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The Effects of Irreversibilities on the Optimal Timing of Pollution Control Policies

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Abstract

We construct a real option model in which government determines the timing of investment in pollution control and examine how irreversibilities affect the optimal investment decisions. Investment is assumed to be completely irreversible and its benefit (a value of damage parameter) is uncertain. Irreversibility of emissions is also considered. By comparing the optimal policies for various cases, we get the following results. Irreversibility of investment delays the optimal timing of policy implementation and this effect gets stronger as the degree of uncertainty increases. Irreversibility of emissions works for the opposite direction, but it does not depend on the degree of uncertainty.

Keywords: Pollution control, Uncertainty, Irreversibility, Real option

1 Introduction

It is obvious that there is considerable *uncertainty* in the degree of damage on society caused by pollution. IPCC (2001) reports that in business-as-usual scenario the average temperature in 2100 is expected to increase 1.4-5.8 degrees centigrade over 1990, and the sea level is expected to rise 9-88 cm. This is an example of great uncertainty in future predictions. Because the scale of damage is uncertain, the optimal pollution control is unknown at present. In many environmental issues, the relationship between economic activities and pollution emissions and a process of pollutant accumulation are uncertain as well.

The level of uncertainty, however, is expected to decrease as more information becomes available in the future. This process is called *learning.* Learning can be roughly divided into two groups: exogenous learning and endogenous learning. In exogenous learning, information can be obtained regardless of the actions of agents and uncertainty will be resolved all at once or gradually in the future. On the other hand, in endogenous learning, agents' actions such as R&D and observation have an impact on how fast information will be accumulated. This type of learning seems more realistic. By considering endogenous learning, we can regard a way of obtaining information as a strategy.

There are a number of studies focusing on exogenous learning such as Yohe and Wallace (1996), Ulph and Ulph (1996), and Kolstad (1996a, 1996b). Assumptions of their models

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are all different. The contents of Kolstad (1996a, 1996b) will be considered later. Yohe and Wallace (1996) simulated an integrated assessment model, which takes into account the threshold of carbon dioxide concentration. Through the simulation they demonstrated that, under the assumption that every uncertain parameter concerning global warming will become clear by 2020, a carbon tax that is supposedly necessary in the near future to minimize the long-term costs should be considerably lower. Regarding the problem of climate change, adjustments in emissions will become possible after uncertainty is resolved. Therefore, a drastic control of emissions is not necessary during a learning period. On the contrary, Ulph and Ulph (1996), by analyzing the two-period game model, numerically demonstrated that such a theory would not work when there are multiple agents. According to their results, the equilibrium control rate in the near future would be adversely increased due to learning. Additionally, in an asymmetric game with different parameters for each country, some countries would face a decline in utility because of learning.

There are only a few papers that directly deal with endogenous learning. Kelly and Kolstad (1999) analyzed an empirical model for climate change, based on the assumption that an agent will obtain information on uncertain parameter values through Bayes learning. They concluded that if observation were the only method, the learning of a parameter (a sensitivity parameter linking the global temperature rise to the greenhouse gas concentration) would take an extremely long time (90 to 160 years). They also stated that the most appropriate environmental policy should be sensitive to the amount of information, and discussed a tradeoff between the amount of emissions control and the lack of information. In studies of exogenous learning, it is typically assumed that most uncertainty will be resolved in approximately 30 years. However, this assumption may not be realistic when there are not any active investments made for learning.

Irreversibility plays an important role in decision-making when uncertainty and learning are present. Decision-making is called irreversible when a decision considerably reduces the scope of choices available for future decision-making (Henry, 1974). Normally discussed irreversibility issues in environmental problems involve pollutant emissions or accumulation, investment for environmental preservation, and environmental damage.

In broad terms, irreversibility of emissions means impossibility to collect released pollutants. While the stock of pollutant may be absorbed through the natural process to a certain degree, the limit of such effect is clearly indicated by steadily increasing time-series changes in atmospheric concentrations of carbon dioxide and sulfur oxide. To artificially reduce emissions below zero is extremely difficult and costly. It may be too late if emissions are reduced after serious damage becomes apparent. Arrow and Fisher (1974) described the value of flexible policies that consider irreversibility of land development and keep lands undeveloped until benefits of land development and preservation are known. This value is defined as the "quasi-option value." In terms of pollution, it supports increased control of pollutants today (Chichilnisky and Heal, 1993). However, Epstein (1980) and Ulph and Ulph (1997) indicated that whether or not current control should be increased depends on the shape of cost and damage functions when there is irreversibility of emissions.

In existing literature, irreversibility of emissions is formulated as the following two assumptions: (a) emissions cannot be negative, and (b) there is no natural absorption. It can be said that the assumption by Arrow and Fisher (1974) is a modification of (a). Kolstad (1996a, 1996b) and Ulph and Ulph (1996) adopted assumption (a), and Ulph and Ulph (1997), Narain and Fisher (1998), and Pindyck (2000, 2002) adopted the both assumptions (a) and (b).

