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Subsidy Policy for Automation and Economic Growth*

Kenichiro Ikeshita†

Abstract

In recent years, although rapid progress in digital technology has made it easier to automate production processes, it has raised concerns that machines may deprive workers of employment. However, the effects of automation-oriented policies on economic growth have not been thoroughly studied. This study examines the effect of task automation on economic growth and its policy implications by incorporating a task-based approach into a neoclassical growth model. I find that the aggregate production function becomes asymptotically linear if firms' cost-minimization behavior determines the number of automated tasks. Consequently, a higher subsidy rate for automation increases the returns on machines (capital) and generates perpetual economic growth. For factor prices, the rental prices of capital and wages converge to constant values along the equilibrium path. In addition, while an increase in the subsidy rate for automation raises the long-term rental price of capital, it does not affect the long-term wage rates.

Keywords: Automation, Task, Neoclassical growth model, AK model, Subsidy policy

1. Introduction

In recent years, technological innovations have been rapid in areas, such as Artificial Intelligence (AI) and industrial robotics. These technological innovations raised concerns that machines will deprive workers of their jobs, as production processes become increasingly automated. Frey and Osborne (2017) report that within 10–20 years, approximately 47% of the U.S. labor force will be replaced by machines. Brynjolfsson and McAfee (2014), Ford (2015), and Davenport and Kirby (2016) also point out that owing to the rapid development of digital technologies, machines are gaining an advantage in jobs traditionally performed by humans. For example, economists reported a decline in the percentage of labor income to total income (labor share) in many developed countries. They interpret this decline in labor share as evidence that workers are having increasing difficultly competing with machines.

By contrast, some argue that automation through digital innovation can be an important source of

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economic growth. In Japan, where the population is expected to decline, digitalization and automation using robots are considered necessary to increase productivity and achieve economic growth. Hence, national and local governments are implementing various subsidy policies to promote the use of information technology (IT) and the social implementation of robots. For example, to encourage automation and the use of IT by Small and Medium Enterprises (SMEs), SME Support Japan offers "manufacturing subsidies" and "IT introduction subsidies." In addition, the Tokyo Metropolitan SME Support Center provides subsidies to promote productivity improvements through the use of the Internet of Things (IoT) and robots.[1)]

What is the impact of automation on economic growth? Do automation subsidy policies increase productivity and promote economic growth? To answer these questions, I examine the effect of task automation on economic growth and its policy implications by incorporating a task-based approach into a neoclassical growth model.

The task-based approach analyzes the labor market structure by considering a worker's job as a collection of tasks. A task-based approach distinguishes between tasks and skills. Each task is performed by a worker or machine based on its characteristics.

Task analysis was developed based on both economic growth theory and labor economics. In economic growth theory, Zeira's (1998) pioneering work introduces a task model, while Acemoglu and Zilibotti (2001) incorporate endogenous technological progress into the task-based approach and analyze productivity differences across countries. More recently, Acemoglu and Restrepo (2018) develop a general equilibrium model that integrates task models and directed technical change to explore the impact of automation on the labor market and economic growth. Nakamura and Zeira (2024) analyze "technological unemployment" due to automation using an endogenous growth model with a task-based approach. Hémous and Olson (2022) examine the relationship between automation and income inequality by constructing a growth model that distinguishes between skilled and unskilled workers.

In the field of labor economics, Autor et al.'s (2003) pioneering study proposes a model that analyzes routine and non-routine tasks separately. Their study finds that progress in IT expands inequality by encouraging the automation of routine tasks. In addition, they use U.S. data to show that advances in IT polarized the U.S. labor market by reducing labor demand for routine tasks. Acemoglu and Autor (2011) identify the problems with the skill premium model while providing a detailed explanation of the advantages of the task-based approach.

The model deployed in this study combined the task-based approach presented by Acemoglu and Autor (2011) with the Ramsey model. Therefore, my model is considered a simplified version of the model analyzed by Acemoglu and Restrepo (2018). However, two major differences exist between their study and mine. First, according to Acemoglu and Restrepo (2018), the number of automated tasks is technologically constrained

1) For details on these projects, see SME Support Japan (2024) and Tokyo Metropolitan SME Support Center (2020).

and technological progress implies that more tasks are technologically automated. On the other hand, this study analyzes the case in which the firm's cost-minimization behavior determines the number of automated tasks.

