

Design Consideration of Waste Dumping on Inclined Surface with Limited Area Based on Probabilistic Stability Analysis of Numerical Simulations: A Case Study

Bat-Erdene, Bugunei

Department of Earth Resources Engineering, Kyushu University

Kawano, Koki

Department of Earth Resources Engineering, Kyushu University

Sasaoka, Takashi

Department of Earth Resources Engineering, Kyushu University

Hamanaka, Akihiro

Department of Earth Resources Engineering, Kyushu University

他

<https://hdl.handle.net/2324/7364868>

出版情報 : Mining. 5 (3), pp.44-, 2025-07-10. Multidisciplinary Digital Publishing Institute : MDPI

バージョン :

権利関係 : © 2025 by the authors.



Article

Design Consideration of Waste Dumping on Inclined Surface with Limited Area Based on Probabilistic Stability Analysis of Numerical Simulations: A Case Study

Bugunei Bat-Erdene *, Koki Kawano, Takashi Sasaoka, Akihiro Hamanaka  and Hideeki Shimada 

Department of Earth Resources Engineering, Kyushu University, Fukuoka 819-0395, Japan

* Correspondence: bat24r@mine.kyushu-u.ac.jp

Abstract

A case study of designing a waste dump was conducted for the iron mine located in the Bulacan area, Philippines. Iron ore mines generate a relatively high amount of waste, and at the study mine, the constrained waste dumping area of 3 hectares necessitated a higher dump design, leading to potential stability issues. Additionally, the waste dump is projected to be situated on an inclined surface; subsequently, there is a concern about dump stability. Therefore, this study aims to find the optimum waste dump design by assessing its stability, and a geometrical configuration was conducted to optimize the bench parameters. Numerical modeling of the finite difference method (FDM) was used to estimate the distribution of the Factor of Safety by simulating several models. Models with steeper base inclinations ($>12^\circ$) demonstrate progressive instability, as demonstrated by pre-assessment. The statistical analysis results show that the total model simulations with a 45-degree slope angle have a significantly high probability of failure of 38.2%. Whereas models with 35-degree and 40-degree slope angles have probabilities of failure calculated as 0.3% and 6.5%, respectively. Therefore, results suggest that the general slope angle should be kept at 40 degrees or less. Moreover, the results show that an average of 0.02 points drops in FoS for each 2.5 m of increment in dump height. Regarding geometrical setup, four benches with 7.5 m of berm would be preferable for the waste dump design of the case study. Overall, the effect of an inclined surface as a base was discussed, the effect of a gradual increase in dump height was outlined, and the significance of the dump slope angle on dump design was highlighted.

Keywords: waste dump; dump height; inclined surface; numerical modeling

Academic Editors: Mohammad H.B. (Farzine) Nasser and Bibhu Mohanty

Received: 20 May 2025

Revised: 20 June 2025

Accepted: 9 July 2025

Published: 10 July 2025

Citation: Bat-Erdene, B.; Kawano, K.; Sasaoka, T.; Hamanaka, A.; Shimada, H. Design Consideration of Waste Dumping on Inclined Surface with Limited Area Based on Probabilistic Stability Analysis of Numerical Simulations: A Case Study. *Mining* **2025**, *5*, 44. <https://doi.org/10.3390/mining5030044>

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

In an open-pit mining operation, the overburden, which consists of waste and uneconomic rocks, must be removed to access valuable mineral resources. During this removal process, the waste material is deposited near the mining area to form a waste dump [1]. Iron ore mines, in particular, generate a relatively large amount of waste. Due to the land-use limitation of the project, the constrained waste dumping area (3 hectares) necessitated a higher dump design at the study mine, leading to potential stability issues. As a result, concerns regarding potential dump failure arise, making it essential to design a waste dump that ensures safety and stability.

One of the primary engineering challenges in open-pit mining operations is the safe deposition of a large volume of dump material within a limited dump site area [1,2]. A

limited dumping area often necessitates the design of a higher waste dump, which, in turn, can adversely affect the stability of the waste rock dump [2,3]. Therefore, a thorough stability analysis is essential to ensure the safe and effective design of the waste dump.

On the other hand, waste rock dumps typically consist of a wide range of soils and crushed rock materials, including silty and sandy gravels, as well as boulders [3,4]. The large variety of particle sizes, including very coarse grains, and the wide size distribution within a waste dump, create challenges in characterizing and determining the geotechnical parameters of waste dump materials [3]. To address this challenge, this study proposes the utilization of the Monte Carlo method by considering specific ranges of geotechnical parameters and simulating multiple numerical models.

A slope failure in a waste rock dump, regarded as a significant geohazard, may result in injury or death of mine workers, destruction of equipment, and loss or delay in production [1]. Therefore, this study aims to design a safe and stable waste dump, which is critical to avoiding potential adverse impacts and minimizing the risk of failure.

In a broader concept, this study considers the global challenges involved in open-pit iron ore mining operations. Some research assesses surface disturbance of the iron ore basin [5] and explores a new strategy of the restoration for land surfaces disturbed by mining [6]; both studies highlight the importance of land-use limitations for mining operations [5,6]. The importance of the study would be emerged by demonstrating the possibility of the waste dump design without disturbing a large surface area. Therefore, this study aims to overcome engineering challenges, particularly considering a large volume of waste dumping in a constrained area, which can be common in certain regions, and the findings can be applied to other regions with comparable geological formations and land-use constraints.

Several studies have been conducted on the stability of waste dumps using numerical modeling and empirical methods. Some of this research focuses on the effect of dump height on stability [7–9], demonstrating the adverse impact of high dumps, assessing the critical dump height, and highlighting its significance. Additionally, some studies highlight the effect of dump slope on stability [10,11], indicating that steeper slopes significantly influence dump stability. Moreover, many studies have focused on the stability of coal mine waste dumps, as they generate a substantially higher amount of waste; however, the dumping methods used are unique [12–15]. In contrast, very few studies have been conducted on the waste dumps of iron mines [11,16,17], and another study focused on the stability assessment of an iron ore mine waste dump considering dynamic analysis [18]. Therefore, further research is needed to investigate iron ore mine dumps while considering the unique characteristics of iron mining. This study analyzes the stability of iron mine waste dumps on inclined terrain, taking into account the overall volume of the dump due to the limited area designated for the waste site.

Several methods have been developed to analyze the stability of both mine slopes and waste dump slopes. The limit equilibrium method is among the most widely used techniques for slope stability analysis [19–21]. Numerical modeling is also widely employed for slope stability analysis, with the most common methods being the finite element method [20–22], the finite difference method [23–25], and the discrete element method [26,27]. Another study compares the Strength Reduction Method based on the numerical modeling of the Finite Difference Method and the Finite Element Method [28]. With rapid advancements in computational efficiency, the Finite Difference Method and Finite Element Method are preferred in numerical modeling, particularly for calculating the Factor of Safety [19,24].

While this study focuses on optimizing waste dump stability through geometric design parameters, there are several critical factors that influence the dump slope stability, and they

must be addressed. Those factors include hydrogeological aspects (pore pressure effects, rainfall infiltration), dynamic loading (blast vibrations and seismic activity/earthquakes), and material degradation (weathering effects and erosion) [18,29,30]. These critical aspects will be considered for further analysis to establish a holistic stability assessment for the iron ore mine waste dump.

Additionally, it is crucial to note that this study represents a pre-feasibility level analysis; it involves certain limitations and includes some assumptions to simplify the numerical models. Overall, this study evaluates and specifically focuses on the gradual increase in dump height, inclined terrain, general dump slope angle, and bench configurations, calculating the Factor of Safety as an indicator of dump stability to determine the optimal design for the waste dump in a case study of an iron mine.

2. Materials and Methods

2.1. Study Area

The mine site is located in the Philippines, on the island of Luzon, within the Municipality of Dona Remedios Trinidad, Province of Bulacan (Figure 1). It is situated at a central geographic position of 15°03'30" North latitude and 121°08'00" East longitude, with an elevation ranging from 430 to 515 m above mean sea level. The area is part of the Central Luzon East Basin stratigraphy, predominantly composed of Paleogene volcanics and sediments. It lies within the Bayabas Geologic Formation [31], where lithologies mainly consist of andesitic to basaltic compositions, interspersed with sandstone–siltstone intercalation layers.

Waste material, including soil and rock, will be deposited adjacent to and behind the active excavation area, following the natural topography of the Bayabas Geologic Formation, which exhibits a gradient ranging from 18 to 30%. The waste materials are anticipated to consist of sediments, pyroclastics, tuffs, tuffaceous materials, sandstones, siltstones, and agglomerates.

A considerable amount of waste is generated from iron ore mines, with the quantity depending on the thickness of the overburden layers [11]. In this case, the mining project's 10-year production target is projected to generate a total of 695,721.75 metric tons of waste, comprising both overburden and material from gaps between ore beds. Therefore, the total volume is approximately 358,000 cubic meters, assuming an average density of 1.94 tons/m³ (1940 kg/m³), estimated from overburden material samples. The estimated stripping ratio is approximately 2.09:1, with an annual production rate of 90,000 metric tons of iron concentrate, which dictates the pit limit.

The topography of the area is believed to have developed through a combination of volcanic deposition, differential erosion, sedimentation, magmatic intrusion, and regional tectonic activity. This geological setting is characteristic of island arc environments located between convergent plate boundaries—specifically, between an oceanic plate and a continental plate—within a tropical climate. Given the natural constraints on available land for waste disposal, the designated area for waste dumping is currently limited to approximately 3 hectares (Figure 2). Consequently, the design of the waste disposal area must prioritize geotechnical safety, considering the limitations imposed by both natural and anthropogenic factors.

Additionally, according to the Philippine Atmospheric, Geophysical and Astronomical Services Administration, the Philippines has a mostly tropical rainforest climate. In particular, Bulacan province, where the study area is located, also falls under a tropical rainforest climate [32]. This climate is characterized by high temperatures, high humidity, and significant rainfall throughout the year. The high precipitation adversely affects the slope stability [29]; therefore, the effect of rainfall should be considered when designing the waste dump.