Irreversibility of investment means that resources used for investment in pollutant control cannot be used for any other purposes. Investment to modify plant facilities into energy-efficient ones or to install scrubbers in power plants is irreversible. A significant share of the investment costs for abatement becomes sunk cost. If we consider this factor, in order to avoid unnecessary investment, it is desirable to wait until when enough scientific knowledge is accumulated and information on damage becomes evident and apply appropriate policies after that, instead of excessively reducing pollutants.

Irreversibility of damage refers to the occurrence of irreversible damages such as extinction of certain species and serious health damage. Narain and Fisher (1998) considered situations where catastrophic damages occur with a positive probability and reduce the level of social welfare to zero thereafter. They came up with the results that if catastrophic risk is avoidable, that is, the probability of a catastrophe depends on the stock of pollution control capital, the effects of irreversibility of emissions will get stronger, although their arguments are somewhat ambiguous. We will not explore the effects of irreversibility of damage in this paper.

Kolstad (1996a) used a general two-period exogenous learning model that assumes that learning about the impact of pollution occurs between periods 1 and 2, and explained that irreversibility of emissions and that of investment would affect current pollution control in opposite directions. Kolstad (1996b) simulated an empirical model of learning associated with climate change to demonstrate that irreversibility of investment would have a greater impact than irreversibility of emissions. He emphasized the risk of a hasty implementation of permanent environmental measures. In these studies irreversibility of investment was defined as the low depreciation rate of pollution control capital.

Pindyck (2000, 2002) developed a model of irreversible investment for pollution control when there is uncertainty in both environmental damage and the pollutant accumulation process. He described this as an optimal stopping model using the *real option* approach. He developed a framework to calculate the optimal timing for the implementation of pollution control investment, specified functional forms, and demonstrated a couple of numerical examples through calculations. Real option is an application of an option pricing theory that is typically used in financial engineering, and is a method to figure out values of investment when future benefit is uncertain (Dixit and Pindyck, 1994). Pindyck (2000) analyzed a wide range of optimal policies based on various assumptions using the continuous time learning model, and showed that irreversibility of investment would delay the optimal control timing. It was also shown that irreversibility of emissions would make the optimal control timing earlier. However, his analyses focused on irreversibility of investment, and irreversibility of emissions was not mentioned much. Pindyck (2000) dealt with uncertainty in damage and in the accumulation process separately. Pindyck (2002, Section 3) constructed a model that considers both aspects simultaneously.

We examine roles of irreversibility of emissions and that of investment, using the framework of Pindyck's model. In the next section a pollutant control model is introduced, and a solution applying specific cost and damage functions is derived. In Section 3, we first analyze the model with reversible emissions, and then the model with reversible investment to investigate the effects of irreversibilities on the optimal policies. Section 4 provides a summary and conclusions. We get the following results. Irreversibility of investment delays the optimal timing of policy implementation and this effect gets stronger as the degree of uncertainty increases. Irreversibility of emissions works for the opposite direction, but it does not depend on the degree of uncertainty.

2 Timing of Pollutant Control

2.1 The Model

In this section we introduce the analysis of pollutant control with a single agent by a simplified version of the model developed by Pindyck (2000).

A benevolent governmental institution is regarded as one risk neutral decision-making agent. Specific pollutants that are emitted as a result of economic activity will be focused on. Pollutants accumulate in the atmosphere, and society is damaged by the stock of pollutants. Damage is measured in a monetary value. However, a parameter for the scale of damage is uncertain. Let $E(t)$ and $M(t)$ denote the amount of pollutant emitted and the stock of pollutant accumulated in the atmosphere at continuous time t respectively. It is assumed that economy is constant, and the amount of pollutant emissions before implementation of control methods is constant. We set the uncontrolled emissions level as the standard value 1. This means that: $E(t) \in [0, 1]\ \forall t \geq 0$.[1] The stock is assumed to change over time as follows:

$$\frac{dM(t)}{dt} = E(t), \quad M(0) = M_0. \tag{1}$$

[1]The assumption that the emissions level is nonnegative corresponds to irreversibility of emissions (a) described in Section 1.

(1) shows that all emitted pollutants are accumulated in the atmosphere, and that the accumulated pollutants will not be absorbed through natural processes.[2]

Let us assume that the damage is proportional to the stock of pollution, and is expressed as $\theta(t)M(t)$.[3] The damage parameter $\theta(t)$ is a nonnegative uncertain variable that changes over time (a stochastic process), and it is assumed to follow a geometric Brownian motion:[4]

$$d\theta(t) = \sigma\theta(t)dz, \quad \theta(0) = \theta_0, \tag{2}$$

where z represents a Wiener process. In addition, $\sigma(\geq 0)$ is called a volatility parameter. Higher σ means higher variance of $\theta(t)$. Although a value of $\theta(t)$ at each time is observable, the future value of θ always remains uncertain since it is a stochastic process. Damage parameter can move up and down randomly, but there is no trend in that movement. Even if the stock of pollutants is constant, the social impact of pollution is thought to vary because of other random factors. We set (2) as a simplification of these facts.

