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PARAMETER ESTIMATION FOR GENERALIZED MIXED FRACTIONAL STOCHASTIC HEAT EQUATION

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Abstract

We study the properties of a stochastic heat equation with a generalized mixed fractional Brownian noise. We obtain the covariance structure, stationarity and obtain bounds for the asymptotic behavior of the solution. We suggest estimators for the unknown parameters based on discrete time observations and study their asymptotic properties.

Key Words and Phrases: Stochastic partial differential equation; Generalized mixed fractional Brownian motion; Parameter estimation.

1. Introduction

For modeling fluctuations in movements of stock prices, Brownian motion has been used traditionally as the driving force for modeling log returns. It was suggested by some that the driving force may be chosen as a fractional Brownian motion to model long range dependence. Bjork and Hult (2005) and Kuznetsov (1999) observed that the use of fractional Brownian motion for modeling fluctuations in movement of stock prices is not justifiable as it allows arbitrage opportunities. To avoid this problem, Cheridito (2000, 2003) suggested the use of a mixed fractional Brownian motion (mfBm) as a suitable model for the driving force. The mixed fractional Brownian motion is a Gaussian process which is a mixture of a Brownian motion and an independent fractional Brownian motion. Option pricing for processes driven by mfBm with superimposed jumps is investigated in Prakasa Rao (2015). Pricing geometric Asian power options under mixed fractional Brownian motion environment is studied in Prakasa Rao (2016). Kallianpur and Xiong (1995) discussed the properties of solutions of stochastic partial differential equations (SPDEs) driven by an infinite-dimensional Brownian motion. They indicate that such SPDEs can be used for modeling the study of neuronal behavior in neurophysiology and for building stochastic models for turbulence. Parametric estimation for SPDEs driven by fBm is discussed in Chapter 8 of Prakasa Rao (2010). Generalized mixed fractional Brownian motion is a finite mixture of independent fractional Brownian motions. It is known that this process is a centered Gaussian process which is self-similar in a suitable sense and not Markov. Properties of these processes are studied in Chapter 3 of Mishura and Zili (2018). Long range dependence property of time-changed Generalized mixed fractional Brownian motion is investigated in Prakasa Rao

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(2025). Ergodic properties of the solution of a fractional stochastic equation driven by a mixed fractional Brownian motion are discussed in Avetisian and Ralchenko (2020). Parameter estimation for a mixed fractional stochastic heat equation has been investigated in Avetisian and Ralchenko (2023). Parametric estimation for SPDEs driven by infinite dimensional mixed fractional Brownian motion is investigated in Prakasa Rao (2022). Parametric estimation for stochastic parabolic equations driven by an infinite dimensional mfBm are studied in Prakasa Rao (2023). Our aim in this paper is to study problems of parameter estimation in a stochastic heat equation driven by a Generalized mixed fractional Brownian motion based on discrete time observations. Our techniques are analogous to those in Avetisian and Ralchenko (2023).

We study parameter estimation for a stochastic heat equation of the form

$$(\frac{\partial u}{\partial t}-\frac{1}{2}\frac{\partial^2 u}{\partial x^2})(t,x)=\sigma_1\dot{W}_x^{H_1}+\sigma_2\dot{W}_x^{H_2}, t>0, x\in\mathbb{R} \tag{1}$$

under the condition

$$u(0,x)=0, x\in\mathbb{R}. \tag{2}$$

We denote the space of real numbers by $\mathbb{R}$ for typographical convenience. The process on the right hand side of the equation (1) is a generalized mixed fractional noise. It consists of two independent fractional Brownian motions W^{H_1} and W^{H_2} with Hurst indices $0<H_1,H_2<1$ and the parameters σ_1 and σ_2 are positive constants. We investigate the problem of estimating the parameters H_1,H_2,σ_1 and σ_2 based on discrete observations of the solution $u(t,x)$ of the equation (1).

2. Preliminaries

Suppose that $W^{H_i}=\{W_x^{H_i},x\in\mathbb{R}\}$, $i=1,2$ are two independent two-sided fractional Brownian motions with Hurst indices $H_i, i=1,2$ respectively. Let G be the Green's function of the heat equation given by

$$\begin{aligned} G(t,x) &= \frac{1}{\sqrt{2\pi t}}\exp\{-\frac{x^2}{t}\}, \text{ if } t>0,\\ &= \delta_0(x) \text{ if } t=0. \end{aligned}$$

The random field $\{u(t,x),t\geq 0,x\in\mathbb{R}\}$ defined by

$$u(t,x)=\sigma_1\int_0^t\int_{\mathbb{R}}G(t-s,x-y)dW_y^{H_1}ds+\sigma_2\int_0^t\int_{\mathbb{R}}G(t-s,x-y)dW_y^{H_2}ds \tag{3}$$

is called a solution of the stochastic partial differential equation (SPDE) defined by (1) and (2).