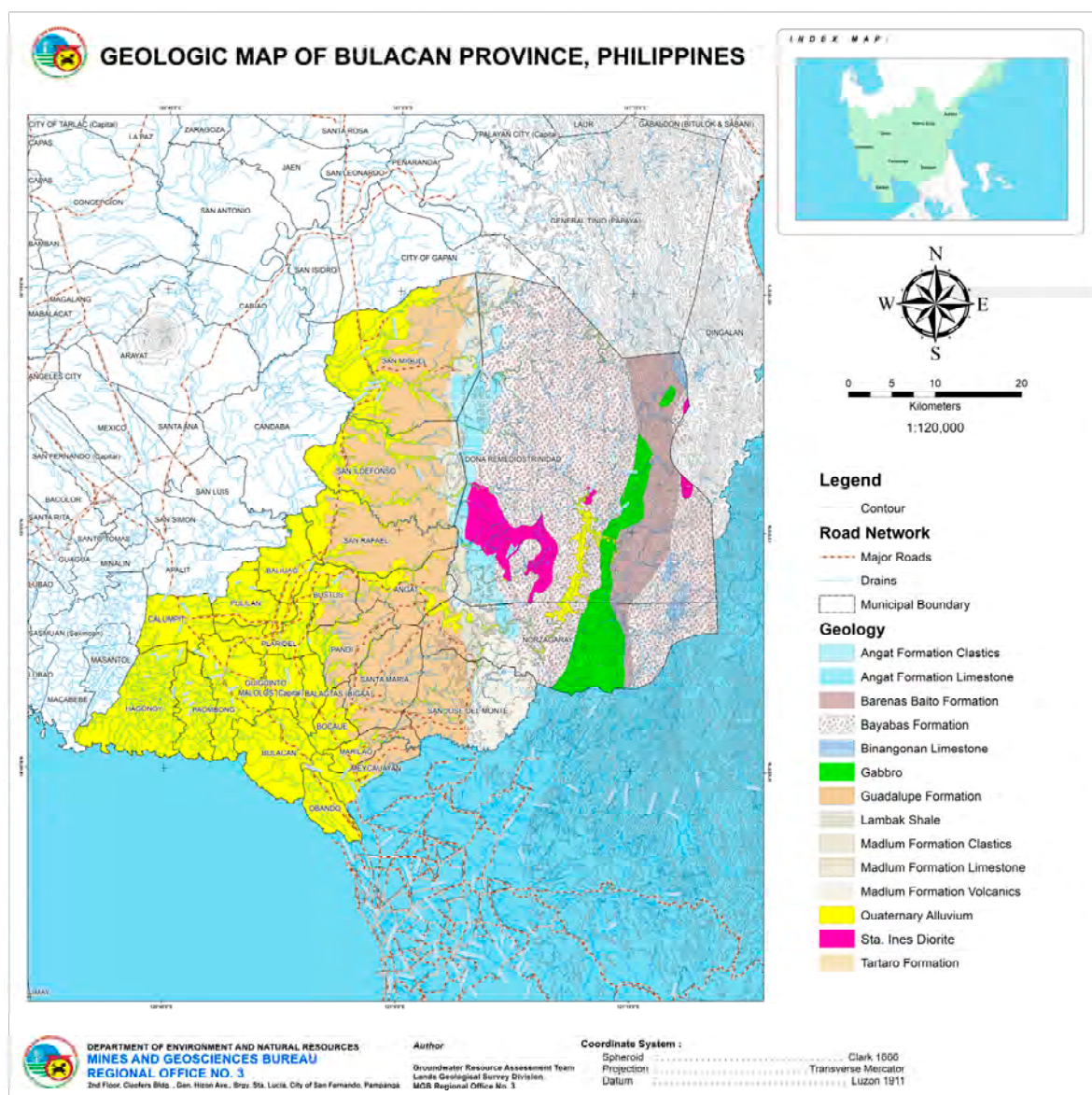


Figure 1. Geological map of the province of Bulacan [31].

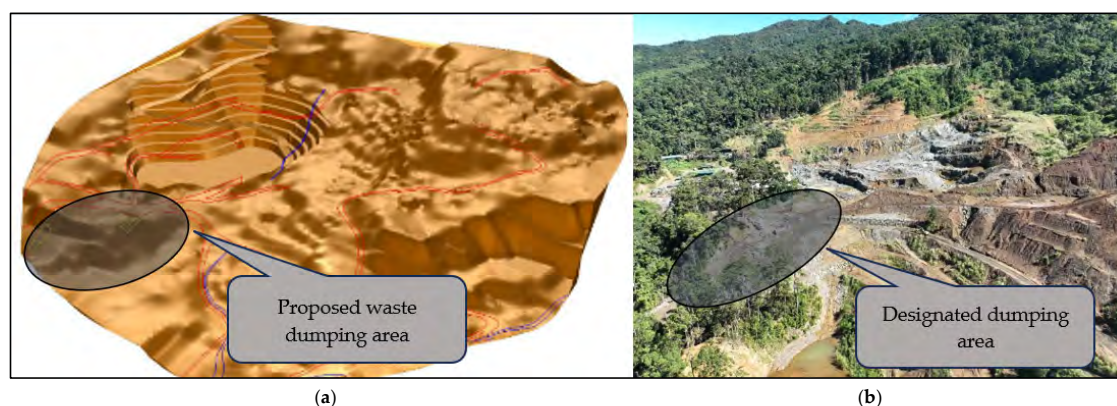


Figure 2. Designated area for waste dump (a) project area for the mine waste dump, and (b) aerial view of the current dump establishment.

Overall, the geological properties and topographical characteristics highlight the general rock composition of the overburden material, providing insight into the volume of overburden and the limitations of land use.

2.2. Material Properties of the Waste Dump

The properties of waste dump material play a crucial role in stability assessments. To obtain the necessary data for numerical modeling, laboratory testing of site samples and in situ testing should be conducted [33]. Since the waste dump site was newly established, obtaining representative samples directly from the site was not possible. Therefore, the geotechnical properties of the waste dump material were determined by considering a range of mine waste material properties based on a literature review of reference values.

Therefore, considering the properties of the overburden material, a distinctive approach was employed to estimate the strength parameters. Two soil samples were collected from the overburden material at the mining area, which is expected to be deposited in the dump site. First, the basic properties of the soil samples were determined, followed by their classification. Strength parameters were then selected based on the soil classification and reference values suggested in various studies.

Based on the test results and in accordance with the Unified Soil Classification System (USCS) standard, both samples were classified as SM—silty sand [34]. Table 1 presents the basic parameters and classification of the soil samples.

Table 1. Soil sample test results and classification.

Samples	Soil Sample 1	Soil Sample 2
Size distribution	Gravel 11.94%	Gravel 4.45%
	Sand 67.23%	Sand 65.33%
	Fines 20.83%	Fines 30.22%
Classification	Silty sand (SM)	Silty sand (SM)
Wet density {kg/m ³ }	1920	1960
Dry density {kg/m ³ }	1620	1620
Moisture content {%}	18.24	20.88

Since these samples were collected from the mining area rather than directly from the dump site, it is important to note that they represent the characterization of the overburden materials. Moreover, in the study area, overburden, blasted rock materials, and previously mined loose materials will be deposited together in the waste dump. Consequently, once the mixed waste materials begin to be dumped, the properties of the waste dump material are expected to change.

The strength properties of the soil based on reference values [35–37] are listed below in Table 2. When assuming geotechnical parameters, a certain range can be pre-determined (in our case, the range was determined with reference values), then the normal distribution is created within this range [37] for selecting random variables from the probability distribution. So, the following ranges of cohesion and friction angles, which were determined by the reference value ranges of 20–50 kPa [36] and 27–34 degrees [35,37], were considered for further detailed probabilistic analysis of slope failure. The strength parameters for the model are selected by considering the average value of reference values along with the standard deviations.

Certain strength parameters, such as Young's modulus and Poisson's ratio of dump materials, have minimal impact when estimating the Factor of Safety (FoS) using the strength reduction technique [2,22,24]. Assuming nominal values for these geomechanical properties does not significantly affect FoS results [22]. Therefore, nominal average values of Young's modulus (15 MPa) and Poisson's ratio (0.3) were assumed for the numerical models and treated as non-variable parameters.

Table 2. Strength properties of SM soil and parameters used for the models.

Properties	Compacted SM	Saturated SM	Pre-Assessment	Statistical Analysis
Young's Modulus {MPa}	7–30 [36]		15	15
Poisson's Ratio	0.2–0.3 [35]		0.3	0.3
Cohesion {kPa}	20 [36]	50 [36]	35	35 ± 7.5
Friction Angle {degree}	34 [35,37]	27–34 [35,37]	31	30.5 ± 1.75
Density {kg/m ³ }			1940	1940

Moreover, for the pre-assessment of the effect of terrain inclination, strength parameters of cohesion at 35 kPa and a friction angle of 31 degrees were selected as mean deterministic values. Subsequently, an average density of 1940 kg/m³ was used for both the pre-assessment and detailed analysis.

The geomechanical parameters of the base formation, on which the waste will be deposited, are essential for numerical modeling [33]. Based on the field study and geological structure of the mining area, the base rock mass was found to consist of sandstone and siltstone. Rock samples were collected from the underlying geological formation at the projected dump site, and their strength parameters were determined through laboratory testing. The average values obtained from these tests are presented in Table 3 and were used in the numerical models for the base rock.

Table 3. Strength parameters of the base rock mass.

Properties	Sandstone/Siltstone
Young's Modulus {GPa}	3.38
Poisson's Ratio	0.18
Cohesion {MPa}	12.9
Friction Angle {degree}	25.4
Density {kg/m ³ }	2780

In this case study, it is assumed that the waste dump is situated on the rock formation by stripping the topsoil. Although some portions of the base rock formation consist of soil-like materials, it was assumed that the dump is fully situated on the rock base for simplification of the numerical model. The strength parameters shown in Table 3 represent the intact rock formation beneath the designated dump site; therefore, they correspond to relatively high values. Since the failure is expected on the dump slope and the study focuses more on the dump structure itself, the effect of the base rock geotechnical properties is not significant in the numerical model.

2.3. Numerical Modeling for Factor of Safety Calculation

A Factor of Safety (FoS) is defined as the ratio of the resisting forces (which prevent slope failure) to the disturbing forces (which promote slope failure). It is a significant and commonly used factor that indicates the stability of the slope [38,39]. A FoS below 1 means the slope is unstable, and vice versa [38]. The Shear Strength Reduction Method (SSR) is a technique used to compute the Factor of Safety (FoS) for slopes in numerical modeling. It involves gradually reducing the shear strength of the rock or soil until the system fails [19,20,39]. In this study, as an output parameter, the Factor of Safety of the waste dump is estimated with the strength reduction (SSR) method by taking certain ranges of geomechanical parameters into account.

For models with relatively simple geometric shapes, the FDM model can serve as an efficient tool, offering reliable results with reduced computational complexity [24]. Numerical modeling of the FDM method was used in this study; the distribution of FoS has been estimated by simulating the models. To facilitate a comparative assessment and validate the consistency between the FDM and FEM models, the FEM analysis was performed after the evaluation of the FDM model results.

The FEM and FDM models can yield different results depending on mesh discretization and boundary conditions [28]. The FEM approach can be particularly valuable in cases involving complex geometries, due to its flexibility in mesh generation and possibility of unstructured meshes [22]. Therefore, the FEM method was utilized for the evaluation of geometrical setups for the bench and the bench berm by considering the irregular shape of the topographical map.

