

Numerical investigation of diffusion effect of non-condensable gas on cavitating flows around isolated hydrofoil based on mass transfer model with its advection

Yan, Longlong

School of Intelligent Manufacturing Engineering, Shanxi Institute of Science and Technology

Tsuda, Shin-ichi

Department of Mechanical Engineering, Graduate School of Kyushu University

Watanabe, Satoshi

Department of Mechanical Engineering, Graduate School of Kyushu University

Gao, Bo

School of Energy and Power Engineering, Jiangsu University

<https://hdl.handle.net/2324/7440640>

出版情報 : Physics of Fluids. 38 (8), pp.083365-, 2026-08-27. AIP Publishing

バージョン :

権利関係 : This article may be downloaded for personal use only. Any other use requires prior permission of the author and AIP Publishing. This article appeared in Longlong Yan, Shin-ichi Tsuda, Satoshi Watanabe, Bo Gao; Numerical investigation of diffusion effect of non-condensable gas on cavitating flows around isolated hydrofoil based on mass transfer model with its advection. Physics of Fluids 1 August 2026; 38 (8): 083365. and may be found at <https://doi.org/10.1063/5.0337383>



Numerical investigation of diffusion effect of non-condensable gas on cavitating flows around isolated hydrofoil based on mass transfer model with its advection

Longlong Yan (闫龙龙), ^{1, a)} Shin-ichi Tsuda (津田 伸一), ^{2, b)} Satoshi Watanabe (渡邊 聡), ^{2, c)}
Bo Gao (高波) ^{3, d)}

AFFILIATIONS

¹School of Intelligent Manufacturing Engineering, Shanxi Institute of Science and Technology, Jincheng, 048011, China

²Department of Mechanical Engineering, Graduate School of Kyushu University, 744, Motooka, Nishi-ku, Fukuoka, Japan

³School of Energy and Power Engineering, Jiangsu University, Zhenjiang 212013, China

^{a)} Electronic mail: 1045593274yll@sina.com

^{b)} Author to whom correspondence should be addressed: tsudashin@mech.kyushu-u.ac.jp

^{c)} Electronic mail: fmnabe@mech.kyushu-u.ac.jp

^{d)} Electronic mail: gaobo@ujs.edu.cn

ABSTRACT

It is known that non-condensable gas (NCG) sometimes plays a non-negligible role in cavitating flows. In general, NCG can be present in a liquid in the following two forms, dissolved in a liquid or trapped in free nuclei. Currently, the effect of the dissolved NCG on a cavitation process, especially for the diffusion effect, has been usually neglected in most of numerical cavitation models because the diffusion is generally slow compared to the flow convection. Nevertheless, the diffusion effect should be paid attention to in some special cases such as the cavitating performance of pumps operated at off-design conditions, especially in the case of low flow rates. To achieve a breakthrough in predicting cavitating flows, our group have proposed a simple mass transfer model across liquid-gas interface based on a well-known homogeneous cavitation model by Sauer and Schnerr (SS model), called as SS-NCG model, and also coupled an additional advection-diffusion equation of the NCG concentration for a two-dimensional Computational Fluid Dynamics (CFD) simulation around a hydrofoil. In the present study, a three-dimensional delayed detached eddy simulation (DDES) coupled with the cavitation model has been employed to investigate the detailed influence of NCG on cavitation. By comparing three typical cases, i.e., no dissolved oxygen (SS model), low and high amount of dissolved oxygen cases (SS-NCG model), the unsteady and statistical features of the NCG influences are indicated. The results show that the presence of NCG will enhance the cavitation degree and may lead to the occurrence of gaseous cavitation. From the perspective of unsteady characteristics, the cavitation process is stabilized and damped by NCG. The NCG concentration in cavitating regions is statistically lower than that in the bulk liquids and the concentration difference is increasing in the case with a higher value of NCG content. In addition, it is found that the mechanism of the cavitation process considering NCG and not is quite different. With the presence of NCG, cavitation can be regarded as an interaction between the varying

pressures inside and outside the bubbles, where the NCG mainly affects the cavitation dynamics through its modulation of the local compressibility inside the bubbles. In contrast, the cavitation process without NCG is only dominated by the local static pressure.

Keywords: cavitation, non-condensable gas, mass diffusion

Nomenclature table:

Symbols

c_l	NCG concentration in bulk liquid
c_b	NCG concentration at bubble interface (liquid side)
$c_{l\infty}$	NCG concentration at inlet boundary
C	Chord length of the hydrofoil
C_L	Lift coefficient
C_D	Drag coefficient
C_p	Pressure coefficient
$\mathfrak{D}$	Gas diffusion coefficient
f	Frequency
F_L, F_D	Lift and drag forces
F_1, F_2	Blending functions
H	Henry's constant
k	Turbulent kinetic energy
L_t	Turbulent length scale
$\dot{m}_v, \dot{m}_g$	Mass transfer rate (vapor, NCG)
$\dot{M}_g$	Diffusion flux of NCG at bubble interface
M_g, M_l	Molar mass of NCG and liquid

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

Symbols

M	Molar weight ratio M_g/M_l
m	Mass of NCG inside a bubble
m_g, m_l	Mass of NCG and liquid
n	Number density of bubbles per unit mixture volume
n'	Number density of bubbles per unit liquid volume
n_g, n_l	Amount of substance of NCG and liquid
p	Static pressure
p_v	Saturation vapor pressure
p_g	Partial pressure of NCG inside bubble
p_{ref}	Reference pressure
r	Bubble radius
R_g	Specific gas constant of NCG
Re	Reynolds number
S	Span length of the hydrofoil
S_t	Strouhal number
$\bar{s}_{ij}$	Strain rate tensor of the filtered velocity field
T	Temperature
t	Time
u_i, u_j	Velocity components
U_0	Inlet velocity
V	Control volume within the liquid phase

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

Symbols

V_c	Cavity volume
x_i, x_j	Spatial coordinates
x_g	Mole fraction of NCG at bubble interface on the liquid side
y^+	Non-dimensional wall distance
α	Angle of attack
α_l	Volume fraction of liquid
α_v	Total cavity volume fraction (vapor + NCG)
κ	Interfacial curvature
μ_l, μ_v, μ_g	Dynamic viscosity of liquid, vapor, NCG
μ_m	Mixture dynamic viscosity
μ_t	Turbulent eddy viscosity
ν_m	Mixture kinematic viscosity
ν_t	Turbulent kinematic viscosity
ρ_l, ρ_v, ρ_g	Density of liquid, vapor, NCG
ρ_m	Mixture density
σ	Cavitation number
σ_t	Surface tension
Δ	Length of the largest side of each computational cell
τ	Characteristic collapse time
ω	Specific dissipation rate in turbulence model

I. Introduction

Cavitation is an important two-phase flow phenomenon in the field of hydrodynamics, especially for hydraulic machinery such as pumps, water turbines, ship propellers, and so on. Cavitation can be regarded as the breakage of a liquid medium when the local pressure drops below saturated pressure [1]. In many cases, the occurrence of cavitation is usually associated with noise, vibration, erosion, and decrease of performance or efficiency of those hydraulic machines [2].

Since cavitation was first observed, the nature and characteristics of the phenomenon have been extensively studied over a hundred years [3] for bubble dynamics, cavitation patterns, cavitation instabilities, etc. For the inception of cavitation, there are two prerequisite conditions, i.e., the presence of bubble nuclei generally containing non-condensable gas (NCG) as weak points in a liquid, and the opportunity for the liquid pressure to drop below the vapor pressure. Particularly, in the case of liquid water, the presence of bubble nuclei containing air as the NCG has been used to reconcile theoretical and experimental estimates of the tensile strength of water, and the amount of air is known to have significant influences on the cavitation inception [4]. In addition, the NCG can be present in the following two forms, dissolved in a liquid or trapped in free bubble nuclei believed to remain stable for a long time [5], and it can behave differently from vapor in response to pressure variations because the precipitation or dissolution of NCG as the mass transfer is slower than the evaporation or condensation as the phase change. However, the detailed mechanisms whereby NCG modifies the process of the mass transfer in cavitating flows remain poorly understood [6].

In the 1960s, Holl [7] studied the effect of NCG on the occurrence of cavitation. Within his research, two types of cavitation, i.e., vaporous cavitation and gaseous cavitation, are distinguished. The results indicate that the difference between the desinence-cavitation number for the two types of cavitation is proportional to the dissolved air content. Brennen [8] analyzed the dynamic balances of dissolved air in a natural super cavitation behind a spherical headform and evaluated the mass transfer rate into the cavity theoretically by employing a turbulent boundary-layer model with a cavity surface velocity given by the potential flow. Just recently, Amini et al. [4] illuminated the role of the dissolved gas content in water for the behavior of tip vortex cavitation (TVC). They measured the inception and desinence thresholds of TVC under various gas content and observed that the desinence-cavitation number is significantly higher than the inception cavitation number resulting in a large hysteresis. It is argued that the delayed desinence is a consequence of the convective diffusion of dissolved air across the cavity interface [4]. Besides, our group has also observed the hysteresis phenomenon of sheet cavitation in an experiment for a two-dimensional convergent-divergent nozzle [9].

In practical engineering applications, a finite amount of NCG is always present in operating liquids unless the liquid is specially treated and deaerated. In the field of aeronautics industry, jet fuels are a complex multicomponent mixture and usually contain multiple types of dissolved gas due to their complex hydrocarbon nature [10]. The aircraft fuel pumps are operated in various environments in terms of temperature and pressure due to the different geographical locations and altitudes. According to Henry's law, the dissolved gas in the jet fuels may get released when the atmospheric pressure decreases, i.e., the out-gassing phenomenon. It is anticipated that the out-gassing phenomenon should affect the cavitation process to some extent. However, there have been few experimental studies concerning the influences of NCG content in cavitation performances of hydraulic machinery up to now due to the difficulties in accurately measuring the NCG content.

Magne et al. [11] recently examined the effect of the dissolved gas content on the cavitating and non-cavitating performance of an axial inducer by representing the jet fuel out-gassing with a water-CO₂ mixture. The experimental results show that the non-cavitating performance of the inducer is independent of the CO₂ content in the liquid while the cavitating performances are significantly affected by the gas content. Firstly, the head drop of the inducer occurs at a higher suction pressure when increasing the dissolved gas content for all the tested rotation speeds and flow coefficients. Furthermore, the degree of cavitation development is significantly different, especially around the pre-rotation region, under different gas contents near the head drop points of the cavitation performance curves. In addition, similar conclusions are obtained in a numerical investigation of the non-condensable gas effect on the cavitation performance of a centrifugal pump [12]. The results show that the critical cavitation number will decrease with the reduction of the NCG content. Recent studies on cavitation instabilities in marine propulsion systems have highlighted the importance of understanding the fundamental mechanisms governing cavitating flows. For instance, Wang et al. [13] demonstrated that tip-vortex cavitation instability is an important source of broadband pressure fluctuations, vibration, and noise. They further reported that the predicted pressure fluctuations were overestimated, which was attributed to the assumption of pure vapor collapse and the neglect of the damping effects of NCG in the cavitation model. This finding suggests that NCG can play an important role in cavitation dynamics, which is a significant problem in fluid engineering.

In addition, significant deviations from a similarity rule for cavitating flows in pumps are known to occur at times [14]. It is argued that the geometrical similarity and identical operation liquids, i.e., at the same temperature, gas content and nucleus spectrum, are the two prerequisites for applying the model laws. For the geometrically similar pumps working at a room temperature, the thermodynamic effect can be neglected and the effect of water quality is believed to be the main reason to cause the deviations from the similarity rule. In this case, the law of similarity employing the cavitation number of $\sigma = 2(p_{ref} - p_v)/\rho V_{ref}^2$, where p_{ref} and p_v are the reference pressure and saturated pressure, V_{ref} and ρ are the reference flow velocity and density of the working liquid, for cavitating flows does not well hold. As evidence, our recent experimental results concerning a converging-diverging nozzle show that the monitoring pressure coefficients do not collapse into one, which implies that the flow similarity based on the cavitation number does not well hold even though the gas content is little [9]. From a theoretical point of view, the diffusion process of NCG cannot be ignored if hydraulic machinery is working at off-design conditions. The cavitating recirculation regions, or cavitating separating regions, will appear when the hydraulic machinery is working at off-design conditions. In these low velocity regions, it is not surprising that there will be enough time for the mass exchange of NCG via the interfaces of bubbles. In this case, the diffusion of NCG has become an undeniable factor. Therefore, a rational approach to evaluate the influences of the diffusion process of NCG in cavitating flow is necessary to accurately predict hydraulic machinery performance at off-design conditions.

According to the analysis of the three aspects mentioned above, it can be found that the diffusion process of NCG plays a crucial role in cavitating flows in certain situations. Nevertheless, although previous studies have demonstrated the influence of NCG content on cavitation behavior, the quantitative coupling mechanism between NCG diffusion and hydraulic cavitation dynamics remains insufficiently understood. In particular, it is still unclear how the transport and redistribution of NCG affect the local pressure evolution relating to cavity growth and collapse, cavity unsteady shedding characteristics, and the occurrence of gaseous-like cavitation under high dissolved-gas

conditions.