Let $C(E)$ denote the cost of reducing emissions to the level E. Investment in pollution control is assumed to be irreversible and all costs needed for investment become sunk cost. The cost function satisfies: $C(1) = 0, C' < 0, C'' \geq 0$. It is assumed that investment is made only once, but the timing of investment is arbitrary, and such timing is a controllable variable determined by the agent.

Under this setting, the optimal amount of pollution control and timing must be determined by the agent. It is assumed that the agent takes measures to reduce emissions to the level E at the time when the value of the damage parameter θ reaches a certain value of θ^* or higher. This value θ^* is called a *critical value.*

The objective function for the agent is expressed as:

$$W = \mathcal{E}\left[-\int_0^\infty \theta(t)M(t)e^{-rt}dt - C(E)e^{-rT}\right]. \tag{3}$$

(3) shows that the objective function is the sum of the present value of benefit minus investment cost. Here, $r(>0)$ is the discount rate and $\mathcal{E}[\,\cdot\,]$ is an operator indicating an expected value at time 0. A variable T is the time when abatement investment is implemented. In other words, it is when θ reaches a value of θ^* or higher for the first time. As shown in (2), a value of θ is uncertain, thus T is also a random variable.

2.2 Optimal Policy

We calculate the values of θ^* and E that maximize the objective function (3) under the constraints (1) and (2), taking a typical real option approach.

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[2]This assumption corresponds to irreversibility of emissions (b).

[3]Models may use the expression $-\theta(t)M(t)$ to represent "benefit" gained by pollution.

[4]Usually the equation used to describe geometric Brownian motion takes a form: $d\theta = \alpha\theta dt + \sigma\theta dz$, where α is a drift parameter. However, in this paper we set $\alpha = 0$.

Let $W^N(\theta_0, M_0)$ denote the value function for the "no-adopt" region where no policies are taken within a society. Then we have

$$\begin{aligned} W^N(\theta_0, M_0) &= \mathcal{E}\left[-\int_0^\infty \theta(\tau)M(\tau)e^{-r\tau}d\tau\right] = -\int_0^{dt}\theta_0 M_0 e^{-r\tau}d\tau + \mathcal{E}\left[-\int_{dt}^\infty \theta(\tau)M(\tau)e^{-r\tau}d\tau\right] \\ &= -\int_0^{dt}\theta_0 M_0 e^{-r\tau}d\tau + e^{-rdt}\mathcal{E}[W^N(\theta(dt), M(dt))], \end{aligned} \tag{4}$$

where dt shows an infinitesimal time. Since $\theta(dt) = \theta_0 + \sigma\theta_0 dz$, and $M(dt) = M_0 + E(0)dt = M_0 + dt$, Taylor series expansion of W^N to the second order gives

$$\begin{aligned} W^N(\theta(dt), M(dt)) &= W^N(\theta_0, M_0) + W_\theta^N d\theta + W_M^N dM \\ &\quad + \frac{1}{2}\{W_{\theta\theta}^N (d\theta)^2 + 2W_{\theta M}^N d\theta dM + W_{MM}^N (dM)^2\} \\ &= W^N(\theta_0, M_0) + \sigma\theta_0 W_\theta^N dz + W_M^N dt \\ &\quad + \frac{1}{2}\{(\sigma\theta_0)^2 W_{\theta\theta}^N (dz)^2 + 2\sigma\theta_0 W_{\theta M}^N dtdz + W_{MM}^N (dt)^2\}, \end{aligned}$$

where $W_\theta^N, W_M^N, W_{\theta\theta}^N, W_{\theta M}^N$, and W_{MM}^N are partial derivatives, i.e. for example, $W_M^N = \dfrac{\partial W^N}{\partial M}$, and $W_{\theta\theta}^N = \dfrac{\partial^2 W^N}{\partial \theta^2}$. Applying $\mathcal{E}[dz] = 0$ and approximation rules by Ito's lemma: $(dt)^2 \approx 0, dtdz \approx 0$, and $(dz)^2 \approx dt$, we have

$$\mathcal{E}[W^N(\theta(dt), M(dt))] = W^N(\theta_0, M_0) + \left(W_M^N + \frac{1}{2}(\sigma\theta_0)^2 W_{\theta\theta}^N\right)dt. \tag{5}$$

Substitution of (5) into (4) leads to:

$$(1 - e^{-rdt})W^N = -\int_0^{dt}\theta_0 M_0 e^{-r\tau}d\tau + e^{-rdt}\left(W_M^N + \frac{1}{2}(\sigma\theta_0)^2 W_{\theta\theta}^N\right)dt. \tag{6}$$