First, the FDM model was established, and a pre-assessment was conducted using model variations with different inclinations and heights. After evaluating the effect of inclination, the average slope angle of the case study area was selected for further analysis. Models yielding critical FoS values were identified, and a suitable height range was defined. A base model was then developed based on the pre-assessment results. Subsequently, statistical analysis was performed through numerical simulations, and the results were interpreted using a statistical approach. Based on the findings from the FDM analysis, the optimum slope angle and the corresponding safe dump height were determined. These design parameters were then applied to develop FEM models for comparison. Finally, the FoS results from both methods were briefly compared to assess whether the FDM and FEM analyses produced consistent outcomes.

The shape and geometry of the waste dump, the geomechanical properties and density of the waste material, and the strength parameters of the base rock are critical factors influencing dump stability [1,40]. It is important to note that, for simplification in numerical modeling, the study assumes homogeneous waste material properties. This assumption may lead to an overestimation of actual field conditions, as in reality, the mine waste consists of heterogeneous materials and a wide range particle size distribution.

The geometrical parameters of the base model are illustrated along with the 3D model view in Figure 3. The mean height is calculated as the arithmetic average of the maximum height (H_1) and minimum height (H_2), based on the trapezoidal shape illustrated in Figure 3. In terms of design considerations for a waste dump situated on inclined terrain, the length, width, and mean height are critical parameters influencing stability. These dimensional parameters are interdependent when evaluating models with a fixed dump volume; that is, modifying one parameter often necessitates adjustments to the others. This interrelation is especially important when designing and establishing a new waste dump in areas with limited available space, as is the case in this study.

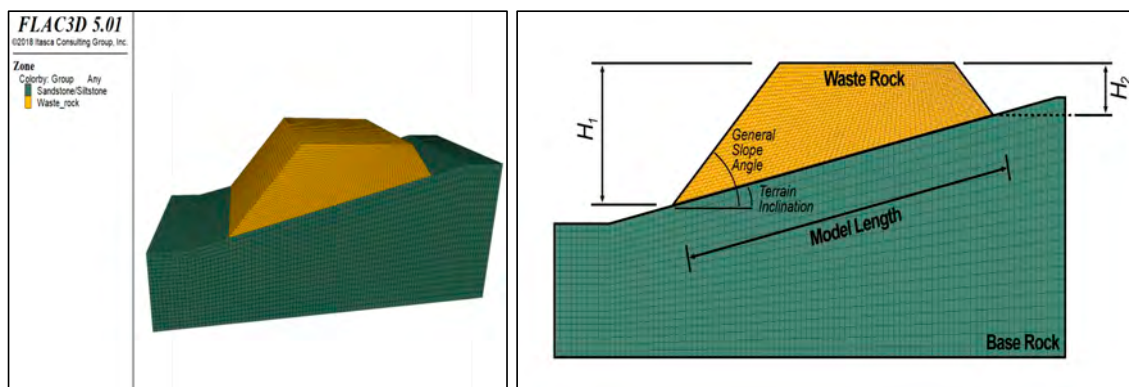


Figure 3. The 3D model view and the geometrical parameters.

2.4. Probabilistic Analysis with the Monte Carlo Approach

The probabilistic method is an effective approach for slope stability analysis [41], particularly when combined with numerical simulations. The normal distribution, also referred to as the Gaussian distribution, is the most commonly used probability density function in such analyses [42]. Many geotechnical variables tend to follow this distribution, especially when influenced by additive, non-dominant random effects [38]. Therefore, unless there is a compelling reason to choose an alternative distribution, geotechnical probabilistic analyses typically assume normal distribution [38].

In this study, random variables were generated in FLAC3D (Version 5.01) using the *gauss_dev* keyword, which assumes a normal (Gaussian) distribution and produces random values based on the specified mean and standard deviation [39]. It is recognized that empirical evidence often indicates an inverse relationship between the friction angle and cohesion, whereby an increase in cohesion corresponds with a decrease in the friction angle [3,38]. However, in this study, the friction angle and cohesion were treated as independent parameters for the purpose of simplifying the analysis.

The Monte Carlo simulation technique is widely used in probabilistic slope stability analysis [38,43]. It allows for the incorporation of uncertainties associated with the geomechanical properties of rock and soil materials [25,43]. The method involves random sampling from predefined probability distributions, and when a sufficient number of model iterations are performed—such as in the calculation of safety factors—it can generate a distribution of possible outcomes, thereby providing a probabilistic measure of slope stability [38]. Since the actual geotechnical parameters are not possible to obtain at this stage, there is an uncertainty regarding the geotechnical properties of the waste material. To overcome the uncertainty in geotechnical properties, the Monte Carlo method was utilized. Even though the reference values were assumed for the analysis, these reference values have a certain range. So, instead of defining a deterministic value from these reference values, a normal distribution was created covering the range of reference values, and random values are selected from this distribution with the Monte Carlo approach. Therefore, this study is based on numerical modeling combined with the Monte Carlo method.

To account for the uncertainties affecting waste dump stability, geomechanical properties such as cohesion and friction angle of the waste material are treated as random variables in the analysis [43]. In this study, the Monte Carlo simulation technique was employed to generate cohesion and friction angle values within the assumed parameter ranges. This approach created 10,800 total iterations with random values, and FoS was calculated for each iteration. The number of unstable iterations ($FoS < 1$) divided by the total iterations and multiplied by 100% gives a probability of failure, which is a commonly used approach for estimating the failure probability in numerical modeling [38]. The probabilistic approach is utilized to analyze the output of the numerical simulations.

3. Results

In this section, the effect of surface inclination on dump stability was evaluated. Additionally, the influence of the gradual increase in dump height and the geometrical parameters of the model was assessed. A pre-assessment was first conducted, and models yielding Factor of Safety (FoS) values near 1 were identified and subsequently carried forward for detailed analysis.

3.1. The Effect of Terrain Inclination

The topography and inclination of the surface where the waste will be dumped directly affect the overall stability of the waste dump [41,44]. Waste dumps disposed of on a mildly inclined surface are generally more favorable and stable than those on steeper slopes [2,16].

To evaluate the effect of terrain inclination on FoS, four variations of models are created with the inclinations of 0, 6, 12, and 18 degrees.

Furthermore, models are divided into two sub-groups, “A” and “B” model groups. The “A” model group has identical model lengths with different heights, whereas the “B” model group includes variable lengths with the same mean height. Both model groups have a variation in inclination. Therefore, considering these waste dump design parameters, eight models with the same volume were created. Model parameters for pre-assessment and the results of FoS for eight models are given below in Table 4.

Table 4. The 3D models for pre-assessment and model dimension parameters.

Model No.	Terrain Inclination (Degrees)	Height H ₁ (m)	Height H ₂ (m)	Mean Height (m)	Length (m)	Width (m)	Volume (m ³)	General Slope Angle (Degrees)	FoS
Model 1A	0	30	30	30	150	100	~358,000	45	1.32
Model 2A	6	38.5	22.7	30.6	150	100	~358,000	45	1.18
Model 3A	12	49.3	17.9	33.6	150	100	~358,000	45	1.05
Model 4A	18	63.5	17.1	40.3	150	100	~358,000	45	0.94
Model 1B	0	45	45	45	125	100	~358,000	45	1.12
Model 2B	6	51.5	38.5	45	126	100	~358,000	45	1.03
Model 3B	12	58.9	31.1	45	133	100	~358,000	45	0.99
Model 4B	18	66.9	23.1	45	142	100	~358,000	45	0.93

For simplicity, and due to uniform conditions along the width, variations in dump width were not evaluated. In the numerical models, the dump slope angle was fixed at 45 degrees. A width of 100 m was assumed to facilitate volumetric modeling, and both parameters were treated as non-variable design inputs during the pre-assessment phase.

The Factor of Safety (FoS) was calculated for each model to evaluate the influence of terrain slope inclination. The results indicate that models with gentler terrain exhibited higher FoS values—for example, Model 1A had an FoS of 1.32, and Model 1B had an FoS of 1.12. In contrast, lower FoS values were observed on steeper terrain inclinations (e.g., 12° and 18°), where the models showed a higher likelihood of failure, with FoS values falling below 1.

When comparing models such as 1A and 1B, and 2A and 2B, it is evident that the effect of mean dump height is more pronounced on mildly inclined surfaces. Furthermore, Models 4A and 4B both yielded the similar FoS values of 0.93 and 0.94, despite differing design parameters: mean heights of 40.3 m and 45 m, and lengths of 142 m and 150 m, respectively. This suggests that at higher inclinations, the influence of design dimension parameters becomes less significant compared to mildly inclined terrains, as the effect of slope inclination dominates. Additionally, the “A” group models exhibit a marked decrease in FoS values—from 1.32 at 0° inclination to 0.94 at 18°—demonstrating the combined impact of increasing dump height and terrain inclination. Specifically, comparing Models 1A and 4A reveals an approximate 29.5% reduction in FoS, underscoring the substantial influence of both dump height and terrain slope on stability. In contrast, the “B” group models show a more modest decrease in FoS (from 1.12 to 0.93), reflecting primarily the effect of terrain inclination.

The shear behavior of the interface between waste dump materials and the base formation was not explicitly modeled in this study. Future investigations will incorporate interface elements by determining the normal stiffness coefficient. The most dominant and severe mode of waste dump slope failure is progressive slope failure, which is initiated by the development of shear zones, also referred to as shear failure. Dump slope failure occurs as these shear zones propagate and interconnect, influenced by stress redistribution and material weakening [17,22]. Since the base rock possesses significantly higher strength properties than the dump material and given the assumption that the dump rests on intact base rock, failure within the base formation is not anticipated in the numerical

model. Instead, shear failure is expected to occur within the dump material. Therefore, the Maximum Shear Strain Rate was used as the evaluation parameter, and the resulting contours illustrate the potential failure surfaces (Figure 4).

