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Computation and Comparison for Heat and Fluid Flow Using a QUICK and Other Difference Schemes

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Numerical computations were carried out for four sample cases of heat and fluid flow using a QUICK and other finite difference schemes. It is found that for a forced convection a QUICK scheme provides much less grid dependence than other schemes such as a central difference scheme, an Upwind difference scheme, a power law scheme and a hybrid difference scheme. For natural convection of low-Prandtl number fluid, a QUICK scheme gave oscillatory convection while the other schemes did not do for the same numerical conditions.

Introduction

Accurate prediction of complex heat and fluid flow is a topic of great importance in thermal science and engineering. However, it is not easy to make a simulation in detail. General purpose program with a high order accuracy is still required to permit the computation within a shorter time or presently available computing resources. QUICK (Quadratic Upwind Interpolation of Convective Kinematics) scheme has been considered to be one of reliable difference scheme. This work aims to develop a general purpose program to solve for two-dimensional problems of heat and fluid flow using a SIMPLE[1] method with a QUICK[2] scheme for primitive variables. Sample problems were solved and compared with previous works.

Nomenclature

a	= thermal diffusivity [m^2/s]
D	= diameter [m]
d	= diameter [m]
Dt	= dimensionless time step [-]
F	= dimensionless time $(=ta(RaPr)^{1/2}/H^2)$
g	= acceleration constant due to gravity [m/s^2]
H	= height [m]
L	= length [m]
Lr	= re-attachment length [m]
Nu	= Nusselt number [-]
$\overline{Nu}$	= average Nusselt number on a heated wall [-]
Pr	= Prandtl number [-]
Ra	= Rayleigh number $(=\beta gH^3\Delta T/a\nu)$
Re	= Reynolds number [-]
r	= coordinate [m]
S	= generalized source term
T	= temperature [K]
ΔT	= temperature difference $(=T_h-T_c)$ [K]
t	= time [s]
U	= dimensionless x-directional velocity component [-]
u	= x-directional velocity component [m/s]
V	= dimensionless y-directional velocity component [-]
v	= y-directional velocity component [m/s]
x	= coordinate [m]
y	= coordinate [m]
β	= volumetric coefficient of expansion [K^{-1}]
Γ	= generalized diffusion coefficient
ν	= kinematic viscosity [m^2/s]
ρ	= density [kg/m^3]
Φ	= general variable

Finite Difference Equation with QUICK Scheme

In this section presented is a general difference equation with a QUICK scheme. The QUICK formulation is written as a summation of an upwind difference scheme and an additional source term which is similar to that derived by

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Hayase *et al.*[2] except the treatment for the boundary control volumes.

The partial differential equations describing two-dimensional unsteady heat and fluid flow may be expressed in a general form as follows:

$$\frac{\partial}{\partial t}(\rho\Phi)+\frac{\partial}{\partial x}(\rho u\Phi)+\frac{\partial}{\partial y}(\rho v\Phi)=\frac{\partial}{\partial x}(\Gamma\frac{\partial\Phi}{\partial x})+\frac{\partial}{\partial y}(\Gamma\frac{\partial\Phi}{\partial y})+S \quad (1)$$

where S may be linearized as $S = S_c + S_p\Phi$.

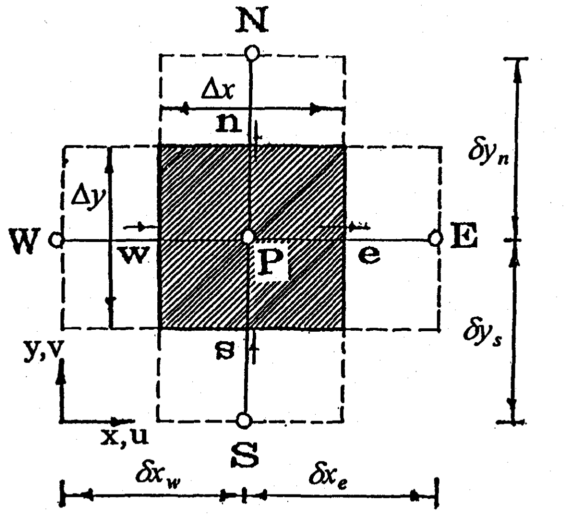

Fig. 1 Control volume specification

General differential equation (1) may be discretized on the non-uniform staggered grids by integrating it over the finite volume as shown in Fig.1 and a general finite difference equation is obtained:

$$a_P\Phi_P = a_E\Phi_E + a_W\Phi_W + a_N\Phi_N + a_S\Phi_S + b + S_{ad}^* \quad (2)$$