Dividing both sides of (6) by dt and taking the limit of $dt \to 0$, we get

$$rW^N = -\theta_0 M_0 + W_M^N + \frac{1}{2}\sigma^2\theta_0^2 W_{\theta\theta}^N. \tag{7}$$

(7) is called the *Bellman equation.*

We assume the emissions level after control to be E, and let $W^A(\theta_0, M_0, E)$ denote the value function for the "adopt" region where policies are taken immediately. In this case,

$$W^A(\theta_0, M_0, E) = \mathcal{E}\left[-\int_0^\infty \theta(\tau)M(\tau)e^{-r\tau}d\tau\right] - C(E),$$

and using the same procedure as above we can show that W^A satisfies:

$$r(W^A + C(E)) = -\theta_0 M_0 + EW_M^A + \frac{1}{2}\sigma^2\theta_0^2 W_{\theta\theta}^A. \tag{8}$$

General solutions of (7) and (8) are

$$W^N(\theta, M) = A\theta^\beta + A'\theta^{\beta'} - \frac{\theta M}{r} - \frac{\theta}{r^2}, \tag{9}$$

$$W^A(\theta, M, E) = B\theta^\beta + B'\theta^{\beta'} - \frac{\theta M}{r} - \frac{\theta E}{r^2} - C(E), \tag{10}$$

where A, A', B, and B' are real constants. In (9) and (10) above, θ_0 and M_0 are written as θ and M for the sake of simplification. We omit subscripts hereafter when there is no possibility of confusion. We can derive the values of β and β' as solutions of a fundamental quadratic $\frac{\sigma^2}{2}x(x-1) = r$ regarding x, which we can obtain by assuming $W^N = A\theta^x$ and comparing coefficients, thus

$$\beta = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2r}{\sigma^2}}, \quad \beta' = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2r}{\sigma^2}}.$$

It is easy to see that $\beta > 1$ and $\beta' < 0$. The first and second term of the right-hand side of (9) are *option values*, i.e. the values of option for society to implement the control policy at some time in the future. There should be no option values in W^A because it is the value function when policies are immediately taken, hence $B = B' = 0$. The third term of the right-hand side of (9) coincides with that of (10), and represents benefit caused by the initial stock (its absolute value represents damage due to the initial stock). The fourth term of each function represents benefit accrued by continuing emissions.

Value functions must satisfy the following boundary conditions:

$$W^N(0, M) = 0, \tag{11}$$

$$W^N(\theta^*, M) = W^A(\theta^*, M, E^*), \tag{12}$$

$$W^N_\theta(\theta^*, M) = W^A_\theta(\theta^*, M, E^*). \tag{13}$$

We can see from (2) that $\theta(t) = 0$ holds for any $t (\geq 0)$ when $\theta_0 = 0$. Therefore, (11) follows from a definition of the value function. (12) is referred to as the "*value matching condition.*" It says that the choice to implement or not to implement control should be indifferent when the damage parameter reaches the critical value. (13) is called the "*smooth pasting condition.*" It means that, when θ is indicated as a horizontal axis, graphs of W^N and W^A must have a smooth contact (not having a kink) if a decision is optimal at the critical value.[5] Here E^* is the optimal amount of emissions, i.e.

$$E^* = \arg\max_{E \in [0,1)} W^A(\theta^*, M, E).$$

Coefficients A and A' of (9) should be determined so that the value functions satisfy these boundary conditions, and the values of θ^* and E^* are obtained at the same time.

To reach a concrete solution, the cost function $C(E)$ is specified as a linear function: $C(E) = c(1 - E)$. It follows that

$$\frac{\partial W^A}{\partial E} = -\frac{\theta}{r^2} + c.$$

Note that since this value does not depend on the amount of emissions E, the optimal emissions should be $E^* = 0$ if control is to be implemented. We can derive $A' = 0$ from

[5] Refer to the Chapter 4, Appendix C (pp.130-132) in Dixit and Pindyck (1994) for details of the smooth pasting condition.

(11). Next, substitution of (9), (10), $B = B' = 0$, and $E^* = 0$ into (12) and (13) leads to the following equations:

$$A\theta^{*\beta} - \frac{\theta^*}{r^2} + c = 0, \tag{14}$$

$$\beta A \theta^{*\beta-1} - \frac{1}{r^2} = 0. \tag{15}$$

From (14) and (15), we get

$$\theta^* = \frac{\beta c r^2}{\beta - 1},\ A = \left(\frac{\beta-1}{c}\right)^{\beta-1}\left(\frac{1}{\beta r^2}\right)^{\beta},$$

and the optimal investment time is $T = \inf\{t \mid \theta(t) \geq \theta^*\}$. Value functions can be expressed as:

$$W^N(\theta, M) = \left(\frac{\beta-1}{c}\right)^{\beta-1}\left(\frac{1}{\beta r^2}\right)^{\beta}\theta^{\beta} - \frac{\theta M}{r} - \frac{\theta}{r^2}, \tag{16}$$

$$W^A(\theta, M, E^*) = -\frac{\theta M}{r} - c. \tag{17}$$

The main results mentioned above can be summarized as follows. The optimal emissions of society are

$$E^* = \begin{cases} 1 & (\theta_0 < \theta^*), \\ 0 & (\theta_0 \geq \theta^*), \end{cases}$$

and a critical value is

$$\theta^* = \frac{\beta c r^2}{\beta - 1}. \tag{18}$$

(18) shows that higher marginal control cost and higher discount rate both raise the critical value. This fact seems intuitively evident.

