Analytical Solution of Rotating MHD Casson Fluid Flow in Moving Vertical Channel with the Effect of Suction: Centrifugal Pump

Mohamad Riduan Hashim
Centre for Mathematical Sciences, College of Computing and Applied Science, Universiti Malaysia Pahang Al-Sultan Abdullah

Zulkhibri Ismail
Centre for Mathematical Sciences, College of Computing and Applied Science, Universiti Malaysia Pahang Al-Sultan Abdullah

Ahmad Qushairi Mohamad
Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia

Laila Amera Aziz
Centre for Mathematical Sciences, College of Computing and Applied Science, Universiti Malaysia Pahang Al-Sultan Abdullah

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Mohamad Riduan Hashim[1], Zulkhibri Ismail[1], Ahmad Qushairi Mohamad[2], Laila Amera Aziz[1]
[1]Centre for Mathematical Sciences, College of Computing and Applied Science, Universiti Malaysia Pahang Al-Sultan Abdullah, Lebuhraya Tun Razak, 26300 Gambang, Pahang, Malaysia,
[2]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310, Johor Bahru, Johor, Malaysia.
Corresponding author: zulkhibri@umpsa.edu.my

Abstract: *Casson fluid, a type of non-Newtonian fluid, demonstrates unique behaviour by acting as either an elastic solid or a liquid under certain conditions. This fluid has numerous real-world applications, such as simulating blood flow in arteries (biomedical engineering), manufacturing chocolates and syrups (food industry), formulating cosmetics, optimizing printing and inking processes, and improving solar energy harvesting systems. However, the flow characteristics of Casson fluid under the combined effects of rotating magnetohydrodynamics (MHD) and suction in a channel remain unexplored. To address this gap, the present study derives theoretical solutions for Casson fluid flow influenced by rotating MHD within a vertical channel incorporating suction. The governing equations are converted into dimensionless forms using suitable variables, with initial and boundary conditions duly considered. Analytical solutions are obtained via the Laplace transform method, and velocity profiles both real and imaginary are graphically represented to assess the influence of key dimensionless parameters. The results reveal that the fluid's real velocity decreases with higher values of the magnetic, rotation, suction, and Casson fluid parameters. Conversely, real velocity profiles exhibit an upward trend over time. Meanwhile, the imaginary velocity decreases with stronger magnetic and suction effects but rises in response to higher Casson fluid parameters, rotation, and time.*

Keywords: Casson fluid; Magnetohydrodynamics (MHD); Suction; Laplace transform; Rotating.

1. INTRODUCTION

Fluid dynamics plays a pivotal role in numerous natural and industrial processes, encompassing both environmental systems and physiological functions in living organisms. Fluids that follow the rule of Newton's law of viscosity are classified as Newtonian fluids, characterized by a linear relationship between shear stress and shear rate. Conversely, fluids that deviate from this linearity are referred to as non-Newtonian fluids [1]. The rheological complexity of non-Newtonian fluids, including time-dependent and nonlinear stress–strain relationships, distinguishes them markedly from their Newtonian counterparts. Owing to these intricate behaviours, non-Newtonian fluids are extensively employed in diverse engineering applications such as biomedical devices, chemical processing, and industrial manufacturing systems. A comprehensive understanding of their flow characteristics is essential for the optimal design and operation of engineering components such as pipelines, pumps, heat exchangers, and mixing units [2]. To accurately capture the complex flow behaviour of non-Newtonian fluids, several constitutive models have been developed, including the Bingham plastic, power-law, Walter-B, viscoplastic, Casson, and Oldroyd-B models [3–7]. Among these, the Casson fluid model has attracted considerable research interest due to its ability to describe fluids exhibiting yield stress. This model is particularly effective in representing materials such as blood, inks, chocolate, and certain polymers. Its practical significance is evident in a variety of engineering systems, including polymer melt processing, textile coating, magnetohydrodynamic (MHD) pumps and motors, aerodynamic heating applications, and the purification of mineral oils [8]. As such, the study of Casson fluids continues to evolve, with a growing emphasis on understanding their thermofluidic behaviour under diverse boundary and flow conditions.