To figure out the fundamental mechanisms of the diffusion effect of NCG on cavitating flows, we have carried out a series of experimental investigations [9][15]. However, the elucidation of the gas diffusion influences is limited by various factors due to the difficulty of controlling the operating liquid quality, such as accurate measurement of the NCG content, etc. [9]. It is even more challenging to measure NCG in bubbles. In this situation, computational fluid dynamics (CFD) provides a powerful approach to give more insights into the mechanisms behind the complex phenomenon. For the numerical simulation of cavitating flows, turbulence and cavitation model are the two crucial aspects to improve the simulation results. For the turbulence model, the Reynolds Averaged Navier–Stokes (RANS) approach is the most widely used method due to its cost-effective and robust performance. Nevertheless, the well-known flaw that overestimates the turbulence viscosity when coupled with the current cavitation models makes RANS failed to accurately predict complex cavitating flows [16]. In recent decades, the large eddy simulation (LES) approach, as a more physical and high-accuracy method, has gradually attracted attention in simulating cavitating flows [17]. However, the demand of grid quality for an effective LES simulation is extremely high, especially around wall regions due to the demand for high resolution of computational grids not only along the wall-normal direction but also along the stream-wise and span-wise directions. Therefore, a hybrid RANS/LES model, delayed detached eddy simulation (DDES), is employed in the present study [18]. Concerning the cavitation model, it can be generally divided into two categories, i.e., barotropic models and phase transport models. Since the importance of NCG is gradually recognized, several cavitation models considering the effects of NCG content have been developed in recent years. Mithun et al. [19] and Vasilakis et al. [20] proposed three-phase cavitation models based on the barotropic equation of state for modeling the pressure-density relation of each phase. Singhal et al. [21] developed the so-called full cavitation model, which accounts for the NCG effect as well as other important factors. Lu et al. [22] introduced NCG into Schnerr-Sauer model by assuming that the liquid releases the portion of NCG when the liquid vaporizes while the NCG will remain gaseous when the vaporized liquid again condenses to water. In their models, the diffusion process of NCG is not taken into account. Very recently, Malekshah [23] proposed a modified cavitation model based on the phase transport model, Zwart-Gerber-Belamri (ZGB) cavitation model, and the so-called merging theory, which models the diffusion process by the merging gas bubbles and vapor bubbles. However, the physical mechanism of the merging process seems not convincing. On the other hand, several new cavitation models based on coupled Eulerian-Lagrangian approach were developed in the last decade [24] [25][26][27]. To sum up, though several kinds of cavitation models taking the NCG effects into account are available, to the best of our knowledge, there is no cavitation model which can reasonably model the diffusion process of NCG in a hydraulic cavitation until now. To achieve a breakthrough in predicting cavitating flows, a modified cavitation model based on the well-known Schnerr-Sauer cavitation model [28], the SS-NCG model, has been recently developed by our group [29]. The inclusion of NCG is achieved by solving an additional advection-diffusion equation of NCG concentration, which is closer to the real physical process.

Owing to the complexity of engineering flows, simplified configurations are widely adopted to establish a fundamental understanding of the underlying physical mechanisms. Accordingly, the present study focuses on cavitating flow around a two-dimensional hydrofoil to systematically investigate the influence of NCG diffusion. Different from our previous 2D work [29], which mainly

focused on the model development and preliminary validation under a conventional condition, the present study significantly extends the SS-NCG framework in both physical scope and numerical methodology. A special inflection-point condition is investigated to analyze unique phenomena associated with cavitation instabilities. Moreover, the SS-NCG cavitation model is coupled for the first time with a three-dimensional DDES framework to resolve complex unsteady cavitating structures. The novelty of the present work lies in revealing the mechanism by which NCG diffusion modifies cavitation dynamics, including the stabilization and damping of high-frequency fluctuations, and the coexistence of cavitation collapse and gaseous-like cavitation during the same shedding process.

In section 2, the mathematical methods including the theories of the SS-NCG model are outlined. In Section 3, the numerical results of the three typical cases, i.e., SSmodel, LowDO, and HighDO, are compared with some experimental results. Then the unsteady and statistical characteristics of the cavitating flow considering NCG are analyzed on the basis of the numerical results. At last, some important conclusions will be summarized.

II. Mathematical methods

To model the influence of NCG in cavitation processes, a new homogeneous cavitation model based on Schnerr-Sauer (SS) cavitation model, called SS-NCG model in the present paper, has been constructed [29]. It should be noted that the homogeneous-flow assumption has been widely applied to various simulations of cavitating flow. The basis of this assumption would be that hydraulic cavitation typically occurs in an accelerated flow field, and the slip velocity ratio against the liquid velocity is expected to be small due to the higher liquid velocity. In fact, some cavitation simulations based on the homogeneous assumption successfully predict specific cavitation instabilities (e.g., alternate cavitation or rotating cavitation) in a cavitating inducer [30] as a turbomachine. Therefore, taking account of the NCG influence in the framework of homogeneous cavitation model is significant. On the other hand, it has been pointed out that the homogeneous-flow assumption may underestimate the lift force of a cavitating hydrofoil as pointed out by Kato [31] in a benchmark test using various solvers. This means that the distribution of cavities on the suction side of a hydrofoil does not reproduce the experimental state, in which the simulations mostly underestimate the cavity length. Therefore, for more precise simulations, it is necessary to develop and apply the cavitation model that can consider not only the influence of NCG but also the slippage. However, taking account of both influences is more complex because the assumption of S-S model that the number of bubbles per unit liquid volume kept constant is not satisfied in the slipped velocity field, and the number density must be additionally solved. Therefore, in this paper, the homogeneous-flow assumption was employed as a preliminary stage before including the influence of the slip velocity. Besides, the present model assumes that, in cavitating regions, the vapor phase and released NCG share the same cavity volume, whereas outside the cavitating region the NCG remains fully dissolved in the liquid phase and therefore does not contribute to an independent gaseous volume fraction. Figure 1 illustrates an image of a typical cavitating flow around a hydrofoil, consisting of an attached sheet cavity and a detached cloud cavity. As mentioned in Introduction, NCG can be present in a liquid or in bubble nuclei. When the fluid flows around a leading edge of a hydrofoil, a local static pressure on the suction side decreases. At the same time, the volume of the bubble nuclei containing NCG in the fluid, e.g., NCG nuclei in Fig. 1, will begin to increase and reach cavitation

inception. With the growth of such cavitation bubbles, the partial pressure of NCG, p_g , will decrease, then the chemical equilibrium of NCG between inside and outside the bubbles will be destroyed. In this paper, the concentration of mass of NCG per unit volume of water as the bulk liquid is denoted by c_l [kg/m³] while the NCG concentration at the bubble interface on the liquid side is denoted by c_b [kg/m³]. During the growth, with the decrease of the partial gas pressure inside a bubble, c_b becomes generally lower than c_l , and the dissolved NCG in the liquid will diffuse to the bubble. On the other hand, the NCG in the cavitation bubbles will diffuse to the liquid when the bubbles collapse in a pressure recovery region.

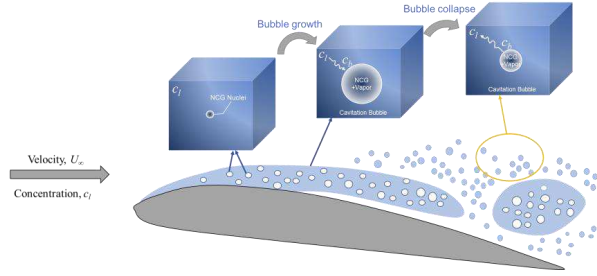


FIG. 1. Image of typical cavitating flow around a hydrofoil with NCG diffusion effect.

A. Governing equations

With the homogeneous flow assumption that both bubbles and the surrounding liquid have the same velocity, the mass and momentum conservation equations for the mixture consisted of liquid, vapor, and NCG are given as,

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m u_i)}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial (\rho_m u_i)}{\partial t} + \frac{\partial (\rho_m u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[(\mu_m + \mu_t) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \sigma_t \kappa \frac{\partial \alpha_l}{\partial x_j}, \quad (2)$$

where u_i , u_j are the velocity vectors, and ρ_m , μ_m are the mixture density and the viscosity. p is the static pressure, and μ_t is the turbulent eddy viscosity. σ_t is the surface tension, and κ is the interfacial curvature between liquid phase and vapor phase with gas. α_l is the volume fraction of the liquid phase. On the basis of the void fraction α_v by the vapor with the gas phase in bubbles, the definition of ρ_m and μ_m can be given as,

$$\rho_m = (\rho_v + \rho_g)\alpha_v + \alpha_l \rho_l, \quad (3)$$

$$\mu_m = (\mu_v + \mu_g)\alpha_v + \alpha_l \mu_l, \quad (4)$$

$$\alpha_v = 1 - \alpha_l, \quad (5)$$

where the subscripts m , v , and l represent the mixture phase, vapor phase, and liquid phase, respectively, and the subscript g represents the gas. Note that, α_v represents the total cavity volume fraction occupied collectively by vapor and NCG.

Since the unknown density of NCG, ρ_g , and the vapor volume fraction α_v are introduced, the phase transport equations as well as the ideal gas law are established for the closure,

$$p_g = \rho_g R_g T, \quad (6)$$

$$\frac{\partial (\rho_v \alpha_v)}{\partial t} + \frac{\partial (\rho_v \alpha_v u_i)}{\partial x_i} = \dot{m}_v, \quad (7)$$

$$\frac{\partial(\rho_g \alpha_v)}{\partial t} + \frac{\partial(\rho_g \alpha_v u_i)}{\partial x_i} = \dot{m}_g, \quad (8)$$

where R_g is the specific gas constant and T is the operating temperature of the liquid. $\dot{m}_v$ and $\dot{m}_g$ represent the rate of the mass transfer due to the phase change, i.e., cavitation and diffusion process of NCG, respectively. Equations (7) and (8) are the mass conservation equations for the species of liquid and NCG, respectively. In the present study, all the processes in a cavitating flow are assumed to be isothermal because the analysis is done for cavitation in water at the room temperature where the thermodynamic effect [32] can be neglected. Some previous studies [33] considered two types of bubbles (or bubble nuclei), one composed solely of vapor and the other composed solely of NCG. Consequently, two distinct bubble volume fractions are introduced in such models. In addition, Malekshah and Wróblewski [23] introduced a merging model between the two types of bubbles to simulate cavitation development while accounting for NCG effects. In contrast, in the present model, all bubbles are assumed to contain both saturated vapor and NCG, which coexist within the same bubble volume. Therefore, the volume fractions occupied by vapor and NCG are always equal. Since actual bubbles, including nuclei, can contain both vapor and NCG, we believe that this modeling approach is more physically realistic, and thus such a merging process is unnecessary. The assumption that the vapor inside a bubble always remains saturated is not valid when considering strongly non-equilibrium phenomena such as bubble collapse associated with cavitation erosion. However, the objective of the present study is to capture cavitating flow structures at the hydrofoil scale. Therefore, the above assumption is considered acceptable for the present problem. It should also be noted that the density of NCG inside a bubble may exceed the saturated vapor density. Such conditions can still be represented appropriately, provided that the ideal-gas assumption remains valid.

Using Eqs. (1) and (3), the divergence of the velocity can be obtained as,

$$\frac{\partial u_i}{\partial x_i} = \frac{1}{\rho_m} \left\{ \rho_l \frac{D\alpha_v}{Dt} - \frac{D}{Dt} (\rho_v \alpha_v + \rho_g \alpha_v) \right\}. \quad (9)$$

Substituting Eqs. (9) into Eqs. (7) and (8), the source terms $\dot{m}_v$ and $\dot{m}_g$ can be obtained as,

$$\dot{m}_v = \frac{\rho_v}{\rho_m} \left\{ (\rho_l + \rho_g \alpha_v) \frac{D\alpha_v}{Dt} - \alpha_v \frac{D\rho_g \alpha_v}{Dt} \right\}, \quad (10)$$

$$\dot{m}_g = \frac{1}{\rho_m} \left[\rho_g (\rho_l \alpha_v - \rho_v \alpha_v) \frac{D\alpha_v}{Dt} + \{ \rho_l (1 - \alpha_v) + \rho_v \alpha_v \} \frac{D\rho_g \alpha_v}{Dt} \right]. \quad (11)$$

Note that the density of the vapor and the liquid is assumed to be constant during the derivation, while the NCG density in the vapor phase is variable.