We now present a numerical example to understand the characteristics of the solution visually. Suppose that $\sigma = 0.2, r = 0.04, c = 100$, and $M_0 = 0$. Then through simple calculations we get $\beta = 2.0$, and $\theta^* = 0.32$. Figure 1 shows graphs of W^N and W^A as functions of the initial value of damage parameter θ_0. Value function for the adopt region W^A becomes a constant function, since the initial stock is assumed to be zero. The agent should implement control policies when θ first reaches 0.32. Note that the smooth pasting condition holds at P.

Let us consider the case where the investment is too early and the critical value θ' is smaller than θ^*. Solution of differential equations (7) and (8) under the boundary condition that $W^N = W^A = -c$ for $\theta = \theta'$ leads to the value function W^N in Fig. 2, and it is clear that the welfare is lower than the optimal case. In the case where the investment is too late and the critical value θ'' is greater than θ^*, we have a solution depicted in Fig. 3 and the welfare is lower than in Fig. 1 again. Thus it is shown that the investment decision at the critical value θ^* is optimal. In both cases the smooth pasting condition does not hold and value functions have kinks at points P' and P''.

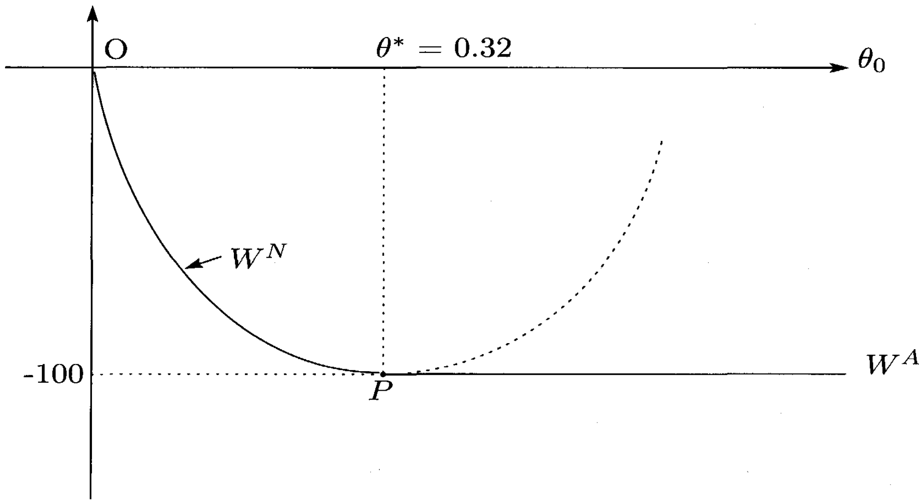

Fig. 1 Value function

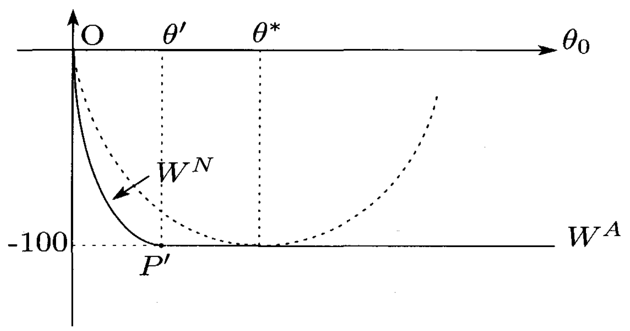

Fig. 2 Case of early investment

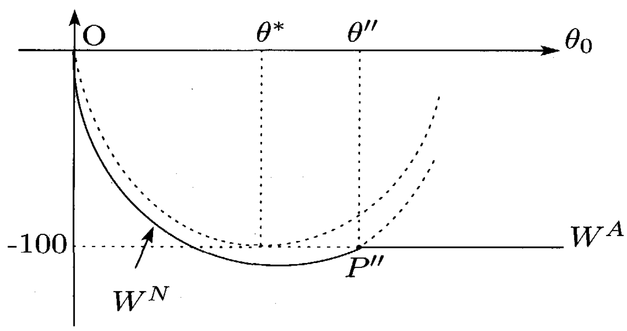

Fig. 3 Case of late investment

3 Effects of Irreversibilities

3.1 Irreversibility of Investment

In order to see how irreversibility of investment affects decision-making, let us examine the optimal decision-making for a model with reversible investments. When investment is reversible it is possible to remove the equipment for abatement without any additional costs and resources used for investment can be redirected to other purposes. This means that even if we implement abatement policies and lower emissions now, we can increase the emissions and get back to the original state in the future.

