Visualization of Theoretical Frameworks in Fundamental Electromagnetics Education

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Visualization of Theoretical Frameworks in Fundamental Electromagnetics Education

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Abstract

The paper presents an innovative approach to the visualization of theoretical frameworks in electromagnetics education, aimed at enhancing learning outcomes. Relationships among physical quantities and fundamental equations in electromagnetics are visually represented. Through this method, connections between electrostatic, magnetostatic, and time-varying fields are systematically mapped out, with an emphasis on duality, unit consistency, and dimensional analysis. The visualization framework is designed to aid students in understanding Maxwell's equations by organizing complex concepts into a two-dimensional structure that highlights the relations on key variables and fundamental equations. A "Trinity Memorization" strategy is also proposed to strengthen long-term memory retention, integrating mathematical consistency, theoretical rationality, and dimensional coherence. The layout is governed by rules based on the properties of physical quantities, such as their association with electric or magnetic fields, and whether they are 'intensive-like' or 'extensive-like' variables. Symmetry and duality in the equations are also emphasized, allowing learners to better grasp the interconnected nature of the principles. It is argued that this method not only facilitates memory retention but also improves the understanding of electromagnetic field dynamics, providing a more intuitive approach to learning.

1 Introduction

Electromagnetics is a discipline that describes the interactions of electric and magnetic fields and provides fundamental theoretical foundations for many fields in physics and engineering. Therefore, it is widely studied in the early years of science and engineering programs at many universities. At Kyushu University, it serves as a core discipline subject in the general education curriculum for science students. The author has been responsible for teaching "Fundamental Physics II" (electromagnetics section, now renamed Fundamentals of Electromagnetics) since 2014 and has been engaged in the **visualization of the theoretical framework of electromagnetics.** Since July 2023, significant progress has been achieved in this endeavor. This paper aims to explain the results

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of this progress. The approach for constructing two-dimensional visualization, as of July 2024, has already been reported regarding the research aspects of data analysis methods[1]. Therefore, this paper focuses on the details of the framework diagram, its descriptive methodology, and its relationship with learning methods.

The framework diagram primarily uses the differential form of Maxwell's equations as the starting point for conceptualizing the framework. To confirm this uniqueness, this paper investigates multiple elementary and secondary-level electromagnetics textbooks[2–11]. Upon reviewing the explanations of Maxwell's equations in each textbook, the mainstream approach positions Maxwell's equations as an endpoint (or intermediate step), with only textbooks[6, 7] treating them as the starting point. The most common order of explanation for Maxwell's equations follows: Gauss's law for the electric field $E \rightarrow$ Gauss's law for the electric flux density (electric displacement) $D \rightarrow$ Gauss's law for the magnetic flux density $B \rightarrow$ Ampère's law and magnetic field intensity $H \rightarrow$ Faraday's law of electromagnetic induction $\rightarrow$ Ampère-Maxwell's law. Given that the historical development of electromagnetics traces the progression from Coulomb's and Biot-Savart's laws to Maxwell's equations, it is mainstream to present theoretical explanations in accordance with this historical perspective. However, there is also an argument that prioritizing the memorization of Maxwell's equations facilitates better learning outcomes in understanding the theoretical framework.[6,7]

Maxwell's equations, which are a cornerstone of elementary electromagnetics, have undergone continuous modifications in notation since their publication in 1862. Given the author's limited expertise, thoroughly analyzing the original texts and comprehensively covering the various representations (e.g., E-H formulation, unit systems other than SI) is challenging. Therefore, a summarized explanation[12] was referenced during the writing of this paper. Based on this background, particularly in the context of delivering short-term lectures on the fundamentals of electromagnetics, this study attempts to visualize the theoretical framework as a means to help learners accurately and durably memorize numerous emerging physical variables and fundamental equations before advancing to a deeper understanding. A similar attempt has been made by Hosokawa[11], and the approach of using visually accessible maps has been proposed as an effective method. However, the visualization of the theoretical structure discussed in this paper has been recognized as having a unique perspective. As in the referenced textbooks, this visualization is primarily based on the SI unit system and the commonly adopted EB formulation of the fundamental equations.

2 Motivation for Visualization of the Theoretical Framework

2.1 Quantitative Complexity of electromagnetics Fundamentals

First, the required memory capacity for mastering the fundamentals of electromagnetics is examined from a quantitative perspective. The physical variables used in electromagnetics include both vectors and scalars. These variables include $\mathbb{A}$, $\mathbb{B}$, $\mathbb{D}$, $\mathbb{E}$, $\mathbb{F}$, $\mathbb{H}$, $\mathbb{I}$, $\mathbb{M}$, $\mathbb{P}$, $\mathbb{m}$, $\mathbb{p}$, $\mathbb{j}$, $\mathbb{c}(\mathbb{v})$among the vector quantities (additionally, $\mathbb{r}$, $\mathbb{s}$, $\mathbb{S}$, $\mathbb{n}$, etc., used in spatial integrals; in this paper, double-struck font is used for vectors) and scalar values obtained through spatial integration, such as C, L, R, q, Φ, P, U, $V(\phi)$, and material coefficients σ, ε, μ. Additionally, variables such as Φ for linked magnetic flux, P for Joule heat, U for energy, various energy densities u, Joule heat density p, and the torque vector $\mathbb{T}$ exist. Furthermore, some notations are easily confused, such as charge density (and resistivity) ρ. In this paper, to avoid confusion, the Poynting vector is denoted as $\mathbb{S}_P$ instead of $\mathbb{S}$, surface charge density as ρ_S instead of σ, resistivity as ρ_R instead of ρ, and line

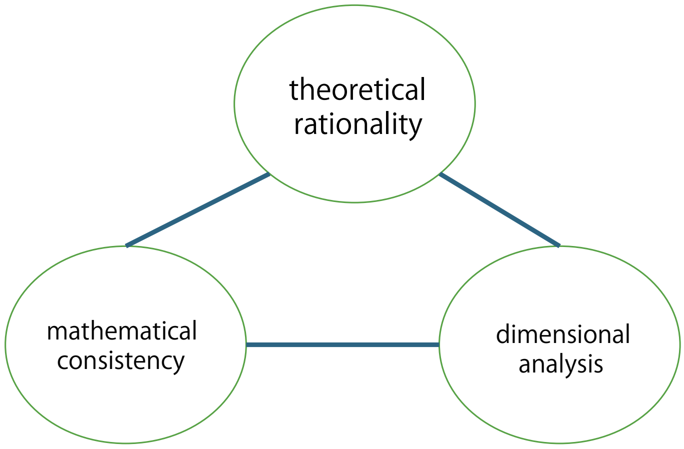


Figure 1: Concept of Trinity Memorization (*"TM"*)

current density as $\mathbb{j}_L$ instead of κ, which increases the number of distinct variables. Even with this distinction, these physical variables amount to 37 different types.

Moreover, electromagnetics is characterized by the heavy use of symbols in its unit system, including [A], [V], [H], [F], [C], [Wb], [T], [W], [N], [J], [Ω], [S], in addition to [s], [m], and previously used [℧]. These 15 unit symbols are combined with the 37 aforementioned physical variables. Furthermore, to describe the relationships among these 37 physical variables in a theoretical framework, more than 50 fundamental equations are required. When considering the problem-solving process in electromagnetics, it becomes necessary to accurately recall these relationships and combine them appropriately. This necessitates the memorization of over 100 individual elements for essential understanding.

2.2 Strategies for Enhancing Long-Term Memory

Considering this perspective, accurately memorizing information of this scale is extremely difficult when it comes to long-term retention, even if it may be feasible for short-term memory. One of the significant aspects of learning electromagnetics is associating vector analysis with real-world phenomena. However, the volume of information has already reached a level that is challenging to cover within short-term courses, often necessitating a reduction in the emphasis on understanding vector analysis.

In advanced learning, integrating reasoning with memorization is recommended over simple rote learning. A method frequently emphasized involves linking three different types of information. When information is retained using two paired elements (e.g., a mathematical definition and conceptual understanding of a phenomenon), no mechanism exists to correct recall errors, as only a 50% probability ensures correctness. However, by linking three types of information, even if one is incorrect, the remaining two can compensate and facilitate error correction. This design principle is also applied in error correction methods used in computer storage systems.[13]

In physics, a third piece of information that can be effectively utilized is the concept of units (i.e., dimensional analysis). While this follows the standard dimensional analysis approach, electromagnetics and electrical engineering employ a slightly modified unit system that enhances this three-element memory strategy. Additionally, the "mathematical definitions" mentioned earlier can be further extended in electromagnetics, where concepts such as "duality," commutative laws, and symmetry further reinforce theoretical and mathematical consistency.

Figure 1 illustrates this concept. In this paper, this approach is referred to as "Trinity Memorization” (三位一体記憶, hereafter *“TM”*). *“TM”* is also incorporated as a strategic component in the visualization of the framework.

2.3 System Visualization as a Learning and Memory Strategy

“TM” is an approach focused on individual point memorization; however, by associating multiple points (fundamental equations), a reinforcing framework is established. Strengthening the memory of a single point through correction simultaneously reinforces and supplements related information, making it possible to systematically organize and enhance retention. Typically, such memory networks are naturally constructed within learners’ memory in a bottom-up manner through learning. The resulting structures are often complex, and their visualization is not always guaranteed.

In this study, the author discovered that the theoretical framework of electromagnetics fundamentals could be systematically organized within a highly constrained two-dimensional framework. Significant progress has been made in visualizing the theoretical framework constructed and organized by pioneering physicists. Similar approaches have been attempted elsewhere[11, 14], and in that sense, this is not the first of its kind. However, the layout of the framework diagram itself is unprecedented, leading to an ongoing verification process through its publication on Wikimedia Commons since July 5, 2023, along with the following description:[14]

> This is an illustration of the theoretical system and laws in the study of electromagnetics. Centering on Maxwell’s equations, the important formulas and laws to be dealt with are illustrated under the system and organized to facilitate learning as a theory and law system. Since it is structured like a mandala, we have named it Madalat of electromagnetics. The initial EDBH-MaxwellEquations configuration will be available on 2020/10/6 `https://prezi.com/snfxjsxasaui/` and further details on 2020/11/24 `https://prezi.com/thk167yns8dx/`, both on prezi.com and have been used in lectures. The original idea for this diagram is also owned by the contributor.

