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Second-price Auctions with Information Acquisition Costs

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Abstract

We analyze bidders' information acquisition behaviors in second-price private value auctions. In our model, each bidder lacks knowledge for their own valuation of the object for sale and can only discover it by incurring a cost. We investigate the pure strategy equilibrium of information acquisition, providing conditions for the existence of equilibrium and proposing an algorithm for discovering all equilibria. Our findings suggest that the number of informed bidders remains constant across all equilibria. Furthermore, we demonstrate that the expected revenue is a decreasing function of the number of uninformed bidders.

Keywords: Acquisition costs, Equilibrium, Information acquisition, Second-price auction

JEL Classification: C62 , C72 , D44 , D82

1 Introduction

Evaluating the valuation of an asset comes with associated costs. For example, when acquiring a company, the buyer must estimate its future value in order to determine its current market price. When considering the purchase of underground resources for private use, it is crucial to engage experts in estimating the extractable quantities. In sports, signing a player often entails incurring expenses to predict their future performance. When the expenses incurred for these evaluations are substantial, buyers may opt to forego expensive assessments. Instead, they may rely on existing information to make decisions. Conversely, when the cost of obtaining information is low, there is an incentive to acquire it and base bidding decisions on the acquired valuation.

Furthermore, the incentive for information acquisition depends on other auction participants. This is because the incentive for information acquisition (or the value of information) differs when everyone else acquires information compared to when no one else acquires it. In this sense, information acquisition behavior can be viewed as strategic decision-making.

In this study, we analyze information acquisition behaviors in sealed-bid second-price auctions. We assume that each bidder lacks the knowledge required to make their private valuation of the object being auctioned. In the standard auction model, bidders are privy only to their own valuations and remain unaware of those of other bidders. In our model, each bidder has the option of acquiring information concerning their own valuation by incurring a cost. Acquiring this information models, for example, commissioning experts to estimate the object's worth. Meanwhile, each bidder remains uninformed about other bidders' valuations and, furthermore, does not know which other bidders have acquired information. Following the information acquisition decision, each bidder determines their bids.

In our model, bidder i's valuation is drawn from distribution function F_i. Our equilibrium concept is based on the bidders' mutually dependent incentives for information acquisition. We establish the conditions for the existence of an information acquisition equilibrium (IAE) and demonstrate the process of finding such an equilibrium by introducing an algorithm. Furthermore, we illustrate that the expected revenue of an auctioneer increases with the increasing number of informed bidders.

Our study relates to various strands of literature on second-price auctions. Cao et al. (2017) provided conditions for the existence of a unique equilibrium in second-price auctions, assuming that bidders are informed about their valuations and participation costs and that the joint distributions of these valuations and costs across bidders are not necessarily identical.[1] Tan and Yilankaya (2006) offered conditions for the existence of an equilibrium in second-price auctions with bidder participation costs. They showed that if bidders are symmetric, the concavity of the distribution function for valuations serves as a sufficient condition for the uniqueness of equilibrium. They also investigated the condition in a special case involving asymmetric bidders. While Cao et al. (2017) and Tan and Yilankaya (2006) established the existence of an equilibrium in second-price auctions with participation costs by using fixed-point theorems,

[1]Moreover, Bobkova (2024) assumed that in a two-bidder auction, a valuation consists of a common value component, which is relevant to both bidders, and two private value components, each relevant to the respective bidder. Furthermore, Milgrom and Weber (1982) demonstrated that sellers can increase expected revenue by providing signals, such as expert appraisals of the quality of items for sale, in auctions with affiliated values. Shi (2012) conducted a study on optimal auctions with information acquisition, focusing on mechanism design.

we provide a constructive algorithm for finding equilibria, focusing on information the acquisition behavior of bidders in second-price auctions.[2]

Compte and Jehiel (2007) studied information acquisition behavior by comparing dynamic auction formats (e.g., ascending-price or multistage) with sealed-bid formats, where valuations were discretely distributed on a finite set. In contrast, our model assumes that valuations are distributed over a continuous real interval. Moreover, we introduce the concept of an information acquisition equilibrium and provide a condition for an equilibrium to exist in a second-price auction.[3]

The remainder of this paper is organized as follows. Section 2 introduces the proposed model and definitions. Section 3 presents the conditions for the existence of an equilibrium, properties of the equilibrium concept, and a constructive algorithm for finding an equilibrium. Section 4 analyzes the relationship between the expected revenue and the number of informed/uninformed bidders. Section 5 provides a summary and additional remarks. The proofs are provided in Section 6.

2 Preliminaries

2.1 Model

There is one indivisible object for sale, worth zero to the seller, and there are n risk-neutral buyers, i.e., bidders. Let $N = \{1, ..., n\}$ be the set of all bidders with $n \geq 2$. The auction format is a sealed-bid second-price auction. For every $i \in N$ and $j \in N$, the valuations $v_i, v_j \in V = [\underline{v}, \overline{v}]$ are assumed to be independently drawn from distribution functions $F_i : V \to [0, 1]$ and $F_j : V \to [0, 1]$, respectively. For every $i \in N$, let f_i be the corresponding density function and EV_i denote the expected valuation: $EV_i := \int_{\underline{v}}^{\overline{v}} \{v_i \cdot f_i(v_i)\} dv_i$.

Each bidder's decision-making involves two stages:

1. deciding whether to acquire information, and
2. determining the value of their bid in a second-price auction.

Each bidder $i \in N$ does not know their own valuation v_i of the object prior to making a bid. They independently decide whether to acquire information about v_i. Bidders can be heterogenous in terms of their information acquisition costs while those costs are common knowledge. Once a bidder incurs his information acquisition cost, he is fully informed of his own valuation, privately. If bidder i decides to acquire information, i becomes aware of the value of v_i and incurs an acquisition cost $c_i \in C = [\underline{c}, \overline{c}]$.[4] Even after bidder i acquires information, the bidder does not know the valuation v_j of any other bidder j, and furthermore, does not have information about which other bidder

[2] For example, Cao et al. (2017) adopted the Schauder-Tychonoff fixed-point theorem, which states that any continuous mapping from a nonempty compact convex subset of a locally convex topological space to itself has a fixed point.

[3] Therefore our research is also closely related to the information structure in an auction. Hausch and Li (1993) analyzed each bidder's entry decision and information acquisition costs in a common-value auction. Stegeman (1996) considered bidders with private valuations for an object being auctioned and the costs associated with making a bid. Persico (2000) examined a model in which costs varied based on the quality of information in affiliated value auctions.

