusions and thus changed implicitly and significantly the traditional Aristotelian meaning of science as demonstrative knowledge.

As mentioned in section 2, the phenomenal relationships [Step (4)] are established by taking a mathematical detour [Steps (1)-(3)]. From the standpoint of causation, however, Galileo's theoretical procedure seemed to avoid the essential problem: what is the cause of the acceleration of natural fall? It is generally accepted among historians of science that Galileo changed the form of question from "why?" to "how?"(42) However, from an Aristotelian point of view, Galileo could only offer the formal cause, ignoring other causes. It is true that Aristotelians would have accused of Galileo for having engaged only in the trivial mathematical matters of falling bodies instead of having tried to answer the causal questions of falling bodies. Indeed, we find that he refrained from investigating the causal aspect of this phenomenon: "The present does not seem to me to be an opportune time to enter into the investigation of the cause of the acceleration of natural motion, concerning which various philosophers have produced various opinions, ..."(43) But it seems to me that Galileo's banishment of the search for causes from his mathematical physics was intended not to be eternal, but to be temporary as revealed by the underlined part of the quotation above. We have already seen revealed in his rhetorical strategy that Galileo held deep in his mind the Aristotelian ideal of demonstrative knowledge. As for this change of question-style, we should interpret Galileo's attitude to be ambivalent rather than definite.

Before going into the examination of the genesis of Galileo's conception of mathematical physics, we cannot escape the problem of medieval influence on Galileo. As for a relation between Galileo's theory of motion and medieval kinematics, my conclusion is that he received no influence of theoretical significance from medieval scholastic endeavors. It is true that he inherited and used such words of scholastic origin as *impetus, velocitas, gradus velocitatis* and so forth. The use of medieval vocabulary, however, does not prove that he was under the influence of medieval kinematics, since this fact simply tells that nobody can work in an intellectual void. Rather we have to pay attention to the function that the relevant words, concepts, and rules are supposed to have in a particular system, as well as to their metamorphoses from one system to another. Galileo's gradual sophistication of the concept of instantaneous velocity will offer a good example on this point, but a detailed explanation is well beyond the scope of this paper.

Instead, let us take an example of "the Merton rule," which has often been said to be an instance of medieval influence on Galileo. In the Oresmean version of this rule, the instantaneous velocity was unquestionably supposed to be proportional to the time elapsed ($v \propto T$).(44) If Galileo had known this rule, there could have been no such struggle on his part that we have seen in section 2.2. Even if we concede that some other version than Oresme's had been available to Galileo, which presented at any rate two alternatives, ($v \propto S$) or ($v \propto T$), as to the "*uniformiter difformis motus,*" he could have been easily familiar with ($v \propto T$) as the other alternative, the consequence of which is again contradictory to the actual process of Galileo's acquisition of ($v \propto T$). Anyway, it is manifest to me that any assumption of possible influence on Galileo of medieval kinematics stands on so shaky foundations that it may legitimately be eliminated.

Finally I would like to think about the historical origin of Galileo's conception of mathematical physics. Many scholars interested in Galileo have offered their interpretations on this particular topic. I am afraid that Galileo's lines, "Nature is written in the language of mathematics," became too famous to conceal his reality. From these lines, different scholars have read different implications: for example, the predominance of Platonic mathematicism as in Koyré, the emergence of modern scientific questions of "How?" instead of "Why?" as in Mach, and the methodological importance of experimentation as in Drake.(45) I do not think that they were entirely wrong, and I frankly admit that each has its own advantage as an *ex post facto* interpretation of the victory of Galilean mathematical physics. What really constitutes a historical problem, however, is to set and solve a question: What did these lines really mean in Galileo himself? The lines insisted neither on the mathematical structure inherent in the reality of nature, nor on the legitimate reading of the Book of Nature, since he neither elaborated the ontological theory of nature and epistemological theory of human understanding in general(46) nor did he speak anywhere of legitimate reading thereof itself except for the relative superiority of his in regard to the other ways of reading.

I would like to understand Galileo's famous lines as his manifestation of his innermost decision and hope to read the Book of Nature. As we have seen in the preceding section, he had *no* warrant to read the Book of Nature that way, being surrounded by many theoretically insurmountable difficulties. Nevertheless, with his mathematical arsenal, Galileo dared to step in further to a realm where Aristotelian physics had made itself quite at home. To put this in other terms, a Paduan professor of mathematics crossed over the traditional disciplinary boundary and intruded into physics. Whether or not "nature is written in the language of mathematics" was not a matter of logical persuasion, but rather a matter of practice. The question must have been solved by working out in a particular field of research. Moreover, it was his "lucky" success in a particular field, the kinematics of natural fall, that brought home to Galileo the ultimate legitimacy to his way of reading the Book of Nature and therefore to his conception of mathematical physics.