Generally, in the process of learning, it is beyond debate that the most desirable outcome is the natural construction of such information memory networks through repeated learning in a bottom-up manner. However, when it is possible to present a framework as the final learning outcome in advance (though this does not apply to all subjects), a top-down learning approach that allows learners to visualize the learning goal can also be effective. Regardless of the dichotomy between bottom-up and top-down approaches, the framework has been published to expand learners’ options while simultaneously undergoing validation.

2.4 Selection Criteria for Visualization Methods

The visualization methods adopted in this paper and in the Wikimedia Commons images[15] are explained. In electromagnetics, both vector fields, such as electric and magnetic fields, and scalar fields, such as electric potential and energy, coexist. Therefore, in this paper, vectors are represented using double-struck font. Arrows with only coefficients appended indicate a relationship where the variable at the arrowhead equals the variable at the arrow tail multiplied by the coefficient. In the framework diagrams where colors are available[15], categorization is represented using font color and glow effects (upper blue-toned fonts represent electric fields, lower green-toned fonts represent magnetic fields, Ampère-related quantities have a red glow, and Volt-related quantities have a yellow glow). Closely related physical variables are grouped together: the upper-right region pertains to dielectric media, the lower-right to magnetic media, and the right-central region to conductive media.

In the center of the diagram, distributed vector quantities such as densities are placed, while their integral-based macroscopic variables are positioned on the outer edges.

The complexity of visualization needs to be adjusted according to the learner's level of knowledge and experience. As shown in Section 3, understanding the framework is facilitated by considering the square-shaped frame enclosed by $\mathbb{EDBH}$ at the center, then moving inward to $\rho, \mathbb{E}, \phi$, and $\mathbb{A}$, followed by summarizing the characteristics of each medium (dielectric, magnetic, and conductive), and finally extending the relationships of linear elements to the outer periphery in a stepwise manner.

2.5 Comparison with Existing Approaches

The following points highlight the uniqueness and advantages of this structured learning approach in organizing the fundamentals of electromagnetics.

Selection of Fundamental Equations with Structural Awareness: More than 50 fundamental equations appear in electromagnetics, and each can be expressed in various derivative forms. Additionally, different textbooks prioritize different equations as the most fundamental. For example, Gauss's law for the electric field can be formulated using $\mathbb{D}$, $\mathbb{E}$, integral form, and charge versus charge density representations. The framework diagram enhances understanding of duality and symmetry in the theoretical structure, aiding learners in determining which equations should be considered fundamental and which should be treated as derivative forms.

Visualization of Problem-Solving Algorithms: The step-by-step approach for combining fundamental equations and reasoning in problem-solving can be illustrated as a guided sequence, allowing learners to grasp the process intuitively.

Memory Reinforcement and Correction: Similar to mind maps and concept maps, bidirectional connections enhance mutual reinforcement, contributing to long-term retention of learning outcomes. Exercises where learners reproduce the framework diagram from memory without reference materials can be effective in establishing structured recall.

Practical Application of *"TM"*: Learning the visualization method proposed in this study provides an opportunity to apply similar techniques to other areas of study. "Trinity Memorization" (termed *"TM"*) is effective in physics through dimensional analysis, while other subjects may require different fundamental elements. By addressing this adaptation, learners' ability to self-correct and refine their understanding is expected to improve.

3 Explanation of the Visualization Results

In this chapter, after discussing the rules for system visualization, the discussion will expand to concrete examples of visualization in electromagnetics. From this chapter onward, numerous unique terms (denoted in bold and underlined) will be introduced. It should be noted here that these terms are further elaborated and organized in Section4.6.

3.1 Layout Rules for the Visualization

The approach for the visualization and the methodology for data analysis in visualization research are described in [1]. To summarize, as previously mentioned, the attributes of 37 physical variables were organized, and layout rules were provisionally set to map them two-dimensionally. The rules were then optimized based on the dualization, generalization, and factorization of fundamental equations used to describe the relationships between variables. Ultimately, four categories were adopted for attribute classification:

cat.①: Whether the variable pertains to (static) electric fields, (steady current) magnetic fields, or dynamic fields ($\partial/\partial t \neq 0$).

cat.②: Based on unit decomposition, whether the variable includes ampere (denoted as *"w/A"*), volt (denoted as *"w/V"*), or neither.

cat.③: Whether the variable exhibits intensive-like properties, akin to intensive-variables in thermodynamics (termed as *"intensive"*), or extensive-like properties, akin to extensive-variables in thermodynamics (termed as *"extensive"*).

cat.④: Whether the variable represents a differential physical quantity describing localized distribution or an integral physical quantity obtained through spatial integration.

Regarding units, considering the *"TM"* framework, a subset-based dimensional analysis approach is adopted to enhance memory retention. Non-prioritized units are excluded from core thought processes and fundamental memory, serving only as filters at the frontend (reading) and backend (final result description). Internal reasoning is conducted solely using prioritized units. The prioritized units include [A], [V], [s], [m], [Ω], [℧], [J], [W], and [N]. By reconstructing the units to encompass the relationships among physical variables in this manner, *"TM"* functions more effectively.

As shown in [1], the cat.② attribute corresponds to the conjugate relationship in thermodynamics. Specifically, the product of *"w/A"* and *"w/V"* includes dimensions such as power [W] and energy [J]. Furthermore, for variables with differing cat.③ attributes, their units also reflect this distinction; for instance, *"intensive"* variables typically include [m^{-1}], while *"extensive"* variables include [m^{-2}]. Correspondingly, *"intensive"* variables are often integrated using line integrals, whereas *"extensive"* variables are typically integrated using surface integrals.

3.2 Visualization Structure of the Electromagnetic Theory Framework (inside *"shell"*)

The visualization of the theoretical framework is highly complex, and presenting the entire structure as learning material at once may impose a psychological burden on learners. Therefore, a sequential approach is taken, explaining the structure step by step while referring to the rules outlined in the previous section.

Figure 2 illustrates the central framework, a square-shaped arrangement of $\mathbb{E}$, $\mathbb{D}$, $\mathbb{B}$, and $\mathbb{H}$ (hereafter referred to as the *"shell"*), with complexity increasing from (a) to (c).

Figure 2(a) shows the fundamental physical variables arranged according to categories cat.①–cat.③ and includes the four differential forms of Maxwell's equations. In the layout rules, the horizontal axis is organized based on the cat.② attribute: the right side contains physical variables including [A] (ampere) denoted as *"w/A"*, while the left side contains physical variables including [V] (volt) denoted as *"w/V"*, with the center containing variables that include neither. The vertical axis is arranged based on the cat.① attribute: the upper section corresponds to electric fields (electrostatics), the lower section to magnetic fields (steady currents), and the central region to variables involving time variations such as $\partial/\partial t \neq 0$.

With this arrangement, $\mathbb{E}$, $\mathbb{D}$, $\mathbb{B}$, and $\mathbb{H}$ occupy the positions shown in the figure, each associated with one Maxwell's equation, and their spatial differential forms (rot, div) are positioned diagonally. The relative positioning of $\mathbb{E}$ to $\mathbb{D}$ and $\mathbb{B}$ to $\mathbb{H}$ creates a line-symmetric arrangement, which also reflects their conjugate relationships (hereafter referred to as *"counterpart"*).

Similarly, for cat.③, diagonally opposite variables share the same attribute such as electric **field intensity** $\mathbb{E}$ and magnetic **field intensity** $\mathbb{H}$ are *"intensive"*, while electric **flux density** $\mathbb{D}$

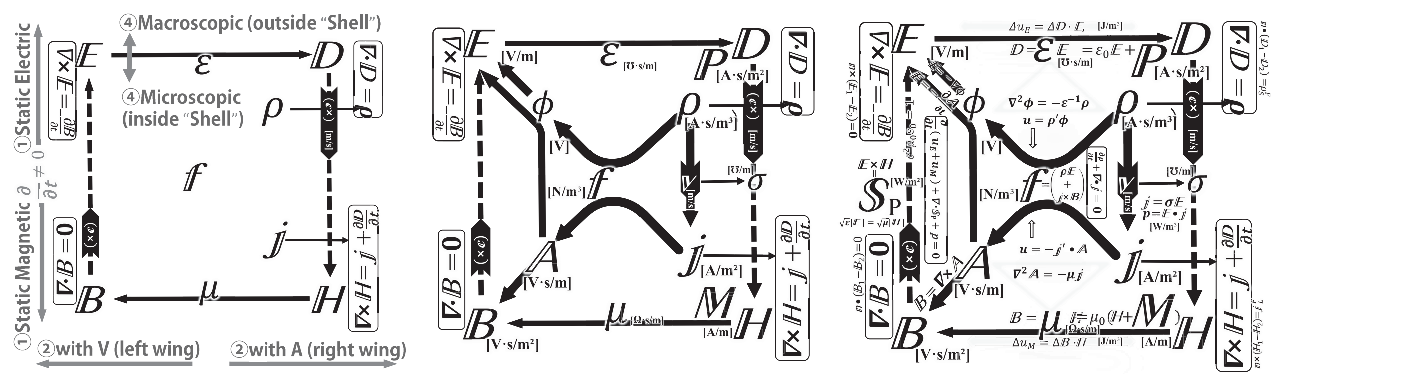
and magnetic **flux density** $\mathbb{B}$ are *"extensive"*. Along each edge of the *"shell"*, clockwise arrows primarily represent conversions, with variables (sequentially ε [℧·s/m], $\mathbb{c}$ [m/s], μ [Ω·s/m], $\mathbb{c}$ [m/s]) acting as modifiers that indicate multiplicative scaling. The variable $\mathbb{c}$ is specifically limited to its role in electromagnetic wave propagation, and completing a full cycle confirms the relationship $\varepsilon\mu c^2 = 1$. Through this arrangement, it becomes evident that multiplying or dividing by ε or μ switches physical variables between the left and right sides, altering the *"w/A"*⇔*"w/V"* attribute, while simultaneously switching the *"intensive"*⇔*"extensive"* attribute due to the layout. The fact that Maxwell's equations are written vertically indicates, according to the layout rules, that these equations do not involve changes in cat.② attributes. This results from separating ε and μ from Maxwell's equations.