[4] The results provided in this paper can be generalized to the model in which a bidder i does not know the value of c_i but knows the distribution of c_i. See Abe, Onda, and Yamato (2024).

j has acquired information. We use q_i to represent bidder i's decision on information acquisition, which can be either "Yes" or "No." Let $q_i \in \{1, 0\}$, where 1 represents "Yes" (or "acquire").

2.2 Information acquisition equilibrium

Since the second stage is a second-price auction, bidding truthfully is a weakly dominant strategy for bidders. When a bidder does not acquire information, bidding the expected valuation is a weakly dominant strategy. Therefore, our main task is to determine which subset of the bidders will choose to acquire information in the first stage. This is equivalent to finding a Nash equilibrium in a complete information setting.

We suppose that each bidder i chooses q_i to maximize their ex ante expected payoff. Bidder i's ex ante expected payoff depends on who else acquire information. For each $i \in N$, let $M \subseteq N \setminus \{i\}$ represent a set of uninformed bidders except for i. Selecting M is equivalent to choosing $q_{-i} \in \{1, 0\}^{N \setminus \{i\}}$ by setting $M(q, i) = \{j \in N \setminus \{i\} | q_j = 0\}$.

Let $EU_i^{\text{Yes}}(N, M)$ represent bidder i's ex ante expected payoff when selecting $q_i = 1$ for a set $M \subseteq N \setminus \{i\}$ of uninformed bidders. Similarly, let $EU_i^{\text{No}}(N, M)$ denote the ex ante expected payoff that bidder i anticipates by choosing $q_i = 0$. The following definition of equilibrium states that no bidder has an incentive to switch their information acquisition decision q_i.

Definition 1. *Profile $q \in \{1, 0\}^N$ is an information acquisition equilibrium (IAE) if for every $i \in N$,*

- $EU_i^{Yes}(N, M(q, i)) \geq EU_i^{No}(N, M(q, i))$ *if* $q_i = 1$, *and*
- $EU_i^{Yes}(N, M(q, i)) \leq EU_i^{No}(N, M(q, i))$ *if* $q_i = 0$.

Note that we focus on pure strategy equilibria. The first line requires that, considering the choices made by other bidders, any bidder opting for "Yes" has no incentive to switch their choice to "No," while the second line requires the opposite scenario for those initially selecting "No." In other words, an IAE can be viewed as a Nash equilibrium for bidders' information acquisition because all bidders select their best responses to opponents' choices.

Since there is a one-to-one mapping between $M \subseteq N$ and $q \in \{1, 0\}^N$ given by $M(q) = \{j \in N | q_j = 0\}$, we can define an IAE in terms of M as well as q. For every $i \in N$, we define the following function s_i: for every $M \subseteq N \setminus \{i\}$,

$$s_i(N, M) := EU_i^{\text{Yes}}(N, M) - EU_i^{\text{No}}(N, M).$$

Function s_i enables us to define bidder i's best response function explicitly as follows: for every $M \subseteq N \setminus \{i\}$,

$$BR_i(N, M) = \begin{cases} \{1\} & \text{if } s_i(N, M) > 0, \\ \{1, 0\} & \text{if } s_i(M, M) = 0, \\ \{0\} & \text{if } s_i(N, M) < 0. \end{cases}$$

Therefore, an IAE is equivalently defined in terms of $(s_i)_{i \in N}$ and M as follows: $M \subseteq N$ is an IAE if

- for every $i \in N \setminus M$, $s_i(N,M) \geq 0$ and
- for every $i \in M$, $s_i(N, M \setminus \{i\}) \leq 0$.

The first (second) line requires that all bidders who choose Yes (No) continue choosing Yes (No).

Now,we present explicit formulas for EU_i^{Yes} and EU_i^{No}. Lemma 1 indicates that bidder i's decision depends on the distribution functions $(F_j)_{j\in N}$ of all bidders and bidder i's acquisition cost c_i. In the following formulas, for each $X \subseteq N$, let $\Pi_{j\in X} F_j(h) = 1$ if X is an empty set.

Lemma 1. *Let $i \in N$ and $M \subseteq N \setminus \{i\}$. Let $EV_M := \max_{j\in M} EV_j$. We have*

$$EU_i^{Yes}(N,M) = \begin{cases} \int_{\underline{v}}^{\overline{v}} \Pi_{j\in N\setminus\{i\}} F_j(h) - \Pi_{j\in N} F_j(h)\ dh - c_i & \text{if } M = \emptyset, \\ \int_{EV_M}^{\overline{v}} \Pi_{j\in N\setminus(M\cup\{i\})} F_j(h) - \Pi_{j\in N\setminus M} F_j(h)\ dh - c_i & \text{if } M \neq \emptyset, \end{cases}$$

and

$$EU_i^{No}(N,M) = \begin{cases} \int_{\underline{v}}^{EV_i} \Pi_{j\in N\setminus\{i\}} F_j(h) dh & \text{if } M = \emptyset, \\ 0 & \text{if } M \neq \emptyset \text{ and } EV_i \leq EV_M, \\ \int_{EV_M}^{EV_i} \Pi_{j\in N\setminus M} F_j(h) dh & \text{if } M \neq \emptyset \text{ and } EV_i > EV_M. \end{cases}$$

Lemma 1 shows that the distributions $(F_j)_{j\in N}$ and the acquisition costs $(c_j)_{j\in N}$ influence the existence and characteristics of IAEs. In the next section, we investigate the conditions on $(F_j)_{j\in N}$ and $(c_j)_{j\in N}$ that ensure the existence of IAEs.

3 Equilibrium analysis

3.1 Existence and multiplicity

According to Lemma 1, IAEs depend on $(F_j)_{j\in N}$ and $(c_j)_{j\in N}$. We consider the following three cases for $(F_j)_{j\in N}$ and $(c_j)_{j\in N}$:

i. Different $(F_j)_{j\in N}$ and different $(c_j)_{j\in N}$,
ii. Different $(F_j)_{j\in N}$ but a common c^*,
iii. A common F but different $(c_j)_{j\in N}$.