We attempt to solve for the value function and the critical value in the same framework as the previous section. As has been argued above, the optimal emissions level would be zero if control policies are to be implemented, so we need to consider only two states; $E = 0$ and $E = 1$. Policies of abating and increasing emissions should have a common critical value because of the assumption of no costs. In contrast to the previous analyses, even in the state $E = 0$ there exists the value of option to implement the policy of increasing emissions in the future. Thus $B = B' = 0$ no longer holds in the solution (10). Note that if the initial value of damage parameter θ_0 is extremely large and close to infinity, there would be no value of the option to increase emissions. Therefore $\lim_{\theta \to \infty} B\theta^{\beta} + B'\theta^{\beta'} = 0$ should hold and we have $B = 0$, since $\beta > 1$ and $\beta' < 0$.

Let W^{RN} and W^{RA} denote the value functions in the case of reversible investment. To determine all unknown variables (θ^*, A, B') and solve for the value functions, we have to impose another boundary condition:

$$W^{RN}_{\theta\theta}(\theta^*, M) = W^{RA}_{\theta\theta}(\theta^*, M, E^*), \tag{19}$$

which requires the continuity of the second derivatives of W^{RN} and W^{RA}. (19) is called the "*super contact condition*" (Dumas, 1991), and this is also a condition for maximizing the value function at the critical value. Substituting $B = 0$ and $E^* = 0$ into (9) and (10) and applying conditions (11)–(13), and (19), we get

$$W^{RN} = \frac{c^{\beta'}}{(\beta - \beta')r^{2\beta}}\theta^{\beta} - \frac{\theta M}{r} - \frac{\theta}{r^2}, \tag{20}$$

$$W^{RA} = \frac{c^{\beta}}{(\beta - \beta')r^{2\beta'}}\theta^{\beta'} - \frac{\theta M}{r} - c, \tag{21}$$

and in this case the critical value is $\theta^* = cr^2$.

We have already solved the critical value as $\dfrac{\beta c r^2}{\beta - 1}$ when investments are irreversible. Since $\dfrac{\beta}{\beta - 1} > 1$, this result shows that the existence of uncertainty increases the critical value. When irreversible investment for pollution control is to be taken under uncertainty government should delay the timing of applying control policies. Moreover, it turns out that the value function always takes a greater value in the case where investment is reversible than in the case where it is irreversible, i.e.

$$W^{RN} > W^{N} \quad (0 < \theta < cr^2), \tag{22a}$$

$$W^{RA} > W^{N} \quad \left(cr^2 \leq \theta < \frac{\beta c r^2}{\beta - 1}\right), \tag{22b}$$

$$W^{RA} > W^{A} \quad \left(\theta \geq \frac{\beta c r^2}{\beta - 1}\right). \tag{22c}$$

See Appendix for derivations of (22a)-(22c). This difference comes from the value of option to increase emissions.

Let us see the numerical example with the same parameter set as in Section 2. Figure 4 shows graphs of W^{RN} and W^{RA}. The critical value in the reversible investment case is 0.16, that is half of the value in the case of Fig. 1, and the smooth pasting condition holds at point Q. Also shown in Fig. 4 are graphs of W^N and W^A (broken lines). It can be seen that for the region $\theta_0 > 0$, value function in the case of reversible investment is always greater than that in the case of irreversible investment.

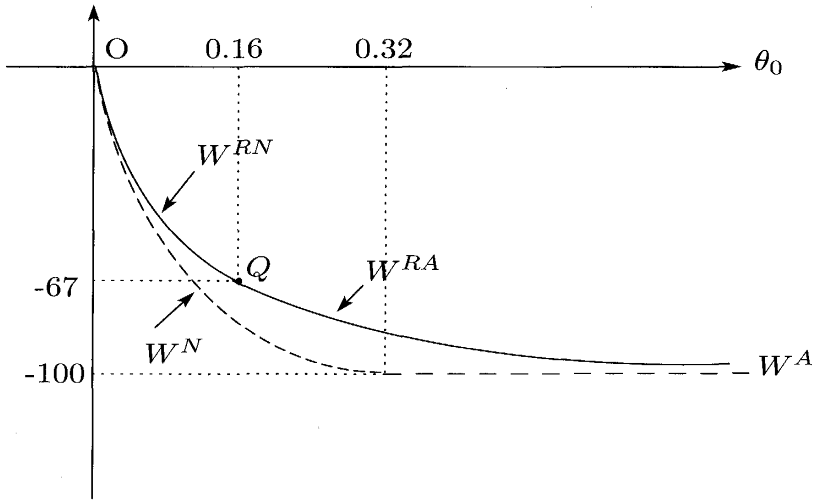

Fig. 4 Comparison of value functions

Next let us examine the optimal policies for a deterministic model to see the effects of uncertainty. We solve the following problem assuming that the initial value of the damage parameter and the stock of pollutant are θ_0 and M_0 respectively:

$$\max_{E} \ W(\theta_0, M_0, E) = -\int_0^{\infty} \theta_0 M(t) e^{-rt} dt - c(1-E), \tag{23}$$

$$\text{s.t.} \ \frac{dM(t)}{dt} = E, \ \ M(0) = M_0. \tag{24}$$

The damage parameter remains the same forever: $\theta(t) = \theta_0 \ \forall t \geq 0$. Since there is no uncertainty, we can decide the optimal action at time 0. From (24), $M(t) = M_0 + Et$. Substituting this into (23), we obtain

$$W(\theta_0, M_0, E) = -\frac{\theta_0 M_0}{r} - c + E\left(c - \frac{\theta_0}{r^2}\right),$$

which shows that the optimal emissions are

$$E^* = \begin{cases} 1 & (\theta_0 < cr^2), \\ 0 & (\theta_0 \geq cr^2). \end{cases}$$

As a result, the critical value is cr^2. When the value of volatility parameter σ increases, β will decrease, and the value of $\dfrac{\beta}{\beta - 1}$ will increase. Therefore, the greater the degree of uncertainty, the higher the critical value will be. The above discussion can be summarized in the following proposition.

Proposition 1 *In a real option model of irreversible pollution control under uncertainty, irreversibility of investment raises a critical value, delays the optimal timing of policy implementation, and lowers the social welfare. This effect gets stronger as the degree of uncertainty increases.*

3.2 Irreversibility of Emissions

Now we relax an assumption of irreversible emissions and assume that a fraction of the stock of pollutants is absorbed by natural processes. In this case, the stock evolves as:

$$\frac{dM(t)}{dt} = E(t) - \delta M(t), \quad M(0) = M_0, \tag{25}$$

where δ represents the absorption rate.

Assuming irreversible investment, we get a critical value $\theta^* = \frac{\beta cr(r+\delta)}{\beta - 1}$ through the same procedure as Section 2. This value is greater than $\frac{\beta cr^2}{\beta - 1}$, since $\delta > 0$. In other words, irreversibility of emissions lowers the critical value and advances the control policies.

This effect, however, does not depend on the degree of uncertainty. We solve for the optimal decision-making in a deterministic model as in Section 3.1. If $E(t) = E$ (constant) is assumed in (25), we have

$$M(t) = \frac{E}{\delta} + \left(M_0 - \frac{E}{\delta}\right) e^{-\delta t}. \tag{26}$$

Substituting (26) into (23) leads to the result that the critical value is $cr(r+\delta)$, which is $\frac{r+\delta}{r}$ times as cr^2 in the case of irreversible emissions. The ratio remains the same regardless of the degree of uncertainty. The reason for this seems to be that irreversibility of emissions is represented only by the absorption rate in the model. On the other hand, uncertainty plays a crucial role in deciding the benefit of delaying investment, so the effect of irreversibility of investment depends on how uncertain future will be.

Proposition 2 *In a real option model of irreversible pollution control under uncertainty, irreversibility of emissions lowers a critical value and makes the optimal timing of policy implementation earlier. This effect does not depend on the degree of uncertainty.*

4 Conclusions

In this paper we have examined how uncertainty and irreversibility interact in the decision-making of pollutant control investment using the real option theory. It is assumed that the damage is a linear function of the stock of pollutant, and the cost of control investment is a linear function of abatement of emissions.

The optimal decision is to bring the emissions level down to zero when an uncertain damage parameter exceeds a certain point (critical value). Considering the case of reversible investment, we have shown that irreversibility of investment delays the optimal timing of policy implementation and lowers the social welfare. This effect gets stronger as the degree of uncertainty increases. On the other hand, irreversibility of emissions advances the optimal timing of policy implementation. This effect, however, does not depend on the degree of uncertainty.

It becomes evident from these results that irreversibility of investment gets more important than that of emissions as the degree of uncertainty increases. Therefore the government should exercise greater care in the implementation of irreversible policies. Policies with relatively weak irreversibilities should be given priorities.

We must note that these model analyses have some problems. While it can be seen from the models that the emissions level should be zero when control is to be implemented, it is unrealistic to completely eliminate the emissions of specific pollutants. The results of the models are based on the assumption that cost function is linear regardless of the scale of the control rate, but actual cost function is predicted to be nonlinear for certain range of the control rate. Therefore, it may be more appropriate and desirable to implement a discontinuous control up to a certain point and shift to a gradual reduction after that point. Additionally, this paper assumes that damage would be proportional to the stock. An analysis of the case of a nonlinear damage function will be necessary.

Future research topics may include the numerical analyses of more complicated scenarios and the examination of a game situation where multiple agents determine the timing of the investment in pollution control as a strategy.