For example, in the top-left equation, $\nabla\times\mathbb{E} = \partial\mathbb{B}/\partial t$, its right-hand side clearly corresponds to $\mathbb{B}$, which is positioned in the lower-left region (*"w/V"*) of the diagram. It is evident from both theoretical reasoning and unit consistency that $\mathbb{H}$, positioned at *"counterpart"*, should not be included in this equation. Similarly, in the Maxwell's equations on the right side, physical quantities such as charge density ρ [A·s/m^3] and current density $\mathbb{j}$ [A/m^2], which serve as sources for electric and magnetic fields and belong to *"w/A"*, are appropriately positioned inside the *"shell"* on the right side, as shown in (a).

Finally, at the center, the generalized Lorentz force is selected as the fundamental expression of electromagnetic phenomena. Instead of using the integral form $\mathbb{F}$[N], it is represented in the differential form per unit volume as $\mathbb{f}$ [N/m^3]. This replacement follows the rule that the interior and surrounding areas of the *"shell"* are reserved for differential physical quantities in cat.④. However, the primary reason for this selection is to ensure that $\mathbb{f}$'s equation is enclosed within the framework of Figure 2, maintaining symmetry in its notation.

In Figure 2(b), the charge velocity $\mathbb{v}$ [m/s], the electrical conductivity of the conductor σ [Ω/m], polarization $\mathbb{P}$, and magnetization $\mathbb{M}$ are placed inside the *"shell"*. Additionally, the units of physical variables are specified. Since $\mathbb{P}$ and $\mathbb{M}$ have the same units as $\mathbb{D}$ and $\mathbb{H}$, respectively, they are positioned in close proximity to those variables.

Next, in order to describe the Lorentz force $\mathbb{f}$ acting on ρ and $\mathbb{j}$, the scalar potential field ϕ [V] and the vector potential field $\mathbb{A}$ [V·s/m] produced by ρ and $\mathbb{j}$ are positioned at their *"counterpart"* locations, as cat.② classifies them as *"w/V"*. The curved arrows indicate that ρ and $\mathbb{j}$ form the



(a) (b) (c)

Figure 2: Stepwise Explanation of the Framework Structure. (a) Fundamental variables and the layout rules of Maxwell's equations. (b) Arrows added to illustrate the relationships between physical variables, indicating how they interact. (c) Fundamental equations corresponding to the arrows between physical variables are incorporated, providing a complete framework.

potential fields ϕ and A through $\mathbb{f}$. Furthermore, the relationship $\mathbb{j} = \rho\mathbb{v}$ is depicted between ρ and $\mathbb{j}$, where $\mathbb{v}$ [m/s] represents the carrier drift velocity.

Figure 2(c) illustrates the result of associating and positioning the fundamental equations along the arrows between each physical variable, nearly completing the description within the *"shell"* (its four edges and interior). The central variable $\mathbb{f}\,[\mathrm{N/m^3}]$ is represented by the equation $\mathbb{f} = \rho\mathbb{E} + \mathbb{j}\times\mathbb{B}$, whose right-hand side comprises the sums of variable pairs $\rho, \mathbb{E}$ and $\mathbb{j}, \mathbb{B}$, located near their *"counterpart"* positions in Figure 2(c), following a transformation referred to later as *"conjugate"* type. This duality in mathematical expressions and spatial arrangement, along with unit consistency, reinforces both theoretical principles and the *"TM"* framework.

The Lorentz force $\mathbb{F}$ frequently appears in dual expressions involving electric and magnetic fields, such as $\mathbb{F} = q\mathbb{E} + q\mathbb{v}\times\mathbb{B} = q\mathbb{E} + I\,\mathbb{L}\times\mathbb{B}$, which can be easily derived by multiplying the aforementioned $\mathbb{f}$ by an infinitesimal volume dv.

Regarding the three media, the equations $\mathbb{D} = \varepsilon\mathbb{E}$ and $\mathbb{B} = \mu\mathbb{H}$ in dielectrics and magnetic materials become non-dual when incorporating polarization $\mathbb{P}$ and magnetization $\mathbb{M}$, yielding $\mathbb{D} = \varepsilon_0\mathbb{E} + \mathbb{P}$ and $\mathbb{B} = \mu_0(\mathbb{H} + \mathbb{M})$. However, since $\mathbb{P}$ and $\mathbb{M}$ have the same units as $\mathbb{D}$ and $\mathbb{H}$, respectively, only quantities of the same units can be added, maintaining consistency. In conductors, the carrier drift velocity $\mathbb{v}$ [m/s] is linked to electrical conductivity σ, leading to the differential form of Ohm's law, $\mathbb{j} = \sigma\mathbb{E}$. Consequently, for all three media, the equations $\mathbb{D} = \varepsilon\mathbb{E}$, $\mathbb{B} = \mu\mathbb{H}$, and $\mathbb{j} = \sigma\mathbb{E}$ exhibit an identical duality.

Additionally, the equations $\Delta u_E = \Delta\mathbb{D}\cdot\mathbb{E}$, $\Delta u_M = \Delta\mathbb{B}\cdot\mathbb{H}$, and $p = \mathbb{j}\cdot\mathbb{E}$ are annotated alongside, and the nature and combination of these equations will be discussed in the following section.

The upper-right region of the *"shell"* aggregates descriptions related to dielectric media, while the lower-right region contains descriptions for magnetic media. The right-central region near the vertical middle of the diagram consolidates information related to conductive media. The sections near the vertical center contain descriptions related to electromagnetic induction associated with $\partial/\partial t \neq 0$, the Poynting vector $\mathbb{S}_P$ related to light, as well as the continuity equation representing charge conservation (involving ρ and $\mathbb{j}$) and the energy conservation equation (including $\mathbb{S}_P, u_E, u_M$ and p). These last two equations are aligned vertically because the cat.② attributes of their constituent terms are identical.

3.3 Classification of Boundary Conditions and Fundamental Equations

The fundamental equations written in Figure 2 can be classified into four categories based on their properties (which will increase to five categories in the next section). The horizontally written equations mainly describe relationships between physical constants that are *"counterpart"* to each other, differing in nature from the vertically arranged equations mentioned earlier, where constituent elements are not separated into left and right components. Notably, all equations centrally written in the horizontal direction form pairs. When classified into the first and second categories, they take the following forms:

$$u = \mathbb{X}\cdot\mathbb{Y} \tag{1}$$

$$\mathbb{Y} = \eta\mathbb{X} \tag{2}$$

where $\mathbb{X}$ and $\mathbb{Y}$ are pairs of variables placed separately on the left and right, either as *"counterpart"* or in proximity. Equation (1) belongs to the first type, referred to as the *"conjugate"*-type, while equation (2) belongs to the second type, referred to as the *"constitutive"*-type. In the *"conjugate"*-type, the right-hand side consists of a product representing a conjugate relationship, ensuring that the left-hand side variable always contains units related to energy [J] or associated units [N], [W]. In the *"constitutive"*-type, a coefficient η defines the relationship between two *"counterpart"* variables, and naturally includes [Ω] (or [℧]) in its unit.

Since *"conjugate"* and *"constitutive"*-type equations exist in pairs for variables like $\mathbb{X}$ and $\mathbb{Y}$, one can infer that for the $\mathbb{E} \to \mathbb{D}$ system in the framework diagram, the *"constitutive"*-type equation $\mathbb{D} = \varepsilon\mathbb{E}$ is accompanied by the *"conjugate"*-type equation $\Delta u_E = \Delta\mathbb{D}\cdot\mathbb{E}$, which follows the direction of $\longrightarrow$. These pairs maintain duality in descriptions related to dielectric materials (around ε), magnetic materials (around μ), and conductors (around σ). Additionally, within the *"shell"*, there are triangular or inverted-triangular configurations like $\varepsilon \Leftrightarrow \rho \Leftrightarrow \phi$ and $\mu \Leftrightarrow \mathbb{j} \Leftrightarrow \mathbb{A}$, where similar *"conjugate"*–*"constitutive"*-pairs exist.

Poisson's equations $\nabla^2\phi = -\varepsilon^{-1}\rho$, $\nabla^2\mathbb{A} = -\mu\mathbb{j}$ are classified as *"constitutive"*-type equations containing the ∇^2 operator, while potential energy expressions derived from fields $u = \phi\rho'$, $u = -\mathbb{A}\cdot\mathbb{j}'$ belong to the *"conjugate"*-type.

The third type of fundamental equation formally resembles the *"constitutive"*-type and is associated with arrows linking physical variables, but unlike the second type, it does not connect conjugate variables, meaning there is no paired *"conjugate"*-type equation. For example, the relationships $\mathbb{A}$ [V·s/m] and $\mathbb{B}$ [V·s/m^2] through $\mathbb{B} = \nabla\times\mathbb{A}$, and ϕ [V] and $\mathbb{E}$ [V/m] through $\mathbb{E} = -\nabla\phi$ link variables within *"w/V"*, whereas the relationship between ρ [A·s/m^3] and $\mathbb{j}$ [A/m^2] through $\rho\mathbb{v} = \mathbb{j}$ connects variables belong *"w/A"*.

The fourth type includes continuity equations that describe conservation laws, such as $\partial\rho/\partial t + \nabla\cdot\mathbb{j} = 0$, which is defined as the *"continunity"*-type. Beyond charge and energy conservation laws, Maxwell's equations and boundary condition equations written alongside them also fall into this *"continunity"*-type category.

$$\mathbb{n}\times(\mathbb{E}_1 - \mathbb{E}_2) = 0 \quad [\mathrm{V/m}] \quad (\because \nabla\times\mathbb{E} = -\partial\mathbb{B}/\partial t) \tag{3}$$
$$\mathbb{n}\cdot(\mathbb{D}_1 - \mathbb{D}_2) = \rho_S^F \quad [\mathrm{A{\cdot}s/m^2}] \quad (\because \nabla\cdot\mathbb{D} = \rho) \tag{4}$$
$$\mathbb{n}\cdot(\mathbb{B}_1 - \mathbb{B}_2) = 0 \quad [\mathrm{V{\cdot}s/m^2}] \quad (\because \nabla\cdot\mathbb{B} = 0) \tag{5}$$
$$\mathbb{n}\times(\mathbb{H}_1 - \mathbb{H}_2) = \mathbb{j}_L^F \quad [\mathrm{A/m}] \quad (\because \nabla\times\mathbb{H} = \mathbb{j} + \partial\mathbb{D}/\partial t) \tag{6}$$

These equations follow Miyazoe's notation[2], where $\mathbb{n}$ represents the normal unit vector at the material boundary, and ρ_S^x and $\mathbb{j}_L^x$ denote the surface charge density and line current density at the boundary, respectively.