In the cases (i) and (ii), the existence of an IAE is not guaranteed.[5] However, we have a positive result for the third case. We suppose that $F_i =: F$ for every $i \in N$. Then, from Lemma 1, it follows that

$$s_i(n,m) = \begin{cases} \int_{\underline{v}}^{\overline{v}} \{(1-F(h))F^{n-1}(h)\} dh - \int_{\underline{v}}^{EV} \{F^{n-1}(h)\} dh - c_i & \text{if } m = 0, \\ \int_{EV}^{\overline{v}} \{(1-F(h))F^{n-m-1}(h)\} dh - c_i & \text{if } m = 1, ..., n-1. \end{cases}$$

The following proposition shows that $s_i(n,m)$ increases monotonically with the number m of uninformed bidders.

[5] For case (i), let $N = \{1,2\}$, $V = [0,1]$, $F_1(v_1) = v_1^2$, and $F_2(v_2) = -v_2^2 + 2v_2$. Consider c_1 and c_2 with $\frac{17}{810} < c_1 < \frac{210}{810}$ and $\frac{10}{810} < c_2 < \frac{17}{810}$. This example has no IAE. For case (ii), let $N = \{1,2\}$, $V = [0,1]$, $F_1(v_1) = v_1^2$, and $F_2(v_2) = -\frac{1}{2}v_2^2 + \frac{3}{2}v_2$. Now, consider $c_1 = c_2 = 0.034$. There is no IAE in this example.

Proposition 2. *Let $n \geq 2$. Suppose that $F_i =: F$ for every $i \in N$. For every $i \in N$ and $m = 0, ..., n-2$, $s_i(n,m) < s_i(n, m+1)$.*

Proposition 2 suggests that the incentive to acquire information, $s_i(n,m) = EU_i^{\text{Yes}}(n,m) - EU_i^{\text{No}}(n,m)$, increases as more bidders decide not to acquire information. Moreover, the following corollary shows that the incentive to acquire information, $s_i(n,m)$, is uniquely determined by the number of informed bidders $n-m$.

Corollary 3. *Let $N = \{1, ..., n\}$, $N' = \{1, ..., n'\}$, and $N \subseteq N'$. Suppose that $F_j =: F$ for all $j \in N'$. For every $i \in N$, $m = 0, ..., n-2$, and $m' = 0, ..., n'-2$, we have the following:*

i. if $n - m = n' - m'$, then $s_i(n,m) = s_i(n', m')$;
ii. if $n - m < n' - m'$, then $s_i(n,m) > s_i(n', m')$.

Statement (i) follows immediately from the explicit formula of $s_i(n,m)$, and statement (ii) is proven in the same manner as the proof of Proposition 2. Statement (i) shows that the incentive to acquire information, $s_i(n,m)$, can be viewed as a function of the number of informed bidders, $n-m$. Furthermore, statement (ii) suggests that, with a slight abuse of notation, the function "$s_i(n-m)$" is a decreasing function of $n-m$. In other words, the more bidders are informed, the fewer benefits a bidder obtains from becoming an additional informed bidder.

We again fix $n \geq 2$. For every $m = 0, ..., n-1$, we define bidder i's best response function as introduced in Section 2:

$$BR_i(n,m) = \begin{cases} \{1\} & \text{if } s_i(n,m) > 0, \\ \{1, 0\} & \text{if } s_i(n,m) = 0, \\ \{0\} & \text{if } s_i(n,m) < 0. \end{cases}$$

Proposition 4 shows the existence of IAE by constructing an IAE.

Proposition 4. *Let $n \geq 2$. Suppose that $F_i =: F$ for every $i \in N$. At least one IAE exists.*

The following steps construct an IAE, where we omit n from $BR_i(n,m)$ and write $BR_i(m)$ for simplicity.[6]

1. For every $z = 0, ..., n-1$, define $m^*(z) := |\{i \in N | BR_i(z) = 0\}|$.
2. Let z^* be the minimal $z \in \{0, ..., n-1\}$ satisfying $z \geq m^*(z)$. If $n = m^*(n-1)$, let $z^* := n$.
3. Define $\Delta := \{i \in N | BR_i(z^* - 1) = 0 \text{ and } BR_i(z^*) = 1\}$.
4. Choose arbitrary $z^* - m^*(z^*)$ bidders from Δ. Let $\Delta^* \subseteq \Delta$ denote the set of the selected bidders.
5. Set $q_i^* := 0$ for every $i \in \Delta^*$ and $q_i^* := BR_i(z^*)$ for every $i \in N \setminus \Delta^*$.

We apply the algorithm to study the following four-bidder example. Let $N = \{1, 2, 3, 4\}$, $V = [0, 1]$, and $F(v) = v$ for every $v \in [0, 1]$. For every $i \in N$, we have $s_i(0) = \frac{11}{320} - c_i$, $s_i(1) = \frac{11}{192} - c_i$, $s_i(2) = \frac{16}{192} - c_i$, $s_i(3) = \frac{24}{192} - c_i$. Consider, for example, $c_1 = \frac{23}{192}$, $c_2 = \frac{15}{192}$, $c_3 = \frac{15}{192}$, $c_4 = \frac{10}{192}$.

[6] For all $i \in N$, let $BR_i(-1) = \{0\}$ and $BR_i(n) = \{1\}$ for extreme cases.

For bidder 1, we observe that $s_1(0) < s_1(1) < s_1(2) < 0 < s_1(3)$, implying that: $BR_1(0) = BR_1(1) = BR_1(2) = 0$ and $BR_1(3) = 1$. Similarly, we determine BR_2, BR_3, and BR_4, and Table 1 summarizes BR_1 to BR_4. As defined in Step 1, $m^*(z)$ denotes the number of the bidders who choose 0 for each $z = 0, ..., 3$. In this example, the minimum z that satisfies $z \geq m^*(z)$, i.e., z^*, is 2. Therefore, the profile $(BR_i(z^*))_{i=1,...,4} = (0, 1, 1, 1)$ consists of the best responses that every bidder makes when the number of the bidders, except for oneself, who choose No is two ($= z^*$). However, since only one bidder chooses No in the profile $(BR_i(z^*))_{i=1,...,4} = (0, 1, 1, 1)$, i.e., $m^*(z^*) = 1 \neq 2 = z^*$, this profile is not an IAE. In other words, each bidder's choice $BR_i(z^*)$ is a best response to z^* but not to $m^*(z^*)$. To fill the gap between z^* and $m^*(z^*)$, we arbitrarily choose $z^* - m^*(z^*)$ bidders from the bidders who have an incentive to switch their choices in $(BR_i(z^*))_{i=1,...,4}$. In this example, bidders 2 and 3 benefit by changing their choices from Yes in $z^* = 2$ to No. Hence, considering Step 3, let $\Delta = \{2, 3\}$. In Step 4, we select one bidder from Δ as $z^* - m^*(z^*) = 1$. Let $\Delta^* = \{2\}$ without loss of generality.