Descartes criticized Galileo on account of having constructed his science without foundations and only seeking reasons for particular effects.(47) However, what Descartes developed by his reduction of physics into mathematics through his unique concept of matter (*res extensa*, or in our terms in section 2.1, P=p) was nothing but mathematical physics *without* mathematical formulations (except for a few instances). In order for mathematical physics to be the "paradigm" of future generations, of crucial importance was the particular success, or in Kuhnian terms, the presentation of an "exemplar." In this sense the formation of mathematical physics must be credited to Galileo, not to Descartes.

In this connection, I would like to put special emphasis on the historical significance of Archimedes to Galileo. To speak of "Archmedeanism" in Galileo is far more important than to speak of "Platonism" or "Aristotelianism" in him.(48) What I want to say is not a mere fact that reference to the name of Archimedes is found throughout his writings, often with applause (for example, "the *divine* Archimedes"), from his first treatise (*La Bilancetta*) to his last one (*Discorsi*).(49) More significantly, it can be said that Archimedes was of double importance to Galileo. One is concerned with the theoretical inputs of the former's statics into the latter's kinematics, as shown in Galileo's *De motu*. It was the Archimedean way of understanding the laws of the lever and buoyancy that provided Galileo, from the earliest phase of his career, with the key to his new theory of motion in opposition to the Aristotelian one. The other is relevant to the methodological aspect of science. One of the greatest difficulties for Galileo was related to the *ex suppositione* argument as we have seen in section 2.1. In our terms in that section, his adoption of (A2) instead of (A1) found its validation, in the last analysis, in the fact that it was practiced by Archimedes himself. And this way of validation had also been employed by Galileo in his early writing.(50) On this point, he was surprisingly consistent from beginning to end. Galileo did not commit himself to any version of ontology and epistemology,(51) nor did Archimedes seem to do so. It seemed sufficient for Galileo to have Archimedes as an "exemplar" of the scientific research program without recourse to any philosophical doctrine for the justification of his approach. Therefore, it seems to me to be necessary and sufficient to say that Galileo was an Archimedean.

Notes

(1) Consulting the standard edition of Galileo, that is, *Le Opere di Galileo Galilei,* 20 vols., ed.A.Favaro, Firenze, 1890-1909; 4^a ed. 1968 (hereafter *Opere*), I used the English translations of his two masterpieces done by S. Drake: *Dialogue Concerning the Two Chief World Systems,* trans.S.Drake, Berkeley: Univ. of California Press, 1967 (hereafter *TCWS*) and *Two New Sciences,* trans.S.Drake, Madison, 1974 (hereafter *TNS*). The citation is from *Opere* VIII, p.190; *TNS*, p.147.

(2) *Ibidem.*

(3) Aristotle, *Metaphysica,* II-3.

(4) *Opere* VII, p.229; *TCWS,* p.203.

(5) *Ibidem.*

(6) *Opere* VII, p.233; *TCWS,* p.207.

(7) *Ibidem.*

(8) *Opere* VII, p.236; *TCWS,* p.210.