As pointed out by Avetisian and Ralchenko (2020), the stochastic integrals in (3) exist as path wise Riemann-Stieltjes integrals since the Green's function is Lipshitz continuous and the sample paths of the fractional Brownian motion W^{H_i} are Holder continuous up to order $H_i, i=1,2$. We will now derive some properties of the solution $u(t,x)$.

THEOREM 2.1. *Let $u(t,x), t \in [0,T], x \in \mathbb{R}\}$ be a solution to the equations (1) and (2) as defined by (3). Then the following properties hold.*
(i)For $0 \leq t, s \leq T$ and for $x, z \in \mathbb{R}$,

$$\begin{aligned} Cov(u(t,z), u(s,x+z)) &= Cov(u(t,0), u(s,x)) \\ &= \frac{1}{\sqrt{2\pi}} \int_0^t \int_0^s (q+r)^{-\frac{3}{2}} \int_{\mathbb{R}} (\sigma_1^2 H_1 |y|^{2H_1-1} + \sigma_2^2 H_2 |y|^{2H_2-1}) \\ &\quad \times (sign\ y)(y-x) \exp\{-\frac{(y-x)^2}{2(q+r)}\} dy\, dq\, dr. \end{aligned} \tag{4}$$

(ii) For any fixed $t_1, \ldots, t_n \in [0,T]$, the multivariate process $\{(u(t_1,x), \ldots, u(t_n,x)), x \in \mathbb{R}\}$ is a centered stationary Gaussian process.
(iii) The variance of $u(t,x)$ is given by

$$Var(u(t,x)) = E[u(t,x)]^2 = \sigma_1^2 v_t(H_1) + \sigma_2^2 v_t(H_2), t > 0, x \in \mathbb{R} \tag{5}$$

where

$$v_t(H) = c_H t^{H+1} \quad and \ c_H = \frac{2^{H+1}(2^H-1)\Gamma(H+\frac{1}{2})}{\sqrt{\pi}(H+1)}. \tag{6}$$

(iv)For $t, s \in [0,T]$ and $x > 0$, the covariance function admits the following upper bound:

$$|Cov(u(t,0), u(s,x))| \leq C_{H_1,H_2} ts(\sigma_1^2 x^{2H_1-2} + \sigma_2^2 x^{2H_2-2}) \tag{7}$$

where C_{H_1,H_2} is a positive constant depending on H_1 and H_2.
(v) For $t, s \in [0,T]$ and $x \in R$,

$$\begin{aligned} Cov(u(t,x), u(s,x)) &= \frac{\sigma_1^2 2^{H_1}\Gamma(H_1+\frac{1}{2})((t+s)^{H_1+1} - t^{H_1+1} - s^{H_1+1})}{\sqrt{\pi}(H_1+1)} \\ &\quad + \frac{\sigma_2^2 2^{H_2}\Gamma(H_2+\frac{1}{2})((t+s)^{H_2+1} - t^{H_2+1} - s^{H_2+1})}{\sqrt{\pi}(H_2+1)}. \end{aligned} \tag{8}$$

(vi) For fixed $t > 0$, the process $\{u(t,x), x \in \mathbb{R}\}$ is ergodic.

Proof: Since the processes W^{H_1} and W^{H_2} are independent fractional Brownian motions, it is easy to see that

$$Cov(u(t,x), u(s,z)) = \sigma_1^2 Cov(u_1(t,x), u_1(s,z)) + \sigma_2^2 Cov(u_2(t,x), u_2(s,z)) \tag{9}$$

where

$$u_i(t,x) = \int_0^t \int_R G(t-s, x-y) dW_y^{H_i} ds, i = 1, 2.$$

The properties (i)-(v) follow from the results in Avetisian and Ralchenko (2020,2023) for fractional Brownian motion as noise. For any fixed $t \in [0,T]$, the process $\{u(t,x), x \in \mathbb{R}\}$ is a stationary Gaussian process . From Proposition 4 in Avetisian and Ralchenko (2020), it follows that the covariance function $R(t,x) = Cov(u(t,0), u(t,x))$ of the process will satisfy the inequality

$$|R(t,x)| \leq C_{H_1}\sigma_1^2 t^2 x^{2H_1-2} + C_{H_2}\sigma_1^2 t^2 x^{2H_2-2}, x > 0.$$

Since $0 < H_1, H_2 < 1,$, it follows that $R(t,x) \to 0$ as $x \to \infty$. This in turn implies that the process $\{u(t,x), x \in R\}$ is an ergodic process for any fixed $t > 0$.