Table 1 An example of best response functions.

z	$BR_1(z)$	$BR_2(z)$	$BR_3(z)$	$BR_4(z)$	$m^*(z)$
3	1	1	1	1	0
2	0	1	1	1	1
1	0	0	0	1	3
0	0	0	0	0	4

In Step 5, we replace bidder 2's choice $BR_2(z^*) = 1$ in $(BR_i(z^*))_{i=1,...,4} = (0, 1, 1, 1)$ with 0, which yields $q^* := (0, 0, 1, 1)$. In the profile q^*, the number of the bidders i with $q_i^* = 0$ is 2 ($= z^* = m^*(z^*) + |\Delta^*|$). Hence, for the bidders with $q_i^* = 1$, their choice $q_i^* = 1 = BR_i(z^*)$ is a best response in profile q^*. Moreover, for the bidders with $q_i^* = 0$, their choice $q_i^* = 0 = BR_i(z^* - 1)$ becomes a best response in profile q^* because we have $BR_i(z) = 0 \Rightarrow BR_i(z - 1) = 0$ for every $z = 1, ..., n - 1$ (Proposition 2). Therefore, $q^* := (0, 0, 1, 1)$ is an IAE. Since the choice of $\Delta^* \subseteq \Delta$ is arbitrary in Step 4, $(0, 1, 0, 1)$ is another IAE.

Remark 1. *Note that low-cost (high-cost) bidders do not necessarily choose to acquire (skip) information. For instance, when $c_1 = c_2 = \frac{15}{192}$ and $c_3 = c_4 = \frac{14}{192}$ in the preceding example, bidders 1 and 2 are high-cost bidders, and the others are low-cost bidders, but every bidder has the same BR. Therefore, in some equilibria, high-cost bidders acquire information, and low-cost bidders do not.*

As demonstrated in the above example, the uniqueness of IAE is not necessarily guaranteed.[7] In general, we have the following result on the multiplicity of IAEs. For every $q \in \{1, 0\}^N$, let $m(q) = |\{j \in N | q_j = 0\}|$.

Proposition 5. *Let $n \geq 2$. Suppose that $F_i =: F$ for every $i \in N$. If $s_i(n, m) \neq 0$ for every $i \in N$ and $m = 0, ..., n - 1$, then $m(q) = m(q')$ for all IAEs q and q'.*

[7] An anonymous referee pointed out that multiple IAEs occur when the symmetry of acquisition costs results in symmetric BRs. Therefore, the multiplicity can be viewed as a "knife-edge" case. Although this is beyond the scope of this article, extending bidders' information acquisition behavior to mixed strategies may play a key role in clarifying the properties of IAEs.

Given that $s_i(n,m) = EU_i^{\text{Yes}}(n,m) - EU_i^{\text{No}}(n,m)$, the condition $s_i(n,m) \neq 0$ implies that bidder i has a strict incentive to choose either Yes or No for every m. Therefore, Proposition 5 suggests that if every bidder is assumed to have strict preferences between Yes and No for each m, then the number of bidders opting for No is the same among all IAEs. This result shows that there can be multiple IAEs, while the number of uninformed bidders is unique.

Note that if there are $i \in N$ and $m = 0, ..., n-1$ such that $s_i(n,m) = 0$, Proposition 5 does not necessarily hold.[8] Moreover, the converse of Proposition 5 does not hold.[9] Proposition 5 suggests the following result.

Remark 2. *If $s_i(n,m) \neq 0$ for every $i \in N$ and $m = 0, ..., n-1$, then by repeating the algorithm of Proposition 4, we find all IAEs.*

We obtain an IAE for each arbitrarily selected Δ^* in Step 4. Therefore, by repeating Steps 4 and 5, multiple IAEs are obtained. Let $\mathcal{I}$ denote the set of all IAEs in the auction, and $m^* := m(q)$ for all $q \in \mathcal{I}$ as described in Proposition 5. If there is an IAE $q' \notin \mathcal{I}$ such that $m(q') \neq m^*$, then q' contradicts Proposition 5. Additionally, if $m(q') = m^*$ and there is $i \in N \setminus \Delta$ such that $q'_i \neq q_i$ for any IAE $q \in \mathcal{I}$, then q'_i deviates from i's best response, as $N \setminus \Delta$ is the set of bidders whose choice is the best response to m^*.

The result that the number of uninformed bidders is unique for all IAEs enables the auctioneer to evaluate the auction in terms of the expected revenue since the expected revenue depends on the number $m(q)$ of uninformed bidders, which will be discussed in the next section.

4 Expected revenue

Proposition 5 enables us to determine a unique value of $m := m(q)$ for all IAEs q, representing the number of uninformed bidders in the auction for a given distribution F and acquisition cost profile $(c_j)_{j \in N}$. This unique m determines the expected revenue the auctioneer anticipates from the auction. We use ER to denote the expected revenue, and the following result provides an explicit formula for ER that determines the expected revenue for each possible $m = 0, ..., n$.

Proposition 6. *Let $n \geq 2$. Suppose that $F := F_i$ for every $i \in N$. Let $m = 0, ..., n$ denote the number of uninformed bidders. It holds that*

$$ER(n,m) =$$
$$\begin{cases} \overline{v} + \int_{\underline{v}}^{\overline{v}} \{(n-1)F^n(v) - nF^{n-1}(v)\}dv & \text{for } m = 0, \\ \overline{v} - \int_{\underline{v}}^{EV} F^{n-1}(v)dv + \int_{EV}^{\overline{v}} \{(n-2)F^{n-1}(v) - (n-1)F^{n-2}(v)\}dv & \text{for } m = 1, \\ \overline{v} + \int_{EV}^{\overline{v}} \{(n-m-1)F^{n-m}(v) - (n-m)F^{n-m-1}(v)\}dv & \text{for } m = 2, ..., n. \end{cases}$$

[8] We again consider the four-bidder scenario mentioned earlier, where $V = [0,1]$ and $F(v) = v$ for every $v \in [0,1]$. However, we now consider the following acquisition costs: $c_1 = \frac{25}{192}$, $c_2 = \frac{11}{192}$, $c_3 = \frac{11}{192}$, $c_4 = \frac{1}{192}$. Since $s_2(n,1) = 0$ and $s_3(n,1) = 0$, bidders 2 and 3 have $BR_2(1) = \{1,0\}$ and $BR_3(1) = \{1,0\}$. In this example, there are three IAEs: $(0,1,1,1)$, $(0,0,1,1)$, and $(0,1,0,1)$. The first IAE contains one 0 choice, while the second and third IAEs contain two 0 choices.