B. Cavitation model

In the modelling of the cavitation process considering the diffusion of NCG, Schnerr-Sauer (SS) cavitation model [28] is employed as the basic model. During the dissolution and precipitation process of NCG, the mass of NCG per unit volume of the mixture within the cavitating region can be expressed as,

$$\frac{D\rho_g \alpha_v}{Dt} = \frac{D(mn)}{Dt} = m \frac{Dn}{Dt} + n \frac{Dm}{Dt}, \quad (12)$$

where the parameter m represents the mass of gas inside a bubble, and n represents the number density of bubbles defined per unit volume. Furthermore, according to the formulation of SS model,

another constant n' will be introduced, which is defined as the number of bubbles per unit liquid volume. Thus, the relation between n and n' is given as,

$$n = n'(1 - \alpha_v). \quad (13)$$

The mass of NCG in a bubble can be related to the radius of the bubble, R , by the form

$$\frac{Dm}{Dt} = 4\pi r^2 \dot{M}_g, \quad (14)$$

where $\dot{M}_g$ is the diffusion flux of the NCG at the interface of the bubble.

Substituting Eqs. (13) and (14) into Eq. (12) assuming that n' is kept constant, the relation below is obtained,

$$\frac{D\rho_g \alpha_v}{Dt} = \frac{3\alpha_v}{r} \dot{M}_g - \frac{\rho_g \alpha_v}{1 - \alpha_v} \frac{D\alpha_v}{Dt}. \quad (15)$$

As same as the original SS model, the void fraction α_v can be calculated by the following expression,

$$\alpha_v = n \frac{4}{3} \pi r^3 = \frac{n' \frac{4}{3} \pi r^3}{1 + n' \frac{4}{3} \pi r^3}. \quad (16)$$

From Eq. (16),

$$r = \left(\frac{3\alpha_v}{4\pi n'(1 - \alpha_v)} \right)^{\frac{1}{3}}. \quad (17)$$

Then, according to Eqs. (15) and (16), the material derivative of the NCG density can be given by

$$\frac{D\rho_g}{Dt} = \frac{3}{r} \dot{M}_g - \rho_g \frac{3}{r} \frac{Dr}{Dt}. \quad (18)$$

Substituting Eqs. (15) and (16) into Eqs. (10) and (11), the mass source terms can be rewritten as,

$$\dot{m}_v = \frac{3\alpha_v}{\rho_m r} [\rho_l \rho_v \alpha_l \frac{Dr}{Dt} + \alpha_v (\rho_g \rho_v \frac{Dr}{Dt} - \rho_v \dot{M}_g)], \quad (19)$$

$$\dot{m}_g = \frac{3\alpha_v}{\rho_m r} [\rho_l \alpha_l \dot{M}_g + \alpha_v (\rho_v \dot{M}_g - \rho_g \rho_v \frac{Dr}{Dt})]. \quad (20)$$

Similarly to the original SS model, the present SS-NCG model determines the growth rate of a bubble radius of $\frac{Dr}{Dt}$ based on the Rayleigh-Plesset equation,

$$\rho_l \left[r \frac{d^2 r}{dt^2} + \frac{3}{2} \left(\frac{dr}{dt} \right)^2 \right] = p_v - p + p_g - \frac{2S}{r} - \frac{4\mu_l}{r} \frac{dr}{dt}, \quad (21)$$

where p_v and p_g are the partial pressure of saturated vapor and NCG, respectively. S is the surface tension. Neglecting the second-derivative term on the left-hand side and the last two terms, i.e., surface tension and viscosity effects, on the right-hand side of Eq. (21), the simplified Rayleigh-Plesset equation considering NCG can be written as,

$$\frac{Dr}{Dt} = \text{sign}(p_v + p_g - p) \sqrt{\frac{2}{3} \frac{|p_v + p_g - p|}{\rho_l}}. \quad (22)$$

Using Eqs. (16) and (22), the changing rate of the void fraction is derived as follows,

$$\frac{D\alpha_v}{Dt} = \frac{3\alpha_v(1 - \alpha_v)}{r} \text{sign}(p_v + p_g - p) \sqrt{\frac{2}{3} \frac{|p_v + p_g - p|}{\rho_l}}. \quad (23)$$

As for the diffusion flux $\dot{M}_g$, it can be formulated by a spherically symmetric, one-dimensional diffusion equation of NCG around a spherical bubble. Here, the steady-state solution is adopted to

simplify the diffusion process, and the diffusion flux $\dot{M}_g$ based on the Fick's law is given as follows,

$$\dot{M}_g = \frac{\mathfrak{D}}{r}(c_l - c_b), \quad (24)$$

where $\mathfrak{D}$ is the gas diffusion coefficient. On the basis of Henry's law, the diffusion equilibrium between the liquid and the NCG in a bubble can be given as follows:

$$p_g = Hx_g, \quad (25)$$

$$x_g = \frac{n_g}{n_g + n_l} = \frac{m_g/M_g}{m_g/M_g + m_l/M_l}, \quad (26)$$

where x_g is the molar fraction of NCG at the bubble interface on the liquid side, and H is the corresponding Henry's constant. Here, m_g and m_l are the masses, n_g and n_l are the molar amounts of substance, and M_g and M_l are the molar masses of the NCG and the liquid, respectively. It should be noted that Henry's law in Eq. (25) is only applicable to dilute solutions with relatively low gas solubility. In the present study, oxygen is only slightly soluble in water. Under the experimental conditions, even at saturated dissolved oxygen conditions, the dissolved oxygen concentration is typically on the order of only several mg/L, which is negligibly small compared with the liquid density (about 10^6 mg/L). Therefore, throughout the present simulations, the dissolved gas concentration remains sufficiently low, and deviations from the ideal dilute-solution assumption are expected to be negligible. For dilute solutions containing only a small amount of solute, the above equation can be simplified as:

$$x_g \approx \frac{n_g}{n_l} = \frac{m_g/M_g}{m_l/M_l}. \quad (27)$$

Considering the definitions of the dissolved NCG concentration, $c_b = \frac{m_g}{V}$, and the liquid density, $\rho_l = m_l/V$, x_g can be further written as:

$$x_g = \frac{c_b V/M_g}{\rho_l V/M_l} = \frac{c_b}{\rho_l} \frac{1}{M}. \quad (28)$$

where M is the molar weight ratio of the NCG and the liquid, i.e., $M = \frac{M_g}{M_l}$. V represents a control volume within the liquid phase.

Combining Eqs. (25) and (28) yields

$$c_b = \rho_l \frac{p_g}{H/M}, \quad (29)$$

Also, for obtaining the concentration of NCG in the liquid c_l , the following advection-diffusion equation will be solved,

$$\frac{\partial c_l \alpha_l}{\partial t} + \frac{\partial c_l \alpha_l u_l}{\partial x_i} = \frac{\partial}{\partial x_i} \left(\mathfrak{D} \frac{\partial c_l \alpha_l}{\partial x_i} \right) - \frac{3\alpha_v}{\rho_m r} [\rho_l \alpha_l \dot{M}_g + \alpha_v (\rho_v \dot{M}_g - \rho_g \rho_v \frac{Dr}{Dt})]. \quad (30)$$

The two terms on the left-hand side of Eq. (30) represent the transient term and the convective transport term of NCG in the liquid phase, respectively, while the two terms on the right-hand side correspond to the diffusion term and the interphase mass-transfer source term, respectively. The diffusion term, $\frac{\partial}{\partial x_i} \left(\mathfrak{D} \frac{\partial c_l \alpha_l}{\partial x_i} \right)$, acts as a transport mechanism analogous to the heat-conduction term in an energy equation. Its role is not to directly generate or consume NCG, but to redistribute NCG within the flow field through concentration gradients. In contrast, the second term on the right-hand side of Eq. (30) represents the local source term associated with interphase mass transfer and cavity-

volume variation, which can be rearranged as $-\frac{3\alpha_v}{\rho_m r} \left[(\rho_l \alpha_l + \rho_v \alpha_v) \dot{M}_g - \alpha_v \rho_g \rho_v \frac{dr}{dt} \right]$. In particular, the first part corresponds to the dissolution/precipitation process driven by local nonequilibrium conditions relative to Henry-law equilibrium, whereas the second part accounts for the dilution or accumulation effect caused by transient cavity expansion and collapse. Compared with the diffusion term, the source term generally has a more direct and stronger influence on the local NCG concentration because of the intense nonequilibrium mass transfer occurring near the cavity interface.

In this paper, the diffusion coefficient $\mathfrak{D}$ is assumed to be equal to the effective kinematic viscosity, i.e., both the laminar and the turbulent Schmidt numbers are regarded as unity, while the actual laminar Schmidt number is about 500 as well-known. Of course, the assumption above is generally inappropriate, but in the region where cavitation occurs, the turbulence is expected to be fully developed. According to the paper by Gualtieri [34], the order of the Schmidt number of air in water turbulent flow is about 0.1-1. Therefore, the Schmidt number in this paper was always assumed to be unit. It should be emphasized that the present diffusion model is intended mainly to capture the direct coupling between NCG transport and cavitation dynamics, rather than to establish a highly accurate dissolved-gas diffusion model. An advanced model incorporating a more physically realistic diffusion mechanism is currently under development and will address this issue.

According to the five equations of (1), (2), (18), (23), and (30) with some constitutive ones, a cavitation simulation considering the NCG influence is possible in a closed form.

C. Turbulence Model

For the system closure, the $k - \omega$ SST Delayed DES (DDES) method is employed, which is a modification of the $k - \omega$ SST in RANS method. The governing equations are as follows [35],

$$\frac{\partial k}{\partial t} + \frac{\partial(k\bar{u}_i)}{\partial x_i} = \widetilde{P}_k + \frac{\partial}{\partial x_i} \left[\left(\nu_m + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_i} \right] - \beta^* k \omega F_{DDES}, \quad (31)$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial(\omega\bar{u}_i)}{\partial x_i} = P_\omega + \frac{\partial}{\partial x_i} \left[\left(\nu_m + \frac{\nu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_i} \right] - \beta \omega^2 + 2(1 - F_1) \sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, \quad (32)$$

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, SF_2)}. \quad (33)$$

The bar over the velocity represents the filtered velocity, which corresponds to a time-filtered velocity in unsteady RANS (URANS) regions and a space-filtered velocity in LES regions. The production term in the ω equation, P_ω , is evaluated as

$$P_\omega = \alpha S^2, \quad S = \sqrt{2\bar{s}_{ij}\bar{s}_{ij}}, \quad (34)$$

where $\bar{s}_{ij}$ is the strain rate tensor of the filtered velocity field, $\bar{s}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$. F_1 and F_2 are the following blending functions,

$$F_1 = \tanh(\xi^4), \quad \xi = \min\left(\max\left(\frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{\omega y^2}\right), \frac{4\sigma_{\omega 2} k}{CD_{\omega} y^2}\right), \quad (35)$$

$$F_2 = \tanh(\eta^2), \quad \eta = \max\left(\frac{2\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{\omega y^2}\right), \quad (36)$$

where y stands for the distance to the nearest wall. CD_{ω} is given as

$$CD_{\omega} = \max\left(2\sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10^{-10}\right). \quad (37)$$

The production of the turbulent kinetic energy, $\widetilde{P}_k$, is defined as

$$\widetilde{P}_k = \min(P_k, 10\beta^* k \omega), \quad P_k = \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j}. \quad (38)$$

The model constants, α , β , σ_k , σ_{ω} are calculated with a linear combination between the corresponding coefficients for the $k-\omega$ and $k-\epsilon$ models as follows,

$$\phi = F_1 \phi_1 + (1 - F_1) \phi_2, \quad (39)$$

where ϕ_i denotes either α_i , β_i , σ_{ki} , $\sigma_{\omega i}$. The values of the constants are given as follows according to Ref. [36], $\alpha_1 = \frac{5}{9}$, $\alpha_2 = 0.44$, $\beta_1 = 0.075$, $\beta_2 = 0.0828$, $\frac{1}{\sigma_{k1}} = 0.85$, $\frac{1}{\sigma_{k2}} = 1$, $\frac{1}{\sigma_{\omega 1}} = 2$, $\frac{1}{\sigma_{\omega 2}} = 0.856$, $\beta^* = C_{\mu} = 0.09$.

The function F_{DDES} in Eq. (31) is given as follows.

$$F_{DDES} = \max\left(\frac{L_t}{C_{DDES} \Delta} (1 - F_s), 1\right), \quad (40)$$

where $L_t = \sqrt{k}/(\beta^* \omega)$ is a turbulent length scale, $\Delta = \max(\Delta x_1, \Delta x_2, \Delta x_3)$ is the length of the largest side of each computational cell, and $C_{DDES} = 0.61$ as a constant value. F_s is also a blending function to avoid unsteady solutions near wall regions, chosen as either F_1 or F_2 .

D. Experimental and Computational Setups

In this paper, some experimental data previously conducted in a closed-loop cavitation tunnel in our group (Kyushu University) [37-40] are used. The tunnel has a rectangular test section with a height of 200 mm and a span of 81.5 mm. The test object, a two-dimensional Clark Y-11.7% hydrofoil, is installed at the center between the top and the bottom wall. The chord length of the hydrofoil is $C=100$ mm, and the span length is $S=81$ mm. Note that it has a tip clearance of 0.5 mm in the spanwise direction, which was neglected in the present simulations for computational simplification as this work is intended as a preliminary exploratory study of NCG effects on cavitation dynamics. Nevertheless, recent investigations by the authors indicate that tip-clearance

flow and the associated tip-leakage cavitation can influence cavitation evolution, three-dimensional flow structures, and shedding dynamics, especially during cavity collapse and regeneration. Therefore, the tip-clearance effect should be considered in future studies. In the experiments, the gas content in the water is evaluated using the amount of dissolved oxygen (DO), and three types of water quality is defined here, i.e., high DO (about 80% of the saturated level in atmosphere), medium DO (about 50%), and low DO (about 20%). A detailed description of the experimental apparatus and process can be seen in Ref. [37-40].