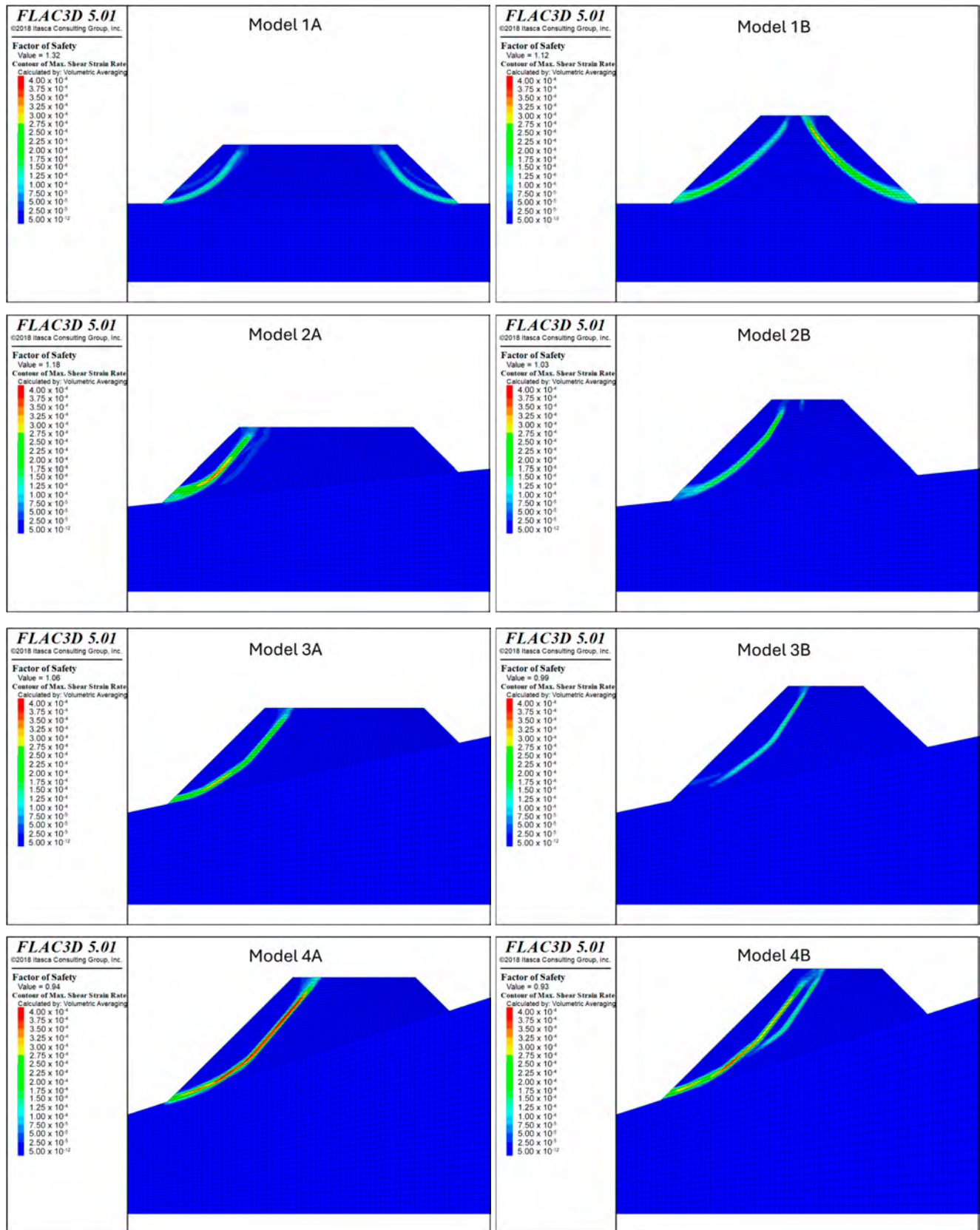


Figure 4. The Maximum Shear Strain Rates of the pre-assessed models.

The results indicate that failure surfaces propagate more extensively as the terrain inclination increases, and that mean dump height influences the extent of failure propagation, as illustrated in Figure 4. Specifically, the shear strain rate intensifies with increasing inclination; the contours, using a consistent strain magnitude range, depict the progression of failure. These figures effectively highlight the potential shear failure surfaces and the expected formation of shear zones within the model.

When comparing Models 1A and 2A, as well as 3B and 4B, it is visually evident that the potential failure surface extends further in Models 2A and 4B. Additionally, a secondary failure surface develops behind the primary failure surface in these models, indicating that a greater volume of material is prone to sliding at higher dump inclinations. Although the Factor of Safety (FoS) is similar for Models 4A and 4B, which share the same terrain inclination, the effect of the height difference is clearly reflected in the extent of failure propagation.

For enhanced stability, waste dumps should preferably be sited on topographically undulating surfaces with as mild inclination as possible [16]. According to the results, for the case study mine, it is preferable to avoid steeper foundations ($>12^\circ$), since models with steeper base inclinations ($>12^\circ$) demonstrate progressive instability, as demonstrated by FoS reductions within each group of models.

Overall, the findings corroborate previous studies that highlight the adverse effect of inclined terrain on dump stability [2,16]. Therefore, terrain inclination significantly influences dump stability, and other design parameters should be carefully assessed and optimized accordingly.

3.2. The Effect of the General Dump Slope

After investigating the effect of terrain slope, the main numerical model was established, and further analysis was conducted. Two-dimensional (2D) models were employed for statistical analysis due to their efficiency in computational time and storage, as well as the necessity to perform numerous iterations.

Previous studies have demonstrated that dump height significantly affects stability, with increased height leading to greater settlement and a higher likelihood of instability in waste dumps [2,9,45]. In this case study, considering the shape factor and total volume of the dump model, optimizing the mean height is the most practical approach, given that the dump will be situated on an inclined surface. Once the mean height is optimized, adjustments to other dump design dimensions can be made accordingly.

According to the topographical map and field study, the inclination of the base terrain of the waste dump was estimated at around 6–18 degrees. Therefore, the slope of the base was selected as the average value of 12 degrees for simplification in the numerical model. As indicated in the pre-assessment, the models with 12 degrees of inclined base (Model 3A and 3B) with mean heights of 33.6 and 45 m are close to the critical state of failure, as FoS are estimated at 1.05 and 0.99. Therefore, it is important to analyze the model heights covering these heights.

As the mining operation progresses, more waste is generated, and the waste dump heightens progressively [2]. To investigate the gradual increase in the dump height, the probability of failure is calculated for each 2.5 m increase (from 30 m to 50 m) with different dump slope angles (for 35, 40, and 45 degrees). The effects of the mean height and general dump slope have been assessed by considering the following randomly selected parameters within a certain range based on a normal distribution (Table 5). Four hundred iterations were executed for each parameter variation (3×9 variations), with a total of 10,800 model runs to account for statistical accuracy. For statistical parameter analysis, the following

input data are randomly generated with the FLAC3D program's built-in random number generator tool.

Table 5. Input parameters and results of FoS values used for the statistical analysis.

	Maximum Value	Minimum Value	Mean Value	Standard Error	Standard Deviation	Distribution
Cohesion {kPa}	62.64	5.60	35.10	0.076	7.89	Normal
Friction angle {degrees}	36.54	23.72	30.57	0.162	1.68	Normal

For further analysis, the cohesion of 35 ± 7.5 (kPa) and friction angle of 30.5 ± 1.75 (degrees) ranges were selected so that the mean value plus and minus two times the standard deviation (with 95% confidence level assuming Gauss/normal distribution) would cover the reference value ranges of 20–50 kPa and 27–34 degrees accordingly. Cohesion value and friction angle were randomly selected by using the Monte Carlo method within these ranges and assigned to the relevant zones of the model for each model run.

After assigning the input parameters to the models, the calculation of the FoS was executed for each iteration. As a result, the maximum and the minimum calculated FoS values are 0.59 and 1.73, respectively, and the mean FoS value was estimated as 1.17 with a standard deviation of 0.16, indicating a relatively wide range of distribution. Figure 5 shows the overall frequency of FoS results with an indication of the probability distribution for the case study model.

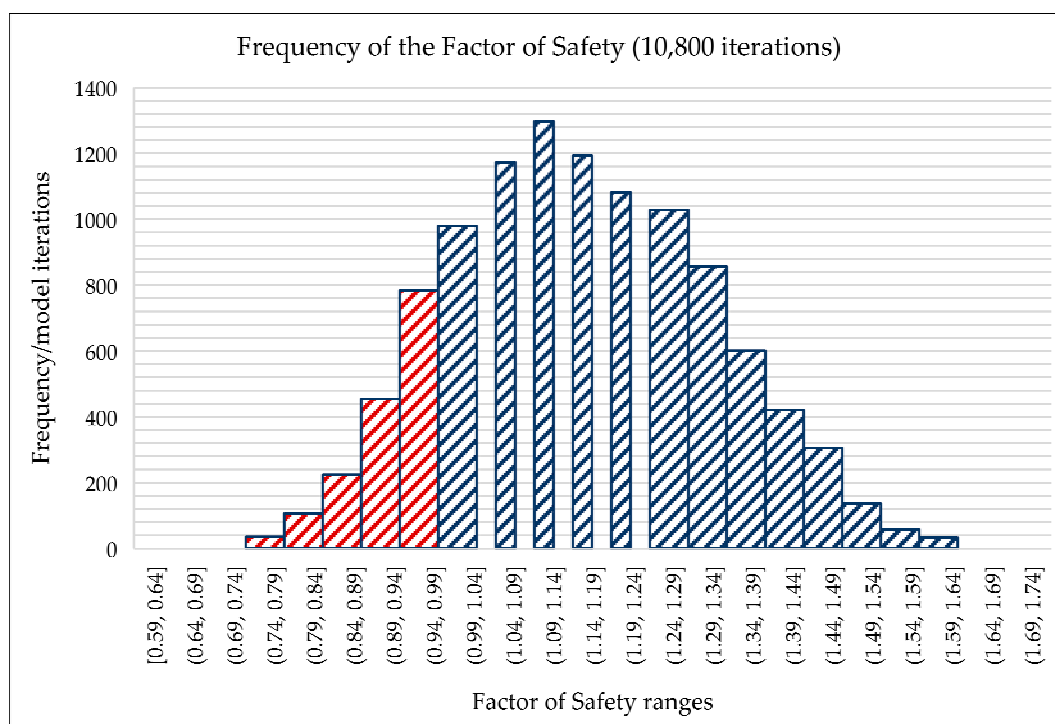


Figure 5. The frequency of FoS values and the resulting distribution of FoS.

The probability of failure (PoF) was calculated by dividing the number of unstable models by the total number of model runs. The overall PoF was estimated at 15.0%, based on 1619 unstable iterations (FoS < 1) out of 10,800 simulations, as highlighted in red in Figure 5 and the stable iterations are indicated with blue.

In statistical analysis, relative frequency refers to the proportion of times a particular value occurs within a dataset relative to the total number of observations [42]. Accordingly,

relative frequency was used to represent the resulting distributions. The overall probability distribution is presented in Figure 6, where a normal distribution curve was fitted to the relative frequency data for density visualization.