The second significant difference is this study's focus on transition dynamics. Acemoglu and Restrepo (2018) analyze only the balanced growth path and do not thoroughly discuss the characteristics of transition dynamics. By contrast, this study examines a simplified model to clarify the characteristics of transition dynamics and policy effects. This study considers these two differences and reveals the features of transition dynamics that Acemoglu and Restrepo (2018) overlook. Consequently, this study demonstrates a novel relationship between automation and economic growth.

In addition, this study is closely related to analyses of "automation capital." In particular, Prettner (2019) focuses on the process of economic growth by distinguishing between automation capital, which is perfectly substitutable with the labor force, and ordinary capital.[2)] However, Prettner (2019) does not use a task-based approach. On the other hand, this study uses a task model to describe the task automation process more explicitly. Consequently, the results of the analysis are similar to those of Prettner (2019), even if tasks have different labor productivity. Thus, this study complements the analyses of Acemoglu and Restrepo (2018) and Prettner (2019) regarding capital accumulation.

The analysis yields the following three main results. First, the aggregate production function becomes asymptotically linear if the firm's cost-minimizing behavior determines the number of automated tasks. In a typical neoclassical growth model, capital accumulation reduces the marginal productivity of capital. In this model, as capital (machines) accumulates, more tasks become automated because capital becomes cheaper. This effect increases the demand for capital and decreases the demand for labor. Consequently, the marginal productivity of capital becomes asymptotically constant, implying that the aggregate production function is linear.

Second, if capital productivity and subsidy rates for automation are sufficiently high, the economy will achieve sustainable growth even without perpetual technological progress. The direct cause of this outcome is that, as mentioned earlier, the production function becomes linear and the structure of the model is close to the AK model. In economic growth theory, various externalities cause the production function to be AK-shaped, generating sustained growth. In this study, task automation creates an AK-shaped production function. This finding implies that automation is a pathway to achieving long-term growth. Moreover, a higher subsidy for automation leads to a higher demand for machines, which raises the rental price of capital. This effect promotes capital accumulation and increases the economic growth rate. In other words, automation subsidies are effective in stimulating economic growth.

Third, the rental prices of capital and wages converge to constant values on the equilibrium path. In

2) Several studies also investigate robotic capital. Robotic capital is characterized by its strong substitutability for human labor. For more about robotic capital, See Berg et al. (2018) and Lankisch et al. (2019).

addition, while an increase in the subsidy rate for automation increases the long-term rental price of capital, it does not affect the long-term wage rates. In this study, as capital accumulates in the transition path, more tasks are automated and the demand for capital increases, while the demand for labor declines (i.e., tasks previously performed by labor are mechanized). Consequently, a decrease in the rental price of capital and an increase in wages were suppressed. In the long run, the rental price of capital converges to its lower bound, and the wage converges to its upper bound.

The remainder of this paper is organized as follows. Section 2 presents the model and clarifies the structure of the production process and factor prices. Section 3 derives the economic equilibrium path and shows the conditions for sustainable economic growth. Finally, I present my conclusions in Section 4.

2. The Model

In this section, I analyze a neoclassical growth model combined with the task-based approach of Acemoglu and Autor (2011). Its production structure is the same as that of Ikeshita et al. (2023). Specifically, all tasks are potentially automatable and firms' cost minimization determines whether to adopt automation technology. However, this study differs from Ikeshita et al. (2023) in that it (i) introduces an optimal choice of consumption and savings by households and (ii) analyzes the effects of government subsidy policies on the equilibrium path.

2.1 Households

First, I describe household behavior using the simple Ramsey model. Households determine their consumption paths to maximize the sum of their discounted utilities. Let $c(t)$ denote the per-capita consumption at time t. The household's objective is to maximize the total utility U, expressed as

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)t}, \tag{1}$$

where $\rho > 0$ is the subjective discount rate, $n > 0$ is the rate of population growth, and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution. I assume that $\rho > n$, as in the usual neoclassical model.

