Stochastic Homogenization of Generalized Parabolic and Elliptic Equations: The Effects of Lower-Order Terms

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論 文 名　：Stochastic Homogenization of Generalized Parabolic and Elliptic Equations: The Effects of Lower-Order Terms
（一般化された放物型方程式および楕円型方程式の確率的均質化：低次項の影響に関する研究）

区　　分　：甲

論　文　内　容　の　要　旨

Homogenization theory plays a crucial role in understanding the macroscopic behavior of heterogeneous media with microscopic structures. In this thesis, we investigate the homogenization of both parabolic and elliptic partial differential equations with random coefficients, with a particular focus on the influence of lower-order terms. The work is divided into two main parts: the first part presents a qualitative analysis for parabolic equations, extending homogenization theory by deriving key estimates such as the Caccioppoli inequality and Meyers estimate, which are fundamental tools for leading to weak convergence of solutions in H^1. The second part provides a quantitative analysis for elliptic equations, focusing on convergence rates under the spectral gap assumption, which provides a quantitative framework for analyzing random fluctuations. We show that lower-order terms significantly affect the rates in $d \geq 3$, reducing the convergence from $O(\epsilon)$ to $O(\epsilon^{d/(d+2)})$ in unbounded domains, while in bounded domains the rate remains $O(\epsilon^{1/2})$, dominated by boundary effects. This thesis not only establishes the fundamental theory for homogenization with lower-order terms but also provides precise quantification of their effects on convergence rates, contributing to both theoretical understanding and practical applications of homogenization theory.