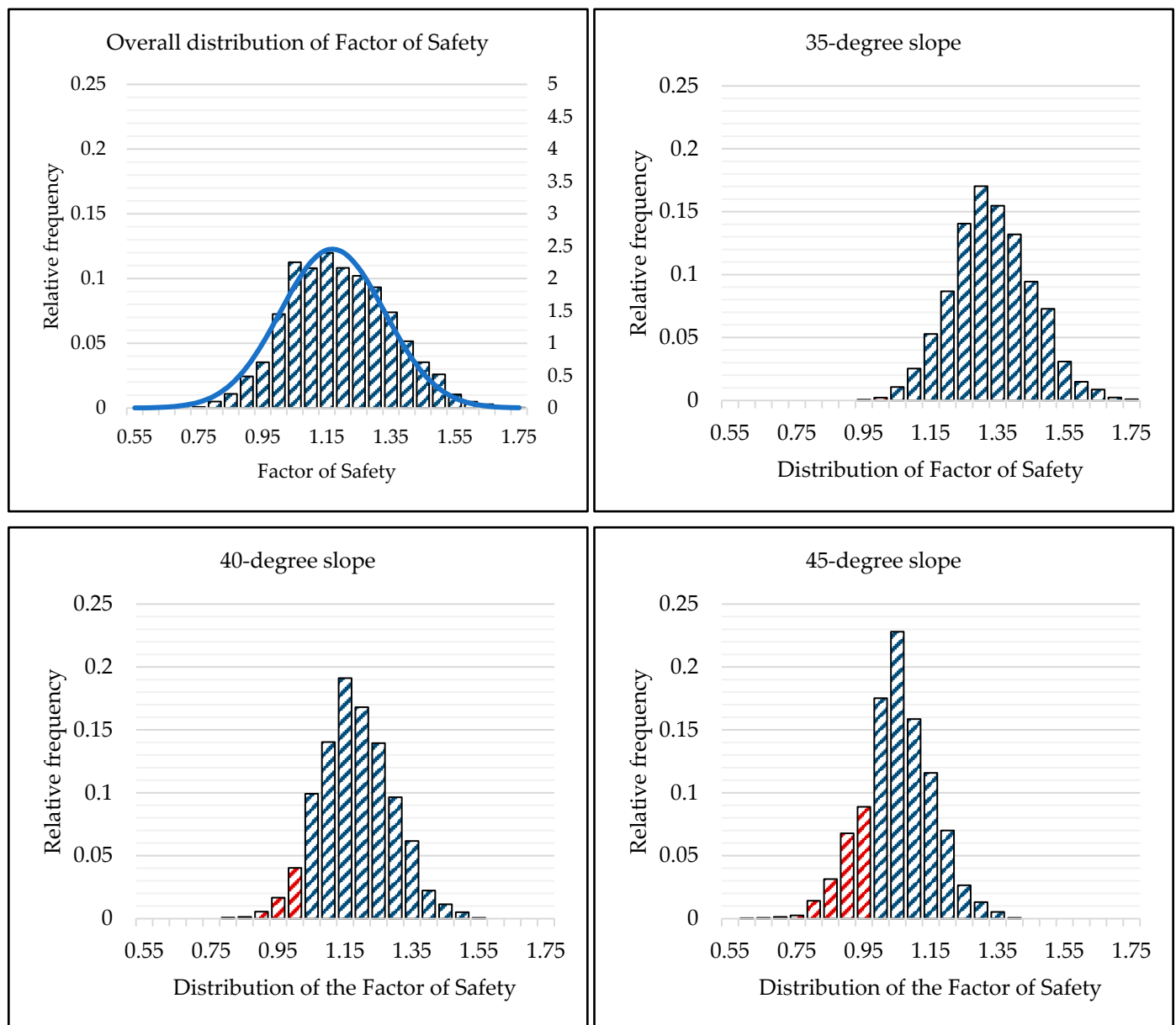


Figure 6. The overall distribution of the Factor of Safety and the resulting probability distribution (the frequency of failed models shown in red) for each slope angle.

As shown in Figure 6, the three histograms represent the distribution of the Factor of Safety (FoS) for slopes with increasing inclinations of 35°, 40°, and 45°. Statistically, these graphs reveal a clear trend: as the slope angle increases, the central tendency (mean and mode) of the FoS distribution shifts leftward, indicating lower FoS values, and the probability of failure ($\text{FoS} < 1.0$) correspondingly increases. The comparison was conducted over the same FoS value range. Overall, the statistical interpretation confirms that slope stability decreases, and the risk of failure rises with increasing slope steepness, as evidenced by the downward shift in FoS distributions and the expanding left-tail region representing FoS values below 1.0.

When comparing different general slope angles of the dump, a significant difference in stability is observed. Models with a 45-degree slope angle exhibit a mean Factor of Safety (FoS) of 1.02 and a probability of failure of 38.2%, indicating that this slope is neither optimal nor acceptable for design purposes. In contrast, models with slope angles of 35° and 40° have considerably lower probabilities of failure, at 0.3% and 6.5%, respectively. The mean FoS values for these slopes are 1.31 for 35° and 1.16 for 40°, reflecting more stable and acceptable design parameters. The probability distributions corresponding to each slope angle are presented in Figure 6.

It can be concluded that maintaining a lower general slope angle is preferable for dump stability. However, reducing the slope angle necessitates adjustments to other geometric parameters and dimensions of the dump, which may have adverse effects due to site limitations, such as increased dump height and reduced bench width. Therefore, the optimal slope angle should balance these competing factors. In this case study, a slope angle of less than 40 degrees is considered a reasonable design criterion.

3.3. The Effect of a Gradual Increase in the Mean Dump Height

To assess the effect of the gradual increase in mean dump height, the probability of failure was calculated for each 2.5 m increment in height, as presented in Table 6. The 2.5 m increment was chosen for practical reasons, reflecting the expected leveling interval of approximately 2 to 2.5 m.

Table 6. The Probability of Failure and the mean Factor of Safety for each height increment of 2.5 m.

Mean Dump Height {m}	35° Slope		40° Slope		45° Slope	
	PoF	The Mean FoS	PoF	The Mean FoS	PoF	The Mean FoS
30	0.3%	1.40	2.0%	1.24	14.3%	1.10
32.5	0.0%	1.38	1.8%	1.23	19.0%	1.08
35	0.0%	1.35	4.3%	1.19	21.3%	1.06
37.5	0.3%	1.33	5.3%	1.17	29.5%	1.04
40	0.3%	1.31	5.3%	1.16	37.5%	1.02
42.5	0.5%	1.30	8.0%	1.14	44.3%	1.01
45	0.5%	1.28	10.5%	1.12	53.5%	0.99
47.5	0.0%	1.26	10.5%	1.11	57.8%	0.97
50	0.8%	1.22	10.8%	1.10	67.0%	0.96

The following graph shows a correlation between gradual increments in dump height and the probability of failure, as well as the mean FoS values (Figure 7). The results indicate that the probability of failure increases with rising dump height, a trend that is more pronounced at steeper slope angles. For the 40° slope, the probability of failure rises from 2.0% at 30 m to 10.8% at 50 m, demonstrating a clear correlation between dump height and failure risk. Furthermore, for the 45° slope, the probability of failure increases sharply from 14.3% at 30 m to 67.0% at 50 m, confirming that dump height has a significant impact on stability at steeper slopes.

On the other hand, at a gentle slope of 35°, the PoF remains consistently low (below 1% across all heights) while the average Factor of Safety gradually decreases from 1.40 at 30 m to 1.22 at 50 m, indicating a gradual decrease in the stability indicator. Overall, the results demonstrate that both higher dump heights and steeper slopes significantly compromise the stability of the waste dump.

Figure 7b shows a very consistent and gradual decrease in the mean FoS with increasing dump height. The results show that an average of 0.02 points drops in FoS for each height increment. When the mean FoS reaches near a unit, the probability of failure

increases drastically, indicating the significant effect of gradual dump height increase on the stability of the dump. It also brings the necessity of the FoS calculation for different levels when establishing the waste dump. In other words, it is important to check the decrease rate of the FoS as the dump progresses.

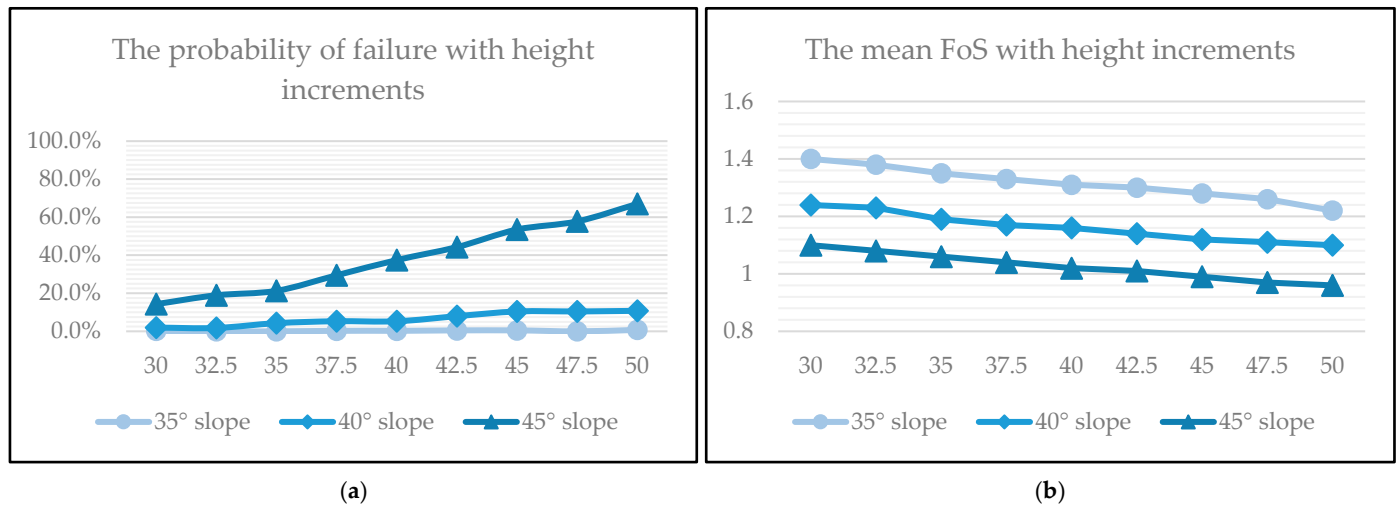


Figure 7. Correlations between (a) gradual increments in dump height and the probability of failure, and (b) gradual increments in dump height and the mean FoS values.

Even though the threshold value for the FoS is 1.0, depending on the design purpose a higher FoS value should be considered [38]. According to the standard set by the Republic of the Philippines Department of Public Works and Highways Office of the Secretary, the Factor of Safety shall be at least 1.2 as mentioned in the DPWH Standard Specification for Item 522a-Protection Systems for Unstable Slope [46]. Therefore, for the case study mine, at the 40-degree slope, the mean dump height should be kept under 35 m; moreover, for the 35-degree slope, it would be sufficient to keep it under 50 m.

3.4. The Effect of Bench Parameters

The FEM analysis was conducted using a base model (Figure 8) derived from a topographic survey of the designated area, featuring a 40° general dump slope with a mean height of 35 m ($H_1 = 57.809$ m, $H_2 = 12.192$ m). To ensure consistency, the same range of maximum shear strain contours was applied across models with one to five benches.

The results revealed that slope inclination influences failure propagation, leading to a secondary failure surface behind the primary slip zone. This phenomenon, observed in both FDM and FEM analyses, suggests that stress redistribution weakens the material behind the initial failure plane, increasing the risk of progressive slope collapse. The presence of a secondary failure surface implies that traditional single-surface stability analyses may underestimate risk, particularly in steep, multi-bench slopes. This aligns with previous studies on strain-softening behavior in waste dumps, where localized shear zones trigger additional failures [2,11].