Households earn asset income from their own assets and wage income by supplying their labor force while spending on consumption, paying a lump-sum tax to the government, and saving the rest. These savings result in an increase in assets. Therefore, the household's budget constraint is

$$\dot{a}(t) = (r(t) - n)a(t) + w(t) - \pi(t) - c(t), \tag{2}$$

where $a(t)$ is the amount of assets per capita, $r(t)$ is the interest rate, $w(t)$ is the wage rate, and $\pi(t)$ is the amount of taxation per capita. Taxes collected by the government are a source of subsidies for firms.

In this economy, households determine the path of per-capita consumption to maximize Eq. (1) under the two constraints of Eq. (2) and the No-Ponzi Game conditions. As is well known, the per-capita consumption path satisfies the following Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}[r(t) - \rho]. \tag{3}$$

However, the path of assets per capita must satisfy the following transversality condition:

$$\lim_{t\to\infty}\left\{a(t)\exp\left[-\int_0^t [r(s) - n]ds\right]\right\} = 0. \tag{4}$$

Equation (4) shows that the No-Ponzi Game condition holds with equality on the optimal consumption path.

2.2 Production

Next, I describe firms' production activities. In this study, final goods are produced by combining various tasks. For simplicity, this study assumes that the final good is the numeraire and its price is unity. The production function of the final good is given in Cobb-Douglas form:[3)]

$$Y = \exp\left(\int_0^1 \ln y(i)\, di\right), \tag{5}$$

where $y(i)$ is the amount of task i.

This model assumes that human labor or capital (machines) can perform tasks. While a task *automated* when it is produced by capital, it is *not automated* when it is performed by human labor. To incorporate the process of automation into the model, I assume that the task production function is perfectly substitutable between capital and labor, as follows:

$$y(i) = A_K k(i) + A_L \gamma(i) l(i), \tag{6}$$

where $k(i)$ and $l(i)$ are the amount of capital and labor used for the production of task i. A_K and A_L represent the productivity parameters for capital and labor, respectively. $\gamma(i)$ represents the task-specific productivity. I assume that the function $\gamma:[0,1] \to \mathbb{R}^+$ is continuously differentiable and strictly increasing. The latter assumption implies that labor has a comparative advantage in the production of higher-indexed tasks.

In the following, I describe the behavior of firms based on the two production functions in Eqs. (5) and (6). I assume that all markets are perfectly competitive. Therefore, the price of a task is equal to the unit cost of task production. Let the rental price of capital be R. If the production of task i is automated, then the cost of producing one unit of task i is R/A_K from Eq. (6). If the task is not automated, then the unit cost is $w/A_L\gamma(i)$ because the task is performed using labor.

The government subsidizes the use of each production factor. Specifically, for each unit of production of an automated task, the government subsidizes τ of its unit cost; for each non-automated task, the government subsidizes v of the unit cost of the task. With government subsidies, the effective unit costs of using the

3) Acemoglu and Restrepo (2018) assume both task automation and that old tasks are replaced by new (more complex) tasks. However, to focus on the relationship between automation and capital accumulation, this study focuses only on task automation and ignores the introduction of new tasks into the production process.

two production factors are $(1-\tau)R/A_K$ and $(1-\nu)w/A_L\gamma(i)$, respectively. The producer of a task chooses the lower of these two effective costs as a factor of production. Therefore, the price of task i is given as:

$$p(i) = \min\left[\frac{(1-\tau)R}{A_K}, \frac{(1-\nu)w}{A_L\gamma(i)}\right]. \tag{7}$$

Here, I describe the automated tasks. Since $\gamma(i)$ is an increasing function of i, there exists a threshold of automation, $I \in (0,1)$, such that the two unit costs are equal. Therefore, From Eq. (7), I is determined such that the following condition is satisfied:

$$\frac{(1-\tau)R}{A_K} = \frac{(1-\nu)w}{A_L\gamma(I)}. \tag{8}$$

Figure 1 shows the relationship between I and the factor prices. The line representing $(1-\tau)R/A_K$ is horizontal because it is independent of i, while $(1-\nu)w/A_L\gamma(i)$ is a declining curve with a unique value of I that satisfies Eq. (8). For all tasks such that $i < I$, the task is automated and produced with capital because the unit cost of producing it with automation technology is less than that of producing it with non-automation technology. For tasks such that $i \geq I$, the task is not automated and is produced using labor.