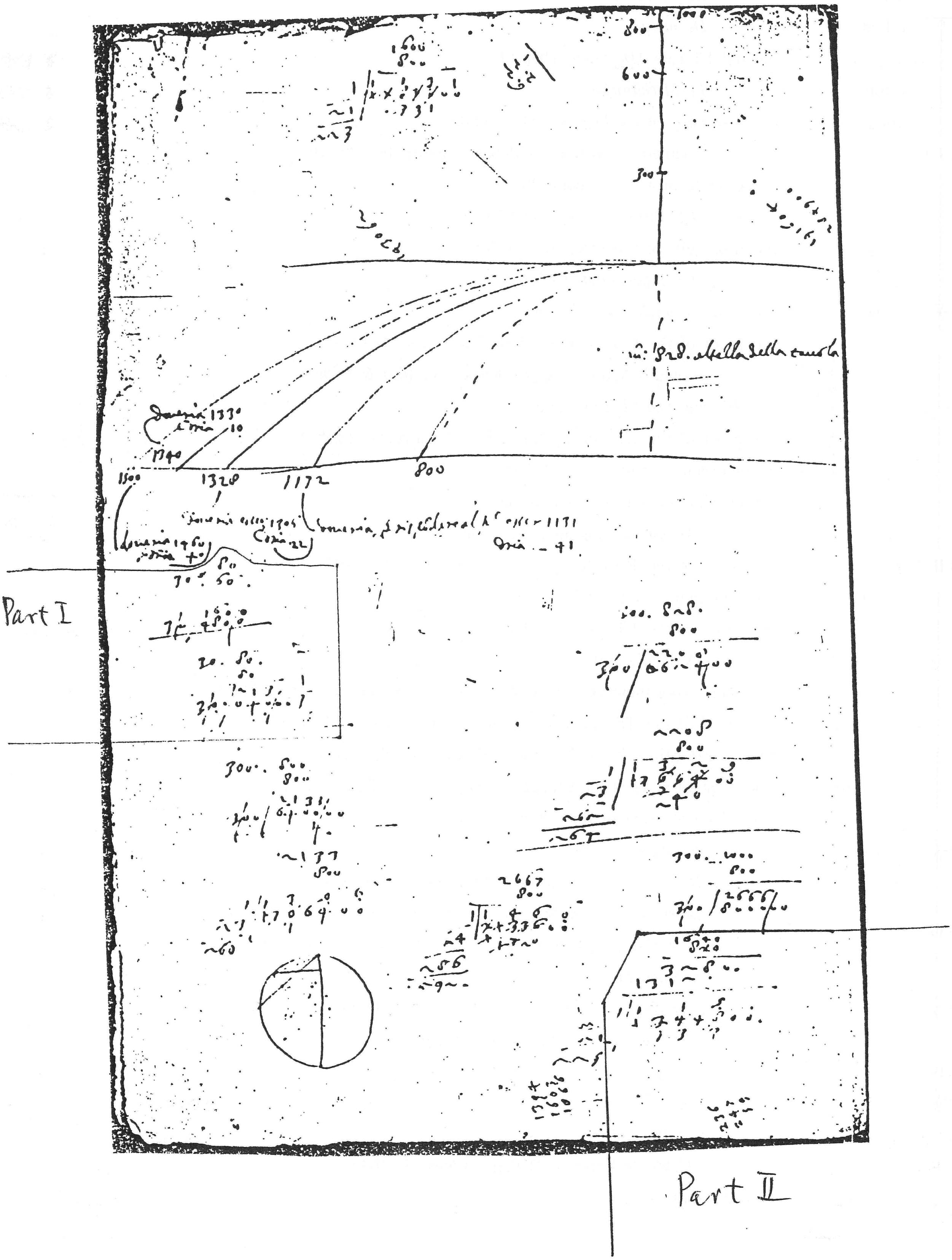
(9) *Cf.* Wallace, William A., *Galileo and His Sources: The Heritage of the Collegio Romano in Galileo's Science,* Princeton