It should be noted that the critical Shear Reduction Factor (SRF) is equivalent to the Factor of Safety (FoS) [47], allowing SRF to be interpreted as FoS. To ensure comparability, the same geomechanical properties used in the pre-assessment were applied in the FEM analysis. The results show that the FoS obtained from the FEM model closely matches the mean FoS from the FDM model, indicating that both methods produce consistent outcomes.

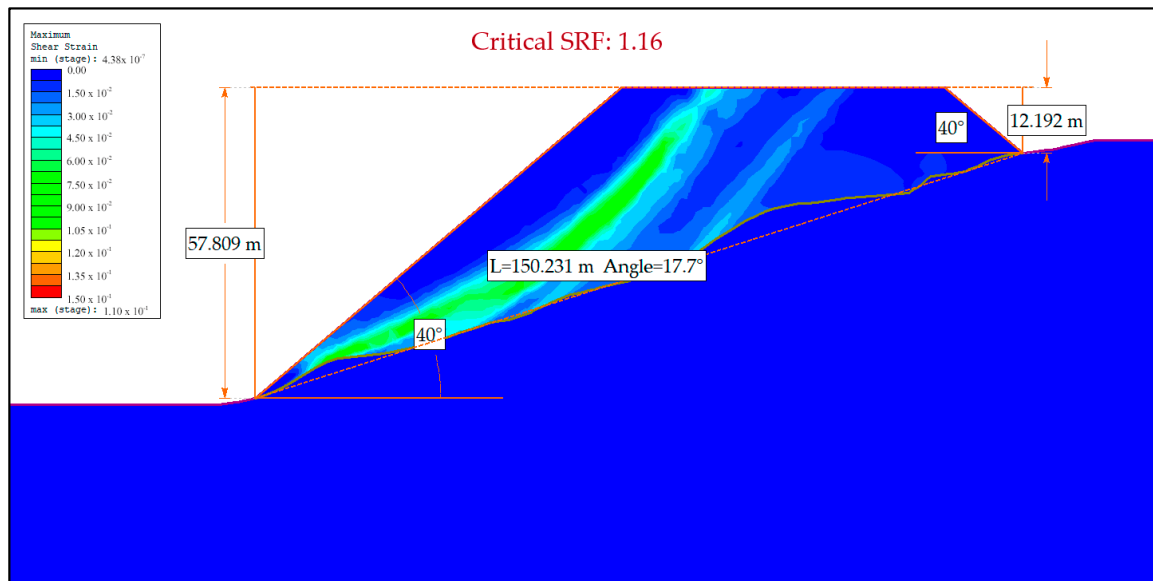


Figure 8. The base model of the FEM analysis with the maximum shear strain contours.

Several bench parameters were considered in the geometric configuration of the FEM analysis, as illustrated in Figure 9. Models with two to five benches were analyzed, with the bench berm set at 10 m. The overall dump slope was maintained at 40° , while the bench slope angles were adjusted according to the respective bench configurations. For instance, a three-benched dump consists of three benches, each with a slope angle of 50° and berm widths of 10 m. Bench heights were equally divided based on the total height H_1 , with consistent slope angles assigned to each bench. Consequently, the bench slope angles were determined as 44° for a two-benched dump, 50° for a three-benched dump, 57° for a four-benched dump, and 64° for a five-benched dump.

Furthermore, berm widths of 5 and 7.5 m were investigated for comparison, as shown in Figure 10. The single-bench dump ($\text{FoS} = 1.16$) exhibited the lowest stability, whereas multi-bench designs improved the FoS to a range of 1.19–1.20. Although multiple-benched dump designs showed similar critical SRF results, the four-benched dump recorded a slightly higher critical SRF (1.20), prompting further analysis of berm width effects on the stability improvement.

Narrow or smaller safety berms are generally difficult to maintain, and wider berms are usually preferred [15]. However, when the overall slope angle is fixed, increasing the berm width results in steeper bench slopes, which adversely affect dump stability. Therefore, an optimum berm width must be determined. The results in Figure 10 suggest that a berm width of 7.5 m provides the optimal balance for waste dump stability in this case study.

There are some studies regarding the sole effects of the bench berm and bench slope [15,33]. Distinctively, in this study, the general slope angle was set constant for FEM analysis since the effect of general slope angle has already been discussed in a previous section. Subsequently, the bench berm and bench slope become dependent on each other, indicating a combined effect. When comparing the four-benched dump models, 7.5 m of berm has a slightly higher FoS value than 5 and 10 m of berm. The results of the bench configuration show that the geometrical parameters have noteworthy effects on the stability of the dump, as FoS can be improved from 1.16 to 1.22.

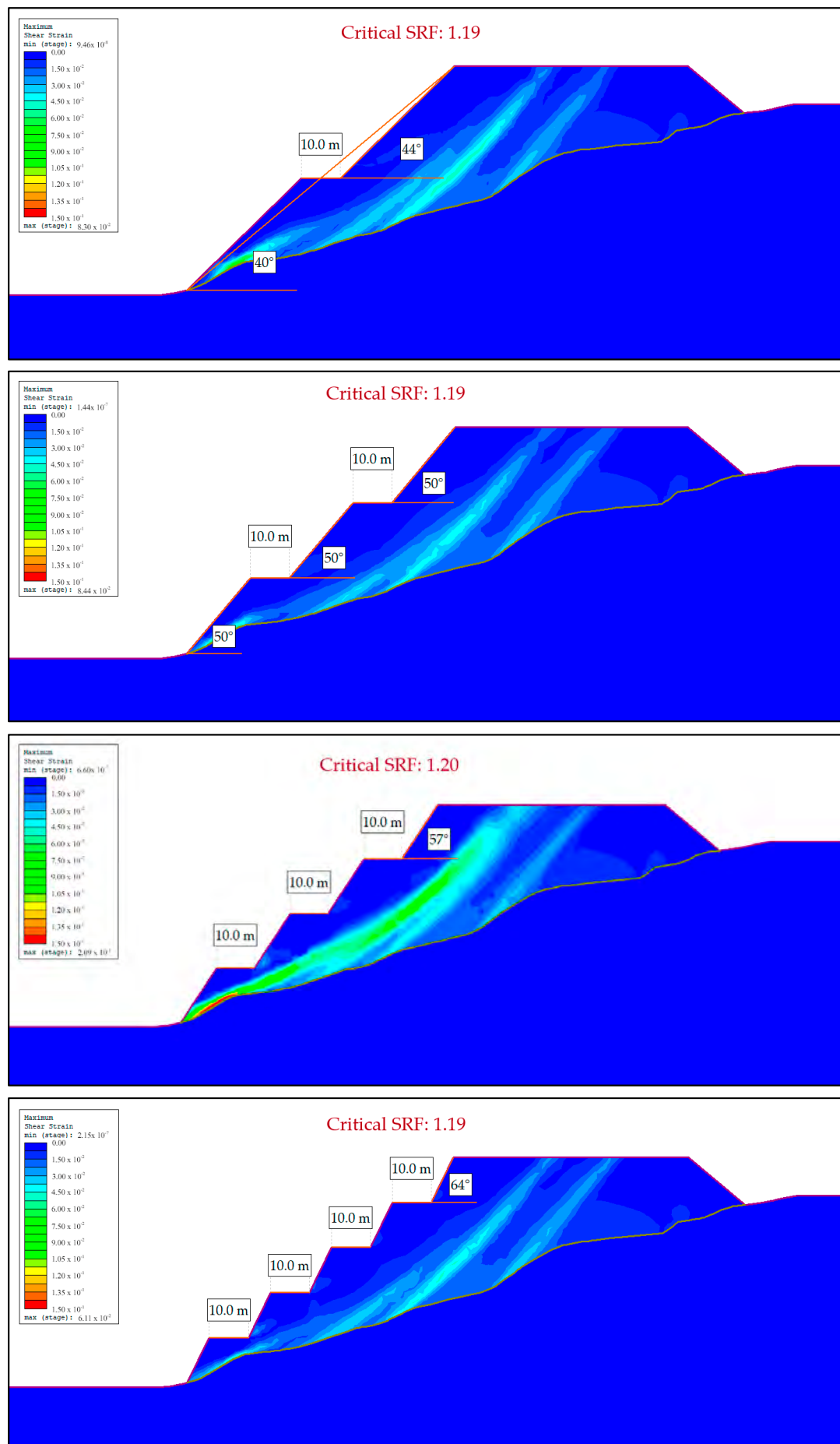


Figure 9. The bench configuration with the results of the maximum shear strain contours.

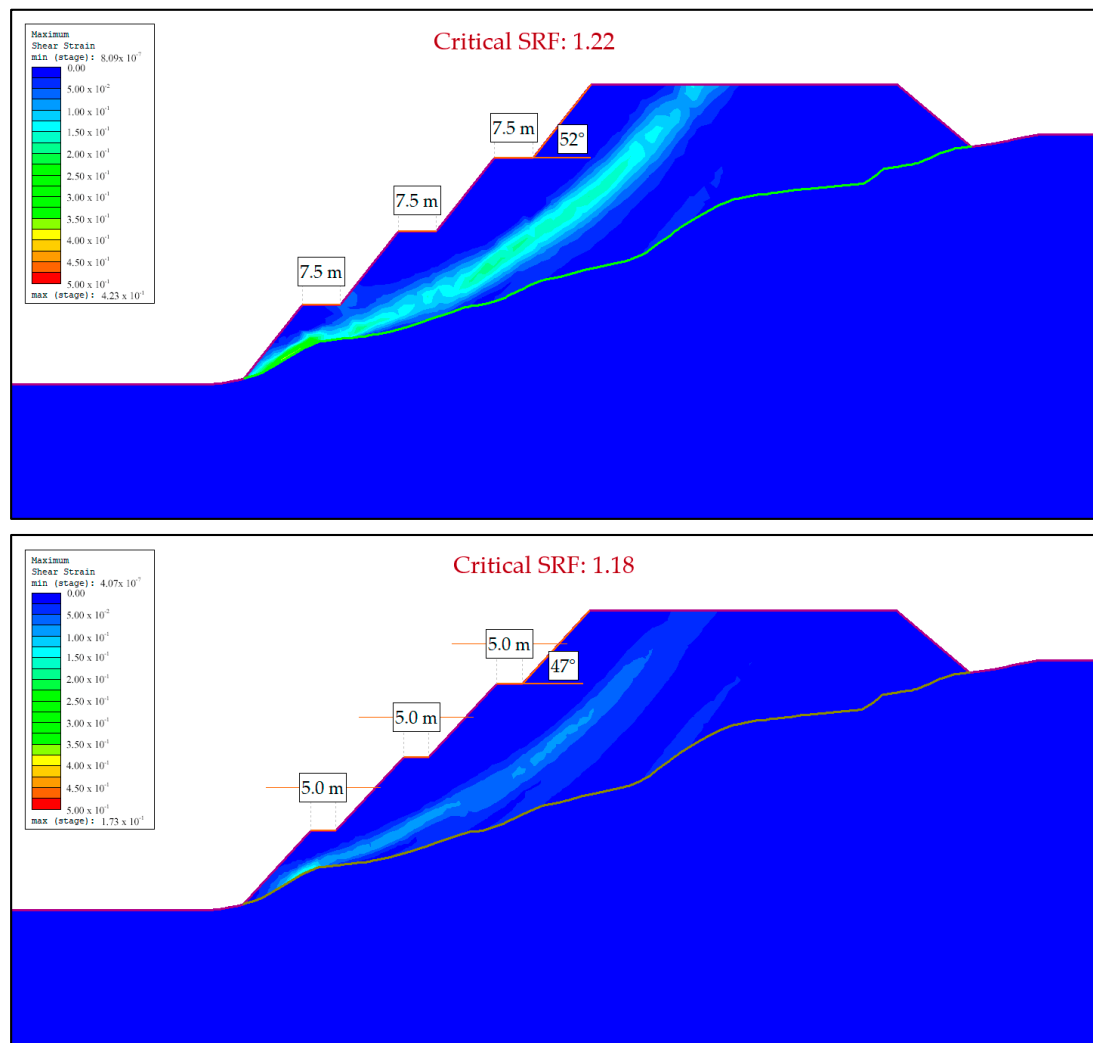


Figure 10. The safety berm configuration of the four-benched dump model.