Building on this foundation, magnetohydrodynamics (MHD) emerges as a critical area of study within fluid dynamics, particularly when examining the interaction between magnetic fields and electrically conducting fluids such as Casson fluids. MHD encompasses the combined influence of electromagnetic forces and fluid motion, which is of paramount importance in various scientific and industrial contexts, including plasma physics, geophysics, and cooling systems in nuclear reactors [9]. The MHD analysis of Casson fluids has become increasingly relevant in biomedical engineering due to the model's applicability to blood flow, particularly in scenarios involving external magnetic control or targeted drug delivery. The yield stress nature of Casson fluids makes them suitable for modelling physiological flows, thus offering insights into circulatory dynamics and potential medical interventions for vascular disorders [10]. Dey et al. [11] conducted a study comparing the non-Newtonian Carreau and Casson blood viscosity models to predict hemodynamics in an idealized eccentric arterial stenosis using the Large Eddy Simulation (LES) technique. Considering these applications, several studies have focused on the MHD flow of Casson fluids under different physical configurations and boundary conditions. Azmi et al. [12] conducted an analytical investigation of the pulsatile flow of a Casson fluid over a vertical cylinder embedded in a porous medium, accounting for the effects of slip velocity and an externally applied magnetic field. In a subsequent study, Azmi et al. [13] derived exact solutions for unsteady free convection flows of Casson fluids within a vertical channel subjected to MHD forces. Extending this work, Anwar et al. [14] analysed the combined effects of thermal radiation and medium porosity on the MHD flow of a Casson fluid past a vertical plate with ramped wall velocity and temperature. Their analytical approach utilized a hybrid method involving the Durbin technique and the Laplace transform. Additionally, Khalid et al. [15] examined the unsteady free convection flow of a Casson fluid with MHD effects over an oscillating vertical plate embedded in a porous medium. Their results demonstrated a significant reduction in fluid velocity due to the combined influence of magnetic field intensity, increased Casson parameter, and porous media resistance.

Rotating fluid dynamics constitutes a fundamental area within fluid mechanics, with significant implications for both natural phenomena and engineered systems. When a fluid is analysed in a rotating frame of reference, the governing equations of motion are altered by additional inertial forces most notably, the Coriolis and centrifugal forces [16]. These rotational influences can substantially modify fluid behaviour, giving rise to distinctive features such as secondary circulations, Ekman layers, and enhanced vorticity generation, which are generally absent in non-rotating systems. The analysis of rotating fluid motion is crucial across a wide range of disciplines [17]. In geophysical and astrophysical contexts, rotating fluid models are employed to describe atmospheric circulation, oceanic currents, and the internal dynamics of planetary bodies. In engineering applications, rotating fluid flows are commonly encountered in systems such as turbines, centrifuges, rotating heat exchangers, and industrial mixing devices. In all these cases, the interaction between rotational motion and fluid flow plays a pivotal role in determining system performance, operational efficiency, and overall stability [18]. Several researchers have contributed to the theoretical analysis of rotating fluid flow. Zin et al. [19] investigated the effects of radiative heat transfer on unsteady magnetohydrodynamic (MHD) free convection flow of a rotating Jeffrey fluid past an infinite vertical plate embedded in a porous medium with ramped wall temperature, obtaining analytical solutions via the Laplace transform method. Similarly, Ismail et al. [20] analysed the combined effects of rotation and MHD on the free convection flow of a second-grade fluid in a porous medium and derived exact solutions using the Laplace transform technique. Mohamad et al. [21] conducted a theoretical investigation on the behaviour of rotating Casson fluid in a moving channel disk configuration, also employing the Laplace transform method for solution. In addition, Rajput et al. [22] examined the effects of rotation and radiation on a Newtonian fluid under the influence of MHD, focusing on unsteady flow past an impulsively started vertical plate with variable wall temperature. Other than that, Rahman and Islam [23] investigated the flow field characteristics and performance of both non-rotating and rotating cylinders. They analysed the flow velocity and behaviour in each case to understand the effects of rotation on the flow dynamics. The Laplace transform method is recognized as a powerful and widely utilized analytical technique for addressing transient flow problems, particularly in the study of Casson fluid dynamics over flat surfaces. Several researchers have employed this method to obtain exact or semi-analytical solutions under varying physical conditions. For example, Bilal et al. [24] analysed the flow of a Casson fluid through a porous medium over a Riga plate, incorporating the effects of thermal radiation and chemical reactions. They derived analytical solutions for the case of an exponentially accelerated inclined vertical plate. In a related context, Abdelhameed [25] investigated the unsteady free convection flow of a Newtonian fluid over an accelerating plate subjected to an external magnetic field. Tassaddiq et al. [26] extended the scope of Casson fluid studies by examining magnetohydrodynamic (MHD) flow with Newtonian heating and porosity over a fixed vertical plate, employing a fractional-order model with Mittag-Leffler memory. Further contributions include the work of Rao et al. [27], who addressed unsteady MHD flow of a Casson fluid through a porous medium over an exponentially accelerated vertical plate, considering heat and mass transfer. Kataria et al. [28] expanded this framework by incorporating the combined influences of chemical reactions, MHD effects, and internal heat generation or absorption. Their study, conducted over an exponentially accelerating vertical plate embedded in a porous medium with ramped wall temperature and surface concentration, demonstrated that fluid velocity increases with heat generation, while magnetic field strength, higher Casson parameter values, and stronger chemical reaction rates lead to flow retardation. Khan et al. [29] examined a similar configuration with the inclusion of Newtonian heating, focusing on a moving vertical plate within a porous matrix. Subsequently, Kataria and Patel [30] analysed heat and mass transfer in a Casson fluid under MHD conditions for an oscillating vertical plate, considering ramped temperature boundary conditions.