These fundamental equations within the *"shell"* can be classified into four categories. Equations associated with arrows are of the *"constitutive"*-type, and those spanning across left and right also have a paired *"conjugate"*-type. Additionally, the *"continunity"*-type includes conservation laws, Maxwell's equations, and boundary conditions, all of which are systematically positioned within the framework.

3.4 Visualization Structure of the Electromagnetic Theory Framework (Peripheral Section)

Outside the *"shell"*, integral variables obtained through spatial integration were arranged based on cat.④, and the fundamental equations connecting them were mapped accordingly. Charge quantity q [A·s] was placed outside the upper part of the *"shell"* corresponding to charge density ρ [A·s/m^3], and current I [A] was placed outside the lower part of the *"shell"* corresponding to current density $\mathbb{j}$ [A/m^2]. These connections (relations) generally involve spatial integration (line, surface, or volume integration). Additionally, $\mathbb{P}$ and $\mathbb{M}$ are connected via spatial integration to the electric dipole moment $\mathbb{p}$ [A·s·m] and the magnetic moment $\mathbb{m}$ [A·m^2], respectively.

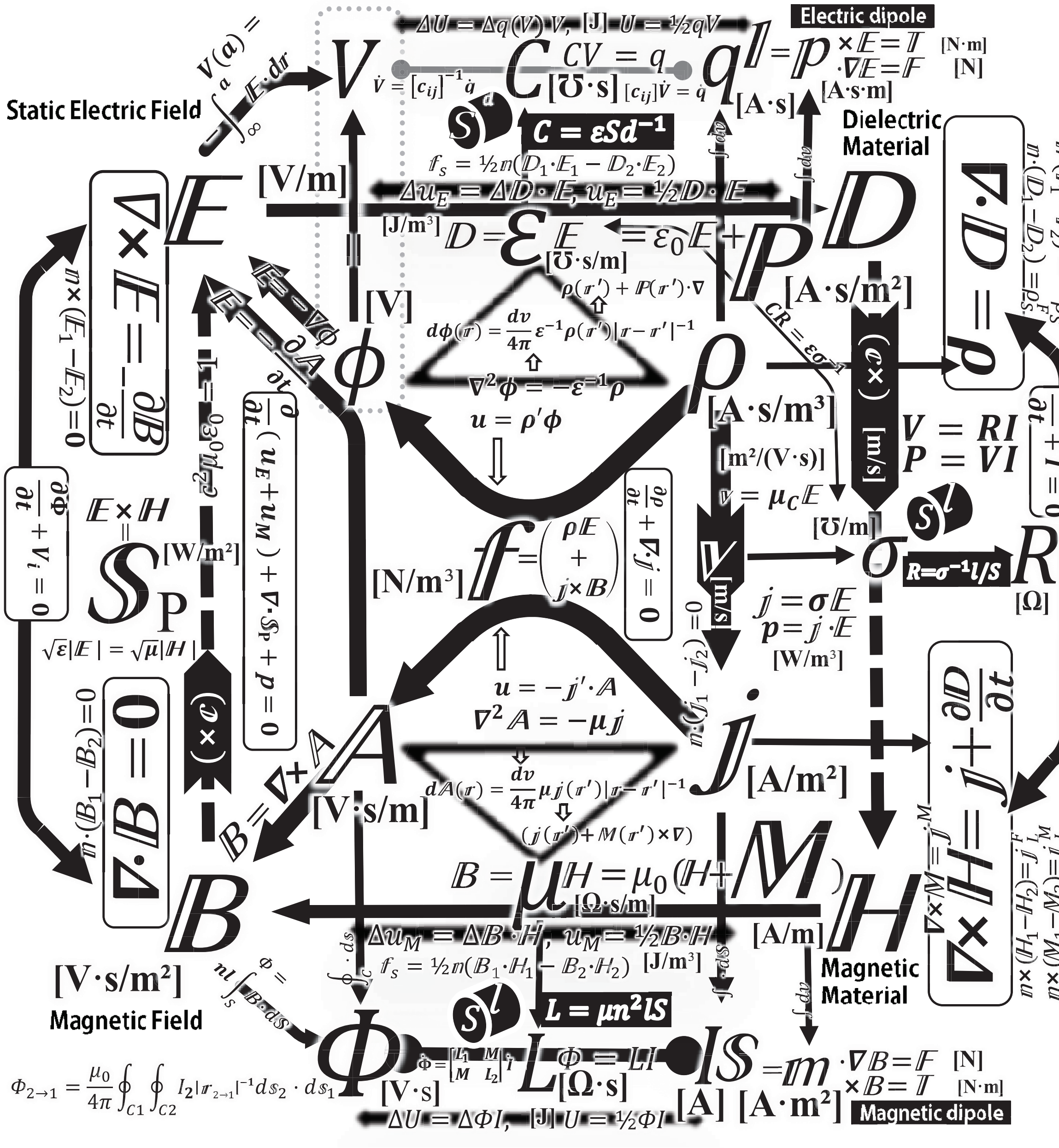


Figure 3: Complete diagram including the periphery outside the *"shell"* for the elementary electromagnetic theory framework. The original design follows CC BY-SA 4.0 of [15]. Outside the *"shell"*, macroscopic physical quantities (such as charge q and current I) obtained through spatial integration are arranged. Relationships derived from integrating the central differential physical quantities (such as charge density ρ and current density $\mathbb{j}$) are represented by vertical arrows ($\uparrow\downarrow$).

Figure 3 illustrates the full theoretical framework of fundamental electromagnetics, covering nearly all basic concepts.

Outside the *"shell"*, integral physical constants are described, and their layout rule follows the spatial integration targets of the differential physical quantities, connecting outward from the *"shell"* via vertical arrows (rightward for conductor-related quantities). The fundamental equations associated with these connecting arrows can be easily derived in the form of spatial integrals, and this type is classified as the fifth category of fundamental equations, referred to as the *"integrate"*-type.

V **(Electric Potential) Reconfiguration and Vertical Symmetry**The potential ϕ is located within the *"shell"*, but placing the same value outside the *"shell"* as the physical constant V (where in a conductor, the potential within an infinitesimal volume and the total potential from spatial integration are equal) ensures that Φ and V correspond to the magnetic field quantities $\mathbb{A}$ [V·s/m] and its integral, the linked magnetic flux Φ [V·s], maintaining the symmetry (duality) between the electric and magnetic field frameworks. The triangular ($\triangleleft$) positional relationships of $\mathbb{E}$, ϕ, and V for the electric field and $\mathbb{B}$, $\mathbb{A}$, and ϕ for the magnetic field are also dual, with the only deviation being that scalars in the electric field correspond to vectors in the magnetic field.

The only asymmetric element is the transition from $\mathbb{A}$ [V·s/m] to $\mathbb{E}$ [V/m] due to electromagnetic induction, but this transition is supported by *"TM"* through the mathematical consistency of Maxwell's equations combined with the vector potential and through dimensional consistency. The electromagnetic potential equation, $\mathbb{E} = -\nabla\phi - \partial\mathbb{A}/\partial t$, clearly visualizes the origins of divergence fields and curl fields. Additionally, the framework helps to systematically consider how the number of turns per unit length, nl, should be introduced into fundamental equations for magnetic fields.

Peripheral Section 1: Lumped Element Components The upper section connects to electrostatic components and electric dipoles, where ϕ [V], ε [℧·s/m], ρ [A·s/m^3], and $\mathbb{P}$ [A·s/m^2] are appropriately spatially integrated to correspond to V [V], C [℧·s], q [A·s], and $\mathbb{p}$ [A·s · m], respectively. If learners understand the scalarization process through spatial integration involving scalor-products, the corresponding relationships (*"integrate"*-type) can be determined almost automatically. The theoretical framework and unit dimensionality provide supplementary guidance on which physical constants are connected. Similarly, the spatial integration leading to the inductive component variables in the lower section can also be derived. However, special attention must be given to nl (n being the number of coil turns per unit length and l being the coil length), which serves as a dimensionless accumulated factor. In the framework diagram, this is introduced during the integration from $\mathbb{B}$ [V·s/m^2] to Φ [V·s]. For the spatial integration related to conductors on the right side, only R appears. Even in these lumped element expansions, the previously mentioned *"conjugate"*—*"constitutive"*-pairs exist, leading to the derivation of the three fundamental relationships: $\Delta U = \Delta q V$ and $q = CV$, $\Delta U = \Delta\Phi I$ and $\Phi = LI$, and $P = VI$ and $V = RI$, which directly connect to elementary learning.

Peripheral Section 2: Moments Moments $\mathbb{p}$ [A·s·m] and $\mathbb{m}$ [A·m^2] are macroscopic physical quantities obtained through integration and are positioned outside the *"shell"*. Their physical definitions and relationships with the differential forms $\mathbb{P}$ [A·s/m^2] and $\mathbb{M}$ [A/m] are reinforced through *"TM"*, ensuring unit consistency.

Equations Related to Moments: There exists a duality between the potential field description, translational force $\mathbb{F}$ [N], and torque vector $\mathbb{T}$ [N·m]. The equations for $\mathbb{F}$ and $\mathbb{T}$ correspond to the *"conjugate"*-type, and it is evident that the vector-product applies to $\mathbb{E}$ and $\mathbb{B}$ rather than $\mathbb{D}$ and $\mathbb{H}$. Furthermore, the relationship between $\mathbb{p}$ and $\mathbb{m}$ and their differential counterparts $\mathbb{P}$ and $\mathbb{M}$ can be easily established through spatial integration, which also connects to Maxwell's stress in unit area and volume.

Supplement to the *"shell"* **Interior:**Within the *"shell"*, Coulomb's law, which directly determines $\mathbb{E}$ and V from charge q, is constructed by integrating the path $q \to \rho \to \mathbb{D} \to \mathbb{E} \to V$ in the framework diagram, making it a complex expression. In the case of $\rho \to V$, Poisson's equation can be derived using the three-dimensional delta function formula $\nabla^2(1/|\mathbb{r}|) = -4\pi\delta(\mathbb{r})$. Similarly, the Biot–Savart law, which determines $\mathbb{B}$ ($\mathbb{A}$) from current I, follows the path $I \to \mathbb{j} \to \mathbb{H} \to \mathbb{B} \to \mathbb{A}$.