Let $\delta > 0$ and define

$$V_N(t) = \frac{1}{N}\sum_{i=1}^{N}[u(t,k\delta)]^2, t > 0, N \geq 1, \tag{10}$$

$$\mu(t) = \sigma_1^2 v_t(H_1) + \sigma_2^2 v_t(H_2). \tag{11}$$

Let

$$\rho_{t,s}^{H_1,H_2}(k) = Cov(u(t,k\delta), u(s,0)) \text{ and } r_{t,s}(H_1,H_2) = 2\sum_{k=-\infty}^{\infty}[\rho_{t,s}^{H_1,H_2}(k)]^2.$$

Theorem 2.2. *(i) For any $t > 0$,*

$$V_N(t) \to \mu(t) \;\; a.s. \;\; as \;\; N \to \infty. \tag{12}$$

(ii) Suppose further that $H_1, H_2 \in (0, \frac{3}{4})$. Then, for any distinct positive $t_1, \ldots, t_n$, the random vector

$$\sqrt{N}(V_N(t_1) - \mu(t_1), \ldots, V_N(t_n) - \mu(t_n))$$

converges in law to a multivariate normal distribution with mean vector 0 and covariance matrix R as $N \to \infty$ where

$$R = ((r_{t_i,t_j}(H_1,H_2)))_{n\times n}.$$

Proof: Since the process $\{u(t,x), x \in \mathbb{R}\}$ is an ergodic process for any $t > 0,$, it follows that

$$V_N(t) = \frac{1}{N}\sum_{k=1}^{N}[u(t,k\delta)]^2 \to E([u(t,0)]^2) \;\; a.s\text{as} \;\; N \to \infty.$$

Note that

$$|\rho_{t_i,t_j}^{H_1,H_2}(k)| \leq C(\sigma_1^2(k\delta)^{2H_1-2} + \sigma_2^2(k\delta)^{2H_2-2})$$

for some constant $C > 0$ depending on H_1 and H_2 and hence

$$(\rho_{t_i,t_j}^{H_1,H_2}(k))^2 \leq C(\sigma_1^4(k\delta)^{4H_1-4} + \sigma_2^4(k\delta)^{4H_2-4})$$

which in turn implies that

$$\sum_{k=-\infty}^{\infty}(\rho_{t_i,t_j}^{H_1,H_2}(k))^2 < \infty$$

since $H_1, H_2 \in (0, \frac{3}{4})$. Following the arguments in the proof of Theorem 1 in Avetisian and Ralchenko (2023), it follows that

$$\sqrt{N}(V_N(t_1) - \mu(t_1), \dots, V_N(t_n) - \mu(t_n))$$

converges in law to a multivariate normal distribution with mean vector 0 and covariance matrix R where

$$R = ((r_{t_i,t_j}(H_1, H_2))_{n \times n}$$

by the Cramer-Wold technique and the multivariate Breuer-Major theorem (cf. Arcones (1994)).

3. Estimation of H_1 given H_2

Let $\delta > 0$. We now consider the problem of estimation of the parameter H_1 given H_2, σ_1^2 and σ_2^2 and the process $\{u(t,x), t \geq 0, x \in \mathbb{R}\}$ is observed at the points, $x_k = k\delta, k = 1, \dots, N$ for fixed $0 < t_1 < t_2 < t_3$. Following the method of moments for estimation of the unknown parameters which consists in equating the sample moments to the population moments and observing that

$$V_N(t) \to \mu(t) = \sigma_1^2 v_t(H_1) + \sigma_2^2 v_t(H_2) \;\; a.s \;\; \text{as} \;\; N \to \infty,$$

we obtain the Equations

$$\begin{aligned} V_n(t_i) &= \sigma_1^2 v_{t_i}(H_1) + \sigma_2^2 v_{t_i}(H_2), i = 1,2,3 \\ &= \sigma_1^2 C_{H_1} t_i^{H_1+1} + \sigma_2^2 C_{H_2} t_i^{H_2+1}, i = 1,2,3. \end{aligned} \tag{13}$$