Despite the extensive analytical efforts dedicated to Casson fluid flow over plates using the Laplace transform and related techniques, limited attention has been given to the application of these methods within channel geometries. This is particularly noteworthy given the practical relevance of such configurations in biomedical and industrial systems. While studies such as those by Bilal et al. [24], Rao et al. [27], and Kataria et al. [28] have provided valuable insights into Casson fluid dynamics over vertical and oscillating plates, the analytical exploration of Casson fluid flow within bounded channels, especially under transient conditions, remains relatively underdeveloped. This research gap highlights the importance of extending analytical methodologies such as the Laplace transform to address the more complex fluid behaviours encountered in channel flows. In response to this gap, recent investigations have begun to focus on Casson fluid flow within vertical channel geometries. For example, Khan et al. [31] analytically studied the effects of thermal radiation on Casson fluid flow within a vertical channel, considering both fixed and oscillating wall configurations. Ahmad Qushairi et al. [32] employed the Laplace transform method to solve the unsteady free convection flow of a Casson fluid within a fixed vertical channel, providing one of the few analytical treatments in this domain. Complementary to these efforts, Mallikarjuna et al. [33] numerically analysed pulsatile Casson fluid flow within a channel, incorporating magnetohydrodynamic (MHD) and slip velocity effects. Bukhari et al. [34] extended this model by eliminating the slip condition and including porosity and thermal radiation, using a finite difference scheme to obtain numerical solutions. Additionally, Divya et al. [35] proposed a mathematical model for MHD Casson fluid flow in a channel with heat and mass transfer, applying the perturbation method for analytical resolution. Sheikh et al. [36] further contributed by investigating Casson fluid flow in a vertical channel under the influence of an external magnetic field, solving the governing equations analytically using the Laplace transform method. Then, Hasim et al. [37] formulate the mathematical models and solution for the effect of accelerated plate on free convection flow in Brinkman type fluid through two vertical channels and solved the solution by using Laplace transform. These recent studies collectively underscore a growing research interest in Casson fluid dynamics within channel geometries and affirm the importance of both analytical and numerical techniques in capturing the complex interactions between flow, heat transfer, and magnetic effects in non-Newtonian fluids.