Appendix

A Derivation of (22a)

We can see from (16) and (20) that $W^{RN} > W^N$ for $\theta > 0$ is equivalent to the following inequality:

$$\frac{c^{\beta'}}{(\beta-\beta')r^{2\beta}} > \left(\frac{\beta-1}{c}\right)^{\beta-1}\left(\frac{1}{\beta r^2}\right)^{\beta}. \tag{A1}$$

Calculations using $\beta' = 1-\beta$ shows that (A1) is rewritten as:

$$\beta^{\beta} > (2\beta-1)(\beta-1)^{\beta-1}. \tag{A2}$$

Taking natural logarithms of both sides of (A2), we obtain

$$\beta\log\beta > \log(2\beta-1) + (\beta-1)\log(\beta-1).$$

Let $f(\beta) = \beta\log\beta - \log(2\beta-1) - (\beta-1)\log(\beta-1)$. Then $f'(\beta) = \log\left(\frac{\beta}{\beta-1}\right) - \frac{2}{2\beta-1}$, $f''(\beta) = -\frac{1}{\beta(\beta-1)(2\beta-1)^2}$. All we need to do is to show that $f(\beta) > 0$ for any $\beta > 1$. Through simple calculations we have $\lim_{\beta\to1+0} f'(\beta) = \infty$ and $\lim_{\beta\to\infty} f'(\beta) = 0$. Clearly $f''(\beta) < 0$, therefore $f'(\beta)$ is a monotonically decreasing function. Hence $f'(\beta) > 0\ \forall\beta > 1$. Since $\lim_{\beta\to1+0} f(\beta) = 0$, it follows that $f(\beta) > 0\ \forall\beta > 1$. Thus (A1) holds, and (22a) has been proved. Q.E.D.

B　Derivation of (22b)

The term $-\dfrac{\theta M}{r}$ is common in W^{RA} and W^{N}, so let us ignore it and treat W^{RA} and W^{N} as functions of θ only;

$$W^{RA}(\theta)=\frac{c^{\beta}}{(\beta-\beta')r^{2\beta'}}\theta^{\beta'}-c,\ W^{N}(\theta)=\left(\frac{\beta-1}{c}\right)^{\beta-1}\left(\frac{1}{\beta r^{2}}\right)^{\beta}\theta^{\beta}-\frac{\theta}{r^{2}}.$$

Note that both W^{RA} and W^{N} are convex functions, that is, $W^{RA\prime\prime}(\theta)>0, W^{N\prime\prime}(\theta)>0$ for any θ. Let $g(\theta)$ denote a function represented by the tangent to the graph of W^{RA} at point $(cr^{2}, W^{RA}(cr^{2}))$. Then

$$g(\theta)=-\frac{\beta-1}{(2\beta-1)r^{2}}(\theta-cr^{2})+\frac{c}{2\beta-1}-c.$$

By convexity of W^{RA},

$$W^{RA}(\theta)\geq g(\theta)\ \ \forall\theta\geq cr^{2}. \tag{A3}$$

In addition, $g\left(\dfrac{\beta cr^{2}}{\beta-1}\right)=W^{N}\left(\dfrac{\beta cr^{2}}{\beta-1}\right)=-c$ holds.

Next, let $h(\theta)$ denote a function represented by the straight line that connects two points; $(cr^{2}, W^{N}(cr^{2}))$ and $\left(\dfrac{\beta cr^{2}}{\beta-1}, W^{N}\left(\dfrac{\beta cr^{2}}{\beta-1}\right)\right)$. Then

$$h(\theta)=-\left(\frac{\beta-1}{\beta}\right)^{\beta}\frac{1}{r^{2}}(\theta-cr^{2})+W^{N}(cr^{2}).$$

According to (A2) in the proof of (22a), $\dfrac{\beta-1}{(2\beta-1)r^{2}}>\left(\dfrac{\beta-1}{\beta}\right)^{\beta}\dfrac{1}{r^{2}}$, which means the absolute value of slope of g is greater than that of h. We already know that $g=h$ at $\theta=\dfrac{\beta cr^{2}}{\beta-1}$. Therefore, we get

$$g(\theta)>h(\theta)\ \ \forall\theta<\frac{\beta cr^{2}}{\beta-1}. \tag{A4}$$

Convexity of W^{N} gives

$$h(\theta)\geq W^{N}(\theta)\ \ \forall\theta\in\left[cr^{2},\frac{\beta cr^{2}}{\beta-1}\right]. \tag{A5}$$

From (A3), (A4), and (A5), we obtain

$$W^{RA}(\theta)\geq g(\theta)>h(\theta)\geq W^{N}(\theta)\ \ \forall\theta\in\left[cr^{2},\frac{\beta cr^{2}}{\beta-1}\right),$$

and thus (22b) has been proved. Q.E.D.

C　Derivation of (22c)

It is clear that (22c) holds, since $\dfrac{c^{\beta}}{(\beta-\beta')r^{2\beta'}}>0$. Q.E.D.

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