When using potentials ϕ and $\mathbb{A}$ for description, these equations transform into Poisson's equation, maintaining a degree of duality between electric and magnetic fields. However, the definition equations transitioning from ϕ and $\mathbb{A}$ to $\mathbb{E}$ and $\mathbb{B}$ are non-dual, and the equations representing $\mathbb{E}$ and $\mathbb{B}$ lack sufficient duality between electric and magnetic fields. Although these equations are fundamental to introductory electromagnetics, from a theoretical framework perspective, they can be understood as a combination of multiple equations under the assumption of electrostatic and magnetostatic conditions. Furthermore, the complexity and lack of duality in these equations indicate that they should be approached with an awareness of their theoretical prerequisites.

By memorizing the layout rules cat.①∼cat.④ and utilizing *"TM"* for self-correction and duality checks in memorization, learners can sufficiently internalize and describe not only the *"shell"* structure shown in Figure 2 but also the surrounding framework structure. Utilizing *"TM"* requires prior learning of fundamental laws and phenomena, but the qualitative aspects of these topics fall within the scope of high school education. Consequently, by combining this with vector calculus, learners can integrate mathematical reasoning, spatial integration and differentiation, and the unit dimensional consistency unique to engineering fields to make *"TM"* functional. Even with an incomplete framework diagram, knowledge can be effectively reinforced if supplemented through *"TM"*-based reasoning.

4 Considerations Based on the Framework Diagram

4.1 Introduction of a Thermodynamic Perspective

The following excerpt from Miyazoe's textbook[2], p.213, is presented:

> ここに述べたいくつかの基礎方程式は同じ形をしている．対応を示せば
>
> **(Traslation)** Several fundamental equations described here share the same form. If correspondences are shown:
>
> $$\begin{array}{ccccc} \mathbb{j} & \mathbb{D} & \mathbb{j}_m & \mathbb{j}_H & \mathbb{B} \\ \mathbb{E} & \mathbb{E} & \mathbb{v} & -\nabla T & \mathbb{H} \\ \sigma & \varepsilon & \rho_m & \kappa & \mu \end{array}$$
>
> となる．基礎方程式が同じ現象は互いにたとえることができる. たとえは親近感を与え, 類推によって記憶や理解を助ける．しかし, 似ていない点まで似ていると早合点したり, 無理に似させようとしてかえって混乱を生じることがある, 特に, 似たことを引合いに出すことは予測や暗示に役立つけれども, 証明にはなっていないことに注意しなければならない.
>
> **(Traslation)** Thus, phenomena governed by identical fundamental equations can be analogized with each other. Analogies provide a sense of familiarity and facilitate memory and understanding through inference. However, one must be cautious not to mistakenly assume that dissimilar aspects are also similar, or to force a resemblance that leads to confusion. In particular, while drawing comparisons can aid in prediction and implication, it should be noted that such analogies do not serve as proof.

This highlights the duality between the laws of electromagnetics and thermodynamics. However, rather than being coincidental, it can be considered as a deliberate design in the theoretical formulation to maintain a consistent structure in quantity definitions. Furthermore, Maeda's textbook[3] states in page 43:

> 電界 $\mathbb{E}$ が式 (1.2) で力によって定義されたベクトルであるのに対して、電束密度 $\mathbb{D}$ は束あるいは流線の概念でとらえるべき物理量である．式 (4.6) によれば, $\mathbb{E}$ と $\mathbb{D}$ は単に定数

だけ違う量にすぎないように見えるけれども，誘電体がある場所に局在したり，不均一な場合には ε は場所の関数となる．

(translation) While the electric field $\mathbb{E}$ is defined as a vector by force in equation (1.2), the electric displacement field $\mathbb{D}$ should be regarded as a physical quantity based on the concept of flux or streamlines. According to equation (4.6), $\mathbb{E}$ and $\mathbb{D}$ may appear to differ only by a constant factor. However, when a dielectric is localized or inhomogeneous, ε becomes a function of position.

.

In this study's framework visualization (framework diagram), the thermodynamic concept of conjugate relationships was further incorporated. By introducing "intensive-like" physical properties (*"intensive"*) and "extensive-like" physical properties (*"extensive"*), analogous to intensive and extensive variables in thermodynamics, the framework diagram structure played a significant role in comprehension. In the vector fields of electromagnetics, $\mathbb{EDBH}$, conventional definitions of intensive variables (which remain unchanged when space is divided) and extensive variables (which change proportionally when space is divided) can not directly apply. The nature of spatial integration/accumulation for intensive and extensive variables is generally non-integrated or integrated, respectively, but in vector fields, the direction of integration affects the result. For example, $\mathbb{E}$ [V/m] accumulates correctly in the series direction, while $\mathbb{D}$ [A·s/m^2] accumulates correctly in the parallel direction.

For theoretical consistency, intensive-like variables (*"intensive"*) were aligned with serial accumulation (line integrals), while extensive-like variables (*"extensive"*) with parallel accumulation (surface integrals), ensuring coherence between variable properties, unit dimensions, and accumulation directionality.

Figure 4 summarizes intensive-like and extensive-like physical properties. This classification is easily understood by considering the integral expressions of Gauss's theorem and Stokes's theorem:

$$\int_{dV} \mathbf{a}\cdot d\mathbb{S} = \int_V \nabla\cdot\mathbf{a}\, dv, \oint_{dS} \mathbf{a}\cdot d\mathbb{s} = \int_S \nabla\times\mathbf{a}\cdot d\mathbb{S}. \tag{7}$$

It should also be noted that in the fundamental equations, medium properties such as ε, μ, and σ are deliberately excluded. However, considering Kirchhoff's laws, the classification also corresponds

	Intensive	**Extensive**
Definition of physical quantity	Accumlate in a line, line-integral	Accumlate in an area, surface integral
Dimensional tendency	[/m]	[/m^2]
Preferred representation	Vortex field	Divergent field
Notation	$\nabla\times\mathbb{X}$	$\nabla\cdot\mathbb{Y}$
Boundary condition	$\mathbb{n}\times(\mathbb{X}_1-\mathbb{X}_2)$	$\mathbb{n}\cdot(\mathbb{Y}_1-\mathbb{Y}_2)$
Related theorem&law	Stokes' theorem, Kirchhoff's voltage law	Gauss' theorem, Kirchhoff's current law
Physical variables	$\mathbb{E}, \mathbb{H}, \mathbb{A}, V$	$\mathbb{D}, \mathbb{B}, \mathbb{j}, I$

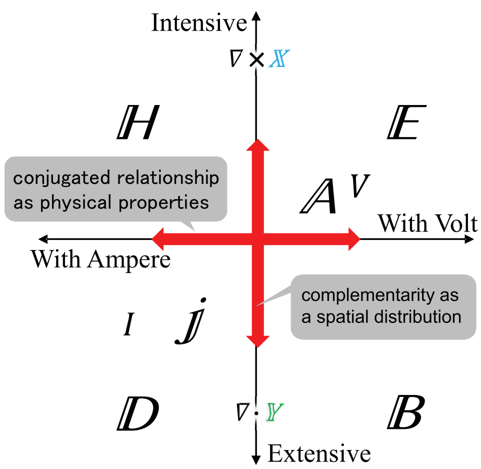


Figure 4: Trends and characteristics of *"intensive"* and *"extensive"* physical properties. (a) summarizes definitions and related concepts. (b) presents these quantities along the vertical axis with *"w/A"* and *"w/V"* on the horizontal axis, illustrating their relationships.

to the relationship between voltage V and current I. With this classification, rather than regarding $\mathbb{E}$ and $\mathbb{D}$ as merely corrected boundary conditions of the same physical quantity, they can be understood as playing roles analogous to voltage and current. As previously discussed, the *"conjugate"*-type equations for spatial energy density $\Delta u_E = \Delta\mathbb{D}\cdot\mathbb{E}$, $\Delta u_M = \Delta\mathbb{B}\cdot\mathbb{H}$ (and in linear conditions, their integrals $u_E = \frac{1}{2}\mathbb{D}\cdot\mathbb{E}$, $u_M = \frac{1}{2}\mathbb{B}\cdot\mathbb{H}$) indicate that *"intensive"* and *"extensive"* variables are not necessarily always in conjugate relationships. This distinction arises because they are also categorized separately under *"w/V"* and *"w/A"*. Another similar variable pair is $\mathbb{A}$ and $\mathbb{j}$. Meanwhile, even the pair of *"intensive"* variables $\mathbb{E}$ and $\mathbb{H}$ form expressions such as $\mathbb{S}_P = \mathbb{E}\times\mathbb{H}\,[\mathrm{W/m^2}]$, $\sqrt{\varepsilon}\mathbb{E} = \sqrt{\mu}\mathbb{H}$. A notable observation is that when determining whether $\Delta\mathbb{D}\cdot\mathbb{E}$ or $\Delta\mathbb{E}\cdot\mathbb{D}$ is correct for small variations, the *"extensive"* side always carries the Δ notation. The *"constitutive"* type fundamental equations, which form *"constitutive"*–*"conjugate"* pairs, include: $\mathbb{D} = \varepsilon\mathbb{E}$, $\mathbb{B} = \mu\mathbb{H}$, $\mathbb{j} = \sigma\mathbb{E}$, and demonstrate simultaneous transitions between *"intensive"*/*"extensive"* and *"w/V"*/*"w/A"* attributes. Organizing the equations such that the left-hand side contains *"extensive"* variables while the right-hand side contains *"intensive"* variables helps prevent memorization confusion. In this context, the electrical conductivity $\sigma\,[\Omega/\mathrm{m}]$ is advantageous for memory retention using *"TM"* and duality principles, as its unit dimension $[\mathrm{m}^{-1}]$ aligns with $\varepsilon\,[\mho{\cdot}\mathrm{s/m}]$ and $\mu\,[\Omega{\cdot}\mathrm{s/m}]$, making it more intuitive than resistivity $\rho_R\,[\Omega{\cdot}\mathrm{m}]$.