U.P., 1984, pp.203-206.
(10) *See* The Fourth Day of *Discorsi,* Proposition 1.
(11) *Opere* VIII, p.274; *TNS,* pp.222-223.
(12) *Opere* VII, p.191; *TCWS,* p.165.
(13) *Opere* VIII, p.274; *TNS,* p.223 with a minor change.
(14) *Ibidem.* Emphasis added.
(15) *Opere* XVIII, pp.12-13. English translation is from S. Drake, *Galileo at Work: His Scientific Biography, Chicago,* 1978, p. 396.
(16) *Opere* VI, p.232.
(17) For a detailed study on this topic, see my article "Galileo's Labyrinth: His Struggle for Finding a Way out of His Erroneous Law of Natural Fall. Part 1," *Historia Scientiarum,* No.48 (1993), pp.169-202, especially pp.171-175.
(18) See Galileo's *De motu* [chapter 14] in *Opere* I, pp.296-302 and its English translation by I.E.Drabkin in *On Motion and On Mechanics,* trans.I.E.Drabkin & S.Drake, Madison, 1960, pp.63-69.
(19) *Opere* X, pp.97-100.
(20) *Opere* X, pp.115-116.
(21) See Galileo's *De motu* [chapter 21] in *Opere* I, pp.328-333 and its English translation by I.E.Drabkin in *op.cit.* [n.18] pp. 100-105.
(22) *Opere* X, pp.248-249. The relevant passages are translated by me in "Galileo's Labyrinth: His Struggle for Finding a Way out of His Erroneous Law of Natural Fall. Part 2," *Historia Scientiarum*, No.49 (1993), pp.28 *ff.*
(23) S.Drake, *Galileo at Work,* pp.262-263.
(24) Ken'ichi Takahashi, "A Historical reconstruction of Galileo's Theory of Motion (1610-1625): An Exodus from His Conceptual Labyrinth," *Bulletin of the Graduate School of Social and Cultural Studies, Kyushu University,* vol.2 (1996), pp.81-93, in Japanese.
(25) *Opere* VII pp.248-256; *TCWS,* pp.221-230.
(26) *Opere* VII pp.128-129; *TCWS,* p.103. Emphasis added.
(27) *Opere* VIII p.296; *TNS,* pp.245-246. Emphasis added.
(28) *Opere* VIII pp.212-213; *TNS,* pp.169-170.
(29) For the details, see S.Drake, *Galileo: Pioneer Scientist,* Toronto: Univ. of Toronto Press, 1990, especially chapters 1 and 7.
(30) For examples, S.Drake, "Galileo's Experimental Confirmation of Horizontal Inertia: Unpublished Manuscripts (Galileo Gleanings XXII)," *Isis*, vol.64 (1973), pp.291-305; *Idem* & MacLachlan, James, "Galileo's Discovery of the Parabolic Trajectory," *Scientific American,* 232 (1975), pp.102-110; Naylor, Ronald.H., "Galileo's Theory of Motion: Process of Conceptual Change in the Period 1604-1610," *Annals of Science,* vol.34 (1977), pp.365-392; Wisan, Winifred L., "Galileo and the Process of Scientific Creation," *Isis,* vol.75 (1984), pp.269-386.
(31) *Opere* VIII p.197; *TNS,* p.154.
(32) *Opere* VIII p.213; *TNS,* p.170 with an addition. Emphasis added.
(33) *Opere* VIII p.205; *TNS,* p.162.
(34) *Ibidem.*
(35) *Ibidem.* Emphasis added.
(36) *Opere* VIII p.207; *TNS,* pp.162 and 163.
(37) *Opere* VIII p.208; *TNS,* p.164. Emphasis added.
(38) *Opere* VII p.170; *TCWS,* p.144.
(39) Detailed examination of these may reveal the fact that Galileo deliberately employed some tacit assumptions. See the relevant discussion immediately below.
(40) In my earlier paper (*op.cit.* [n.17], pp.31*ff*), I wrongly argued for the precedence of fol.116v over fol.179rv which states the GP. But this implicit assumption of the GP in fol.116v shows that this folio was written *after* fol.179rv.
(41) For example, S.Drake, *op.cit.* [n.30]; Shuntaro Ito, *Galileo* in *Man's Intellectual Heritage,* vol.31, Kodansha, 1985 in Japanese.
(42) Mach, Ernst, *Die Mechanik in ihrer Entwicklung;* the Japanese translation (by Yuzuru Fushimi, Kodansha, 1969), p. 115.
(43) *Opere* VIII p.202; *TNS,* pp.158-159. Underline added.
(44) See Clagett, Marshall, *Nicole Oresme and the Medieval Geometry of Qualities and Motions,* The University of Wisconsin Press, 1968, especially III-viii, pp.408*ff.*
(45) Koyré, Alexandre, *Études galiléennes,* Paris, 1939; Mach, Ernst, *op.cit.* [n.42]; S. Drake, *op.cit.* [n.29].
(46) See an excellent paper by Hatfield, Gary, "Metaphysics and the New Science," in Lindberg, David C. and Westman, Robert S. (eds.), *Reappraisals of the Scientific Revolution,* Cambridge Univ. Press, 1990, pp.93-166.
(47) Descartes's letter to Mersenne, 11 Oct. 1638 in *Oeuvres de Descartes* (ed.Ch. Adam & P. Tannery), vol.II, p.380.
(48) Koyré emphasizes the "Platonism" in his *op.cit.* [n.45] while Wallace does the "Aristotelianism" in his *op.cit.* [n.9].
(49) Examples are, *Opere* I p.215 (a divino uomo), p.300 (sub suprahumani Archimedis), p.303 (a divino Archimede); *Opere* VIII p.274 (la sola autorità d'Archimede).
(50) Cf. *De motu,* chap.14 in *Opere* I p.300: His responderem, me sub suprahumani Archimedis (quem nunquam absque admiratione nomino) alis memet protegere.
(51) See Hatfield's article mentioned in n.46.