[9] Even if there are $i \in N$ and $m = 0, ..., n-1$ such that $s_i(m) = 0$, we can have $m(q) = m(q')$ for all IAEs q and q'. Consider $c_1 = \frac{11}{320}$, $c_2 = \frac{25}{192}$, $c_3 = \frac{25}{192}$, $c_4 = \frac{25}{192}$. Bidders 2, 3, and 4 choose No. Bidder 1 has $BR_1(1) = \{1,0\}$. Therefore, this example has a unique IAE $(0,0,0,0)$.

If $m = 0$, all the bidders are informed, and this case is equivalent to a (standard) second-price auction. There is a distinction between the cases $m = 1$ and $m \geq 2$. For $m = 1$, when there is only one uninformed bidder who wins, all informed bidders bid less than EV. Therefore, the uninformed bidder pays the second-highest price, which is determined by one of the informed bidders. For $m \geq 2$, when one of the m uninformed bidders wins, the uninformed bidder pays EV because another uninformed bidder bids EV.

The following proposition illustrates the relationship between the expected revenue $ER(n, m)$ and the number m of uninformed bidders.

Proposition 7. *Let $n \geq 2$. Suppose that $F := F_i$ for every $i \in N$. For $m = 2, ..., n-1$, $ER(n, m) \geq ER(n, m+1)$, where the equation holds only for $m = n-1$.*

Note that the proposition does not necessarily hold for $m = 0$ and 1; it is not guaranteed that $ER(n, 0) > ER(n, 1) > ER(n, 2)$. For example, when $n = 3$ and $F(v) = -v^2 + 2v$ for $v \in [0, 1]$, we observe the following: $ER(3, 0) = \frac{11}{35} = 0.31... < ER(3, 1) = \frac{391}{1215} = 0.32... < ER(3, 2) = \frac{1}{3} = 0.33....$

This proposition suggests that the expected revenue decreases as the number m of uninformed bidders increases. In other words, the expected revenue is an increasing function of the number $n - m$ of informed bidders. In the standard auction model in which information acquisition is not taken into account, the expected revenue increases as the *total* number of bidders increases. However, Proposition 7 shows that in the context of this model, the expected revenue depends especially on the number of *informed* bidders. This result suggests that an auctioneer is likely to improve the revenue from the auction not by inviting more bidders but by increasing the number of informed bidders.

If all bidders follow the same distribution and have a common acquisition cost, then a lower cost leads to more bidders opting to acquire information, which results in higher expected revenue. This is because a lower cost allows every bidder's best response to include more "Yes" choices, thereby decreasing the value of z^*. Therefore, the number of bidders who choose Yes increases. Moreover, the increase in the number of bidders choosing Yes means a decrease in the number of bidders choosing No. Since expected revenue is a decreasing function with respect to the number of uninformed bidders, the expected revenue increases.

5 Conclusion

In this study, we analyzed information acquisition behaviors in sealed-bid second-price auctions. In our model, bidders do not possess prior knowledge for their own valuations of the object being auctioned, and they independently decide whether or not to acquire information about their own valuations by incurring an acquisition cost. We introduced the concept of an Information Acquisition Equilibrium (IAE), which represents a profile of information acquisition decisions made by all bidders. Our main findings can be summarized as follows:

- In cases where bidders have different distribution functions $(F_j)_{j \in N}$, an auction may have no IAE. However, if a common distribution function F is assumed, the existence of IAE is guaranteed, and there is an algorithm for constructively finding all IAEs.

- The uniqueness of IAE is not necessarily achieved, whereas all IAEs share the same number of uninformed bidders.
- The expected revenue of the auction decreases as the number of uninformed bidders increases.

In this study, we have presented a *theoretical* framework for analyzing bidders' behaviors in second-price auctions with information acquisition costs. In contrast, Gretschko and Rajko (2014) conducted an experiment based on the framework of Compte and Jehiel (2007), demonstrating that bidders acquire information more frequently than theoretical predictions suggest in both second- and ascending-price auctions. Cooper and Fang (2008) focused on information about opponents' valuations. Their experiment allowed each bidder to obtain noisy signals about their opponents' valuations in second-price auctions.

Our findings have demonstrated that parameters, including $(F_j)_{j\in N}$ and $(c_j)_{j\in N}$, play a crucial role in determining the proportion of uninformed bidders relative to the total number of bidders. Furthermore, our research indicates that the expected revenue can be considered an increasing function of the number of informed bidders. As a natural progression, our next objective is to conduct experiments that will put these theoretical predictions to the test. This extension contributes to a better understanding of information acquisition behaviors in second-price auctions and provides practical insights for auctioneers seeking to improve auction outcomes.

Moreover, in this paper, the symmetry of F serves as a condition ensuring the existence of pure strategy equilibrium. Further clarification and exploration are needed to provide a weaker condition for F to guarantee the existence of IAEs. Additionally, while we focused on second-price auctions, analyzing information acquisition behaviors in other auction formats (e.g., first-price auctions) and comparing these behaviors with those in second-price auctions is an important topic for future work.