Building upon the recent analytical and numerical investigations into Casson fluid dynamics within vertical channels, attention has increasingly turned to the influence of boundary modifications particularly suction and injection on flow behaviour. Suction plays a critical role in fluid mechanics by extracting fluid through a surface, thereby altering boundary layer characteristics and significantly impacting velocity and thermal distributions. This mechanism has proven effective in numerous engineering applications, including aerodynamic drag reduction, boundary layer control, filtration systems, and thermal regulation in industrial processes. From a theoretical perspective, suction enhances flow stability and promotes efficient heat and mass transfer, making it a valuable control parameter in both Newtonian and non-Newtonian fluid studies. Several recent works have explored suction and injection effects in various non-Newtonian flow configurations [38]. Babu and Reddy [39] examined the unsteady two-dimensional flow of a non-Newtonian fluid over a stretching surface with imposed surface temperature, incorporating the effects of thermal radiation and suction/injection. They employed similarity transformations to convert the governing partial differential equations into a set of ordinary differential equations. Chakradhar et al. [40] analysed the peristaltic motion of Casson fluid within a vertical permeable channel under suction and injection conditions, utilizing the Galerkin weighted residual method. Paul et al. [41] investigated stagnation-point flow of magnetohydrodynamic (MHD) Casson–Williamson hybrid nanofluids over an exponentially stretched cylinder, incorporating viscous dissipation along with suction/injection effects, and solved the transformed ODEs using the boundary value solver bvp4c. Similarly, Ramudu et al. [42] considered steady MHD flow and heat transfer of a shear-thickening Casson fluid over a vertical stretching sheet with a variable heat source/sink, solving the transformed equations numerically via the shooting method in conjunction with a fourth-order Runge–Kutta algorithm. Shehzad et al. [43] focused on mass transfer effects in the MHD flow of a Casson fluid with chemical reactions and suction, performing a parametric analysis based on numerical solutions. While Ismail et al. [44] studied on to derive the theoretical solution for the mathematical models of Brinkman type fluid through vertical channel, incorporating suction and injection effects. They obtained the solution by using Laplace transform method.

Despite these advancements, existing studies predominantly address suction/injection in surface or plate geometries, and to date, no comprehensive analytical treatment has been reported for rotating MHD Casson fluid flow within a vertical channel incorporating suction or injection, particularly via the Laplace transform method. This constitutes a notable gap in the literature. Consequently, the present study aims to provide an analytical investigation of unsteady rotating MHD Casson fluid flow in a vertical channel under the influence of suction and injection at the channel walls. The governing partial differential equations will be solved using the Laplace transform technique, and closed-form expressions for the velocity profiles will be derived. The effects of key parameters including suction/injection rate, Casson parameter, rotating parameter, magnetic field strength, and time will be examined graphically and discussed in detail. The outcomes of this research are anticipated to contribute significantly to the theoretical foundation necessary for practical applications such as flow stabilization in complex fluid systems.

2. MATHEMATICAL FORMULATION

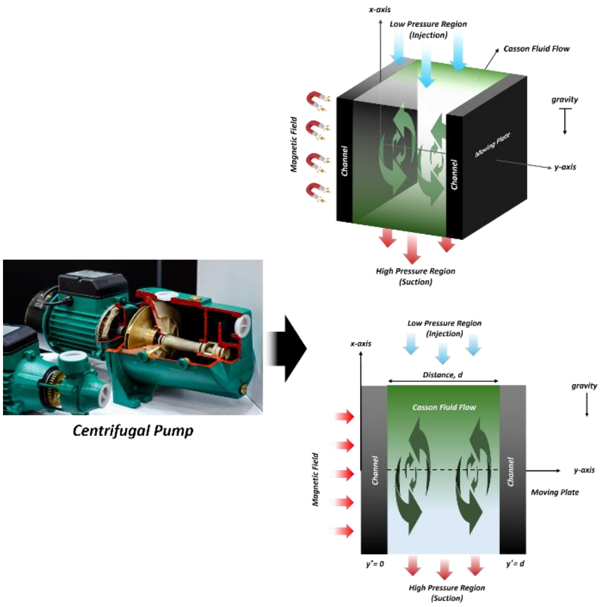


Fig. 1. Physical configuration of the rotating MHD Casson fluid flow in channel

Consider the unsteady flow of rotating incompressible MHD Casson fluid past through two vertical channels with the effect of suction is separated by distance d. The x-axis is in upward direction along the two vertical channel plates and y-axis is in normal direction to the plates. Initially, both the plate and fluid are at rest and at time $t = 0$, the left plate starts to move in its plane with constant velocity U_0 and the other plate remains at rest. Hence, the fluid starts solid body rotation with constant angular velocity Ω parallel to y-axis. A transverse magnetic field of uniform strength B_0 is applied perpendicularly to the x-axis. Physical problem situation can be visualized in Figure 1.