Appendix A: A Historical Reconstruction of Galileo's Theory of Motion

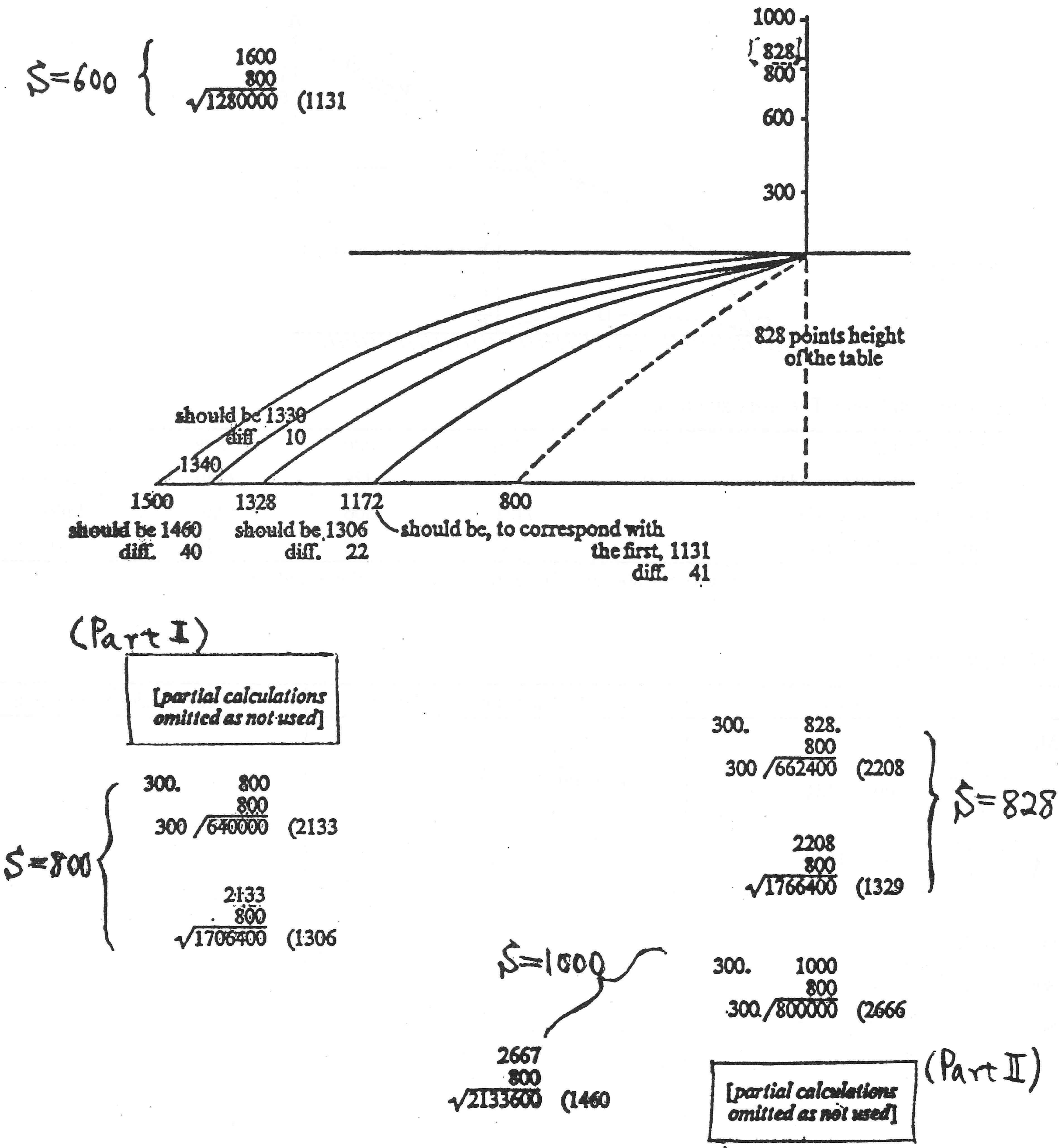
	1586	*La Bilancetta*
	c.1590	*De motu*
		the old LT-rule (*De motu,* chap.14)
	c.1601	*Le Mechaniche*
	1602	the discovery of isochronism of a pendulum
(I)		(Galileo's letter to Guidobaldo, November 29, 1602)
		the Ch-rule proved mechanically
		(*Cf.* Prop.6, coroll.1 & f.151r)
	?	the discovery of the ON- and the TS-rules
		(by experiment, *Cf.*f.107v)
	1604	the false principle $(v \propto S)$ and
		the proof of the ON- and the TS-rules by the Ar-tech
		(Galileo's letter to Sarpi, October 16, 1604 & f.128rv)
	?	the 2V- and the 2D-rules and
		the proof of the LT-rule by the TS- and Ch-rules (f.163v)
	?	an emergence of $(v \propto \sqrt{S})$ (f.152r)
	?	a reconsideration of the V-concept
		(*Mirandum*-fragment on f.164v)
(II)	?	the proof of the RA-rule by the LT- and TS-rules (f.177r)
	c.1609	the emergence of the GP and
		the proof of the LT-rule (f.179rv)
		(Valerio's letter to Galileo, July 18, 1609)
	?	a crucial experiment to decide whether $(v \propto S)$ or $(v \propto \sqrt{S})$ (f.116v)
	ante 1610	the proof of the TS-rule by the Ch- and LT-rules (f.147r), and
		the identification of the RA-rule with the Ch-rule (f.147v)
	ante 1610	a kinematical proof of the Ch-rule by the TS- and LT-rules, and
		a mechanical proof of the Ch-rule (f.172r)
	1610	*Sidereus Nuncius*
	1612	*Discorso intorno alle cose che stanno in su l'aqua*
	1613	*Istoria e dimostrazioni intorno alle macchie solari*
(III)	1618?	fair copies of Galileo's Paduan notes (by Arrighetti & Guiducci)
	1623	*Il Saggiatore*
	1625?	the correct principle $(v \propto T)$ for the proof of the 2D-rule
		(*Cf.* the Second Day of the *Dialogo*)
(IV)	1632	*Dialogo*
	1638	*Discorsi*
	1639	the proof of the GP
(II)		(Galileo's letter to Baliani, August 1, 1639)
	1642	Galileo's death