6 Proofs

Proof of Lemma 1

Proof. $EU_i^{\text{Yes}}(N,M)$ for $M=\emptyset$: Bidder i's ex interim expected payoff for a given $v_i\in V$ is obtained as follows:

$$\begin{aligned}\int_{\underline{v}}^{v_i}(v_i-h)(\Pi_{j\neq i}F_j(h))'dh-c_i &= v_i\int_{\underline{v}}^{v_i}(\Pi_{j\neq i}F_j(h))'dh-\int_{\underline{v}}^{v_i}h(\Pi_{j\neq i}F_j(h))'dh-c_i\\ &= \int_{\underline{v}}^{v_i}\Pi_{j\neq i}F_j(h)dh-c_i.\end{aligned}$$

Therefore, the ex ante expected payoff is expressed as

$$EU_i^{\text{Yes}}(N,M) = \int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_i} f_i(v_i)\Pi_{j\neq i}F_j(h)\ dh\ dv_i-c_i$$

$$= \int_{\underline{v}}^{\overline{v}} \Pi_{j\neq i}F_j(h) - \Pi_{j\in N}F_j(h)\ dh - c_i.$$

$EU_i^{\text{Yes}}(N,M)$ for $M \neq \emptyset$: Let M be the set of m uninformed bidders. Let $EV_M := \max_{j\in M} EV_j$. Bidder i's ex interim expected payoff for a given $v_i \in V$ is $0 - c_i$ if $v_i < EV_M$. If $v_i \geq EV_M$, then the ex interim expected payoff is

$$\int_{EV_M}^{v_i} (v_i - h)(\Pi_{j\in N\setminus(M\cup\{i\})}F_j(h))'dh + \int_{\underline{v}}^{EV_M} (v_i - EV_M)(\Pi_{j\in N\setminus(M\cup\{i\})}F_j(h))'dh - c_i. \tag{1}$$

In a similar manner to the computation for $m = 0$, the first term of (1) is $\int_{EV_M}^{v_i}(v_i - h)(\Pi_{j\in N\setminus(M\cup\{i\})}F_j(h))'dh = (EV_M - v_i)\cdot \Pi_{j\in N\setminus(M\cup\{i\})}F_j(EV_M) + \int_{EV_M}^{v_i} \Pi_{j\in N\setminus(M\cup\{i\})}F_j(h)dh$. The second term of (1) is $\int_{\underline{v}}^{EV_M}(v_i - EV_M)(\Pi_{j\in N\setminus(M\cup\{i\})}F_j(h))'dh = (v_i - EV_M)\cdot \Pi_{j\in N\setminus(M\cup\{i\})}F_j(EV_M)$. Hence, we obtain

$$(1) = \int_{EV_M}^{v_i} \Pi_{j\in N\setminus(M\cup\{i\})}F_j(h)dh - c_i.$$

Given that the ex interim expected payoff for $v_i < EV_M$ is $0 - c_i$, the ex ante expected payoff is

$$\begin{aligned} EU_i^{\text{Yes}}(N,M) &= \int_{EV_M}^{\overline{v}} \int_{EV_M}^{v_i} f_i(v_i)\Pi_{j\in N\setminus(M\cup\{i\})}F_j(h)\ dh\ dv_i - c_i \\ &= \int_{EV_M}^{\overline{v}} \Pi_{j\in N\setminus(M\cup\{i\})}F_j(h) - \Pi_{j\in N\setminus M}F_j(h)\ dh - c_i. \end{aligned}$$

$EU_i^{\text{No}}(N,M)$ for $M = \emptyset$: In this case, i is the unique uninformed bidder. Bidder i's ex interim expected payoff for a given $v_i \in V$ is obtained in a similar manner to EU^{Yes} with $m = 0$: $\int_{\underline{v}}^{EV_i}(v_i - h)(\Pi_{j\neq i}F_j(h))'dh = \int_{\underline{v}}^{EV_i} \Pi_{j\neq i}F_j(h)dh$. Hence, it follows that

$$\begin{aligned} EU_i^{\text{No}}(N,M) &= \int_{\underline{v}}^{\overline{v}} \left(\int_{\underline{v}}^{EV_i} \Pi_{j\neq i}F_j(h)dh\right) f_i(v_i)\ dv_i \\ &= \int_{\underline{v}}^{EV_i} \Pi_{j\neq i}F_j(h)dh. \end{aligned}$$

$EU_i^{\text{No}}(N,M)$ for $M \neq \emptyset$: If $EV_i \leq EV_M$, bidder i's ex interim expected payoff is zero. If $EV_i > EV_M$, then i's ex interim expected payoff is

$$\int_{EV_M}^{EV_i} (v_i - h)(\Pi_{j\in N\setminus M}F_j(h))'dh + \int_{\underline{v}}^{EV_M} (v_i - EV_M)(\Pi_{j\in N\setminus M}F_j(h))'dh \tag{2}$$

The first term of (2) equals $(v_i - EV_i)\Pi_{j\in N\setminus M}F_j(EV_i) - (v_i - EV_M)\Pi_{j\in N\setminus M}F_j(EV_M) + \int_{EV_M}^{EV_i} \Pi_{j\in N\setminus M}F_j(h)\ dh$. The second term of (2) equals $(v_i - EV_M)\Pi_{j\in N\setminus M}F_j(EV_M)$.

Hence, we obtain

$$(2) = (v_i - EV_i)\Pi_{j\in N\setminus M}F_j(EV_i) + \int_{EV_M}^{EV_i} \Pi_{j\in N\setminus M}F_j(h)\ dh.$$

Therefore, if $EV_i > EV_M$, the ex ante expected payoff is

$$\begin{aligned} EU_i^{\text{No}}(N,M) &= \int_{\underline{v}}^{\overline{v}} \left((v_i - EV_i)\Pi_{j\in N\setminus M}F_j(EV_i) + \int_{EV_M}^{EV_i} \Pi_{j\in N\setminus M}F_j(h)dh \right) f_i(v_i)dv_i \\ &= \int_{EV_M}^{EV_i} \Pi_{j\in N\setminus M}F_j(h)dh. \end{aligned}$$

If $EV_i \le EV_M$, then the ex ante expected payoff is zero. □

Proof of Proposition 2

Proof. Let $i \in N$. Function $s_i(n,m)$ is equivalent to

$$s_i(n,m) = \begin{cases} \int_{EV}^{\overline{v}} F^{n-1}(h)dh - \int_{\underline{v}}^{\overline{v}} F^{n}(h)dh - c_i & \text{if } m = 0, \\ \int_{EV}^{\overline{v}} F^{n-m-1}(h)dh - \int_{EV}^{\overline{v}} F^{n-m}(h)dh - c_i & \text{if } m = 1, ..., n-1. \end{cases}$$

For every $m = 1, ..., n-2$, we have $s(n,m+1) - s(n,m) = \int_{EV}^{\overline{v}} F^{n-m-2}(h) - 2 \cdot F^{n-m-1}(h) + F^{n-m}(h)\ dh = \int_{EV}^{\overline{v}} F^{n-m-2}(h) \cdot (1-F(h))^2\ dh > 0$, which does not matter if $n = 2$.