Applying the Boussinesq approximation, the free convective flow is governed by the equation

$$\frac{\partial F^*}{\partial t^*} - 2i\Omega F^* - \mu_0 \frac{\partial F^*}{\partial y} + \frac{\sigma B_o^2 F^*}{\rho} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 F^*}{\partial y^2}, \quad (1)$$

together with imposed initial and boundary conditions

$$\begin{aligned} &F^*(y^*,0) = 0 \ ; \ 0 \le y^* \ge d, \\ &F^*(0,t^*) = U_0 \quad ; \ t^* > 0, \\ &F^*(y^*,t^*) \ \to as\ y^* \to d \ ; \ t^* > 0, \end{aligned} \quad (2)$$

where $F^* = F^*(y,t) = u^*(y^*,t^*) + iw^*(y^*,t^*)$ is a complex velocity, $u^*(y^*,t^*)$ is a primary velocity, $w^*(y^*,t^*)$ is secondary velocity, i is an imaginary number, ρ is density, μ_0 is a constant velocity, σ is electrical conductivity, B_0 is magnetic field, ν is kinematic viscosity and β is Casson parameter. In order to transform the governing equations (1) into dimensionless form, the dimensionless variables are defined as

$$F = \frac{F^*}{U_0},\ y = \frac{y^*}{d},\ t = \frac{\nu t^*}{d^2}. \quad (3)$$

Thus, dimensionless momentum (1) equations can be simplified by using dimensionless equation (2). Then, can be written as

$$\frac{\partial F}{\partial t} - 2i\omega F - S_x \frac{\partial F}{\partial y} + MF = b_1 \frac{\partial^2 F}{\partial y^2} \quad (4)$$

with associated dimensionless initial and boundary conditions,

$$\begin{aligned} &F(y,0) = 0,\ 0 \le y \le 1 \\ &F(0,t) = 1, \qquad t > 0 \\ &F(1,t) = 0, \qquad t > 0 \end{aligned} \quad (5)$$

where $S_x = \frac{v_0 d}{\nu}$ is suction ($S_x > 0$) or injection ($S_x < 0$) parameter, $M = \frac{\sigma \beta_o^2 d^2}{\rho \nu}$ is magnetic parameter, $b_1 = (1+\frac{1}{\beta})$ is constant parameter, β constant Casson fluid parameter, $\omega = \frac{\Omega d^2}{\nu}$ is rotation and t is time parameter.

2.1 Laplace Transform

The following equation shows the Laplace transform solution of the partial differential momentum equation (4) with associated initial and boundary condition (5),

$$b_1 \frac{d^2 \bar{F}(y,q)}{dy^2} + S_x \frac{d\bar{F}(y,q)}{dy} - \left(q - 2i\omega + M\right)\bar{F}(y,q) = 0, \quad (6)$$

and

$$\begin{aligned} &\bar{F}(0,q) = \frac{1}{q}\ , \\ &\bar{F}(1,q) = 0\ , \end{aligned} \quad (7)$$

which is q, is the Laplace transform parameter. Then by using equation (6) and (7) leads to

$$\bar{F}(y,q) = \bar{R}_1(y,q) - \bar{R}_2(y,q) + \bar{R}_3(y,q), \quad (8)$$

where,

$$\bar{R}_1(y,q) = \frac{1}{q} e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \cdot e^{y a_4 \sqrt{b_1}\sqrt{a_5+q}},$$

$$\bar{R}_2(y,q) = \frac{1}{q} \cdot \frac{1}{1-e^{-2a_4\sqrt{a_5+q}}} \cdot e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \cdot e^{y a_4 \sqrt{b_1}\sqrt{a_5+q}},$$

$$\bar{R}_3(y,q) = \frac{1}{q} \cdot \frac{1}{1-e^{-2a_4\sqrt{a_5+q}}} \cdot e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \cdot e^{-y a_4 \sqrt{b_1}\sqrt{a_5+q}}.$$