Figure 2 shows time-averaged lift coefficient curves of the Clark Y-11.7% hydrofoil with the angle of attack of $\alpha=8$ degrees for different DO conditions [40]. The red, green, and blue broken lines indicate each cavitation inception for the high DO, the medium DO, and the low DO, respectively. In Fig. 2(a), the lift coefficients at non-cavitation conditions are different among the three DO conditions, which may be attributed to some slight error of setting the angle of attack in those experiments. Therefore, the lift coefficients are normalized with the non-cavitation lift coefficients, i.e., $C_L/C_{L,non-cav}$, in Fig. 2(b). On the basis of the experimental results, it can be observed that different DO conditions seem to have little effect on the time-averaged C_L . To simplify the analysis, only the two types of water quality, i.e., high DO and low DO, will be further numerically investigated. In addition, there is a notable decrease in the time-averaged lift coefficient for the three different DO conditions from about $\sigma = 1.34$, which corresponds to a development of unsteady cloud cavitation. To figure out the characteristics of this special critical point, the cavitation number, $\sigma = 1.34$, is adopted as a target cavitation number in the following numerical analysis.

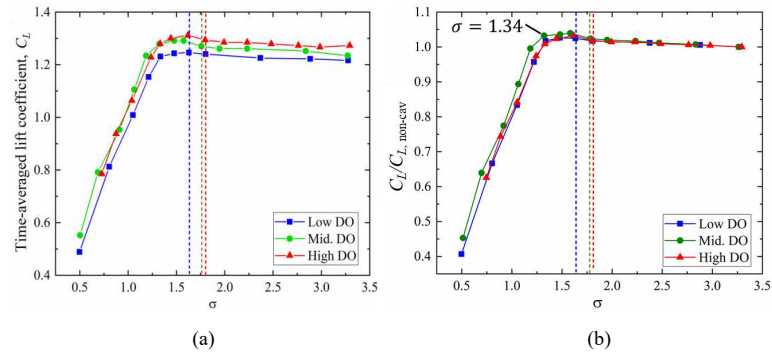


FIG. 2. (a) Time-averaged lift coefficient curves, and (b) normalized time-averaged lift coefficient curves for different DO conditions in the Clark Y-11.7% hydrofoil with $\alpha = 8$ degrees. Reproduced from “Unsteady fluid force and fluid force moment under cavitation on isolated hydrofoil with tip clearance, Master Thesis, Kyushu University, 2018 (in Japanese)”, with the personal permission of the author, Mr. Soichiro Kitamura, who submitted it to Kyushu University as a Master thesis in February, 2018 [40].

On the basis of the experimental results mentioned above, three typical test cases are numerically investigated: (i) No NCG with the original Schnerr-Sauer cavitation model (SS model), (ii) LowDO (20% of the saturated dissolved oxygen in atmosphere), and (iii) HighDO (80% in the same way) with SS-NCG cavitation model. Here, the “No NCG” case does not correspond to a liquid with strictly zero dissolved-gas content. Instead, it represents the baseline S-S cavitation model without explicitly considering NCG diffusion and partial-pressure effects of the gas. The

angle of attack of the hydrofoil for the three cases is $\alpha = 8$ degrees, and the cavitation number $\sigma = 1.34$ is defined at the outlet boundary of the numerical domain as shown later. The geometric configuration of the cavitating flow around the hydrofoil is shown in Fig. 3. Through several trial calculations, the computational domain is reduced slightly compared with the original experimental test section [39, 40] mentioned above. In an LES analysis conducted by Okabayashi et al. [41], the span length was set at $S=0.3C$ to save the computational cost, which is enough to resolve the three-dimensional vortex structures along the spanwise direction. The inlet and outlet sections are $3C$ and $4C$ away from the center of the hydrofoil, respectively. In addition, the height of the test section is also reduced slightly from the experimental one, i.e., $1.6C$ ($=160\text{mm}$). A uniform velocity of $U_0=8.1$ m/s with 5% turbulence intensity is specified at the inlet boundary for each case. No-slip wall condition is applied on the hydrofoil surface while the slip wall conditions are done on the top and bottom walls for simplicity. The outlet pressure of the straight channel is regarded as the reference pressure, p_{ref} , to adjust the outlet cavitation number of $\sigma = 1.34$. In addition, to compare the effects of NCG on the cavitating flows, different NCG concentrations, $c_{l,\infty}$ are imposed at the inlet section, i.e., DO0% (corresponding to the original SS model), DO20%, and DO80%. The working temperature of the liquid is 293K, and the corresponding saturation vapor pressure is $p_v = 2339$ Pa. Other fluid properties and parameters are given in Table I.

Some relevant non-dimensional parameters are defined as follows,

$$Re = \frac{\rho_l U_0 C}{\mu_l} \quad (41)$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho U_0^2 CS}, \quad C_D = \frac{F_D}{\frac{1}{2}\rho U_0^2 CS} \quad (42)$$

$$C_p = \frac{p - p_{\text{ref}}}{\frac{1}{2}\rho U_0^2} \quad (43)$$

where F_L and F_D stand for the lift and drag forces of the hydrofoil, respectively.

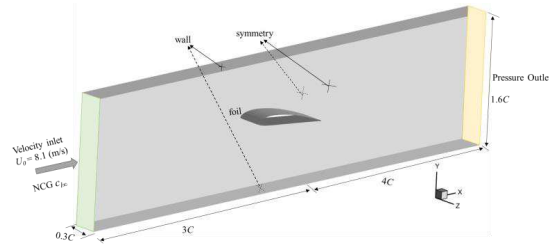


FIG. 3. Schematic diagram of computational domain and boundary conditions.

TABLE I. Fluid properties in the present simulations.

ρ_l [kg/m ³]	ρ_v [kg/m ³]	μ_l [Pa·s]	μ_v [Pa·s]	Henry's constant H [Pa]	molar weight ratio M
998	0.0173	1×10^{-3}	9.7×10^{-6}	7.78×10^9	0.0288

The present calculation is performed using an open-source code, OpenFOAM. Table II lists the adopted discretization scheme for each term in the equations to be solved in this paper. Note that the Multidimensional Universal Limiter with Explicit Solution (MULES) solver [42] is applied to

solve the volume fraction equation in OpenFOAM, which is based on a blending method of the first order and the high order schemes. The PIMPLE algorithm is applied for the unsteady calculation of the discretized equations for both the pressure and the velocity, which allows larger time steps during the calculations. Figure 4 shows the detailed calculation process with the SS-NCG cavitation model. The NCG density in cavitation bubbles, Eq. (17), is solved with the phase transport equation of the liquid volume fraction. Also, the concentration transport equation of the NCG in the liquid phase, Eq. (27), is solved after the calculation of the pressure and velocity.

TABLE II. Summary of discretization schemes.

Discretization		Schemes	Order
Temporal discretization	Transient term	Euler	First order
Spatial discretization	Gradient schemes	Gauss linear	Second order
	Divergence schemes	Gauss linearUpwind	Second order
	Laplacian schemes	Gauss linear corrected	Second order

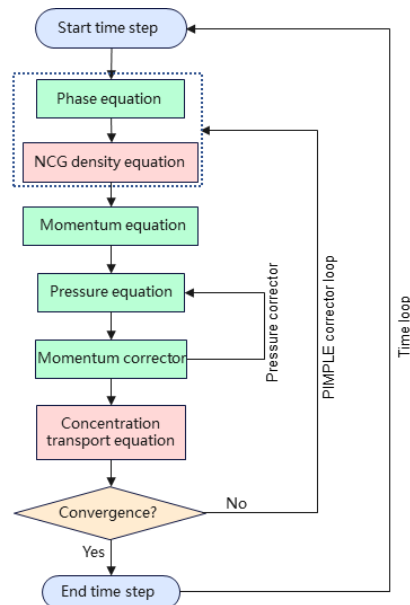


FIG. 4. Flow chart of calculation process with SS-NCG cavitation model.

It is noted that an adjustable time step method in OpenFOAM is employed with the maximum Courant number $C_o=3$ in the present computations. Compared with a fixed time step method, using the adjustable time step enables the time marching of the computation to be efficient while maintaining the computational stability. During the calculation, the mean time step is about $\Delta t = 2.5 \times 10^{-6}$ s. Preliminary tests showed that even a slightly larger time step, such as 5×10^{-6} s, could easily lead to numerical divergence after only a few iterations due to the strong nonlinearity

of the cavitating flow. Therefore, a dedicated time-step independence study was not conducted in the present work. Although the present temporal resolution is considered sufficiently small to capture the dominant unsteady cavitation dynamics and the main Power Spectral Density (PSD) characteristics, the absence of a formal time-step independence verification remains a limitation of this study, particularly regarding the quantitative accuracy of the high-frequency spectral components and small-scale transient structures.

F. Mesh Generation and Validation

To obtain a suitable mesh, four typical sets of structured hexahedral mesh are generated and validated, which are denoted as Mesh 1–4. As shown in Fig. 5, the four meshes share the same topology with the different spatial distributions. On the basis of the trial calculations, a C-type block is generated for a region around the hydrofoil within which most of the cavitation bubbles will appear. In the boundary layer, the height of the first element near the hydrofoil is chosen to satisfy the y^+ -plus (y^+) value in the order of unity for the four meshes. Different from the LES calculation, the values for x -plus and z -plus are not strictly limited. Since the present work mainly focuses on the cavitating regions, the mesh independency analysis is based on the C-type block region depicted in Fig. 5(a). Fig. 6 presents the y^+ distributions on the hydrofoil surface for Mesh 3 at two representative instants obtained from simulations using the original SS model. The results show that the y^+ values within the cavitating region are predominantly in the range of 4.2–6.3, whereas those in the non-cavitating regions are generally below 1. Some spatial variation exists due to the complex cavitation dynamics, and almost uniform low y^+ values are difficult to maintain in cavitating flows because the large density contrast across the liquid–vapor interface leads to significant variations in wall shear stress and, consequently, y^+ values. Nevertheless, the overall y^+ levels indicate an acceptable near-wall resolution for the present DDES simulations. Table III lists the detailed mesh information of the C-type block region. N_x , N_y , and N_z represent the number of the nodes on Edge X, Edge Y, and Edge Z shown in Fig. 5(a), respectively. Besides, the predicted mean lift and drag coefficients, as well as the mean cavity volume, $\bar{V}_c$, and Strouhal number, S_r , are displayed in the table, which are calculated by the original SS model. To further quantify the mesh dependency of the present simulations, an additional mesh-convergence assessment based on the Grid Convergence Index (GCI) methodology was conducted for the four monitored quantities [43][44]. The corresponding estimated GCI values are also summarized in Table III. As shown in the table, the estimated GCI values for $\bar{C}_L$, $\bar{C}_D$ and $\bar{V}_c$ are below approximately 2.5%, 3%, and 2.7%, respectively. The dominant Strouhal number remains nearly unchanged ($S_r \approx 0.47 - 0.48$) for Mesh 2, Mesh 3 and Mesh 4, with the corresponding estimated uncertainty remaining within approximately 2 – 4%. These results indicate that the principal flow quantities exhibit satisfactory convergence behavior with mesh refinement and that an approximately grid-independent state has been achieved for the present simulations. It should be noted that since several quantities become nearly unchanged between the two finest meshes, the formal apparent-order estimation in the standard GCI procedure becomes ill-conditioned. Therefore, the GCI analysis was mainly performed using Meshes 1 – 3, while the Mesh 3 – 4 comparison was used to confirm the plateau convergence behavior. Based on the above analyses, Mesh 3 is employed for the simulation of the three comparative cases as a reasonable compromise between numerical accuracy and computational cost.

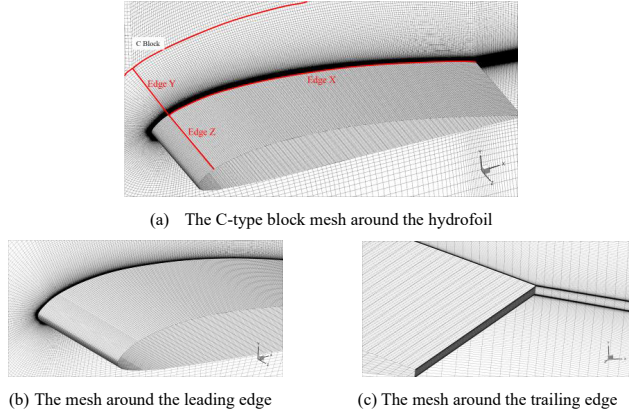


FIG. 5. Sketch of the mesh refinement region in the case of Mesh 3.