In the above equation,

$$a_E = D_e + [\![-F_e,\ 0]\!] \quad (3)$$

$$a_W = D_w + [\![F_w,\ 0]\!] \quad (4)$$

$$a_N = D_n + [\![-F_n,\ 0]\!] \quad (5)$$

$$a_S = D_s + [\![F_s,\ 0]\!] \quad (6)$$

$$b = S_c\Delta x\Delta y + a_P^0\Phi_P^0 \quad (7)$$

$$a_P = a_E + a_W + a_N + a_S - S_P\Delta x\Delta y \quad (8)$$

$$a_P^0 = \frac{\rho_P^0\Delta x\Delta y}{\Delta t} \quad (9)$$

where Φ_P^0 and ρ_P^0 are those at time t, and Φ_P, Φ_E, Φ_W, Φ_N, Φ_S, etc. are those at time $t+\Delta t$. Symbol $[\![A, B]\!]$ represents greater one of A and B. Flow rates F_e, F_w, F_n and F_s through four faces of the control volume in Fig.1 are calculated as follows:

$$F_e = (\rho u)_e\Delta y,\quad F_w = (\rho u)_w\Delta y,$$
$$F_n = (\rho v)_n\Delta x,\quad F_s = (\rho v)_s\Delta x,$$

Diffusion conductance D's are defined as,

$$D_e = \frac{\Gamma_e\Delta y}{(\delta x)_e},\quad D_w = \frac{\Gamma_w\Delta y}{(\delta x)_w},\quad D_n = \frac{\Gamma_n\Delta x}{(\delta y)_n},\quad D_s = \frac{\Gamma_s\Delta x}{(\delta y)_s}.$$

S_{ad}^* in equation (2) is the additional source term for QUICK scheme. When the additional source term $S_{ad}^* = 0$, equation (2) with (3)-(9) becomes a standard first order implicit upwind difference scheme as given in reference [1]. To adopt the QUICK difference scheme for a non-uniform staggered grid system, an additional source term is calculated

$$S_{ad}^* = (S_{ad}^*)_e + (S_{ad}^*)_w + (S_{ad}^*)_n + (S_{ad}^*)_s \quad (10)$$

where

$F_e > 0$,

$$(S_{ad}^*)_e = \frac{F_e}{4}\left[\frac{a^2}{c(c-a)}\Phi_W^* + (4+\frac{a-2c}{c})\Phi_P^* - \frac{a-2c}{a-c}\Phi_E^*\right] \quad (11)$$

$F_e < 0$,

$$(S_{ad}^*)_e = \frac{F_e}{4}\left[\frac{a-2b}{b}\Phi_P^* + (4-\frac{a-2b}{a-b})\Phi_E^* + \frac{a^2}{b(b-a)}\Phi_{EE}^*\right] \quad (12)$$

$F_w > 0$,

$$(S_{ad}^*)_w = \frac{F_w}{4}\left[\frac{-c^2}{d(d-c)}\Phi_{WW}^* - (4-\frac{c-2d}{c-d})\Phi_W^* - \frac{c-2d}{d}\Phi_P^*\right] \quad (13)$$

$F_w < 0$,

$$(S_{ad}^*)_w = \frac{F_w}{4}\left[\frac{c-2a}{c-a}\Phi_W^* - (4+\frac{c-2a}{a})\Phi_P^* - \frac{c^2}{(a-c)a}\Phi_E^*\right] \quad (14)$$

$F_n > 0$,

$$(S_{ad}^*)_n = \frac{F_n}{4}\left[\frac{a^2}{c(c-a)}\Phi_S^* + (4+\frac{a-2c}{c})\Phi_P^* - \frac{a-2c}{a-c}\Phi_N^*\right] \quad (15)$$

$F_n < 0$,

$$(S_{ad}^*)_n = \frac{F_n}{4}\left[\frac{a-2b}{b}\Phi_P^* + (4-\frac{a-2b}{a-b})\Phi_N^* + \frac{a^2}{b(b-a)}\Phi_{NN}^*\right] \quad (16)$$