The infinite geometric series equation is used to solve equation (8), which simplifies the complex equation and makes it easier to find a solution in the t-domain. Hence, the equation (8) becomes,

$$\bar{F}(y,q) = \bar{R}_1(y,q) - \sum_{n=0}^{\infty} \bar{G}_1(y,q) + \sum_{n=0}^{\infty} \bar{G}_2(y,q), \quad (9)$$

where,

$$\bar{R}_1(y,q) = \frac{1}{q} e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \cdot e^{y a_4 \sqrt{a_5+q}},$$

$$\bar{G}_1(y,q) = \frac{1}{q} \cdot e^{\left(-2a_4\sqrt{a_5+q}\right)n} \cdot e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \cdot e^{y a_4 \sqrt{a_5+q}},$$

$$\bar{G}_2(y,q) = \frac{1}{q} \cdot e^{\left(-2a_4\sqrt{a_5+q}\right)n} \cdot e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \cdot e^{-y a_4 \sqrt{a_5+q}}.$$

2.2 Inverse Laplace Transform

From equation (6), the inverse Laplace may be obtained by using the inverse Laplace transform method, and by applying partial fraction decomposition theorem as,

$$F(y,t) = U_1(y,t) - \sum_{n=0}^{\infty} U_2(y,t) + \sum_{n=0}^{\infty} U_3(y,t), \quad (7)$$

where,

$$U_1(y,t) = e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \begin{bmatrix} \frac{1}{2} e^{-ya_4\sqrt{b_1 a_5}} erfc\left(\frac{-ya_4}{2}\sqrt{\frac{b_1}{t}} + \sqrt{a_5 t} \right) \\ + \frac{1}{2} e^{ya_4\sqrt{b_1 a_5}} erfc\left(\frac{-ya_4}{2}\sqrt{\frac{b_1}{t}} - \sqrt{a_5 t} \right) \end{bmatrix},$$

$$U_2(y,t) = e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \begin{bmatrix} \sum_{n=0}^{\infty} \frac{1}{2} e^{(2n-y)a_4\sqrt{b_1 a_5}} erfc\left(\frac{\frac{(2n-y)a_4}{2}}{\sqrt{\frac{b_1}{t}} + \sqrt{a_5 t}} \right) + \\ \sum_{n=0}^{\infty} \frac{1}{2} e^{-(2n-y)a_4\sqrt{b_1 a_5}} erfc\left(\frac{\frac{(2n-y)a_4}{2}}{\sqrt{\frac{b_1}{t}} - \sqrt{a_5 t}} \right) \end{bmatrix},$$

$$U_3(y,t) = e^{\left(-a_4 \frac{S_x}{2}\right)(y)} \begin{bmatrix} \sum_{n=0}^{\infty} \frac{1}{2} e^{(2n+y)a_4\sqrt{b_1 a_5}} erfc\left(\frac{\frac{(2n+y)a_4}{2}}{\sqrt{\frac{b_1}{t}} + \sqrt{a_5 t}} \right) + \\ \sum_{n=0}^{\infty} \frac{1}{2} e^{-(2n+y)a_4\sqrt{b_1 a_5}} erfc\left(\frac{\frac{(2n+y)a_4}{2}}{\sqrt{\frac{b_1}{t}} - \sqrt{a_5 t}} \right) \end{bmatrix}.$$

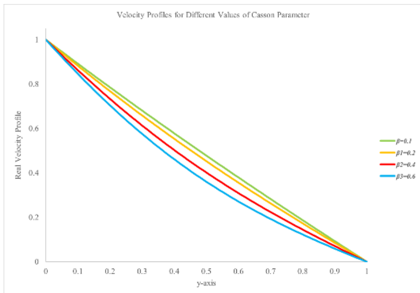
In which, $a_4 = \frac{1}{b_1}$ and $a_5 = \frac{S_x^2}{4b_1} - 2i\omega + M$.