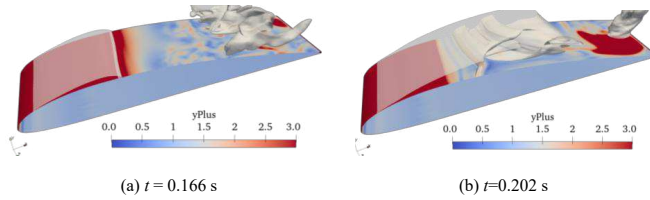


FIG. 6. Distributions of y^+ on the hydrofoil surface at two representative instants for the cavitating case of Mesh 3 (white iso-surface: $\alpha_v=0.4$).

TABLE III. Mesh information in the C-type block region.

	Height of the first layer (mm), Δy	$N_x \times N_y \times N_z$	Total number of computational cells	$\overline{C}_L$	$\overline{C}_D$	$\overline{V}_c$ ($\times 10^{-6}$)	S_t
Mesh 1	0.01	$90 \times 80 \times 40$	1.3 million	1.26	0.16	3.27	0.43
Mesh 2	0.01	$90 \times 80 \times 80$	2.8 million	1.23	0.16	4.32	0.47
Mesh 3	0.01	$160 \times 120 \times 80$	5.1 million	1.20	0.14	4.58	0.48
Mesh 4	0.01	$160 \times 120 \times 120$	8.8 million	1.20	0.14	4.52	0.47
GCI (%)	--	--	--	< 2.5	< 3	< 2.7	2-4

To further assess the applicability of the present numerical framework beyond the specific operating condition ($\sigma = 1.34$) investigated in this study, an additional validation was conducted against the published cavitating Clark Y-11.7% hydrofoil benchmark reported in Ref. [38]. The selected validation case corresponds to a more conventional cavitating condition with $\alpha = 8$ degree, $\sigma = 1.05$, and a dissolved-oxygen concentration of approximately 20%DO (LowDO). The same cavitation model and numerical methodologies adopted in the present study were employed in the simulation, which was performed using a two-dimensional computational mesh.

Figure 7 presents the temporal evolution of the lift coefficient obtained from both the experiment and the numerical simulation. In addition, the comparison of the dominant frequency, mean lift coefficient, and mean drag coefficient between the present simulation and the published

experimental data is summarized in Table IV. The predicted dominant frequency agrees reasonably well with the experimental measurement, with the simulated value of 10.0 Hz compared to the experimental value of 9.8 Hz. Furthermore, the mean lift and drag coefficients are also captured with acceptable agreement. These results indicate that the present numerical framework is capable of reproducing the principal unsteady cavitation characteristics reported in the literature.

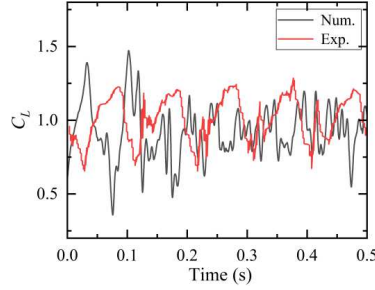


FIG. 7 Time-domain plots of the hydrofoil's lift coefficient.

TABLE IV. Comparison of experimental and numerical results under $\sigma = 1.05$ at LowDO.

	Dominant frequency (Hz)	$\bar{C}_L$	$\bar{C}_D$
Experimental results [38]	9.8	1.03	0.10
Numerical results	10.0	0.92	0.13

Therefore, in the following Section III, although quantitative discrepancies in shedding frequency still exist for the specific operating conditions investigated in the present study, the above benchmark validation further supports the applicability of the present framework for analyzing the relative influence of NCG diffusion on unsteady cavitation dynamics.

III. Results and Discussion

To get a thorough understanding of the characteristics of cavitating flows with NCG, the morphological analysis of cavitation may be a good entry point. In the previous experimental and numerical results, the cavitating flow at the present operating condition is unsteady. Therefore, the evolution of the cavitation structure as well as the cavity volume is shown in Figs. 8 and 9 to describe the unsteady process of the structure qualitatively and quantitatively. The cavity volume V_c can be obtained by integrating the void fraction α_v over the entire computational domain (D), which is defined as,

$$V_c = \int_D \alpha_v dV. \quad (44)$$

The periodic cavitation shedding in the experiment [40] is also presented in Fig. 10, and some discrepancies are confirmed between the numerical and experimental results in Figs. 9 and 10.

Firstly, as for the experimental results shown in Fig. 10, it can be found that the shedding periods of the three cases are quite different. The shedding period in MiddleDO is much smaller than those in HighDO and LowDO. Thus, the complete shedding process is not recorded in the

experiment [40] due to the low save intervals (about 0.025 s). Moreover, an undeniable difference is confirmed between HighDO and LowDO. This phenomenon is not unexpected since the special operating condition, an inflection point on the plots in Fig. 2, is selected in the present study. Arndt et al. found that the spectral characteristics of partial cavitation observed in a symmetric hydrofoil of NACA0015 vary considerably over a range of $1 \leq \sigma/2\alpha \leq 8.5$, and divided it into three regimes [45,46]:

- Regime 1 ($1 \leq \sigma/2\alpha \leq 4$): Type I oscillation with the lower dominant frequency. The Strouhal number, fC/U , is almost constant.
- Regime 2 ($4 \leq \sigma/2\alpha \leq 6$): Type II oscillation with the higher dominant frequency. The Strouhal number, fC/U , almost linearly decreases with the decrease of cavitation number. The frequency well agrees with the shedding frequency of cloud cavities.
- Regime 3 ($6 \leq \sigma/2\alpha \leq 8.5$): A distinct, very low dominant frequency.

Table V summarizes the present simulation conditions with the obtained flow characteristic quantities. Cavitation number is redefined using the time-averaged inlet pressure, σ_{sim} , which is almost equal between the examined three cases. Instead of $\sigma_{\text{sim}}/2\alpha$, $\sigma_{\text{sim}}/2(\alpha - \alpha_0)$ is calculated considering the facts that Clark Y-11.7% hydrofoil is not symmetric and the angle of attack of zero lift force is roughly $\alpha_0 = -3.5^\circ$ according to an experiment [39], and the calculated value of $\sigma_{\text{sim}}/2(\alpha - \alpha_0)$ is about 3.6. In addition, considering the wall effect of the test section, $\sigma_{\text{sim}}/2(\alpha - \alpha_0)$ is further modified with the lift coefficients as follows, i.e., $\sigma_{\text{sim}}/2(\alpha - \alpha_0)_{\text{mod}} = \frac{\sigma_{\text{sim}}}{2(\alpha - \alpha_0)} \frac{\bar{C}_{L_{\text{real}}}}{\bar{C}_{L_{\text{present}}}}$, in which $\bar{C}_{L_{\text{present}}} = 1.04$ and $\bar{C}_{L_{\text{real}}} = 1.17$ are respectively the lift coefficient at the non-cavitation condition calculated in the present domain (Fig. 3) and that in the real domain with the same size as the experimental test section [39, 40] mentioned above. Thus, the characteristic values of $\sigma/2(\alpha - \alpha_0)$ as the numerical and experimental results are respectively $\sigma_{\text{sim}}/2(\alpha - \alpha_0)_{\text{mod}} = 4.05$ and $\sigma_{\text{exp}}/2(\alpha - \alpha_0) = 3.35$ ($\sigma_{\text{exp}} \approx 1.34$ is adopted), indicating that the present condition is fairly close to the boundary between regime 1 and 2. Therefore, even a small deviation of the cavitation number could lead to a significant difference in the dominant frequency, i.e., the differences of shedding period between the three experimental cases would be mainly caused by the partial cavitation instability with some slight difference of the cavitation number, rather than the effect of gas content. Note that the shedding periods or frequencies are calculated on the basis of each corresponding spectra of the lift coefficients as shown in Fig. 12 later.

TABLE V. Numerical conditions and obtained flow characteristic quantities.

Case (DO)	σ_{sim}	$\sigma_{\text{sim}}/2(\alpha - \alpha_0)$	$\sigma_{\text{sim}}/2(\alpha - \alpha_0)_{\text{mod}}$	$\bar{V}_t$ [m ³]	S_t	$\bar{C}_L$
SS model (0%)	1.42	3.6	4.05	4.6×10^{-6}	0.48	1.20
LowDO (20%)	1.42	3.6	4.05	6.4×10^{-6}	0.48	1.17
HighDO (80%)	1.42	3.6	4.05	6.7×10^{-6}	0.48	1.03

Secondly, the dominant frequencies of the three numerical contrast cases are the same, i.e., $S_t=0.48$, which is closer to the result of MiddleDO in the experiment, $S_t=0.61$. The discrepancy of the frequency is mainly attributed to the following two reasons.

- The dominant frequency is highly sensitive to the cavitation number at this special operating working condition as discussed above.
- Some influence of the tip leakage cavitation around the hydrofoil is indicated as shown in

Fig. 10. As mentioned in Section II, the tip clearance is ignored in the numerical simulation to simplify the present analysis. However, the tip leakage cavitation evidently appears in the experiment and would also play a role to affect the shedding process.

In the above analysis, although some differences between the three DO cases are confirmed, the NCG effect on the global large-scale behaviors seems not evident. Therefore, more local small-scale characteristics of the three cases will be further discussed hereinafter.

Returning to Figs. 8 and 9, it can be found that the NCG content has a significant effect on the cavitation morphology, which can be also confirmed in Fig. 10. In the experimental results, many discrete and glassy cavitation bubbles are observed in HighDO conditions, while the more continuous sheet and cloud cavitation appears over the hydrofoil in lowDO and MiddleDO conditions. The special appearance of cavitation in HighDO is attributed to the significant influence of NCG content and suggests that gaseous-like cavitation may partly occur. As for the numerical results, the cavitation appearances are represented by the two different iso-surfaces of void fraction, $\alpha_v = 0.4$ and $\alpha_v = 0.1$, in Fig. 9. The predicted morphology of cavities represented by the two void fraction values in the original SS model, which does not take the NCG effect into account, is almost the same. In contrast, the morphologies are significantly different between the two values when the NCG effect is considered, especially for the HighDO. In the original SS model, when the sheet/cloud cavitation is formed, the bubbles will grow in a low-pressure region around the leading edge of the hydrofoil while they collapse in a pressure-recovering region downstream the low-pressure area. On the other hand, when the NCG is contained in the liquid, it will come out of the solution and diffuse into bubbles with the growth of the bubbles. In the pressure-recovering region, a large equilibrium region where the pressure inside and outside the bubbles is almost equilibrium seems to exist besides a collapsing region of cavitation bubbles as shown later. It is close to an equilibrium that certain bubbles are generated while others collapse, which leads to a low void fraction. The further quantitative discussion will be presented later. In addition, cavitation can occur where a local pressure is higher than the saturation vapor pressure, which is based on Eq. (22). Therefore, the minimum values of cavity volume, V_c , in Fig. 8 in the original SS model are lower than those in the LowDO and HighDO cases. It is also indicated that the presence of the partial pressure of the NCG inside bubbles enhances not only the inception but also the development of cavitation because the mean values of V_c in LowDO and HighDO are higher than that in the original SS model.

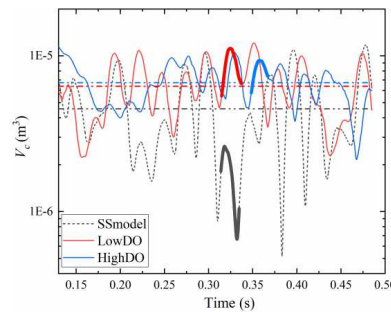


FIG. 8. Time history of the cavity volume V_c (The horizontal dash-dotted lines represent the mean values for the three corresponding cases and the three portions in bold represent the cavitation shedding process)

shown in FIG. 9).

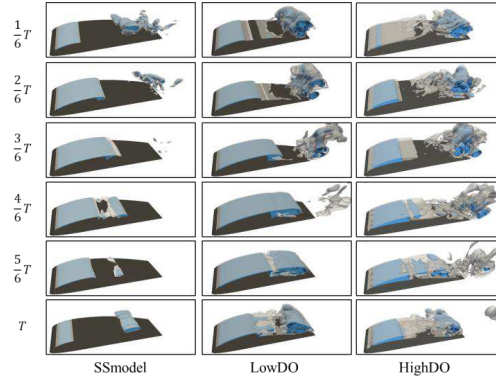


FIG. 9. The periodic cavitation shedding of the three DO cases at $\sigma = 1.34$ as the numerical results (blue iso-surface: $\alpha_v = 0.4$, white iso-surface: $\alpha_v = 0.1$). T is the period of the shedding in each case.