In this framework, the fundamental equations are referenced from textbooks that follow the EB convention. Observing the framework diagram reveals that the dual relationship among $\mathbb{E}$, $\mathbb{D}$, $\mathbb{B}$, and $\mathbb{H}$ varies—sometimes appearing between $\mathbb{E} \Leftrightarrow \mathbb{B}$ and other times between $\mathbb{E} \Leftrightarrow \mathbb{H}$—demonstrating a well-defined structure.

4.2 Framework Diagram as an actual DIKW Learning Model

The DIKW (Data, Information, Knowledge, Wisdom) framework, originally proposed by R. L. Ackoff[16] has been widelyexamined across various disciplines, including corporate management and learning processes. The DIKW (Data, Information, Knowledge, Wisdom) framework, originally proposed by R. L. Ackoff[16], has been widely examined across various disciplines, including corporate management and educational theory

This concept categorizes "knowledge" into hierarchical stages, beginning with **Data**, which is associated with various attributes and analyzed to form **Information**. Within Information, networks are constructed, allowing for the identification of patterns and similarities, leading to what is termed **Knowledge**. At this stage, **Insights** becomes possible, and this is considered a key characteristic preceding the final DIKW stage of **Wisdom**. The specifics of these stages are open to interpretation, and the actual process of intellectual advancement varies by domain, often leading to conceptual explanations. Consequently, educators face challenges in visually presenting what DIKW represents concretely for learners.

This framework conceptualizes "knowledge" as a hierarchical progression, beginning with **Data**, which consists of raw facts that acquire meaning through contextualization and analysis, forming **Information**. Within this stage, structured relationships emerge, enabling the identification of patterns and correlations, which in turn constitute **Knowledge**. At this level, the capacity for **Insight** develops, serving as a distinguishing feature of cognition that precedes the final stage of **Wisdom**. However, the delineation of these stages remains subject to interpretation, as the process of intellectual development varies by domain, often resulting in differing theoretical perspectives. Consequently, a persistent challenge in education lies in effectively visualizing and concretizing the DIKW model to enhance learners' comprehension.

Figure 5 illustrates the steps in constructing the framework diagram from the perspective of DIKW knowledge reconstruction, following the sequence from (1) to (4). Initially, textbooks present

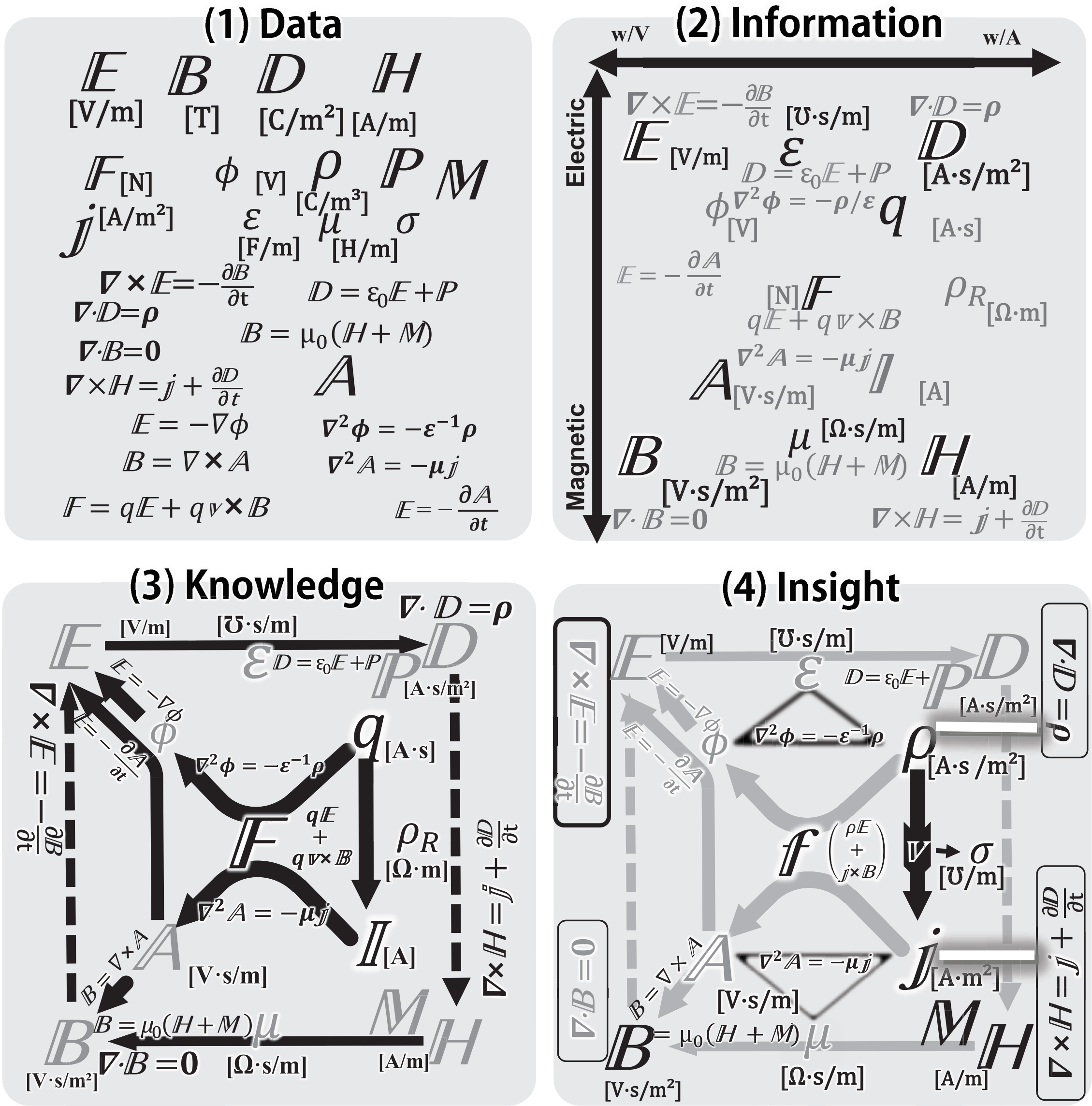


Figure 5: Framework diagram as a DIKW learning model. (1) In the Data stage, physical variables and fundamental equations are merely listed without structure or duality. (2) In the Information stage, each variable is analyzed based on unit dimensions, and attributes such as *"w/A"* and *"w/V"* are used to create layout rules, pairing key Maxwell's equations with physical variables. (3) In the Knowledge stage, variables are interconnected with arrows, and fundamental equations are linked to these connections. (4) In the Insight stage, inconsistencies in the structure become apparent, and refinements are made, such as repositioning ρ and $\mathbb{j}$ to align better with Maxwell's equations.

theoretical explanations, physical phenomena, and fundamental equations sequentially. Step (1) in Figure 5 represents the Data stage, where these elements are recorded as they appear. In step (2), the Information stage, distinctions between electric and magnetic fields and the unit-based categorization of variables as *"w/A"* or *"w/V"* are introduced, creating layout rules that pair fundamental Maxwell's equations with physical variables.

In step (3), the Knowledge stage, physical variables are connected with arrows, and fundamental equations are linked to these connections. Upon analyzing this structure to identify underlying patterns, inconsistencies and areas for refinement emerge, leading to step (4), where Insights (indicated by black or highlighted horizontal lines) are introduced. Several specific improvements are listed below:

- Charge q [A·s] should be replaced with charge density ρ [A·s/m^3], and current I [A] should be replaced with current density vector $\mathbb{j}$ [A/m^2].
- Correspondingly, the Lorentz force should be expressed in its differential form $\mathbb{f}$ [N/m^3] rather than its integral form $\mathbb{F}$ [N].
- The connection between ρ and $\mathbb{j}$ is represented by velocity $\mathbb{v}$ [m/s], and rather than linking $\mathbb{v}$ to resistivity ρ_R [Ω·m], it should be linked to conductivity σ [℧/m].
- The Maxwell's equations $\nabla\cdot\mathbb{D} = \rho$ and $\nabla\times\mathbb{H} = \mathbb{j} + \partial\mathbb{D}/\partial t$ should also be directly connected to ρ and $\mathbb{j}$.

Through these modifications, the Knowledge stage (3) is refined, improving mathematical and structural consistency. The classification of equations in step (4) Insight—such as pairing *"conjugate"* and *"constitutive"*types, ensuring that vertically arranged equations do not cross *"w/A"* and *"w/V"* categories, and recognizing Coulomb's and Biot–Savart's laws as composite equations—can be considered the extraction of Wisdom from the framework diagram. Following Figure 5(4), Wisdom is derived from the framework by extracting predictions and insights. The following are some of the observations made by the author:

Visualization of the Importance of Vector Potential: In the framework diagram, $\mathbb{A}$ is positioned within the *"shell"* and is related to six physical variables (the most among all), indicating its significant importance as a physical variable. The diagram illustrates that without $\mathbb{A}$, the theoretical framework would lose much of its duality. The necessity of $\mathbb{A}$ in the theoretical framework suggests why it was conceptually utilized long before its experimental verification in 1982 by Tonomura[17].

Comparison of Permittivity ε, Permeability μ, Electrical Conductivity σ, and Resistivity ρ_R: In Coulomb's law, ε appears in the denominator, whereas in the Biot–Savart law, μ moves to the numerator. This asymmetry is naturally explained by examining the *"shell"* of $\mathbb{EDBH}$, clarifying the rationale that μ is more fundamental than a coefficient such as $\kappa = \mu^{-1}$. Additionally, the relationship between resistivity ρ_R [Ω·m] and electrical conductivity σ [℧/m] is explicitly shown in terms of their contribution to duality. Poisson's equation also reveals this asymmetry, with the right-hand side expressed as $-\varepsilon^{-1}\rho$ and $-\mu\mathbb{j}$. The term ε^{-1} exhibits an asymmetry, as the transition from ρ to ϕ requires reversing the direction of the arrow associated with ε, which manifests as the coefficient ε^{-1}. To explicitly maintain the duality in equations, the framework diagram adopts the notation $\varepsilon^{-1}\rho$ instead of ρ/ε.