Appendix B1: Folio 116v



Appendix B2: Transcription by Drake

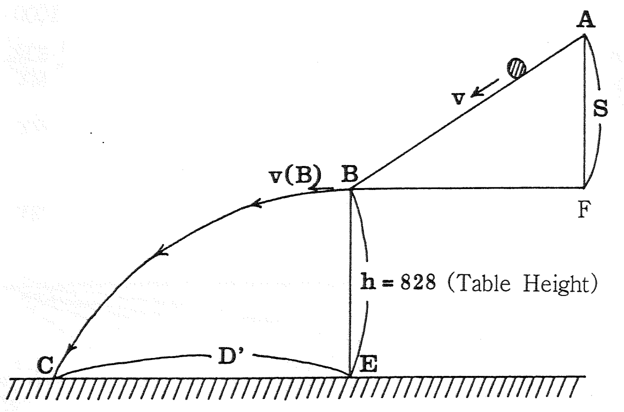
S.Drake, "Galileo's New Science of Motion," in *Reason, Experiment, and Mysticism* (eds. Bonelli & Shea), 1975 (with additions by Takahashi)



English transcription of folio 116ᵛ, showing Galileo's calculations of horizontal distances expected under his mean-proportional rule, using shortest drop as basis. Unused partial calculations are omitted as are trial divisors in root extraction and related remainders. (*Courtesy of Isis*)

Appendix B3: Experiment, Results and Calculations on f.116v

(1) The Experiment of Fol.116v



(2) Experimental and Theoretical Values

Height of Fall (S)	300	600	800	828	1000
Experimental Value (D')	800	1172	1328	1340	1500
Theoretical Value (D)	—	1131	1306	1330	1460
Difference (D' − D)	—	41	22	10	40

Where $S_0=300$ and $D_0=800$ are used as the standard

(3) Calculations

Calculations	The Formula for Calculations	The Basic Assumption
Main Part $\sqrt{\frac{600\times 800}{300}}\times 800=1131$ $\sqrt{\frac{828\times 800}{300}}\times 800=1329$ etc.	$D=\sqrt{\frac{S\times 800}{300}}\times 800$ $D/D_0=\sqrt{S/S_0}$ i.e. $D\propto\sqrt{S}$ $\xrightarrow{D\propto v}$	$v\propto\sqrt{S}$
Omitted Part I $800\times 600\div 300=1600$ $800\times 800\div 300=213[3]\frac{1}{3}$	$D_0\times S\div S_0=D$ $D_0/S_0=D/S$ i.e. $D\propto S$ $\xrightarrow{D\propto v}$	$v\propto S$
Omitted Part II $\sqrt{1640\times 820}=115[9]$	$D=\sqrt{\frac{S\times 820}{300}}\times 820$ for $S=600$	The Change of the Standard $800\rightarrow 820$