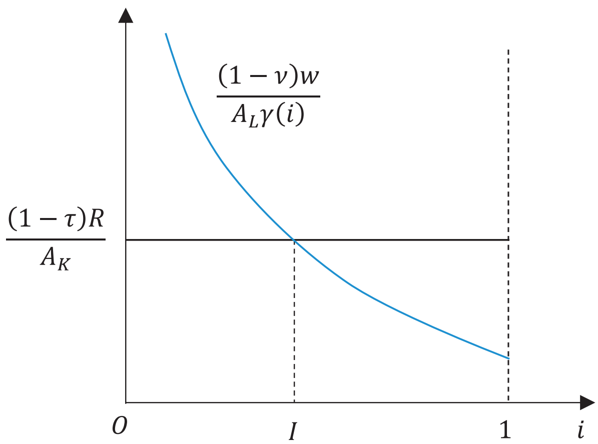

Figure 1: Threshold of Automation and Factor Prices

Next, I consider the demand for capital and labor. Because the production function of the final good is of the Cobb-Douglas type, as given by Eq. (1), the demand function for each task is

$$y(i) = \frac{Y}{p(i)}. \tag{9}$$

When the production of a certain task is automated, the quantity of the task is $y(i) = A_K k(i)$. In addition, Eq. (7) implies that the price of the task is $p(i) = (1-\tau)R/A_K$. By substituting these results into Eq. (9), I obtain the following capital demand for each automated task:

$$k(i) = \frac{Y}{(1-\tau)R}. \tag{10}$$

Similarly, the demand for labor for each non-automated task producer is

$$l(i) = \frac{Y}{(1-\nu)w}. \tag{11}$$

Here, I derive the market-clearing conditions for capital and labor. The total number of automated tasks is I, and the capital demand for each task is given by Eq. (10). Thus, the demand for capital in the economy overall is given by $IY/(1-\tau)R$. Because the capital supply is K, the equilibrium condition for the capital market is

$$K = \frac{IY}{(1-\tau)R}. \tag{12}$$

The equilibrium condition for the labor market is

$$L = \frac{(1-I)Y}{(1-\nu)w}. \tag{13}$$

Here, I calculate how the number of automated tasks is determined in equilibrium. Dividing Eq. (12) by Eq. (13), and substituting Eq. (8) into the result yield

$$\frac{I}{1-I}\gamma(I) = \frac{A_K K}{A_L L}. \tag{14}$$

Figure 2 shows how Eq. (14) determines I. The left-hand side of Eq. (14) is a strongly increasing function with respect to I, whereas the right-hand side represents the relative factor supply (in terms of efficiency units), which is independent of I. Therefore, there is a unique I such that Eq. (14) is satisfied. In addition, an increase in capital K increases the value on the right-hand side of Eq. (14), indicating that it increases the value of I. Intuitively, an increase in capital reduces the rental price of capital. Consequently, more tasks become automated as capital becomes less expensive. Similarly, an increase in A_K, which represents technological progress in automation, encourages task automation.

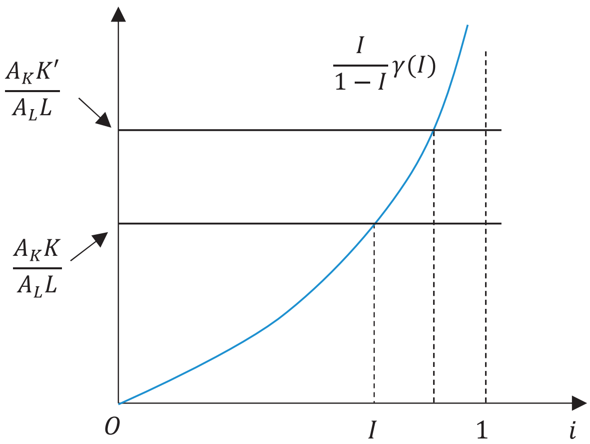

Figure 2: Relative Factor Supply (in Efficiency Units) and Determination of I

Equation (14) also shows that the subsidy rates for each factor of production, τ and ν, do not affect the determination of I. For example, an increase in the subsidy rate for capital utilization increases the demand for capital according to Eq. (10). On the other hand, because the supply of capital is constant in the short run, the rental price of capital also increases by the same proportion[4]. Therefore, the effect of the subsidy rate is completely offset and the subsidy rate does not affect the automation threshold.