Confusion Between Intensive-like and Extensive-like Variables in a Uniform Linear Space: For example, in a vacuum where $\mathbb{D} = \varepsilon_0\mathbb{E}$ with isotropic and constant ε_0 and $\mathbb{P} = 0$, both $\mathbb{E}$ and $\mathbb{D}$ satisfy *"intensive"* and *"extensive"* accumulation conditions despite differing boundary conditions. Under these conditions, the fundamental equation $\nabla\cdot\mathbb{E} = \varepsilon^{-1}\rho$ holds, making the distinction between $\mathbb{E}$ and $\mathbb{D}$ evident only in dimensional analysis rather than in the governing equations. This equation presents a challenge in interpreting its physical meaning since both sides have units of $\varepsilon^{-1}\rho$ [V/m^2] or, upon integration, [V·m], which differs from $\mathbb{D}$.

Fundamental Equations Using the d'Alembertian Operator: Several textbooks[2, 6, 8, 10] introduce the d'Alembertian operator

$$\square^2 = \nabla^2 - c^{-2}\frac{\partial}{\partial t} \tag{8}$$

as a means to simplify Maxwell's equations. In the Lorenz gauge, the vacuum electromagnetic field can be expressed using $\Box^2$ and the electromagnetic potential $\mathbb{A}_4$ as

$$\Box^2 \mathbb{A}_4 = -\mu_0 \mathbb{j}_4. \tag{9}$$

This equation represents a single *"constitutive"* fundamental equation, where the four-dimensional current density $\mathbb{j}_4$ and vector potential $\mathbb{A}_4$ are defined as

$$\mathbb{j}_4 = (j_x, j_y, j_z, c\rho), \quad \mathbb{A}_4 = (A_x, A_y, A_z, c^{-1}\phi). \tag{10}$$

Miyazoe further extended this concept[2], defining the complex operator as

$$\Box = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, j^{-1}c^{-1}\frac{\partial}{\partial t}\right), \tag{11}$$

and representing the current and potential as

$$\mathbb{j}_4 = (j_x, j_y, j_z, jc\rho), \quad \mathbb{A}_4 = (A_x, A_y, A_z, jc^{-1}\phi). \tag{12}$$

This formulation demonstrates that charge conservation (*"continunity"*) is expressed as $\Box \cdot \mathbb{j}_4 = 0$. Observing the remaining *"conjugate"* equation within the framework diagram, the author inferred that a conjugate pair must also exist, leading to the potential energy equation $u = -\mathbb{A}_4 \cdot \mathbb{j}_4$, classified as a *"conjugate"* equation. In fact, this equation is a linear combination of $u = -\mathbb{A} \cdot \mathbb{j}$ and $u = \phi\rho$, confirming the inference. Using Miyazoe's definitions of $\Box$, $\mathbb{A}_4$, and $\mathbb{j}_4$, the fundamental equations in vacuum can be expressed in the following simplified form:

$$\Box^2 \mathbb{A}_4 = -\mu_0 \mathbb{j}_4 \tag{13}$$
$$\Box \cdot \mathbb{j}_4 = 0 \tag{14}$$
$$u = -\mathbb{A}_4 \cdot \mathbb{j}_4 \tag{15}$$

These equations reduce to one *"constitutive"* equation for field description, one *"continunity"* equation for charge conservation, and one *"conjugate"* equation for the poteintial energy associated with the Lorentz force.

4.3 Curriculum Structure for electromagnetics Lectures Using the Framework Diagram

In the Department of Electrical Engineering and Computer Science at Kyushu University, electromagnetics is taught through three different courses: ① *Fundamentals of electromagnetics* in the general education curriculum (one semester, 8 lectures), ② *electromagnetics I, II, III* starting from the second semester of the second year (twice a week, 16 sessions × 3), and ③ *Fundamentals of Electrical Engineering* (effectively 3 sessions) for students outside Group I.

Teaching the physical variables and fundamental equations of electromagnetics efficiently across multiple stages is challenging. As most textbooks illustrate, the curriculum of electromagnetics follows a sequence: ① Electrostatics, ② Dielectrics and Conductors, ③ Magnetostatics, ④ Magnetic Materials, and ⑤ Interaction between Electric and Magnetic Fields. This sequence makes it difficult to divide the subject into discrete segments. Along this sequence, the correct analytical decomposition approach in physics leads **Fundamentals of electromagnetics** to one of its key destinations, Maxwell's equations.

At the same time, several educational aspects must be integrated throughout all five stages: (A) electromagnetics as an **Applied Vector Analysis**, (B) The **Material Science Perspective** of various electromagnetic phenomena (conductors, dielectrics, and magnetic materials), (C) Understanding vector fields as a foundation for **Electromagnetic Wave Engineering**, and (D) The **Circuit Theory Approach**, including spatial integration leading to lumped elements. All these aspects are interwoven across ①–⑤, meaning that only after learning the entire framework can knowledge be fully reconstructed.

In large-scale learning structures like this, a three-session course must selectively extract topics from ①–⑤. However, because Maxwell's equations serve as a primary goal, all necessary components must be included, leading to an overwhelming amount of information. For students without a foundation in vector analysis, introducing content beyond the high school curriculum is particularly challenging. When approximately eight lectures are available, all necessary components can be covered, but it is crucial to actively encourage knowledge reconstruction through assignments to ensure long-term learning retention. On the other hand, in a 48-lecture format, more advanced topics beyond *Fundamentals of electromagnetics* (e.g., the method of images, Maxwell's stress tensor, potential coefficients, Neumann's formula, and composite media including dielectrics and conductors) can be addressed. However, if students have not mastered *Fundamentals of electromagnetics*, comprehending these advanced topics remains difficult. Since a 48-lecture course requires retaining knowledge from six months prior, long-term retention of foundational theories is essential.

Based on these considerations, a curriculum incorporating the staged approach of (A)–(D) was developed using the framework diagram, as shown in Figure 6. Figures 6(a)–(c) focus on understanding electromagnetics as Applied Vector Analysis (A). The aim is to understand and reinforce memory of the *"shell"* of $\mathbb{EDBH}$ with a focus on the differential forms of Maxwell's equations. Maxwell's equations serve as the foundation for problem-solving, deepening the understanding of Helmholtz's theorem, Gauss's theorem, and Stokes's theorem. To grasp $\mathbb{EDBH}$, even though material boundaries do not exist in a vacuum, *"intensive"* and *"extensive"* properties are not omitted, as distinguishing them conceptually is as crucial as distinguishing between current and voltage. Figure 6(b) presents the arrangement and relationships of charge density ρ, current density $\mathbb{j}$, and Lorentz force $\mathbb{f}$. Simultaneously, it separates the concept of potential fields ϕ, $\mathbb{A}$, which generate forces, from charge and current ρ, $\mathbb{j}$, which experience those forces. In Figure 6(c), potentials ϕ and $\mathbb{A}$ are introduced using the Poisson equation, illustrating their similarity in electrostatic and magnetostatic fields. These are then extended to Coulomb's and Biot–Savart's laws (which are nearly identical in form). The final step in (a)–(c) derives the field expressions $\mathbb{E}$ and $\mathbb{B}$ via Coulomb's and Biot–Savart's laws, which lack duality. At this stage, the most fundamental steps are complete. Figure 6(d) explains electromagnetic interactions, Maxwell's equations, the transformation $\mathbb{A} \to \mathbb{E}$, and electromagnetic waves. Figure 6(e) connects electromagnetics with material science by introducing the three media: dielectrics, magnetic materials, and conductors. The concepts of $\mathbb{P}$ and $\mathbb{M}$ are explored, particularly at the atomic level. Figure 6(f) focuses on spatial integration of physical constants and its connection to lumped element components. Once spatial integration (including scalar products) is understood, the final outcome aligns with high school circuit analysis. Figure 6(g) introduces macroscopic moments and, if necessary, provides an introduction to multipole expansion. This curriculum design provides a structured approach to learning electromagnetics using the framework diagram, ensuring that concepts are reinforced in a logical progression while incorporating various perspectives in electromagnetics education.

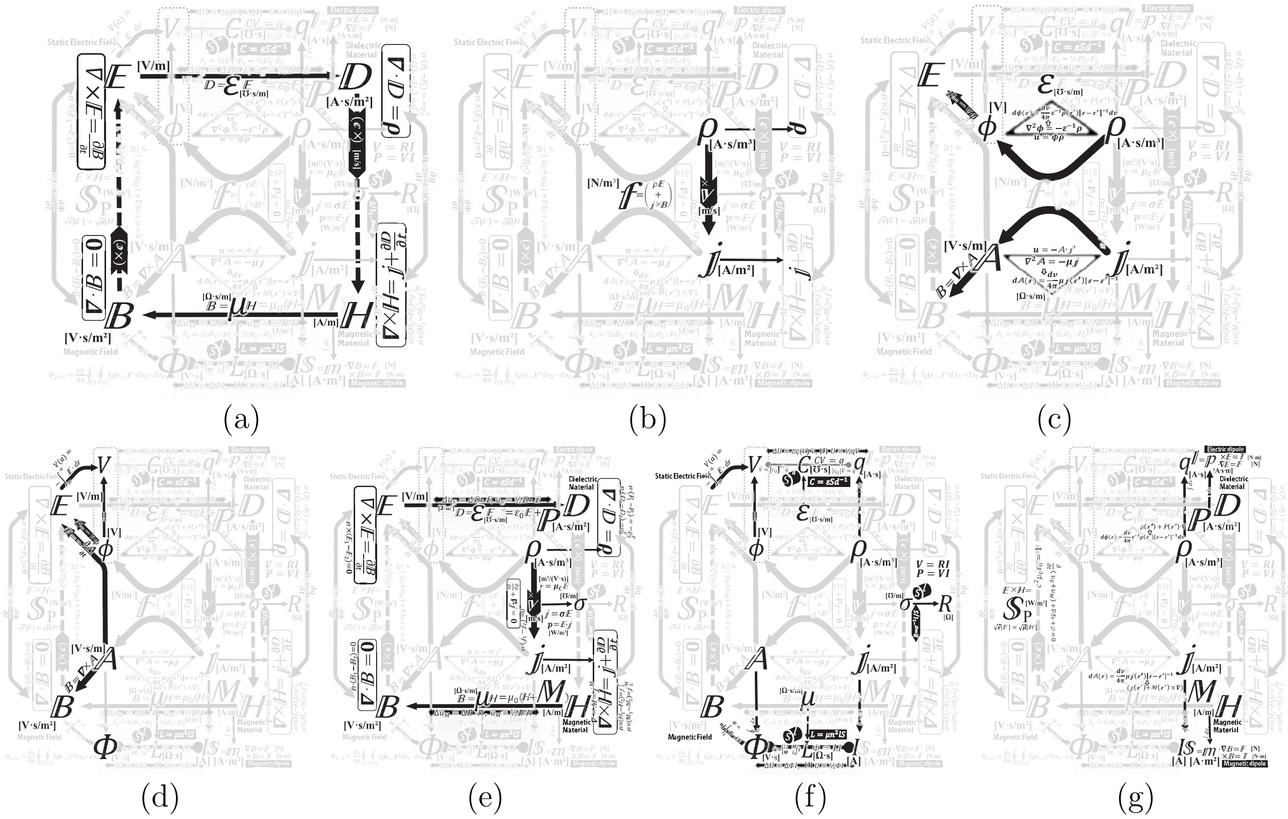


Figure 6: Example of curriculum construction based on the framework diagram. (a) Fundamentals 1: $\mathbb{E}\mathbb{D}\mathbb{B}\mathbb{H}$ and Maxwell's equations, (b) Fundamentals 2: Lorentz force, charge conservation, charge and current, (c) Fundamentals 3: Field representation and potential, (d) Applications 1: Field representation, electromagnetic induction, and electromagnetic potential, (e) Influence of media, (f) Connection to lumped elements and electrical circuits, (g) Moments and the Poynting vector.