For $m = 0$, we have

$$\begin{aligned} & s(n,1) - s(n,0) \\ = & \int_{EV}^{\overline{v}} F^{n-2}(h)dh - \int_{EV}^{\overline{v}} F^{n-1}(h)dh - \int_{EV}^{\overline{v}} F^{n-1}(h)dh + \int_{\underline{v}}^{\overline{v}} F^{n}(h)dh \\ \ge & \int_{EV}^{\overline{v}} F^{n-2}(h)dh - \int_{EV}^{\overline{v}} F^{n-1}(h)dh - \int_{EV}^{\overline{v}} F^{n-1}(h)dh + \int_{EV}^{\overline{v}} F^{n}(h)dh \\ = & \int_{EV}^{\overline{v}} F^{n-2}(h) \cdot (1-F(h))^2\ dh > 0. \end{aligned}$$

□

Proof of Proposition 4

Proof. We omit n from $BR_i(n,m)$ and write $BR_i(m)$ for simplicity. If there are bidders i whose $s_i(z') = 0$ for some $z' = 0, ..., n-1$, then set $BR_i(z') = 1$.

We show that the resulting q^* is an IAE. The function $m^*(z)$ defined in Step 1 is a non-increasing function of z because Proposition 2 shows that $s_i(z)$ is an increasing function of z, which furthermore implies that $BR_i(z) = 1 \Rightarrow BR_i(z+1) = 1$ for every $z = 0, ..., n-2$.

If we have $z^* = m^*(z^*)$, Δ^* in Step 4 becomes an empty set, and hence $q^* = (BR_i(z^*))_{i\in N}$. Step 1 suggests that in profile q^*, there are $m^*(z^*)$ $(= z^*)$ players who choose 0. Therefore, each bidder i with $q_i^* = 1$ chooses her choice $1 = BR_i(z^*)$ as her best response to z^*, i.e., the number of bidders j choosing $q_j^* = 0$ in profile q^*. Moreover, each bidder i with $q_i^* = 0$ chooses $0 = BR_i(z^*) = BR_i(z^* - 1)$, which holds by the monotonicity of BR_i, as his best response to $z^* - 1$, i.e., the number of bidders j choosing $q_j^* = 0$ except for i. Hence, q^* is an IAE.[10]

If we have $z^* > m^*(z^*)$, then Δ^* in Step 4 contains $z^* - m^*(z^*)$ bidders. Since Δ^* is a subset of Δ, Step 3 implies that every $i \in \Delta^*$ has $BR_i(z^* - 1) = 0$ and $BR_i(z^*) = 1$. In view of the definition of $m^*(\cdot)$ in Step 1, $m^*(z^*)$ bidders choose 0 in profile $(BR_i(z^*))_{i\in N}$. As described in Step 5, the $z^* - m^*(z^*)$ bidders in Δ^*, who choose 1 in profile $(BR_i(z^*))_{i\in N}$, switch to 0. Hence, z^* $(= m^*(z^*) + z^* - m^*(z^*))$ bidders choose 0 in profile q^*. Each bidder i with $q_i^* = 1$ chooses her choice $1 = BR_i(z^*)$ as her best response to z^*. Moreover, each bidder i with $q_i^* = 0$ chooses $0 = BR_i(z^*) = BR_i(z^* - 1)$ as his best response to $z^* - 1$. Hence, q^* is an IAE. □

Proof of Proposition 5

Proof. Suppose that $s_i(n, m) \neq 0$ for every $i \in N$ and $m = 0, ..., n - 1$. Assume that there are two different IAEs q and q' such that $m(q) < m(q')$. There is $i \in N$ such that $q_i = 1$ and $q'_i = 0$. Since q is an IAE and $q_i = 1$, we have $s_i(n, m(q)) \geq 0$. In addition, since q' is an IAE and $q'_i = 0$, we have $s_i(n, m(q')-1) \leq 0$. If $m(q) < m(q')-1$, then the monotonicity of s_i (Proposition 2) implies that $0 \leq s_i(n, m(q)) < s_i(n, m(q') - 1) \leq 0$, which is a contradiction. Hence, we obtain $m(q) = m(q') - 1$, which results in $s_i(n, m(q)) = 0$. However, this contradicts the supposition that $s_i(n, m) \neq 0$ for every $i \in N$ and $m = 0, ..., n - 1$. □

Proof of Proposition 6

Proof. We first show that $ER(n, m)$ is determined as follows: for $m = 0$, $ER(n, 0) = n\int_{\underline{v}}^{\overline{v}}\{(v \cdot f(v) - 1)F^{n-1}(v) + F^n(v)\}dv$; for $m = 1$, $ER(n, 1) = EV \cdot F^{n-1}(EV) - \int_{\underline{v}}^{EV} F^{n-1}(v)dv + (n-1)\int_{EV}^{\overline{v}}\{(v\cdot f(v)-1)F^{n-2}(v)+F^{n-1}(v)\}dv$; and for $m = 2, ..., n$, $ER(n, m) = EV \cdot F^{n-m}(EV) + (n - m)\int_{EV}^{\overline{v}}\{(v \cdot f(v) - 1)F^{n-m-1}(v) + F^{n-m}(v)\}dv$.

$m = 0$: The ex interim expected payment of an informed bidder i is $\int_{\underline{v}}^{v_i} h(F^{n-1}(h))'dh = \int_{\underline{v}}^{v_i}(h \cdot F^{n-1}(h))'dh - \int_{\underline{v}}^{v_i} F^{n-1}(h)dh = v_i \cdot F^{n-1}(v_i) - \int_{\underline{v}}^{v_i} F^{n-1}(h)dh$. Hence, the ex ante expected payment of an informed bidder i is $\int_{\underline{v}}^{\overline{v}}\left\{v_i \cdot F^{n-1}(v_i) - \int_{\underline{v}}^{v_i} F^{n-1}(h)dh\right\} f(v_i)dv_i = \int_{\underline{v}}^{\overline{v}} v_i \cdot f(v_i) \cdot F^{n-1}(v_i)dv_i - \int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_i} f(v_i) \cdot F^{n-1}(h)dh\, dv_i = \int_{\underline{v}}^{\overline{v}}\{(v_i \cdot f(v_i) - 1)F^{n-1}(v_i) + F^n(v_i)\}dv_i$. Since $m = 0$, there are n informed bidders. Hence, the (ex ante) expected revenue is $n\int_{\underline{v}}^{\overline{v}}\{(v\cdot f(v)-1)F^{n-1}(v)+F^n(v)\}dv$.