Next, I derive the equilibrium output. For tasks such that $i < I$, production is automated, and the output is represented by $y(i) = A_K K/I$. However, for tasks such that $i \geq I$, production is not automated, and the output is $y(i) = A_L \gamma(i) L/(1-I)$. Substituting these results into Eq. (1) yields the following aggregate output:

$$Y = \frac{(A_K K)^I (A_L L)^{1-I}}{I^I (1-I)^{1-I}} \exp\left(\int_I^1 \ln \gamma(i) di\right). \tag{15}$$

While Eq. (15) resembles a typical Cobb-Douglas production function, I is not a constant parameter, but is determined by Eq. (14). For example, as the amount of capital K, increases, the marginal productivity of capital declines in the usual Cobb-Douglas model. However, according to Eq. (14), an increase in capital quantity leads to an increase in I. This effect reduces the degree of the diminishing marginal productivity of capital and significantly affects the dynamic properties of the model.

Substituting Eq. (14) into Eq. (15) and rearranging the result yields the following expression for the per capita production function:

$$y = (A_K k + A_L \gamma(I) L)\Omega(I), \tag{16}$$

where $k = K/L$ and $y = Y/L$ denote the capital stock and output per unit of labor, respectively. $\Omega(I)$ is defined as $\Omega(I) \equiv \exp\left(\int_I^1 \ln\left(\frac{\gamma(i)}{\gamma(I)}\right) di\right)$.[5] Because Eq. (14) implies the existence of a unique value of $I \in (0,1)$ for any $k > 0$, we represent this (functional) relation as $I = \varphi(k)$. Equation (16) is then represented by the following relationship between k and y:

$$y = \big(A_K k + A_L \gamma(\varphi(k))\big)\Omega\big(\varphi(k)\big). \tag{17}$$

From Eq. (17), we derive the following proposition:

Proposition 1: ***Define a new function*** $\boldsymbol{f}: \mathbb{R}^+ \to \mathbb{R}^+$ ***as*** $\boldsymbol{f(k) \equiv (A_K k + A_L \gamma(\varphi(k)))\Omega(\varphi(k))}$. ***Then,*** $\boldsymbol{f'(k) > 0}$ ***and*** $\boldsymbol{f''(k) < 0}$ ***for any*** $\boldsymbol{k > 0}$. ***Moreover, the following holds for the limit of the marginal productivity of capital:***

$$\lim_{k\to\infty} f'(k) = A_K.$$

4) This is an outcome of the assumption that the production function of the final good is Cobb-Douglas type. I would obtain different results if the model adopts the more general constant elasticity of substitution (CES) production function.

5) $\Omega(I)$ can be interpreted as total factor productivity.

Proof: See Appendix 1.

According to Proposition 1, the marginal product of capital is positive and diminishing while its lower bound is not zero but A_K. In the usual neoclassical production function, an increase in the quantity of capital leads to a diminishing marginal product of capital. However, an increase in capital raises the marginal product of capital through an increase in the number of automated tasks from Eq. (14). Consequently, the effect of the diminishing marginal productivity of capital is weakened and has a positive lower bound.

Figure 3 presents a numerical example. The curve depicted by the solid blue line represents the production function in Eq. (17) (i.e., I is determined from Eq. 14), and the remaining three dotted lines represent Eq. (15) for cases in which I is fixed at $I = 0.2, 0.3$, and 0.4. In Fig. 3, the graph of Eq. (17) represents the envelope of the graph in Eq. (15). Intuitively, Eq. (14) holds because of the producer's choice of whether to use capital or labor to minimize the cost of task production. This result implies that society chooses the level of I to maximize the output of the final good, given K and L. Thus, Eq. (17) becomes the envelope of Eq. (14).