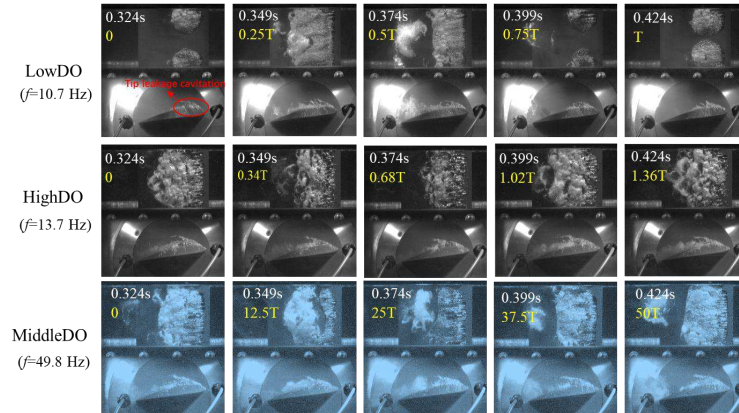


FIG. 10. Snapshots of the cavitation with shedding in the experiment (LowDO: $\sigma = 1.34$; MiddleDO: $\sigma = 1.32$, HighDO: $\sigma = 1.33$). Reproduced from "Unsteady fluid force and fluid force moment under cavitation on isolated hydrofoil with tip clearance, Master Thesis, Kyushu University, 2018 (in Japanese)", with the personal permission of the author, Mr. Soichiro Kitamura, who submitted it to Kyushu University as a Master thesis in February, 2018 [40].

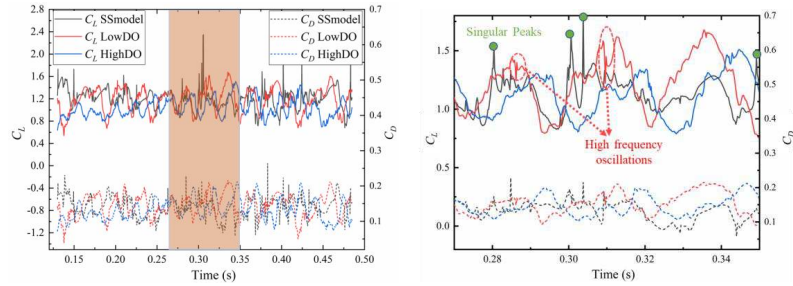
Figures 11 and 12 respectively show the time history of the lift and drag coefficients in the three cases and each corresponding spectrum, obtained in our numerical simulations. The time-averaged lift coefficient $\overline{C_L}$ in the SS model and those in the LowDO and HighDO is in turn 1.20, 1.17, 1.03 while the experimental values in the LowDO and HighDO is 1.06 and 1.01, respectively. Comparing the numerical and experimental results, a qualitative agreement on the degradation of the lift coefficient by the increment of DO is confirmed. In addition, a lower mean value of the lift coefficient in the higher DO is caused by the influence of the dissolved gas. The reason for this

change is that a larger cavity is generated around the hydrofoil in the liquid with a higher concentration of NCG, as mentioned above. Besides, focusing on the high-frequency band of $f > 100$ Hz in Fig. 11 (c), the highest power spectral density among the three cases is the original SS model while the lowest one is the HighDO. Referring to the variation of C_L in the time domain, the most distinct feature is that the singular peaks on C_L and C_D curves in the SS model disappear in the cases of LowDO and HighDO. These singular peaks usually occur due to collapsing of cavitation bubbles, which has been captured by many large eddy simulations [47].

On the basis of the Rayleigh-Plesset equation, a characteristic time τ in the Rayleigh collapse for an initial bubble radius of R_0 under the liquid pressure p_l is given by ignoring the effects of viscosity, non-condensable gas, and surface tension [1],

$$\tau \approx 0.915 R_0 \sqrt{\frac{\rho_l}{p_l - p_v}}. \quad (45)$$

On the other hand, when the NCG influence is taken into account, it will lead to a larger collapse time due to the increase of the pressure inside the bubble. The longer characteristic time in the collapsing causes the disappearance of singular peaks in the C_L and C_D curves observed in the original SS model. Besides, as depicted by the dashed circles in Fig. 11(b), some high-frequency oscillations with relatively larger magnitudes can be observed in the LowDO case while such high frequency fluctuations are remarkably damped in the HighDO. This is because the presence of NCG abates dynamic behaviors since the diffusion rate of NCG is much slower than the rate of liquid-vapor phase change. Therefore, the magnitudes in the higher frequency band with much content of NCG is lower than that with less NCG in Fig. 11 (c). On the whole, in the perspective of unsteady characteristics, the cavitation process is stabilized and damped by the presence of NCG, especially in the high-frequency band.



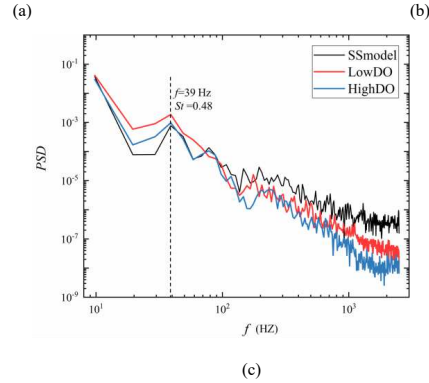


FIG. 11. (a) Time history of lift and drag coefficients, (b) zoom-in of the marked region in (a), and (c) corresponding power spectral density of each lift coefficient of (a), in the three DO cases.

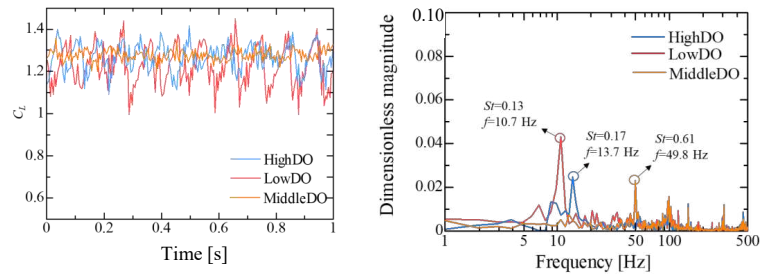


FIG. 12. Time history of lift coefficients and corresponding spectra in the experimental results. Reproduced from "Unsteady fluid force and fluid force moment under cavitation on isolated hydrofoil with tip clearance, Master Thesis, Kyushu University, 2018 (in Japanese)", with the personal permission of the author, Mr. Soichiro Kitamura, who submitted it to Kyushu University as a Master thesis in February, 2018 [40].

Figure 13 shows some instantaneous velocity vectors on the mid-plane of the computational domain (left) and the distributions of the NCG pressure p_g around the hydrofoil (right) during a cavitation shedding process in each case of (a) LowDO and (b) HighDO with (c) the corresponding time histories of the lift and drag coefficients. The six instants in Fig. 13 (c) match to each instantaneous snapshot in Fig. 13 (a) and (b). Note that the vectors are colored by the NCG concentration difference, $c_l - c_{l\infty}$, in which $c_{l\infty}$ is the concentration of the non-condensable gas given at the inlet of the computational domain as a constant value. Evidently, the concentration difference in the cavitating region in the case of HighDO is larger than that in the case of LowDO. Besides, when cavitation inception occurs, an NCG partial pressure p_g inside a bubble starts to decrease due to the increment of the bubble volume, and the NCG concentration at the liquid-side interface of the bubble also decreases according to Henry's law. Hence, the concentration difference of $c_l - c_{l\infty}$ is negatively produced, and the NCG in the bulk liquid will diffuse into bubbles. On the contrary, when cavitation desinence occurs, the NCG in a bubble will diffuse back to the bulk

liquid. Therefore, the NCG partial pressure and concentration in the attached cavitation region near the leading edge of the hydrofoil are lower than that in the detached cavitation region around the trailing edge. These results indicate that NCG continuously accumulates within the sheet-cavitation region during cavity growth owing to diffusion from the surrounding liquid, whereas part of the accumulated gas is subsequently released back into the liquid during cavity collapse.

A re-entrant jet is known as the main cause of the cavity shedding process. The velocity vectors on the mid-plane in Fig. 13 clearly show that the re-entrant jet occurs in the numerical results. In Fig. 13, the red solid points represent the leading edge of the re-entrant jet, and the white face indicates the iso-surface of the void fraction, $\alpha_v = 0.1$. As for the cavity structure represented by the void fraction seems not closely related to the re-entrant jet configuration. The reason might be that the cavitation bubbles with such low void fraction, i.e., bubbles with a relatively small volume, have little effect on the flow field. On the other hand, the re-entrant jet appears to be associated with the NCG concentration. The NCG concentration in the cavitating region upstream the leading edge of the re-entrant jet is relatively smaller than that downstream the jet. The re-entrant jet configuration is extremely complex, and the vortical features are dominant during the shedding process. Therefore, the higher NCG concentration downstream the leading edge of the re-entrant jet may be attributed to the entrainment effect of the vortical flow; the higher NCG region in the main flow are entrained into the detached cavitation regions. Besides, the re-dissolved NCG from bubbles to liquid accompanied by the collapsing can also lead to an increase of the NCG concentration. Compared with the positions of the leading edge of the re-entrant jet in LowDO, i.e., $0.3\text{--}0.75C$, those in HighDO are mainly concentrated in $0.25\text{--}0.4C$, which is further upstream. Thus, the re-entrant jet in the HighDO case exhibits a more upstream leading-edge position compared with the LowDO case, indicating that the NCG content modifies the spatial evolution of the reverse-flow structure.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

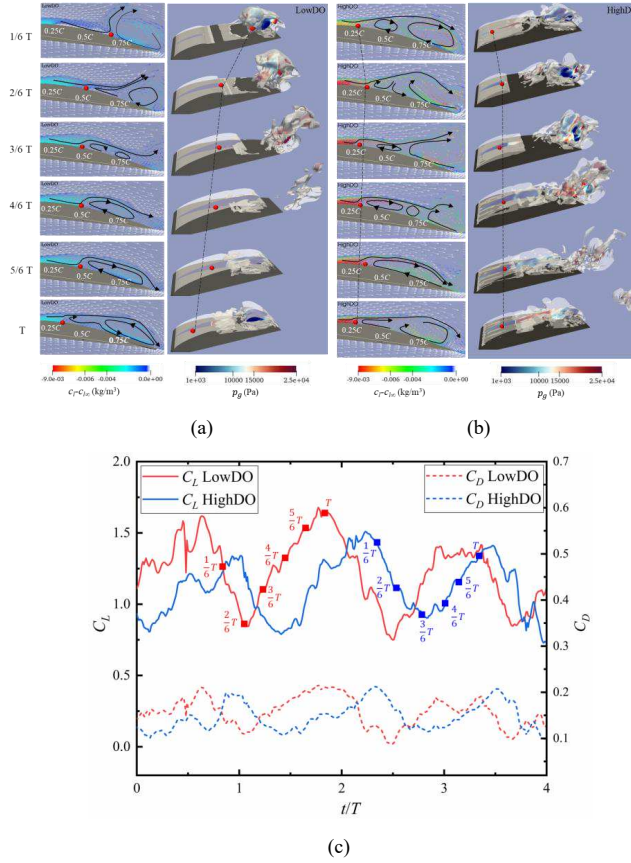


FIG. 13. Distributions of the velocity vector on the mid-plane (left) and NCG pressure p_g (right) during a cavitation shedding process in (a) LowDO and (b) HighDO (The vectors are colored by the NCG concentration difference); (c) time histories of the lift and drag coefficients with the six instants indicated on the lift curves.

To further investigate the characteristics of the cavitating region, a typical instant of HighDO, $1/6T$, is analyzed in detail. As a sample, a streamline passing through the cavitating region on the mid-plane is extracted in Fig. 14 (a). Since the recirculation occurs around $0.3-0.5C$ due to the adverse pressure gradient, the analysis is mainly focused on $0-0.3C$. Figure 14 (b) presents the distribution of the partial pressure of the NCG, p_g , and the total pressure inside the bubble, $p_g + p_v$, as well as the local static pressure, p , along the typical streamline, while Fig. 14 (c) shows the distribution of the pressure difference inside and outside the bubble, $p_g + p_v - p$. As shown in Fig. 15, different from the original SS model, the bubble dynamics using the present cavitation model is not only influenced by the local static pressure and the vapor pressure inside bubbles but also affected by the NCG partial pressure inside the bubbles. In Fig. 14 (b), the change of the partial

pressure of the NCG agrees with that of the local static pressure in the analyzed region. Around the leading edge of the hydrofoil, i.e., Growth region I corresponding to $0-0.125C$, the static pressure decreases dramatically. The total pressure inside the bubbles, $p_g + p_v$, exceeds the local static pressure, and the bubbles gradually grow, i.e., $Dr/Dt > 0$. With the increase of the bubble volume, the partial pressure of the NCG, p_g , decreases. In the collapse region II, $0.125-0.16C$, the total pressure inside the bubbles becomes lower than the local static pressure with the pressure recovery. Thus, the cavitation bubbles shrink, i.e., $Dr/Dt < 0$, and the partial pressure of the NCG increases gradually. As for the equilibrium region III, $0.16-0.3C$, the pressure difference inside and outside the bubbles is much smaller than the other two regions, i.e., the bubbles almost keep a mechanical equilibrium. According to the analyses above, the growth and collapse of cavities mainly depend on the local static pressure and the change of the volume will affect the partial pressure of NCG. Therefore, a similar trend is found between the local static pressure and the partial pressure of the NCG in that cavitating region.