As a consequence, it follows that

$$t_2^{-(H_2+1)} V_N(t_2) - t_1^{-(H_2+1)} V_N(t_1) = \sigma_1^2 C_{H_1} (t_2^{H_1-H_2} - t_1^{H_1-H_2}) \tag{14}$$

and

$$t_3^{-(H_2+1)} V_N(t_3) - t_1^{-(H_2+1)} V_N(t_1) = \sigma_1^2 C_{H_1} (t_3^{H_1-H_2} - t_1^{H_1-H_2}). \tag{15}$$

Taking ratios of the terms on either side of the equations given above, we obtain that

$$\frac{t_2^{-(H_2+1)} V_N(t_2) - t_1^{-(H_2+1)} V_N(t_1)}{t_3^{-(H_2+1)} V_N(t_3) - t_1^{-(H_2+1)} V_N(t_1)} = \frac{(t_2^{H_1-H_2} - t_1^{H_1-H_2})}{(t_3^{H_1-H_2} - t_1^{H_1-H_2})}.$$

Observe that

$$\lim_{H_1 \to H_2} \frac{(t_2^{H_1-H_2} - t_1^{H_1-H_2})}{(t_3^{H_1-H_2} - t_1^{H_1-H_2})} = \frac{\log t_2 - \log t_1}{\log t_3 - \log t_1}$$

by L'Hopital rule. Define the function

$$\begin{aligned} f(H) &= \frac{(t_2^{H-H_2} - t_1^{H-H_2})}{(t_3^{H-H_2} - t_1^{H-H_2})} \;\; \text{if} \;\; H \neq H_2 \\ &= \frac{\log t_2 - \log t_1}{\log t_3 - \log t_1} \;\; \text{if} \;\; H = H_2. \end{aligned} \tag{16}$$

For any fixed H_2, and for $0 < t_1 < t_2 < t_3$, it can be shown that the function $f : \mathbb{R} \to (0,\infty)$ is strictly increasing function in H following arguments in Avetisian and Ralchenko (2023) and hence has an inverse f^{-1}. We define the estimator $\hat{H}_{1N}$ of the parameter H_1 by the equation

$$\hat{H}_{1N} = f^{-1}(\frac{t_2^{-(H_2+1)}V_N(t_2) - t_1^{-(H_2+1)}V_N(t_1)}{t_3^{-(H_2+1)}V_N(t_3) - t_1^{-(H_2+1)}V_N(t_1)}) \tag{17}$$

which will be well-defined for large N. Following the method of proof of Theorem 1 in Avetisian and Ralchenko (2023), we obtain the following result.

THEOREM 3.1. *Suppose $H_1 \in (0,1)$ and $H_1 \neq H_2$. Then the estimator $\hat{H}_{1N}$ is a strongly consistent estimator of H_1 as $N \to \infty$. Furthermore*

$$\sqrt{N}(\hat{H}_{1N} - H_1) \to N(0,\zeta^2) \text{ in distribution as } N \to \infty$$

where ζ^2 depends on t_1, t_2, t_3 and H_2.

Proof: Note that

$$\frac{t_2^{-(H_2+1)}V_N(t_2) - t_1^{-(H_2+1)}V_N(t_1)}{t_3^{-(H_2+1)}V_N(t_3) - t_1^{-(H_2+1)}V_N(t_1)} \to \quad f(H_1) \ a.s \text{ as } N \to \infty.$$

From the continuity of the inverse function f^{-1}, it follows that $\hat{H}_{1N}$ converges a.s. to H_1 as $N \to \infty$. Taking expectations on both sides of the equations (14) and (15) and then taking the ratios, we obtain that

$$\frac{t_2^{-(H_2+1)}\mu(t_2) - t_1^{-(H_2+1)}\mu(t_1)}{t_3^{-(H_2+1)}\mu(t_3) - t_1^{-(H_2+1)}\mu(t_1)} = \frac{(t_2^{H_1-H_2} - t_1^{H_1-H_2})}{(t_3^{H_1-H_2} - t_1^{H_1-H_2})} = f(H_1).$$

Hence

$$H_1 = f^{-1}(\frac{t_2^{-(H_2+1)}\mu(t_2) - t_1^{-(H_2+1)}\mu(t_1)}{t_3^{-(H_2+1)}\mu(t_3) - t_1^{-(H_2+1)}\mu(t_1)}).$$

Therefore

$$\sqrt{N}(\hat{H}_{1N} - H_1) = \sqrt{N}(g(V_n(t_1), V_N(t_2), V_n(t_3)) - g(\mu(t_1), \mu(t_2), \mu(t_3)))$$

where

$$g(x_1, x_2, x_3) = f^{-1}(\frac{t_2^{-(H_2+1)}x_2 - t_1^{-(H_2+1)}x_1}{t_3^{-(H_2+1)}x_3 - t_1^{-(H_2+1)}x_1}).$$

Applying the delta method and observing that $(V_N(t_1), V_N(t_2), V_N(t_3))$ is asymptotically normal after suitable scaling, it can be shown that

$$\sqrt{N}(\hat{H}_{1N} - H_1) \to N(0,\zeta^2) \text{ in distribution as } N \to \infty$$

for some ζ^2 depending on $t_i, i = 1,2,3$ and H_2.. We skip the details.