3. RESULTS AND DISSCUSSION

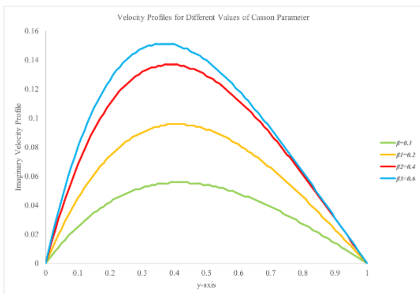
A parametric study was conducted on the flow of rotating MHD Casson fluid through two vertical channels, considering the effects of suction and injection. The problem was solved using the Laplace transform method. Results were then plotted for five key parameters: the Casson parameter, suction parameter, rotation parameter, magnetic parameter, and time. In the plotted graphs, figures (a) and (b) represent the primary and secondary velocity profiles, respectively. Figure 2 illustrates the primary and secondary velocity profiles for various values of the Casson parameter. It is observed that the primary velocity of the fluid decreases, while the secondary velocity increases as the Casson parameter increases, with the rotation, suction, magnetic parameters, and time held constant. This behaviour is attributed to the plastic dynamic viscosity of the fluid near the channel walls, which creates greater resistance to flow. As the Casson parameter increases, this resistance intensifies, resulting in a reduced primary velocity. Conversely, the increase in secondary velocity is due to the Coriolis force acting on the fluid, which redirects part of the flow.

Figure 3 illustrates the influence of the magnetic parameter on the fluid flow. The plotted graph shows that both the primary and secondary velocity components decrease as the magnetic parameter increases, while the Casson, suction, and time parameters are held constant. This behaviour is attributed to the generation of electric currents when an external magnetic field is applied to a moving electrically conducting fluid. The interaction between these induced currents and the applied magnetic field gives rise to a resistive force known as the Lorentz force. This force acts in the direction opposite to the fluid motion, thereby opposing the flow and reducing the fluid velocity. The effect of the Lorentz force is analogous to that of a drag force in resisting motion. Figure 4 illustrates the effect of the rotation parameter on the fluid flow. It is observed that the primary velocity decreases, while the secondary velocity increases with increasing rotational effects. This behaviour is attributed to the Coriolis effect, which arises due to the rotation of the fluid system. The Coriolis force acts perpendicular to the direction of fluid motion and the axis of rotation, thereby reducing the primary flow component and promoting the development of the secondary flow. Essentially, the Coriolis force causes a deflection of moving fluid particles within the rotating frame, leading to a redistribution of momentum and the observed velocity behaviour.

Figure 5 illustrates the effect of the suction parameter on the fluid flow. The plotted graph indicates that the fluid velocity decreases as the suction parameter increases, while the Casson, rotation, magnetic, and time parameters are held constant. This reduction in velocity is attributed to the influence of suction on the pressure gradient within the flow system. Suction alters the boundary layer behaviour by extracting fluid from the flow domain, which effectively increases resistance and reduces the overall flow rate. Last but not least, figure 6 depicts the graph plot on the different values of time parameter shows that the increasing pattern for both primary and secondary velocity profile with a constant value for suction, magnetic, rotating and Casson parameter. Under these conditions, it is found that velocity increase when the value of time t is increased due to the external energy from the plate that supply to the fluid flow which results in increasing fluid particle movement when time is increasing.

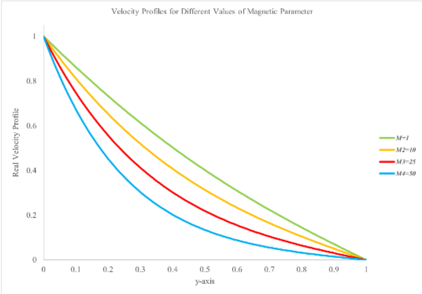


(a)

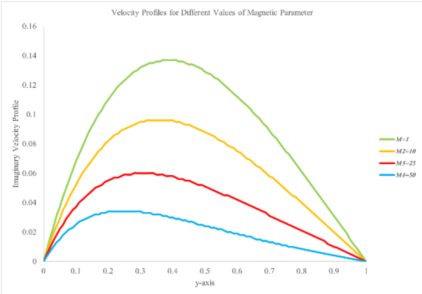


(b)

Fig. 2. Primary (a) and secondary (b) velocity profile for different values of Casson parameters.