As for the detached cavity region, the partial pressure of the NCG is relatively small compared with that in the cavitating region of $0.125-0.35C$. It means that the shed cavitation bubbles continue to grow rather than collapse, which is quite different from our intuition. It indicates that gaseous-like cavitation may occur in the shed cavitating region. Observing other instants of HighDO, both low and high partial pressure p_g can be found in the different region in the detached cavity area. Therefore, both cavitation collapse and gaseous-like cavitation are expected to occur in the detached cavity region during the same cavitation-shedding process. Although it is quite difficult to distinguish gaseous and vaporous cavitation practically, the special cavitation appearance in HighDO shown in Fig. 10 provides some evidence of the above conjecture.

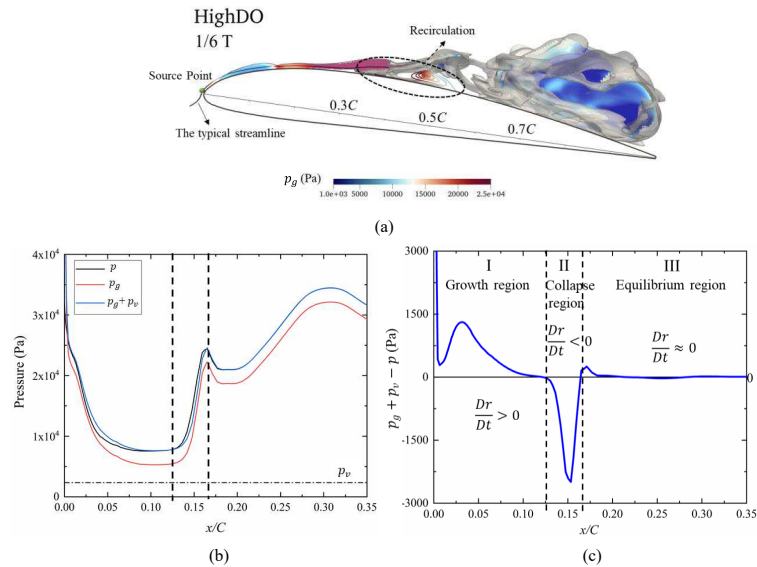


FIG. 14. (a) A selected streamline at the time of $T/6$ in the case of HighDO, (b) Distribution of each pressure along the typical streamline, (c) Distribution of the pressure difference as driving force of

bubble radius change.

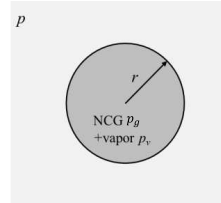


FIG. 15. Pressure equilibrium image of a bubble nucleus in liquid.

Although the previous transient analysis gives the features of a cavitating flow affected by the NCG, it is unsteady with turbulence in nature. Therefore, the flow field quantities are inherently unpredictable, and a statistical analysis aiming at an estimation of the probability of events is useful.

Figure 16 shows time-averaged flow fields on the mid-plane during about 20 times of the oscillation period in the three DO cases. Figure 16 (a) displays distribution of the NCG concentration and that of the NCG partial pressure in the cavitating region in LowDO and HighDO, and Fig. 16 (b) presents the pressure difference inside and outside bubbles, $p_v - p$ for SS model and $p_g + p_v - p$ for LowDO and HighDO, in each cavitating region. In Fig. 16 (a), the NCG concentration at the interface of bubbles is statistically lower than that in the bulk liquid. Besides, the concentration difference is higher as the DO increases, which is significantly important for gaseous cavitation since it defines the mass transfer rate of the NCG within the cavitating region. As for the mean cavity volume, comparing the cavitating region enclosed by the void fraction contour $\alpha_v = 0.1$, it can be observed that the cavitating region by the original SS model without the NCG is smaller than those in the cases of LowDO and HighDO, which evidences some enhancing effect of the NCG on cavitation. Referring to Fig. 16 (b), also from the statistical point of view, it is confirmed that the regions where the pressure difference is positive in LowDO and HighDO are much larger than that by the original SS model. This is the reason why the mean values of the cavity volume V_c in the cases of HighDO and LowDO are larger than that by the SS model as has been shown in Fig. 8. In addition, an equilibrium region that the pressure inside bubbles is very close to the pressure outside is also confirmed in the case of HighDO. In other words, some of the collapsing bubbles may grow again with a high probability in that region, i.e., gaseous cavitation may occur. This is a special feature of HighDO that is significantly different from LowDO and SS model without the NCG. Besides, the reason why the mean value of the cavity volume V_c in HighDO is slightly larger than that in LowDO, as shown in Fig. 8, may be caused by the gaseous cavitation, despite the growth regions in the two cases being similar. Therefore, the different average values of the cavity volume in the three cases in Fig. 8 may attribute to different reasons; the difference with and without the NCG comes from that of the cavity growth region while the difference by the content of the NCG may be from the shrinking rate of bubbles around the trailing edge of the hydrofoil. In the case of the higher content of the NCG, the gaseous cavitation downstream the sheet cavity occurs on the basis of the mass transfer of the NCG from the bulk liquid.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

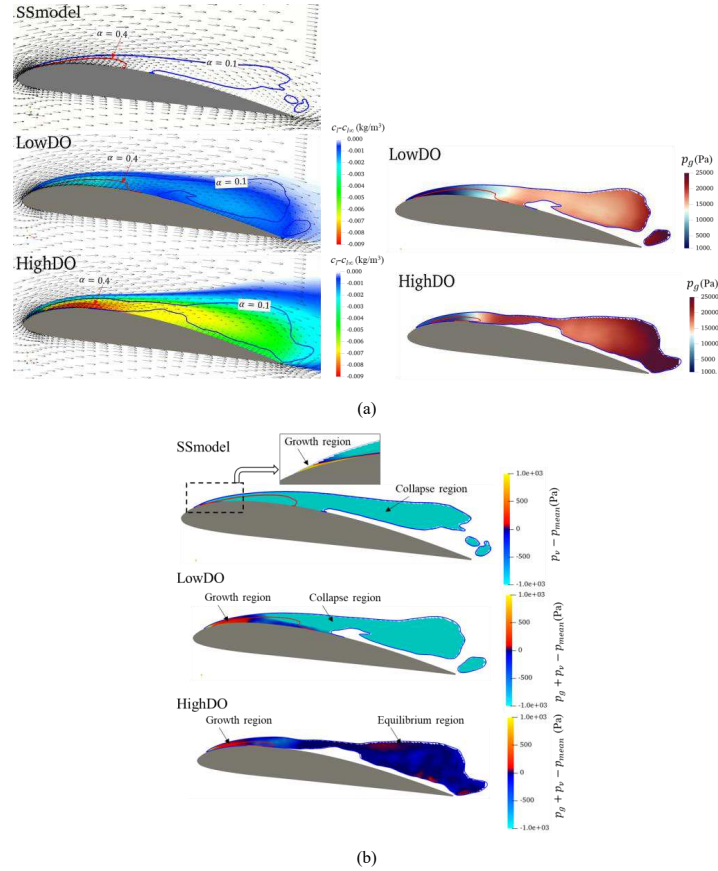


FIG. 16. Time-averaged flow fields on the mid-plane in the three cases (blue contour: $\alpha_v = 0.1$; red contour: $\alpha_v = 0$): (a) distribution of NCG concentration and partial pressure in the cavitating region; (b) pressure difference inside and outside bubbles.

To further investigate the statistical characteristics of the cavitating region quantitatively, the pressure distributions along the streamlines in the three cases extracted from the time-averaged velocity vectors (indicated in Fig. 17 (a)), which pass the same source point near the leading edge of the hydrofoil, are analyzed. Affected by the NCG content, each recirculation region is different with each other while the three streamlines upstream the recirculation areas, i.e., the analyzed segment (about 0–0.4 C) indicated in Fig. 17 (a), are almost coincident. Figure 17 (b), (c), and (d) show the time-averaged void fraction, partial pressure, and pressure difference distribution along the analyzed segment, respectively.

As for the original SS model without the NCG, the void fraction begins to increase at about 0.05 C , which almost agrees with the position that the pressure difference becomes positive in Fig.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

17 (d). On the streamline, it should be noted that the inception of cavitation bubbles does not only require a positive pressure difference between inside and outside bubbles but also require a growth time. Therefore, the position where the pressure difference becomes positive must be slightly further ahead. The void fraction reaches the maximum value at about $0.25C$ and then decreases gradually. Namely, the positive pressure difference makes the bubble nuclei begin to grow, i.e., cavitation starts to occur, and these bubbles keep growing and then collapse with the convection. Therefore, the whole process is only dominated by the local static pressure because the pressure inside the bubbles is kept constant to be the saturation vapor pressure. On the other hand, in LowDO and HighDO, the cavitation process becomes more complex. As seen in Fig. 17 (b), although each variation of the void fraction in LowDO and HighDO appears to be similar to that in the SS model, the mechanism of the bubble growth is quite different. As shown in Fig. 17 (c), not only does the local static pressure change along the streamlines, but the total pressure inside the bubbles also changes. Cavitation with NCG can be regarded as an interaction between the internal cavity pressure and the surrounding liquid pressure, in which the dissolved gas mainly affects the cavitation dynamics through its modulation of the local compressibility and pressure-response characteristics inside the cavity. Besides, along the streamlines, the changing rate of the two pressures, i.e., the internal NCG partial pressure and the external local static pressure, defines the growth and collapse of the bubbles because of the constant vapor pressure. Note that, in the present cavitation around the hydrofoil, the change of the internal NCG pressure is dominated by the change of the cavity volume rather than the mass transfer of the NCG because the vapor cavitation is still dominant even in the HighDO case. Therefore, the pressure inside and outside the bubbles continue to be equilibrium after reaching equilibrium for the first time at around $0.2C$. In addition, affected by the NCG, the variation of the local static pressure is also different between the three cases under the other operating conditions being the same. Inside the growth region in the cases of LowDO and HighDO, about $0-0.2C$, it can be found that the static pressure asymptotically tends to reach a steady value instead of recovering like SS model, which may be attributed to the damping effects of the NCG.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

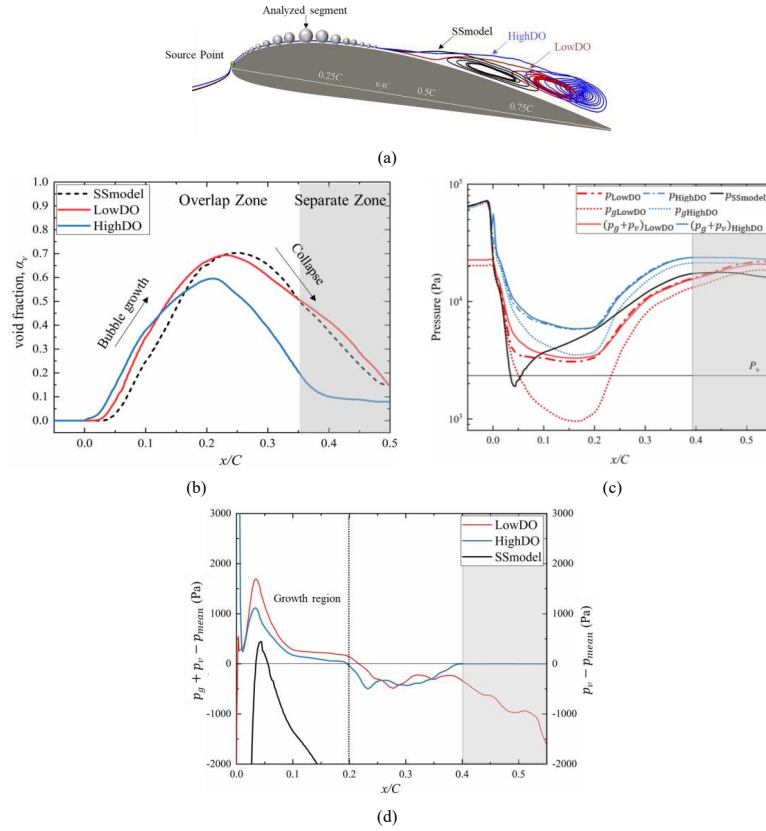


FIG. 17. (a) Streamlines from a source point, (b) the time-averaged void fraction, (c) those of pressure, and (d) those of pressure differences along the streamlines in the three cases.

IV. CONCLUSIONS

In the present work, a three-dimensional delayed detached eddy simulation (DDES) coupled with a recently proposed homogeneous cavitation model, SS-NCG model, is employed to investigate the influence of non-condensable gas (NCG) on cavitation over a two-dimensional Clark Y-11.7% hydrofoil under the present operating condition ($\alpha = 8^\circ$, $\sigma = 1.34$). By comparing three typical cases, i.e., SS model without NCG, LowDO, and HighDO, the unsteady and statistical features of the NCG influence are numerically analyzed. The main conclusions under the present conditions are summarized as follows.