$F_s > 0$,

$$(S_{ad}^*)_s = \frac{F_s}{4}\left[\frac{-c^2}{d(d-c)}\Phi_{SS}^* - (4-\frac{c-2d}{c-d})\Phi_S^* - \frac{c-2d}{d}\Phi_P^*\right] \quad (17)$$

$F_s < 0$,

$$(S_{ad}^*)_s = \frac{F_s}{4}\left[\frac{c-2a}{c-a}\Phi_S^* - (4+\frac{c-2a}{a})\Phi_P^* - \frac{c^2}{(a-c)a}\Phi_N^*\right] \quad (18)$$

where Φ^*'s represent the current values of Φ's in an iterative calculation process. Coefficients *a, b, c* and *d* are grid differences. For Eqs. (11)-(14), these are defined in Fig. 2 (a), and for Eqs. (15)-(18) in Fig. 2 (b).

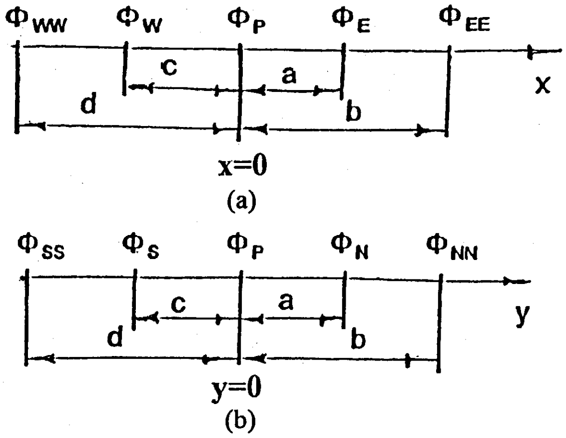

Fig. 2 Definition of *a, b, c* and *d* (d<c<0<a<b)

A special attention has to be paid for the boundary control volumes. The method to deal with boundary control volumes in this paper is different from Hayase's one [2]. A left-hand boundary control volume is taken for example to explain how to deal with a boundary control volume in this paper. When the flow rate through the face of this left-hand boundary control volume F_w<0, $(S^*_{ad})_w$ in Eq. (10) is still calculated by Eq. (14). But, when this flow rate F_w>0, $(S^*_{ad})_w$ here should be set at 0, which is equivalent to an upwind difference scheme herein.

Computed examples

Sample computations were performed for different problems of laminar flow as follows:

(a) Flow in a square cavity with a moving lid.
(b) Axial symmetric flow in a straight tube with a sudden- expansion of a tube diameter.
(c) Natural convection in a square cavity.
(d) Natural convection in a horizontal layer heated from below.

Flow in a Square Cavity with a Moving Lid

The physical situation considered is shown in Fig.3 where the lid of the cavity slides with a constant velocity U_{top} above the cavity and other three sides are fixed. The length of cavity edge is *H*. The fluid flow is presumed to be a two-dimensional laminar flow with constant physical properties.

Numerical computations were carried out for $Re=10^4(Re=U_{top}H/\nu)$. The dimensionless x-directional velocity component $U\,(=u/U_{top})$ and the dimensionless y-directional velocity component $V\,(=v/U_{top})$ are shown in Fig. 4 for $Re=10^4$. This graph is quite similar to

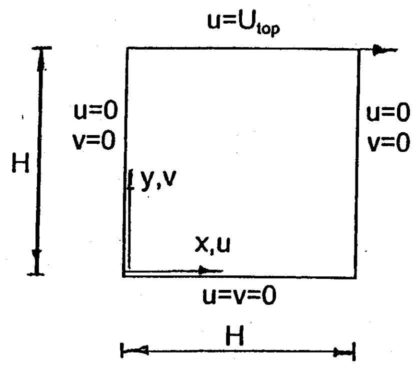

Fig. 3 Flow in a square cavity with a moving lid

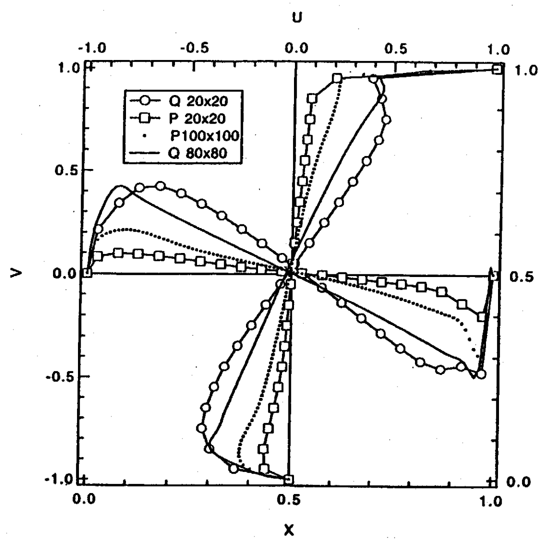

Fig. 4 Centerline velocity profiles for flow in a square cavity with a moving lid ($Re=10^4$, Q and P represent QUICK and power law difference scheme, respectively)