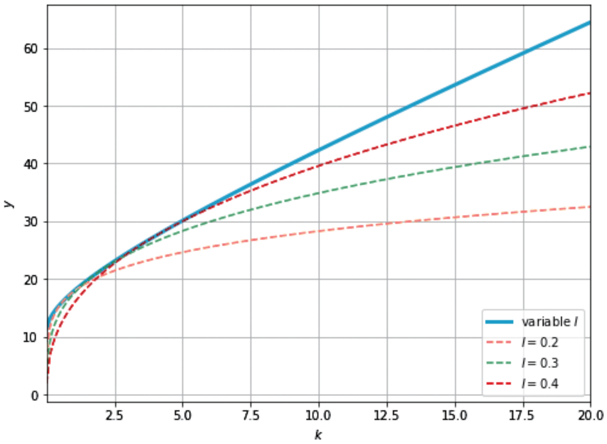

Figure 3: Relationship between Eqs. (15) and (17)
($A_K = 2$, $A_L = 30$, $\gamma(i) = i$)

Next, I derive the equilibrium factor prices. The rental price of capital and wage rate are given as follows:

$$R = \frac{A_K \Omega(I)}{1-\tau} = \frac{A_K \Omega(\varphi(k))}{1-\tau}, \tag{18}$$

$$w = \frac{A_L \gamma(I)\Omega(I)}{1-\nu} = \frac{A_L \gamma(\varphi(k))\Omega(\varphi(k))}{1-\nu}. \tag{19}$$

From Eqs. (18) and (19), I derive the following proposition.

Proposition 2: ***The rental price of capital is a decreasing function of the capital-labor ratio, and the wage is an increasing function of the capital-labor ratio. Moreover, the rental price of***

capital converges to its lower bound, $\boldsymbol{A_K/(1-\tau)}$***, and the wage converges to its upper bound,*** $\boldsymbol{A_L\gamma(1)/(1-\nu)}$ ***as the capital-labor ratio goes to infinity.***

Proof: See Appendix 2.

The same mechanism as in Proposition 1 yields the result of Proposition 2. In a normal neoclassical production function, an increase in capital per worker raises the marginal productivity of labor and leads to a higher wage. In this model, on the other hand, it also encourages task automation and reduces labor demand. Consequently, wage increases are suppressed.

In addition, increases in the government's subsidy rates raise the factor prices in equilibrium. From Eqs. (12) and (13), an increase in the subsidy rate increases the factor demand. However, factor supply is constant in the short run. Consequently, these policies increase factor prices. However, in the long run, a higher subsidy rate for automation increases the return on machines and encourages capital accumulation. I discuss this point in Section 3.

Finally, for the income distribution, Eqs. (18) and (19) give the after-tax capital and labor shares as $(1-\tau)RK/Y=I$ and $(1-\nu)wL/Y=1-I$, respectively. Thus, an increase in capital per-unit of workers, through Eq. (14), raises the capital share while decreasing the labor share. Intuitively, an increase in capital per worker enhances labor productivity while suppressing the increase in wages, thus reducing the labor share. In addition, an increase in A_K decreases the labor share through Eq. (14).

2.3 Government Budget Constraint

In this section, I consider government budget constraints. In each period, the government collects lump-sum taxes from households and uses tax revenue to subsidize firms (i.e., the budget is balanced in each period). For automated tasks, the subsidy per unit of task is given by $\tau R/A_K$. Because the output of a task is $y(i)=A_K K/I$, the total amount of subsidy for an automated task is

$$\int_0^I \frac{\tau R}{A_K}\cdot\frac{A_K K}{I}di=\tau RK. \tag{20}$$

Similarly, the total subsidy for non-automated tasks is

$$\int_I^1 \frac{\nu w}{A_L\gamma(i)}\cdot\frac{A_L\gamma(i)L}{(1-I)}di=\nu wL. \tag{21}$$

From Eqs. (20) and (21), the total subsidy for all task production is $\tau RK+\nu wL$. Assuming a balanced budget, this is equal to the amount of taxes collected from households, $L\pi$. Therefore, the following relationship holds.