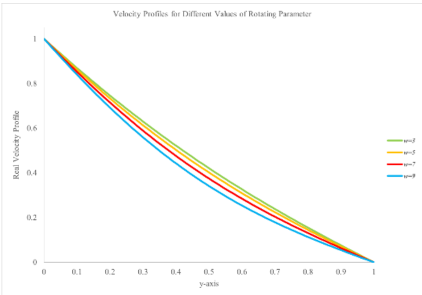


(a)

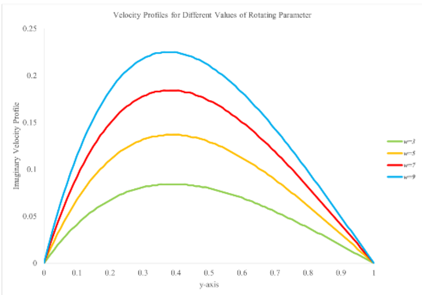


(b)

Fig. 3. Primary (a) and secondary (b) velocity profile for different values of magnetic parameters.

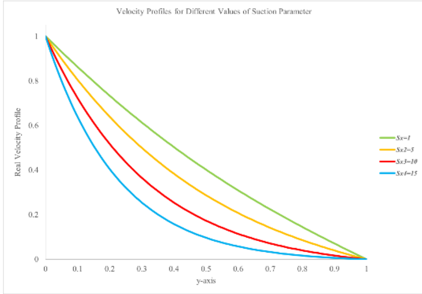


(a)

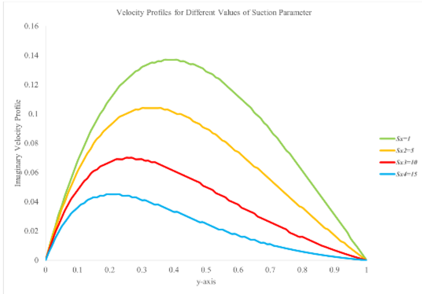


(b)

Fig. 4. Primary (a) and secondary (b) velocity profile for different values of rotating parameters.

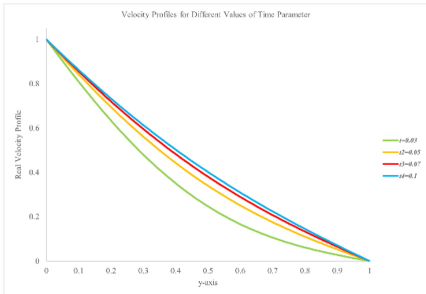


(a)

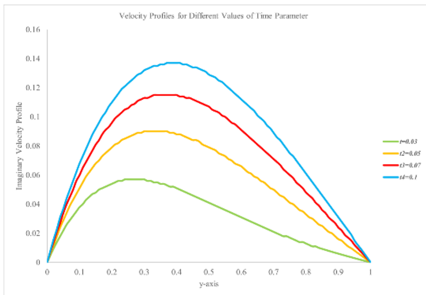


(b)

Fig. 5. Primary (a) and secondary (b) velocity profile for different values of suction parameters.



(a)



(b)

Fig. 6. Primary (a) and secondary (b) velocity profile for different values of time parameters.

4. CONCLUSION

In this paper, the analytical solutions for the flow of a rotating magnetohydrodynamic (MHD) Casson fluid within two vertical channels, incorporating the effects of suction/injection, are thoroughly investigated. The problem is formulated by deriving the dimensionless momentum equations from their dimensional governing equations, which are then solved using the Laplace transform method and its corresponding inverse transform. The obtained analytical solutions are rigorously validated by ensuring they satisfy the prescribed initial and boundary conditions of the problem. To further analyse the behaviour of the system, the results are presented graphically, illustrating the influence of various dimensionless parameters on the flow characteristics. While much of the existing literature on the free convection flow of rotating MHD Casson fluids primarily focuses on scenarios involving a single vertical plate and often neglects the effects of suction, this study distinguishes itself by addressing the more complex configuration of two vertical plates and explicitly incorporating the suction/injection mechanism. The primary objective of this work is to extend the understanding of rotating MHD Casson fluid dynamics by providing analytical solutions for in this specific geometry, thereby contributing to the broader field of non-Newtonian fluid mechanics under the influence of rotating, magnetic fields and suction effects. The findings of this study not only offer theoretical insights but also have potential applications in engineering and industrial processes involving fluid flow control and heat transfer enhancement.

5. REFERENCES

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