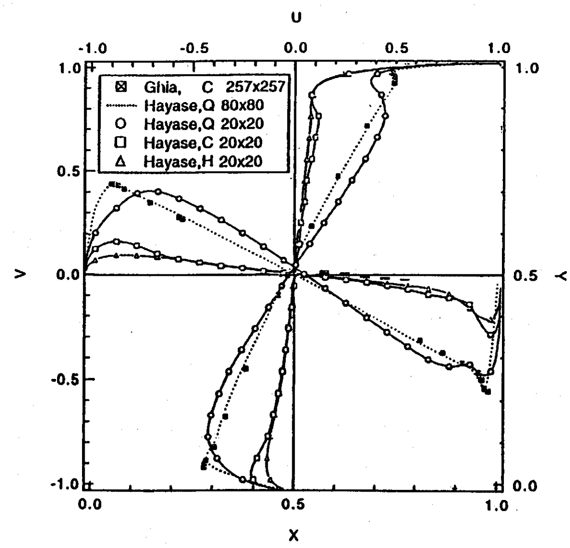

Fig. 5 A graph re-plotted from Fig. 3(c) in reference [2] (C, Q, H represent central, QUICK and hybrid difference scheme, respectively)

Fig.3(c) of Reference [2], where presented are the centerline velocity profiles obtained by Hayase *et al.* [2] and Ghia *et al.*[3] using a QUICK and a central difference scheme with uniform grids of 20×20, 80×80 and 257×257. The profile obtained by Ghia *et al.* using a central difference scheme with 257 × 257 grids may be considered as the most reliable result for comparison. It is shown in Figs. 4 and 5 that the present computational results agree with those by Hayase *et al.* [2]. We can note from Figs.4 and 5 that the QUICK scheme appears to be more reliable than other schemes. For example the profile obtained by a QUICK scheme with 20×20 grids is closer to the standard profile than any other schemes. The profile obtained by a QUICK scheme with 80×80 grids is almost identical to those by the central difference scheme by Ghia with 257×257 grids.

Axial Symmetric Flow in a Straight Tube with Sudden-Expansion of a Tube Diameter

Fig. 6 shows a schematic of the flow in a straight pipe with sudden-expansion of a tube diameter. Fluid with an average velocity U_0 at the inlet with a diameter d enters a straight pipe with a diameter D. The diameter ratio (d/D) is fixed at 0.5. As seen from the diagram, the flow is assumed symmetric in terms of the axis of a tube and fully developed at the further downstream. The length of calculation domain in the stream-wise direction was taken as four times the re-attachment length. For example L/d was fixed at 10 for the case of Re=50 ($Re=U_0 d/\nu$), which was found to be of sufficient length such that the exit boundary conditions had almost no effect upon the re-attachment distance.

The boundary conditions are as follows:

(a) At the inlet, velocity profile is parabolic.

(b) At the outlet, $\partial U/\partial x = 0,\quad V = 0$.

(c) At the wall boundary, $U = 0,\quad V = 0$.

(d) At the symmetric axis, $\partial U/\partial r = 0,\quad V = 0$.

The dimensionless re-attachment length L_r/d determined by various difference schemes are shown in Table 1, where QUICK and UDS (upwind difference scheme) are obtained by the present work by a QUICK scheme and an upwind difference scheme, respectively.