[10] Note that $z^* = 0$ occurs only when $BR_i(z) = 1$ for every $i \in N$ and $z = 0, ..., n - 1$, where the profile $q^* = (BR_i(z^*))_{i\in N}$ consists only of 1. Similarly, we have $z^* = n$ only when $BR_i(z) = 0$ for every $i \in N$ and $z = 0, ..., n - 1$, where the profile $q^* = (BR_i(z^*))_{i\in N}$ consists only of 0.

$m = 1$: The ex interim expected payment of the uninformed bidder is

$$\int_{\underline{v}}^{EV} h(F^{n-1}(h))' dh \,=\, EV \cdot F^{n-1}(EV) - \int_{\underline{v}}^{EV} F^{n-1}(h) dh. \tag{3}$$

Since this is independent of v_i, this is the ex ante expected payment. The ex interim expected payment of an informed bidder i is given as follows: for $v_i \geq EV$,

$$\int_{EV}^{v_i} h(F^{n-2}(h))' dh + \int_{\underline{v}}^{EV} EV(F^{n-2}(h))' dh \,=\, v_i \cdot F^{n-2}(v_i) - \int_{EV}^{v_i} F^{n-2}(h) dh;$$

and zero for $v_i < EV$. Therefore, the ex ante expected payment of an informed bidder i is

$$\begin{aligned} &\int_{EV}^{\overline{v}} \left\{ v_i \cdot F^{n-2}(v_i) - \int_{EV}^{v_i} F^{n-2}(h) dh \right\} f(v_i) dv_i \\ &= \int_{EV}^{\overline{v}} \{(v_i \cdot f(v_i) - 1) F^{n-2}(v_i) + F^{n-1}(v_i)\} dv_i. \end{aligned} \tag{4}$$

Hence, the expected revenue is the sum of (3) and $n-1$ times (4): $EV \cdot F^{n-1}(EV) - \int_{\underline{v}}^{EV} F^{n-1}(v) dv + (n-1) \int_{EV}^{\overline{v}} \{(v \cdot f(v) - 1) F^{n-2}(v) + F^{n-1}(v)\} dv$.

$m = 2, ..., n$: The ex interim expected payment of an uninformed bidder is

$$\int_{\underline{v}}^{EV} \frac{EV}{m} (F^{n-m}(h))' dh \,=\, \frac{EV}{m} F^{n-m}(EV). \tag{5}$$

Since this is independent of v_i, this is the ex ante expected payment of an uninformed bidder. Similar to the case $m = 1$, the ex ante expected payment of an informed bidder i is

$$\int_{EV}^{\overline{v}} \{(v_i \cdot f(v_i) - 1) F^{n-m-1}(v_i) + F^{n-m}(v_i)\} dv_i. \tag{6}$$

Hence, the expected revenue is the sum of m times (5) and $n - m$ times (6): $EV \cdot F^{n-m}(EV) + (n-m) \int_{EV}^{\overline{v}} \{(v \cdot f(v) - 1) F^{n-m-1}(v) + F^{n-m}(v)\} dv$.

Next, for every $t = 0, ..., n-1$, we have

$$t \int_{EV}^{\overline{v}} v \cdot f(v) F^{t-1}(v) dv = \overline{v} - EV \cdot F^t(EV) - \int_{EV}^{\overline{v}} F^t(v) dv. \tag{7}$$

Similarly, we also have

$$n \int_{\underline{v}}^{\overline{v}} v \cdot f(v) F^{t-1}(v) dv = \overline{v} - \int_{\underline{v}}^{\overline{v}} F^n(v) dv. \tag{8}$$

Therefore, by applying (8) to $m = 0$ and (7) to $m = 1, ..., n$ in the equation $ER(n, m)$, we obtain the formula provided in Proposition 6. □

Proof of Proposition 7

Proof. We have $ER(n,m) - ER(n,m+1) = \int_{EV}^{\overline{v}} \{(n-m-1)F^{n-m}(v) - (n-m)F^{n-m-1}(v)\}dv - \int_{EV}^{\overline{v}} \{(n-m-2)F^{n-m-1}(v) - (n-m-1)F^{n-m-2}(v)\}dv = (n-m-1)\int_{EV}^{\overline{v}} \{F^{n-m-2}(v) \cdot (1-F(v))^2\}dv \geq 0$. The equation of the final inequality holds if and only if $m = n - 1$. □

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References

[1] Abe, T., Onda, T., and Yamato, T. (2024). Second-price auctions with information acquisition costs. Working Paper, SSRN.

[2] Bobkova, N. (2024). Information choice in auctions. *American Economic Review* 114(7), 1883-1915.

[3] Cao, X., Tan, G., Tian, G., and Yilankaya, O. (2018). Equilibria in second-price auctions with private participation costs. *Economic Theory* 65, 231-249.

[4] Compte, O. and Jehiel, P. (2007). Auctions and information acquisition: Sealed bid or dynamic formats?. *The Rand Journal of Economics* 38(2), 355-372.

[5] Cooper, D. J. and Fang, H. (2008). Understanding overbidding in second price auctions: An experimental study. *The Economic Journal* 118(532), 1572-1595.

[6] Gretschko, V. and Rajko, A. (2015). Excess information acquisition in auctions. *Experimental Economics* 18, 335-355.

[7] Hausch, D. B. and Li, L. (1993). A common value auction model with endogenous entry and information acquisition. *Economic Theory* 3, 315-334.

[8] Kagel, J. H. and Levin, D. (1993). Independent private value auctions: Bidder behaviour in first-, second- and third-price auctions with varying numbers of bidders. *The Economic Journal* 103 (419), 868-879.

[9] Milgrom, P. R. and Weber, R. J. (1982). A theory of auctions and competitive bidding. *Econometrica* 50 (5), 1089-1122.

[10] Persico, N. (2000). Information acquisition in auctions. *Econometrica* 68 (1), 135-148.

[11] Shi, X. (2012). Optimal auctions with information acquisition. *Games and Economic Behavior* 74 (2), 666-686.

[12] Stegeman, M. (1996). Participation costs and efficient auctions. *Journal of Economic Theory* 71 (1), 228-259.

[13] Tan, G. and Yilankaya, O. (2006). Equilibria in second price auctions with participation costs. *Journal of Economic Theory* 130 (1), 205-219.