4.4 Empirical Evidence of Educational Effectiveness Using the Framework Diagram

Since 2023, lectures incorporating the framework diagram have been conducted. However, as no formal comparative study has been implemented, the effectiveness of these lectures in terms of comprehension has yet to be objectively measured. To address this, an open-ended assignment has been introduced as an applied task: drawing the entire diagram from just memory, without referring to any materials. Completing the full diagram takes approximately 40 minutes, and nearly 70% of students submitted assignments that were almost entirely accurate. This approach has also yielded notable results in graduate school entrance exams for electromagnetics.

The following is the order of description used by the author for constructing the framework diagram:

① Draw the *"shell"* of $\mathbb{E}$, $\mathbb{D}$, $\mathbb{B}$, and $\mathbb{H}$. Add the clockwise arrows ($\rightarrow$) and the medium constants ε, $\mathbb{c}$, μ, and $\mathbb{c}$.

② Write the four differential form Maxwell's equations in a vertical arrangement. Ensure that they are placed diagonally symmetrically while separating the right and left variables without medium constants. Also, include the units of each variable. Even if the units are unclear, the relationships and diagonal symmetry should be used to complete them.

③ Fill in the interior of the *"shell"*. Place ρ, $\mathbb{j}$, and the potentials $\mathbb{A}$ and ϕ, then write Poisson's equation and the potential energy equation (*"conjugate"*–*"constitutive"*-pairs) horizontally. Finally, write the Lorentz force $\mathbb{f}$ at the center. Also, attach one *"constitutive"*-type equation to each arrow connecting the variables.

④ Write $\mathbb{P}$ and $\mathbb{M}$ near $\mathbb{D}$ and $\mathbb{H}$ and complete the definition equations for $\mathbb{D}$ and $\mathbb{B}$ (characteristic equations for the three types of media). Since these equations are also *"conjugate"*–*"constitutive"*-pairs, do not forget to include the energy definition equation. For the third medium, the conductor, write its description on the right side of the vertical center. Again, do not forget to include the paired equations.

⑤ Write the boundary conditions. Use the similarity to Maxwell's equations to list them alongside.

⑥ Write the continuity equations.

⑦ Write Coulomb's law and the Biot–Savart law.

⑧ Write the equations involving spatial integration outside the *"shell"*. The upper section corresponds to the *"constitutive"* type equation $q = CV$, and the lower section corresponds to $\phi = LI$. The connection from inside the *"shell"* is through *"integrate"*-type spatial integration. The nature of the integration can be inferred based on the differences in dimensions. The conjugate type equations corresponding to the upper and lower *"constitutive"*-type equations should also be written. For the third medium, the conductor, write only R from the equations $V = RI$ and $P = VI$ (as V and I are already present, and their relative positions in the framework diagram are distant).

⑨ Since moments are also connected through *"integrate"*-type spatial integration, place $\mathbb{p}$ and $\mathbb{m}$ above and below $\mathbb{P}$ and $\mathbb{M}$, respectively, and describe their relationships (*"constitutive"*-type) with q and I. As conjugate type equations involving $\mathbb{p}$ and $\mathbb{m}$, describe translational force $\mathbb{F}$, torque vector $\mathbb{T}$, and the Coulomb and Biot–Savart equations, which describe potential fields.

⑩ Add the descriptions related to electromagnetic waves on the left side.

4.5 Potential for Application to Other Fields

While the framework diagram method is effective in physics when combined with *"TM"*, further investigation is needed to determine its applicability to other disciplines. However, the creation of diagrams that provide an overview of a theoretical framework is considered beneficial for assessing student comprehension, as noted in Section 4.4 regarding the open-ended assignment. This method facilitates rapid assessment of student understanding.

4.6 Terms Used Exclusively in This Paper

In this paper, **terminology** is deliberately used in itallic for newly introduced concepts, including Trinity Memorization, to emphasize their introduction as novel concepts. In Japanese-language education, replacing these terms with Japanese equivalents can reduce cognitive load for learners by facilitating comprehension. The following is a summary, including their corresponding Japanese equivalents:

Unit Conversions

[℧] = [s]: The unit of electrical conductivity, siemens. The historical unit ℧, previously used, explicitly represents [Ω^{-1}], making it more intuitive for learners. This notation is adopted in this paper.

[A·s] = [C]: The unit of electric charge, coulomb.

[V·s] = [Wb]: The unit of magnetic flux, weber.

[℧·s] = [F]: The unit of capacitance, farad.

[Ω·s] = [H]: The unit of inductance, henry.

[V·s/m^2] = [T]: The unit of magnetic flux density, tesla.

Unit conversions are used for internal reasoning, while standard units are adopted for reading and final documentation.

Concepts Related to Spatial Integration of Physical Quantities and Unit Attributes

***"intensive"*:** A label for physical variables subject to intensive-like properties, like intensive-variables in thermodynamics, that are accumulated along a line and integrated over its length. The unit dimension includes [m^{-1}].

***"extensive"*:** A label for physical variables subject to extensive-like properties, like extensive-variables in thermodynamics, that are accumulated over an area and integrated over a surface. The unit dimension includes [m^{-2}]. The product *"intensive"*· *"extensive"* results in a physical quantity obtained via volume integration.

***"w/A"* = Ampere-inclusive:** A label for physical variables subject to Ampere-inclusive properties, that are assigned a unit which includes ampere. These variables are in a conjugate relationship with *"w/V"*.

***"w/V"* = Volt-inclusive:** A label for physical variables subject to Volt-inclusive properties, that are assigned a unit which includes volt. These variables are in a **conjugate relationship** with *"w/A"*.

Classification of Fundamental Equations (and Layout Rules)

***"constitutive"*= Constitutive Fundamental Equations:** Fundamental equations describing relationships between different physical variables. In this paper, even among *"constitutive"* equations, two subcategories exist depending on whether the linked variable pair forms a conjugate relationship. These equations are always attached to $\Longrightarrow$ links between the variables in the framework diagram. If a paired *"conjugate"* equation exists, the equation is primarily arranged horizontally; otherwise, it is oriented vertically or diagonally.

***"conjugate"* = Conjugate Fundamental Equations:** Equations where the right-hand side consists of the product of conjugate physical variables. The left-hand side represents energy-related quantities (force, torque, power, or energy density). These equations are not attached to arrows in the framework diagram but exist in pairs with *"constitutive"* equations, which dictate their layout. Conjugate relationships are not commonly emphasized in electromagnetics and are introduced as a concept derived from thermodynamics. Exceptions include Lorentz force and moment torque, which do not have a paired *"constitutive"* equation.

"continunity" = **Continuity Fundamental Equations:** Conservation equations structured as sums of multiple terms set to zero. Each term maintains consistent unit dimensions, leading to vertical alignment in the framework diagram. Unlike other equations, these do not necessarily correspond to arrows connecting different physical variables. Although Maxwell's equations and boundary conditions differ in their physical meaning, they share the same structural form and are thus categorized as continuity equations.

"integrate" = **Spatial Integration Fundamental Equations:** Fundamental equations linking variables per unit volume with their integrated counterparts through spatial integration. These equations are attached to arrows in the framework diagram.

Organizing layout rules and classifications for each equation type facilitates accurate reproduction of the framework diagram without omitting fundamental equations.

In addition to these newly introduced English terms, the following two terms are used in this paper:

TM (Trinity Memorization) = 三位一体記憶:

counterpart: A symmetric position in the framework diagram, where an exchange occurs between *"intensive"* and *"extensive"* properties as well as between *"w/A"* and *"w/V"*. This term indicates variable pairs occupying symmetric positions where both exchanges occur simultaneously.

5 Conclusion

A visualization method for the theoretical framework of elementary electromagnetics was proposed to enhance learning effectiveness. This method organizes the relationships between physical quantities and fundamental equations in a two-dimensional structure centered around Maxwell's equations, allowing complex concepts to be understood visually. The visualization process categorizes and arranges physical quantities based on principles such as the characteristics of electric and magnetic fields, dimensional analysis of units, and duality, ensuring that learners can intuitively grasp the essence of physical phenomena. Additionally, Trinity Memorization was introduced to reinforce knowledge retention, providing a learning approach that integrates mathematical consistency, dimensional coherence, and theoretical rationality. Through this framework, learners are expected to achieve long-term knowledge retention and self-correction. Future work will explore the applicability of this approach to other disciplines while further evaluating and refining the educational methodology using the framework diagram.

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This paper presents the results of an attempt to visualize the theoretical framework of elementary electromagnetics. By incorporating the DIKW approach and employing labeling based on four categories, each physical quantity was successfully represented in a two-dimensional network. The opportunity to engage in fundamental electromagnetics education at Kyushu University's Faculty of Arts and Science was instrumental in developing these ideas. Special thanks are extended to Professor Kenji Hayashi of Kyushu University for insights on thermodynamics and to Professor Kazutoshi Kato for the idea of linking "EDBH" with ε, μ, and c in electromagnetics. Gratitude is also expressed to Professor Nobuhiko Sarukura and Mr. Kin'ichi Morita from Osaka University for their valuable feedback and suggestions for improvement.

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