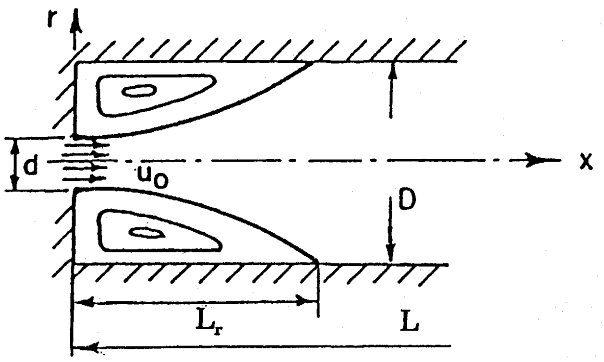

Fig. 6 Axial symmetric Flow in a Straight Tube with Sudden- Expansion of a Tube Diameter

QUICK [4] and HDS [4] (hybrid difference scheme) were obtained by Pollard and Siu [4] by a QUICK scheme and a hybrid difference scheme. Experiment [5] was obtained by Macagno and Hung [5]. It should be noted that Macagno and Hung [5] presented the results in a graph. In Table 1, the data are presented with ranges, which were read from Fig.10 of Reference [5]. The QUICK scheme listed in Table 1 appears to be the most reliable among various schemes, because the computed results with 25×16 grids are very close to the ones obtained with finer grid, 200×100.

Table 1 The dimensionless re-attachment length L_r/d

Scheme	Grid (uniform)	Re=50 L/d=10	Re=100 L/d=18	Re=150 L/d=26	Re=200 L/d=36
QUICK	25×16	2.3324	4.5624	6.8048	9.0594
	200×100	2.3538	4.5772	6.8133	9.0509
UDS	25×16	2.1899	4.2518	6.3367	8.4332
	200×100	2.3329	4.5195	6.7399	8.9528
HDS[4]	25×16	2.15	4.20	6.26	8.34
QUICK[4]	25×16	2.20	4.32	6.45	8.60
Experiment[5]		2.2~2.4	4.3~4.6	6.5~6.8	8.8~9.2

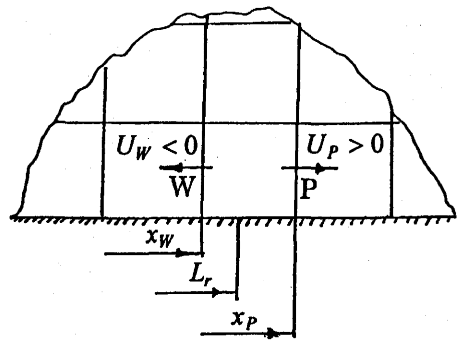

Fig. 7 Scheme for calculation of re-attachment length

In Table 1, the present computational results with a QUICK scheme agree well with the experimental results. However, the QUICK scheme by Pollard and Siu gives smaller results(5%) than the present ones with QUICK . One possible reason is that an interpolation method to calculate the re-attachment length may be different each other (Pollard and Siu [4] didn't describe their scheme). In the present report, the re-attachment length was calculated by the following way. Let W and P in Fig. 7 be inner nodes adjacent to the pipe wall, and the x-directional velocity U_W <0 at point W and U_P>0 at point P. The re-attachment length was calculated by the linear interpolation as follows.

$$L_r = x_w - \frac{U_W(x_P - x_W)}{(U_P - U_W)}$$

Natural Convection in a Square Cavity

The physical situation, shown in Fig. 8, is a two-dimensional square cavity with a vertical wall isothermally heated, and an opposing wall isothermally cooled at a lower temperature, and horizontal top and bottom boundaries are thermally insulated.

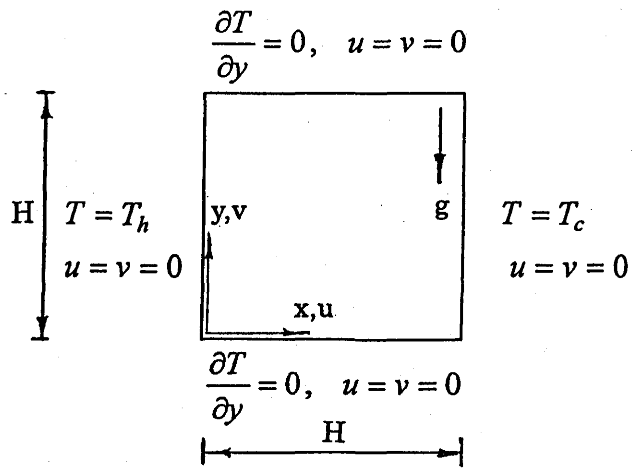

Fig. 8 Natural convection in a square cavity

Sample computations were carried out for Pr=0.7 and 0.025. The computational details, including non-uniform grids, are equal to those by Tagawa and Ozoe[6] except the numerical method and dimensionless equations.