3.5. The Effect of Rainfall

Since the mine is located in a tropical region, it experiences high precipitation, with the highest recorded rainfall reaching up to 1174.7 mm during the rainy months of June to August [44]. Therefore, it is important to consider the effects of intense rainfall that may occur in the study area and in regions with similar climates.

Rainfall can affect the stability of the dump site, as precipitation causes a reduction in the Factor of Safety (FoS) of the waste dump [29]. To evaluate the effect of rainfall, a basic drainage analysis was conducted. Considering the rainy season, the average daily precipitation was calculated as 13.05 mm/day (1174.7 mm over 90 days). According to the field study, the permeability of the soil samples was estimated to be 1.0×10^{-5} m/s.

Based on these input parameters, the influence of rainfall infiltration on waste dump stability was preliminarily assessed by modeling four infiltration scenarios: 0 days (fully drained), 14 days, 28 days, and 42 days of rainfall infiltration. The model used for this analysis (Figure 11, shown with rainfall filtration and drainage boundaries) was the one optimized in the previous section, and the following FoS results were recorded (Table 7).

Results show that as rainfall infiltration increased, a gradual reduction in FoS was observed, decreasing to 1.16 after 14 days and further to 1.13 after 28 days of continuous rainfall. This trend highlights the progressive weakening of slope stability; therefore, high precipitation and high rainfall duration adversely affect the stability of the waste dump. It is also important to mention that a slight recovery is seen at stage 3, possibly due to drainage

and material compaction. Overall, an effective drainage system should be considered when designing the waste dump.

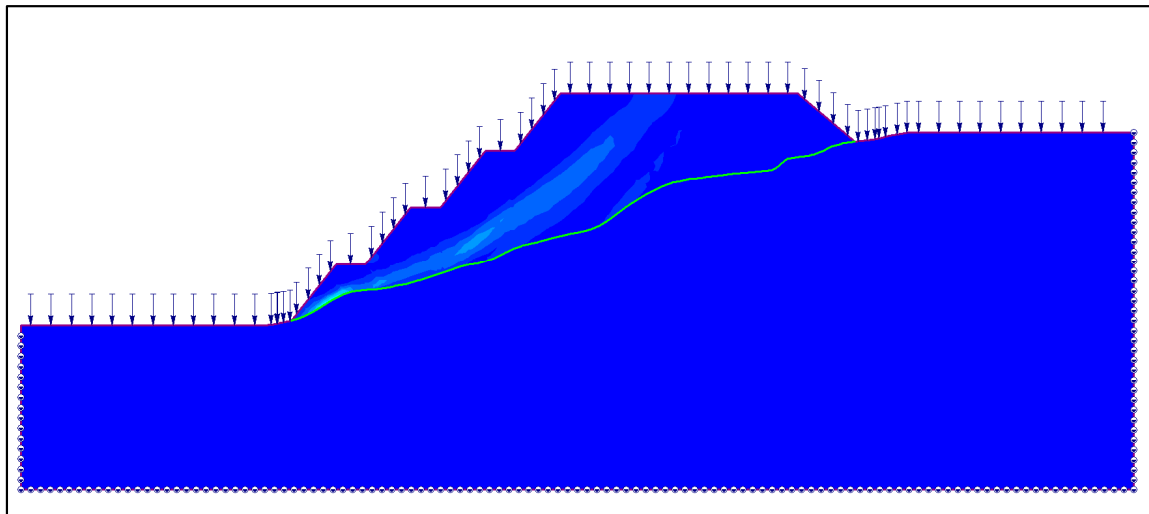


Figure 11. Rainfall analysis model showing the Shear Strain Contours with rain infiltration (shown with arrows) and zero pressure boundaries (illustrated with half-filled circle).

Table 7. The FoS results considering different durations of rainfall.

Stages	Time (Days)	FoS
Initial condition	0	1.22
Stage 1	14	1.16
Stage 2	28	1.13
Stage 3	42	1.17

4. Discussion and Further Research

Overall, the study underlines that the effect of designing a waste dump on an inclined surface is significant in terms of the dump stability; more importantly, more materials are expected to fail when the inclination becomes higher. So, the waste dump should be situated on as mildly inclined a surface as possible. However, in the case study, this condition was unavoidable due to the land-use restrictions. In this case, the dump site should be carefully designed by considering other parameters such as dump height and safety berm.

The statistical analysis results suggest that the general slope angle should be kept at 40 degrees or less, since a 45-degree slope would have a high probability of failure, and the dump height should be kept under a certain level, depending on the slope angle, for design optimization. Moreover, the other dimensions should be modified accordingly. In terms of geometrical setup, four benches with 7.5 m of berm would be preferable for the waste dump design of the case study. A single-benched dump design should be avoided if the operational condition allows it, since it has a relatively lower FoS value, and it would be challenging to maintain as a single-benched high dump from a practical perspective.

As mentioned in the result discussion, it should be noted that the assessment of the gradual increase in the dump height becomes crucial because, after a certain point, the probability of the dump failure increases considerably. Moreover, it is very significant to find the critical point with site-specific input variables.

Even though many studies have been conducted regarding the dump height effect on dump stability [1,2,9], there is still a gap in statistical analysis of the coupled effect of

the inclined surface and gradual increase in the dump, and it is very crucial for assessing the stability of the dump site. Therefore, this study can be helpful when designing the waste dump at the initial stages of the mine development, as results show that insightful information on optimum design parameters can be obtained with very limited initial data. However, a more deterministic analysis can be conducted at the latter stages when more data regarding the geotechnical parameters of the waste dump materials are acquired, and a more precise design can be considered by utilizing the deterministic analysis with certain geotechnical parameters.

A key prerequisite for the viability of the dump is ensuring proper leveling and consolidation, as these factors directly impact the structural stability of the dump. Without adequate compaction and uniform ground preparation, dump instability and potential failures may arise, compromising safety and operational efficiency. This study assumes that appropriate leveling and consolidation measures are in place; therefore, the results may be slightly optimistic or may overestimate the actual field conditions. Therefore, continuous monitoring, coupled with staged implementation of consolidation measures, is essential to mitigate potential risks and optimize the dump's long-term performance. Additionally, there are some countermeasures besides design optimization, such as boulder toe creation and effective drainage systems. These measures are recommended to be utilized in the case study mine.

During the design planning phase of a waste mine dump, it is crucial to establish a robust framework that accounts for both immediate stability and long-term environmental and operational concerns. Moreover, due to the pre-feasibility nature of this study, similar approaches should be applied with caution. While eligibility at an early stage allows for preliminary assessments and conceptualization, further investigations must be conducted as the dump is constructed to verify initial assumptions and adapt to unforeseen geotechnical or environmental conditions.

Since the study was based on literature values, it may not fully cover the actual site-specific geotechnical parameters. For further research, the field study will be conducted to determine the geotechnical properties of waste dump materials. As the waste dump starts to be established, samples will be taken and more detailed geomechanical parameters should be tested, and the effect of corrosion can be assessed by lowering the geomechanical strength properties since the study area has relatively high precipitation.

Further research will be conducted considering the heterogeneity of the waste dump material, since this study covers the homogeneous material with a wide range of properties. A field study will be conducted, samples from dump material will be collected, and grain size distribution will be analyzed. A similar approach of Monte Carlo can be utilized to account for the heterogeneity in terms of the random arrangement of coarse fragments for the results of the waste material size distribution. Moreover, the effect of seismic activities caused by earthquakes and extensive blasts will be investigated since it is one of the significant aspects that influence the dump stability. Therefore, dynamic analysis will be conducted considering the seismic effects on the dump slope stability.

Author Contributions: Conceptualization, B.B.-E.; methodology, B.B.-E.; validation, T.S., A.H. and H.S.; formal analysis, K.K.; data curation, K.K.; writing—original draft preparation, B.B.-E.; writing—review and editing, B.B.-E. and A.H.; visualization, B.B.-E.; supervision, T.S. and H.S.; project administration, H.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: All data and data sets are available per request from the corresponding author.