The special appearance of cavitation at HighDO in the corresponding experiment is attributed to the significant influence of NCG content, which may imply the occurrence of gaseous-like cavitation. Therefore, the presence of NCG will enhance the degree of the development of cavitation in the present hydrofoil flow.

From the perspective of unsteady characteristics, the cavitation process in the present cases is stabilized and damped with the presence of NCG, especially in the high-frequency band. A similar trend is found in the attached cavitating regions between the change of the local static pressure and the NCG partial pressure. On the other hand, as for the detached cavitating regions, both collapsing of cavitation bubbles and gaseous-like cavitation may occur during the same shedding process under the present conditions.

On the basis of the statistical results in the present simulations, the NCG concentration at the interface of bubbles is statistically lower than that in the bulk liquids and the concentration difference increases as the NCG content becomes higher. The mean value of the cavity volume in each DO case may result from the different growth regions and the occurrence of gaseous-like cavitation.

The mechanism of the cavitation process with the NCG and that without the NCG are quite different under the present conditions. With the presence of the NCG, cavitation can be regarded as an interaction between the internal cavity pressure and the surrounding liquid pressure, where the dissolved gas mainly affects the cavitation dynamics through its modulation of the local compressibility inside the cavity. The changing rate of each pressure determines whether the bubbles grow or collapse. In contrast, the cavitation process without the NCG is just dependent on the difference between the local static pressure and the constant vapor pressure.

ACKNOWLEDGMENTS

This work was partially supported by JSPS KAKENHI Grant Number JP20K04269 and Harada Memorial Foundation. One of the authors L. Y. gratefully acknowledges the financial support of the China Scholarship Council (Grants NO. 202208320292) to be a visiting Ph.D. student in Kyushu University. This work was also Supported by the National Natural Science Foundation of China (52506047), the Fundamental Research Program of Shanxi Province (Grants NO. 202403021222363), the Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi (STIP, Grants NO. 2024L457), the Scientific Research Foundation for Advanced Talents of Shanxi Institute of Science and Technology (No. 202505).

AUTHOR DECLARATIONS

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

- [1] Jean-Pierre FRANC, J.-M. MICHEL, *Fundamentals of Cavitation*. 2005. doi: 10.1007/978-94-017-7255-6_1.
- [2] M. Zaresharif, F. Ravelet, D. J. Kinahan, and Y. M. C. Delaure, "Cavitation control using passive

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

- flow control techniques," *Physics of Fluids*, vol. 33, no. 12, p. 121301, 2021.
- [3] C. E. Brennen, *Cavitation and bubble dynamics*. Cambridge university press, 2014.
- [4] A. Amini, M. Reclari, T. Sano, M. Iino, M. Dreyer, and M. Farhat, "On the physical mechanism of tip vortex cavitation hysteresis," *Exp Fluids*, vol. 60, no. 7, pp. 1–15, 2019.
- [5] B. H. Tan, H. An, and C.-D. Ohl, "How bulk nanobubbles might survive," *Phys Rev Lett*, vol. 124, no. 13, p. 134503, 2020.
- [6] S. Mukherjee and H. Gomez, "Understanding how non-condensable gases modify cavitation mass transfer through the van der Waals theory of capillarity," *Appl Phys Lett*, vol. 117, no. 20, p. 204102, 2020.
- [7] J. W. Holl, "An effect of air content on the occurrence of cavitation," *J Fluids Eng*, vol. 82, no. 4, 1960.
- [8] C. Brennen, "The dynamic balances of dissolved air and heat in natural cavity flows," *J Fluid Mech*, vol. 37, no. 1, pp. 115–127, 1969.
- [9] K. Kobayashi, Y. Katayama, S. Watanabe, and S. Tsuda, "Experimental Investigation on Cavity Pressure Inside Sheet Cavitation," *J Fluids Eng*, vol. 143, no. 12, 2021.
- [10] T. Magne, R. Paridaens, S. Khelladi, F. Bakir, P. Tomov, and L. Pora, "Experimental Study of the Hydraulic Performances of Two Three-Bladed Inducers in Water, Water With Dissolved CO₂, and Jet Fuel," *J Fluids Eng*, vol. 142, no. 11, 2020.
- [11] T. Magne *et al.*, "Effect of gas content on the cavitating and non-cavitating performance of an axial three-bladed inducer," *Multiphase Science and Technology*, vol. 32, no. 1, 2020.
- [12] L. Cao, L. Mingming, W. Zhengwei, and Z. Yiyang, "Numerical investigation of the non-condensable gas effect on predicting the cavitation performance of a centrifugal pump," in *IOP Conference Series: Earth and Environmental Science*, IOP Publishing, 2022, p. 12038.
- [13] Wang X, Cheng H, Ji B. Bursting of wetted and cavitating tip vortex around a wake-influenced propeller[J]. *Physical Review Fluids*, 2026, 11(1): 014301.[14] J. F. Gülich, *Centrifugal pumps*, vol. 2. Springer, 2008.
- [15] W. Tsuru, T. Konishi, S. Watanabe, and S. Tsuda, "Observation of inception of sheet cavitation from free nuclei," *Journal of Thermal Science*, vol. 26, pp. 223–228, 2017.
- [16] Y. Longlong, B. Gao, D. Ni, N. Zhang, and W. Zhou, "Numerical Study of Unsteady Cavitating Flow in an Inducer with Omega Vortex Identification," *J Fluids Eng*, 2022.
- [17] L. Yan, B. Gao, D. Ni, N. Zhang, and W. Zhou, "Numerical analysis on the system instability of the cavitating flow around hydrofoil induced by the non-uniform inflow," *International Journal of Multiphase Flow*, p. 104074, 2022.
- [18] L. Deng, Y. Long, and B. Ji, "Delayed detached eddy simulation and vorticity analysis of cavitating flow around a marine propeller behind the hull," *Ocean Engineering*, vol. 264, p. 112442, 2022.
- [19] M.-G. Mithun, P. Koukouvini, I. K. Karathanassis, and M. Gavaises, "Numerical simulation of three-phase flow in an external gear pump using immersed boundary approach," *Appl Math Model*, vol. 72, pp. 682–699, 2019.
- [20] E. S. Vasilakis, N. Kyriazis, P. Koukouvini, M. Farhat, and M. Gavaises, "Cavitation induction by projectile impacting on a water jet," *International Journal of Multiphase Flow*, vol. 114, pp. 128–139, 2019.
- [21] A. K. Singhal, M. M. Athavale, H. Li, and Y. Jiang, "Mathematical basis and validation of the full cavitation model," *J. Fluids Eng*, vol. 124, no. 3, pp. 617–624, 2002.
- [22] N.-X. Lu, G. Bark, and R. Bensow, "Introducing non-condensable gas in unsteady sheet cavitation

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

- modelling,” in *15th Numerical Towing Tank Symposium*, 2012.
- [23] E. H. Malekshah and W. Wróblewski, “Merging theory-based cavitation model adaptable with non-condensable gas effects in prediction of compressible three-phase cavitating flow,” *Int J Heat Mass Transf*, vol. 196, p. 123279, 2022.
 - [24] H. Cheng, X. Long, B. Ji, X. Peng, and M. Farhat, “A new Euler-Lagrangian cavitation model for tip-vortex cavitation with the effect of non-condensable gas,” *International Journal of Multiphase Flow*, vol. 134, 2021, doi: 10.1016/j.ijmultiphaseflow.2020.103441.
 - [25] C. T. Hsiao, J. Ma, and G. L. Chahine, “Multiscale two-phase flow modeling of sheet and cloud cavitation,” *International Journal of Multiphase Flow*, vol. 90, pp. 102–117, 2017, doi: 10.1016/j.ijmultiphaseflow.2016.12.007.
 - [26] S. Yakubov, B. Cankurt, M. Abdel-Maksoud, and T. Rung, “Hybrid MPI/OpenMP parallelization of an Euler–Lagrange approach to cavitation modelling,” *Comput Fluids*, vol. 80, pp. 365–371, 2013.
 - [27] J. Ma, C. T. Hsiao, and G. L. Chahine, *A physics based multiscale modeling of cavitating flows*, vol. 145. Elsevier Ltd, 2017. doi: 10.1016/j.compfluid.2016.12.010.
 - [28] G. H. Schnerr and J. Sauer, “Physical and numerical modeling of unsteady cavitation dynamics,” in *Fourth international conference on multiphase flow*, ICMF New Orleans New Orleans, LO, USA, 2001.
 - [29] Y. Higa, R. Iida, S. Tsuda, R. S. Watanabe, “Numerical Investigation of Cavitating Flow Around a Hydrofoil Considering Advection–Diffusion and Precipitation of Dissolved Gas in Liquid,” *Springer Proc. Phys.*, pp. 73–81, 2024.
 - [30] Campos–Amezcuca R, Bakir F, Campos–Amezcuca A, et al. Numerical analysis of unsteady cavitating flow in an axial inducer[J]. *Applied Thermal Engineering*, 2015, 75: 1302-1310.
 - [31] Kato C. Industry-university collaborative project on numerical predictions of cavitating flows in hydraulic machinery: Part 1—Benchmark test on cavitating hydrofoils[C]//*Fluids Engineering Division Summer Meeting*. 2011, 44403: 445-453.
 - [32] A. J., Stepanoff, “Cavitation Properties of Liquids”, *Trans. ASME J. Eng. Power*, vol. 86, pp. 195-200, 1964.
 - [33] Feng S, He D, Wang G, et al. Suppression effects of non-condensable gas on cavitation in centrifugal pumps: Numerical insight from a modified volume of fluid-cavitation model[J]. *Physics of Fluids*, 2025, 37(10).
 - [34] C. Gualtieri, A. Angeloudis, F. Bombardelli, S. Jha and T. Stoesser, “On the Values for the Turbulent Schmidt Number in Environmental Flows”, *Fluids*. vol. 2, 27 pages, 2017.
 - [35] D. Lindblad, A. Jareteg, and O. Petit, “Implementation and run-time mesh refinement for the $k-\omega$ SST DES turbulence model when applied to airfoils,” *Project work. Chalmers University of Technology*, 2014.
 - [36] F. R. Menter, M. Kuntz and R. Langtry, “Ten years of industrial experience with the SST turbulence model,” *Turbulence, heat and mass transfer*, vol. 4, no. 1, pp. 625–632, 2003.
 - [37] W. Tsuru, Y. Ehara, S. Kitamura, S. Watanabe, and S. Tsuda, “Mechanism of Lift Increase of Cavitating Clark Y-11.7% Hydrofoil,” in *Proceedings of the 10th International Symposium on Cavitation (CAV2018)*, J. Katz, Ed., ASME Press, doi: 10.1115/1.861851_ch135, 2018.
 - [38] Y. Ehara, S. Kitamura, W. Tsuru, S. Watanabe and S. Tsuda, “Unsteady forces and moments acting on a cantilevered rectangular hydrofoil with tip clearance in cavitating conditions”, *IOP Conf. Ser.: Earth Environ. Sci.* 240, 062054, 2019.
 - [39] W. Tsuru, “Numerical Analysis of Cavitating Flow with Homogeneous Model Considering Characteristics of Bubble Nuclei in Liquid”, *Doctoral Thesis in Kyushu University*, 2018 (in

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0337383

- Japanese).
- [40] S. Kitamura, "Unsteady fluid force and fluid force moment under cavitation on isolated hydrofoil with tip clearance," *Master Thesis in Kyushu University*, 2018 (in Japanese).
 - [41] K. Okabayashi, A. Furukawa, T. Otsu, and T. Kajishima, "Large Eddy Simulation Study on the Relationship Between Large-Scale Turbulence Vortices and Unsteady Cavitation Around a Hydrofoil," *J Fluids Eng*, vol. 145, no. 4, p. 41204, 2023.
 - [42] H. Rusche, "Computational fluid dynamics of dispersed two-phase flows at high phase fractions," Ph.D Thesis in University of London, 2002.
 - [43] Celik I B, Ghia U, Roache P J, et al. Procedure for estimation and reporting of uncertainty due to discretization in CFD applications[J]. *Journal of fluids Engineering-Transactions of the ASME*, 2008, 130(7).
 - [44] Roache P J. Perspective: a method for uniform reporting of grid refinement studies[J]. 1994.
 - [45] R. E. A. Arndt, C. C. S. Song, M. Kjeldsen, J. He, and A. Keller, "Instability of partial cavitation: a numerical/experimental approach," *Proceedings of the 23rd Symposium on Naval Hydrodynamics*, 2000.
 - [46] M. Kjeldsen, R. E. A. Arndt, and M. Effertz, "Spectral Characteristics of Sheet/Cloud Cavitation," *J Fluids Eng*, vol. 122, no. 3, pp. 481–487, Sep. 2000, doi: 10.1115/1.1287854. [47] Y. Chen, J. Li, Z. Gong, X. Chen, and C. Lu, "Large eddy simulation and investigation on the laminar-turbulent transition and turbulence-cavitation interaction in the cavitating flow around hydrofoil," *International Journal of Multiphase Flow*, vol. 112, pp. 300–322, 2019, doi: 10.1016/j.ijmultiphaseflow.2018.10.012.