The average Nusselt numbers and pertinent computed details at Pr=0.7 are listed in Table 2 for Ra=10^4 and in Table 3 for Ra=10^6. In these tables, Q and P represent the results obtained using a QUICK scheme and a power law scheme, respectively. U_{max} and V_{max} represent the maximum x-directional dimensionless velocity component U (=uH/a) at a vertical center-line and the maximum y-directional dimensionless velocity component V (=vH/a) at a horizontal center-line. From Tables 2 and 3 the numerical solutions obtained using a QUICK scheme are similar with those using a power law scheme.

Table 2 Computational results for natural convection in a square cavity at Ra=10^4 and Pr=0.7

	Bench mark[7]	Grid 20×20		40×40		80×80	
		Q	P	Q	P	Q	P
$\overline{Nu}$	2.238	2.301	2.299	2.259	2.258	2.248	2.247
Nu_{max}	3.527	3.720	3.726	3.579	3.580	3.541	3.542
$(Y/D)_{max}$	0.143	0.125	0.125	0.138	0.138	0.144	0.144
Nu_{min}	0.586	0.576	0.586	0.583	0.586	0.585	0.585
$(Y/D)_{min}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U_{max}	16.178	16.260	16.134	16.160	16.129	16.183	16.152
$(Y/D)_{max}$	0.823	0.825	0.825	0.813	0.813	0.819	0.819
V_{max}	19.643	19.632	19.525	19.624	19.598	19.626	19.602
$(X/D)_{max}$	0.119	0.125	0.125	0.113	0.113	0.119	0.119

*Q=by a QUICK scheme, P=by a power law scheme

Table 3 Computational results for natural convection in a square cavity at Ra=10^6 and Pr=0.7

	Bench mark[7]	Grid 20×20		40×40		80×80	
		Q	P	Q	P	Q	P
$\overline{Nu}$	8.903	10.297	10.495	9.411	9.410	8.973	8.970
Nu_{max}	18.562	19.323	19.979	20.430	20.662	18.683	18.693
$(Y/D)_{max}$	0.045	0.025	0.025	0.038	0.038	0.031	0.031
Nu_{min}	1.002	0.833	0.832	0.922	1.063	0.965	1.006
$(Y/D)_{min}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0
U_{max}	64.940	68.828	68.633	67.357	66.930	65.639	65.493
$(Y/D)_{max}$	0.850	0.875	0.875	0.863	0.863	0.856	0.856
V_{max}	221.29	240.13	238.24	224.68	223.13	218.26	218.09
$(X/D)_{max}$	0.040	0.025	0.025	0.038	0.038	0.031	0.031

The transient response of the average Nusselt number at Pr=0.025 and Ra=5×10^5 is shown in Fig.9. The average Nusselt mumber computed using a power law scheme gives a constant value after dimensionless time F=20. On the other hand the one obtained using a QUICK scheme is oscillatory. Natural convection of the low-Prandtl number fluid has been known to be oscillatory and the above solution by a power law scheme may not be correct. Thus we can conclude that the QUICK scheme is better than the power law scheme at least for solving the convection of low-Prandtl number fluid. This agrees with the results by Tagawa and Ozoe[6] who used a Utopia scheme.

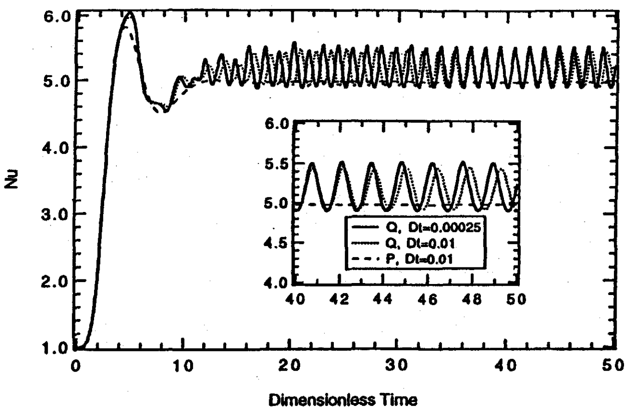

Fig. 9 Transient responses of the average Nusselt number of natural convection in a square cavity (Ra=5×10^5, Pr=0.025, 55×55 non-uniform grids)

Natural Convection in a Horizontal Layer Heated from Below

The schematic is shown in Fig.10 for the present system. This has been called as a Rayleigh-Benard type natural convection. The layer is heated from below at T_h and cooled from above at T_c (T_h>T_c). Both flow and temperature were assumed to be symmetric in terms of the vertical boundaries. The aspect ratio A (horizontal length L/height H) of the domain is set to 4.