Acknowledgments: The author would like to express gratitude to Ore Asia and Mining Development Corporation and Luna Stephanie Ivy Jadormio for providing fundamental information about the study mine and conducting the site investigation.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

FDM	Finite difference method
FEM	Finite element method
FLAC3D	Fast Lagrangian Analysis of Continua in 3 Dimensions
FoS	Factor of Safety
SSR	Shear strength reduction
PoF	Probability of Failure
SRF	Shear Reduction Factor

References

1. Gupta, G.; Sharma, S.K.; Singh, G.S.P.; Kishore, N. Numerical Modelling Based Stability Analysis of Waste Dump Slope Structures in Open Pit Mines—A Review. *J. Inst. Eng.* **2021**, *102*, 589–601. [\[CrossRef\]](#)
2. Chai, S. Maximum height estimation for mine waste dumps. *J. S. Afr. Inst. Min. Metall.* **2022**, *122*, 579–586. [\[CrossRef\]](#)
3. Azam, S.; Wilson, G.W.; Fredlund, D.G.; Van Zyl, D. Geotechnical characterization of mine waste rock. In Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering: The Academia and Practice of Geotechnical Engineering, Alexandria, Egypt, 5–9 October 2009; pp. 3421–3425.
4. Martin, V.; Aubertin, M.; Bussiere, B.; Chapuis, R.P. Evaluation of unsaturated flow in mine waste rock. In Proceedings of the 57th Canadian Geotechnical Conference and the 5th joint CGS-IAH Conference, Quebec City, QC, Canada, 24–27 October 2004.
5. Bazaluk, O.; Petlovanyi, M.; Sai, K.; Chebanov, M.; Lozynskiy, V. Comprehensive assessment of the earth's surface state disturbed by mining and ways to improve the situation: Case study of Kryvyi Rih Iron-ore Basin, Ukraine. *Front. Environ. Sci.* **2024**, *12*, 1480344. [\[CrossRef\]](#)
6. Petlovanyi, M.; Sai, K.; Khalymendyk, O.; Borysovska, O.; Sherstiuk, Y. Analytical research of the parameters and characteristics of new “quarry cavities–backfill material” systems: Case study of Ukraine. *Min. Miner. Depos.* **2023**, *17*, 126–139. [\[CrossRef\]](#)
7. Puja, R.; Anupkumar, G.E.; Gaurav, B. Stability analysis of mine overburden dump and improvement by soil nailing. *Mater. Today Proc.* **2022**, *65 Pt 2*, 735–740.
8. Valenzuela, L.; Bard, E.; Campana, J.; Anabalon, M.E. High Waste Rock Dumps—Challenges and Development. In *Rock Dumps 2008: Proceedings of the First International Seminar on the Management of Rock Dumps, Stockpiles and Heap Leach Pads, Perth, Australia, 5–6 March 2008*; Australian Centre for Geomechanics: Perth, Australia, 2008; pp. 65–78.
9. Tian, Y.; Zhou, W.; Jiskani, I.M.; Cai, Q.; Li, Z.; Lu, X. Stability analysis of varying height waste dump in open-pit mine under particle size gradation and reconstruction effect of waste materials. *Int. J. Min. Reclam. Environ.* **2022**, *36*, 587–604. [\[CrossRef\]](#)
10. Behera, P.K.; Sarkar, K.; Singh, A.K.; Verma, A.K.; Singh, T.N. Dump slope stability analysis—A case study. *J. Geol. Soc. India* **2016**, *88*, 725–735. [\[CrossRef\]](#)
11. Yang, Z.; Liu, X.; Qian, W.; Ding, X.; Ao, Z.; Zhang, Z.; Jiskani, I.M.; Tian, Y.; Xing, B.; Wahab, A. Investigation of Steep Waste Dump Slope Stability of Iron Ore Mine—A Case Study. *Appl. Sci.* **2024**, *14*, 3430. [\[CrossRef\]](#)
12. Kainthola, A.; Verma, D.; Gupte, S.S.; Singh, T.N. A coal mine dump stability analysis—A case study. *Geomaterials* **2011**, *1*, 1–13. [\[CrossRef\]](#)
13. Nguyen, P.M.V.; Wrana, A.; Rajwa, S.; Róžański, Z.; Frączek, R. Slope Stability Numerical Analysis and Landslide Prevention of Coal Mine Waste Dump under the Impact of Rainfall—A Case Study of Janina Mine, Poland. *Energies* **2022**, *15*, 8311. [\[CrossRef\]](#)
14. Enkhbold, B.; Ikeda, H.; Toriya, H.; Adachi, T. Internal waste dump stability analysis: Insights from numerical simulations. *Discov. Geosci.* **2025**, *3*, 23. [\[CrossRef\]](#)
15. Amarsaikhan, T.; Shimada, H.; Wahyudi, S.; Sasaoka, T.; Hamanaka, A. Optimisation of dump bench configuration to improve waste dump capacity of Narynsukhait open pit coal mine. *Int. J. Geosci.* **2018**, *9*, 379–396. [\[CrossRef\]](#)
16. Ravikant; Dash, A.K.; Dewangan, P. Study of the Effect of Floor Inclination on Stability of Waste Dumps—A Case Study of an Iron Ore Mine. In *Sustainable and Innovative Mining Practices (ICSIMP 2023), Proceedings of the International Conference on Sustainable and Innovative Mining Practices, Rourkela, India, 16–18 November 2023*; Springer: Cham, Switzerland, 2025; pp. 280–290.

17. Yang, Z.; Ding, X.; Liu, X.; Wahab, A.; Ao, Z.; Tian, Y.; Bang, V.S.; Long, Z.; Li, G.; Ma, P. Slope Deformation Mechanisms and Stability Assessment under Varied Conditions in an Iron Mine Waste Dump. *Water* **2024**, *16*, 846. [CrossRef]
18. Bao, Y.; Sun, X.; Chen, J.; Zhang, W.; Han, X.; Zhan, J. Stability assessment and dynamic analysis of a large iron mine waste dump in Panzhihua, Sichuan, China. *Environ. Earth Sci.* **2019**, *78*, 48. [CrossRef]
19. Li, H.; Zhang, Z.; Yang, W. Stability analysis of slope based on limit equilibrium method and strength reduction method. *Ann. Chim. Sci. Mater.* **2021**, *45*, 379–384. [CrossRef]
20. Matsui, T.; San, K.C. Finite element slope stability analysis by shear strength reduction technique. *Soils Found.* **1992**, *32*, 59–70. [CrossRef]
21. Fredlund, D.G.; Scoular, R.E.G. Using Limit Equilibrium Concepts in Finite Element Slope Stability Analysis. In Proceedings of the International Symposium on Slope Stability Engineering, IS-Shikoku'99, Matsuyama, Shikoku, Japan, 8–11 November 1999; pp. 31–47.
22. Griffiths, D.; Lane, P. Slope stability analysis by finite elements. *Geotechnique* **2001**, *51*, 653–654. [CrossRef]
23. Soren, K.; Budi, G.; Sen, P. Stability Analysis of Open Pit Slope by Finite Difference Method. *Int. J. Res. Eng. Technol.* **2014**, *3*, 326–334.
24. Cala, M.; Flisiak, J. Slope stability analysis with FLAC and limit equilibrium methods. In *FLAC and Numerical Modelling in Geomechanics*; CRC Press: Boca Raton, FL, USA, 2001; pp. 111–114.
25. Wang, J.; Tan, W.; Feng, S.; Zhou, R. Reliability analysis of open pit coal mine slope. *Int. J. Rock Mech. Min. Sci.* **2000**, *37*, 715–721. [CrossRef]
26. Menglong, D.; Yukun, L.; Yujiao, L. Analysis of development process of landslide accumulation body based on discrete element method. *IOP Conf. Ser. Earth Environ. Sci.* **2018**, *170*, 022028.
27. Chang, C.S. Discrete element method for slope stability analysis. *J. Geotech. Eng.* **1992**, *118*, 1889–1905. [CrossRef]
28. Yang, X.; Yang, G.; Yu, T. Comparison of strength reduction method for slope stability analysis based on ABAQUS FEM and FLAC3D FDM. *Appl. Mech. Mater.* **2012**, *170–173*, 918–922. [CrossRef]
29. Wang, Y.; Han, M.; Lin, X.; Li, D.; Yu, H.; Zhu, L. Influence of Rainfall Conditions on Stability of Slope Reinforced by Polymer Anti-slide Pile. *Front. Earth Sci.* **2021**, *9*, 774926. [CrossRef]
30. Raymond, K.E.; Seigneur, N.; Su, D.; Mayer, K.U. Investigating the Influence of Structure and Heterogeneity in Waste Rock Piles on Mass Loading Rates—A Reactive Transport Modeling Study. *Front. Water* **2021**, *3*, 618418. [CrossRef]
31. Groundwater Resource Assessment Team, Lands Geological Survey Division; Geological Map of Bulacan Province, Philippines. Department of Environment and Natural Resources, Mines and Geosciences Bureau, City of San Fernando, Pampanga, Philippines, 2022.
32. Philippine Atmospheric, Geophysical and Astronomical Services Administration. Available online: <https://www.pagasa.dost.gov.ph/climate/climate-monthly-monitoring-products> (accessed on 16 June 2025).
33. Ozturk, C.A.; Ercelebi, S.; Onsel, I.E.; Ozkan, M. Open pit mine waste dump area design based on stability principles. In Proceedings of the 24th International Mining Congress and Exhibition of Turkey, IMCET'15, Antalya, Turkey, 14–17 April 2015; pp. 570–578.
34. ASTM-D2487-17e1 2017; Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System). ASTM International: West Conshohocken, PA, USA, 2017.
35. Kulhawy, F.H.; Mayne, P.W. *Manual on Estimating Soil Properties for Foundation Design*; Cornell University: Ithaca, NY, USA, 1990.
36. Naval Facilities Engineering Command. *Foundations and Earth Structures*; Department of the Navy: Alexandria, VA, USA, 1986.
37. Obrzud, R.F.; Truty, A. *The Hardening Soil Model—A Practical Guidebook*; Zace Services SA: Lausanne, Switzerland, 2012.
38. Hoek, E. *Practical Rock Engineering*; RocScience: Toronto, ON, Canada, 2007; pp. 19–23.
39. Itasca Consulting Group, Inc. *FLAC3D—Fast Lagrangian Analysis of Continua in Three-Dimensions*, ver. 5.0.; Itasca: Minneapolis, MN, USA, 2012.
40. Gómez, P.; Díaz, M.; Lorig, L. Stability analysis of waste dumps at Chuquicamata Mine, Chile. *Felsbau Rock Mech. J.* **2002**, *20*, 61–68.
41. Sdvyzhkova, O.; Moldabayev, S.; Bascetin, A.; Babets, D.; Kuldeyev, E.; Sultanbekova, Z.; Amankulov, M.; Issakov, B. Probabilistic assessment of slope stability at ore mining with steep layers in deep open pits. *Min. Miner. Depos.* **2022**, *16*, 11–18. [CrossRef]
42. Rice, J.A. *Mathematical Statistics and Data Analysis*, 3rd ed.; Cengage Learning Brooks/Cole: Belmont, CA, USA, 2007; pp. 54–57.
43. Zhou, A.; Huang, X.; Li, N.; Jiang, P.; Wang, W. A Monte Carlo approach to estimate the stability of soil–rock slopes considering the non-uniformity of materials. *Symmetry* **2020**, *12*, 590. [CrossRef]
44. Lin, B. Stability Analysis of Dump Slope: Taking Zijinshan Open-Pit Mine as an Example. *Mob. Inf. Syst.* **2022**, *2022*, 9009528. [CrossRef]
45. Verma, A.K.; Deb, B.; Mukhopadhyay, S.K. Stability analysis of a mine waste dump over an existing dump. *J. Mines Met. Fuels* **2017**, *65*, 41–48.

46. *DPWH Standard Specification for Item 522A; Protection Systems for Unstable Slope, Item 522A*. Republic of the Philippines Department of Public Works and Highways Office of the Secretary: Manila, Philippines, 2014.
47. Rocscience. Available online: <https://www.rocscience.com/help/rs2/documentation/rs2-model/ssr-stability-analysis/shear-strength-reduction-overview> (accessed on 21 April 2025).

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.