4. Estimation of σ_1^2 and σ_2^2 when H_1 and H_2 are known and $H_1 \neq H_2$

Suppose that the Hurst indices H_1 and H_2 are known. We now study the problem of estimation of the parameters σ_1^2 and σ_2^2 based on the discrete set of observations $u(t_i, k\delta), i = 1, 2, , k = 1, \ldots, N$ with $t_1 < t_2$ and a fixed $\delta > 0$. Using the method of moments again, we obtain the equations

$$V_n(t_i) = \sigma_1^2 C_{H_1} t_i^{H_1+1} + \sigma_2^2 C_{H_2} t_i^{H_2+1}, i = 1, 2.$$

Solving these equations, we obtain the estimators

$$\hat{\sigma}_{1N}^2 = \frac{t_1^{-(H_2+1)} V_n(t_1) - t_2^{-(H_2+1)} V_N(t_2)}{C_{H_1}(t_1^{H_1-H_2} - t_2^{H_1-H_2})}$$

and

$$\hat{\sigma}_{2N}^2 = \frac{t_1^{-(H_1+1)} V_n(t_1) - t_2^{-(H_1+1)} V_N(t_2)}{C_{H_2}(t_1^{H_2-H_1} - t_2^{H_2-H_1})}$$

for σ_1^2 and for σ_2^2 respectively. From the almost sure convergence of $V_n(t)$ to $\mu(t)$ as $N \to \infty$, it follows that

$$\hat{\sigma}_{iN}^2 \to \sigma_i^2 \;\; a.s \;\text{ as }\; N \to \infty$$

for $i = 1, 2$ whenever $H_1 \neq H_2$. Observe that the random vector

$$(\sqrt{N}(\hat{\sigma}_{1N}^2 - \sigma_1^2), \sqrt{N}(\hat{\sigma}_{2N}^2 - \sigma_2^2))$$

is a linear function of the random vector

$$(\sqrt{N}(V_n(t_1) - \mu(t_1)), \sqrt{N}(V_n(t_2) - \mu(t_2))$$

with coefficients depending on t_1, t_2, H_1 and H_2. Furthermore the random vector

$$(\sqrt{N}(V_n(t_1) - \mu(t_1)), \sqrt{N}(V_n(t_2) - \mu(t_2))$$

is asymptotically bivariate normal with mean zero and suitable covariance matrix. Hence, it follows that the random vector

$$(\sqrt{N}(\hat{\sigma}_{1N}^2 - \sigma_1^2), \sqrt{N}(\hat{\sigma}_{2N}^2 - \sigma_2^2))$$

is asymptotically bivariate normal with mean zero and suitable covariance matrix Σ.

Following the method of moments, one can obtain alternate set of estimators by observing that, for any $\delta > o$,

$$\frac{1}{N}\sum_{k=1}^{N}[u(t, k\delta)]^4 \to \mu_4 \;\; a.s \;\text{ as }\; N \to \infty$$

where μ_4 is the 4-th central moment of the Gaussian distribution with mean zero and variance $3[\sigma_1^2 v_t(H_1) + \sigma_2^2 v_t(H_2]^2$. This follows from the observation that for a Gaussian distribution with mean zero and variance σ^2, the 4-th central moment is $3\sigma^4$. Let

$$J_N(t) = \frac{1}{N}\sum_{k=1}^{N}[u(t, k\delta)]^4.$$

Solving the Equations

$$V_N(t_1) = \sigma_1^2 C_{H_1} t_1^{H_1+1} + \sigma_2^2 C_{H_2} t_1^{H_2+1}$$

and

$$\sqrt{J_N(t_2)/3} = \sigma_1^2 C_{H_1} t_2^{H_1+1} + \sigma_2^2 C_{H_2} t_2^{H_2+1},$$

we obtain alternate estimators for σ_1^2 and σ_2^2 depending on H_1, H_2 and the choice of t_1 and t_2. These estimators will also be strongly consistent as $N \to \infty$.

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