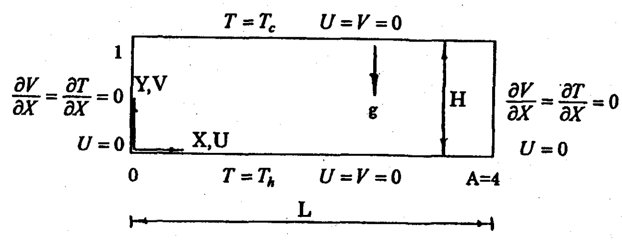

Fig.10 Natural convection in a horizontal layer heated from below

The computations were carried out for Ra=5 × 10^3 and Pr=0.01 using a QUICK scheme and a power law scheme with uniform grids 200 (x-direction) × 50 (y-direction). The transient responses of the average Nusselt number are shown in Fig. 11, in which Q and P represent results by a QUICK scheme and a power law scheme, respectively. For the same case, the computations were performed by Ozoe and Hara [8] with 201 × 51 grids. The average Nusselt number obtained using a power law scheme agrees with that by Ozoe and Hara using a central difference scheme at about $\overline{Nu} = 1.27$. Their calculation gave oscillatory result at Ra=6000 or more. The present result with a QUICK scheme gives an oscillatory value around $\overline{Nu}$=1.68 at Ra=5000.

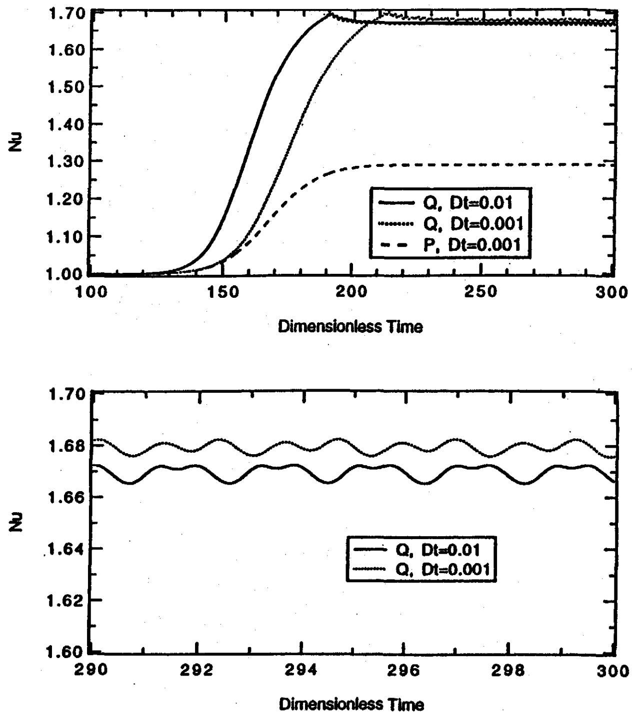

Fig. 11 Transient responses of the average Nusselt number of Natural convection in a layer heated from below (Ra=5 × 10^3, Pr=0.01, 200 × 50 uniform grids)

Ozoe and Hara[8] reported that the average Nusselt number differs extensively depending on the number of roll cells. Number of roll-cells also depends on the magnitude of the grids. Thus, these discrepancies may be clarified with much more detailed computations in future. In any case, a QUICK scheme would provide more reasonable results for the dynamic convection system like a low-Prandtl number convection.

Conclusion

This report presents a number of systems computed with a SIMPLE algorithm with a QUICK scheme. The computations were carried out for four typical problems using various difference schemes and the results obtained are compared with those in available literatures. The conclusions are as follows:

(1) A QUICK scheme gives similar result with other difference schemes for the steady laminar natural convection.
(2) A Quick scheme gives the result in much less grid dependence than those such as a central difference scheme, an upwind difference scheme, a power law scheme and a hybrid difference scheme for the forced convection problems.
(3) A QUICK scheme appears to be suitable to study the oscillatory convection like that of the low-Prandtl number fluid .

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