$$L\pi=\tau RK+\nu wL. \tag{22}$$

3. Equilibrium Path of the Economy

This section derives the economic equilibrium path for the model developed in the previous section. There is a relationship between the rental price of capital and the interest rate, $R(t) = r(t) + \delta$. By substituting this relationship into Eq. (3), I obtain the following consumption growth rate:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left[\frac{A_K \Omega(\varphi(k(t)))}{1-\tau} - \delta - \rho\right]. \tag{23}$$

Substituting the two factor prices, Eqs. (18) and (19), and the government's budget constraint, Eq. (22), into the household's budget constraint, Eq. (2). Using $a(t) = k(t)$, the behavior of the capital per unit of worker is

$$\dot{k}(t) = \left(A_K k(t) + A_L \gamma(\varphi(k(t)))\right)\Omega\left(\varphi\left(k(t)\right)\right) - (n+\delta)k(t) - c(t). \tag{24}$$

Finally, the transversality condition is

$$\lim_{t\to\infty}\left\{k(t)\exp\left[-\int_0^t \left[\frac{A_K \Omega(\varphi(k(s)))}{1-\tau} - \delta - n\right] ds\right]\right\} = 0. \tag{25}$$

The equilibrium path of this economy is characterized by $c(t)$ and $k(t)$, satisfying Eqs. (23)–(25), where $k(0)$ is given historically.

From Proposition 1, because the marginal productivity of capital has a positive lower bound, the economy is expected to achieve sustained growth if A_K is sufficiently large. Therefore, we rewrite the differential equations above using variables that are constant on a balanced growth path. We use $I(t) = \varphi(k(t))$ and $\chi(t) \equiv c(t)/k(t)$. Clearly, $I(t)$ is a state variable because it depends only on $k(t)$. On the other hand, the new variable $\chi(t)$ is a control variable since it is defined using $c(t)$. Using these two new variables, the following system of differential equations can be derived:[6]

$$\frac{\dot{I}(t)}{I(t)} = \frac{1-I(t)}{1+\left(1-I(t)\right)I\dfrac{\gamma'\left(I(t)\right)}{\gamma\left(I(t)\right)}}\left[\frac{A_K\Omega\left(I(t)\right)}{I(t)} - \chi(t) - (n+\delta)\right], \tag{26}$$

$$\frac{\dot{\chi}(t)}{\chi(t)} = \frac{1}{\theta}\left[\frac{A_K\Omega\left(I(t)\right)}{1-\tau} - \delta - \rho\right] - \left[\frac{A_K\Omega\left(I(t)\right)}{I(t)} - \chi(t) - (n+\delta)\right]. \tag{27}$$

Figure 4 shows the phase diagram on the (I, χ) plane based on Eqs. (26) and (27). First, from Eq. (26), the $\dot{I} = 0$ lines are given by the vertical line of $I = 1$ and the curve is defined as

$$\chi = \frac{A_K\Omega(I)}{I} - (n+\delta). \tag{28}$$

6) For the derivation of Eqs. (26) and (27), see Appendix 3.

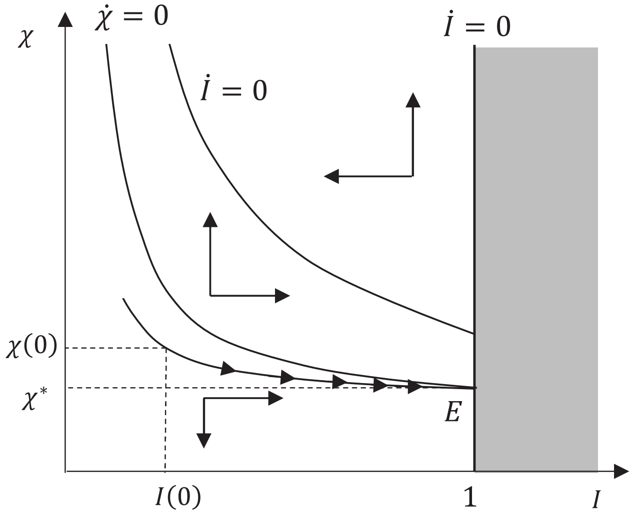

Figure 4: Phase Diagram on the (I, χ) Plane (sustained growth)

The right side of Eq. (28), is a decreasing function of I. However, from Eq. (27), the $\dot{\chi} = 0$ line is represented as a curve, defined as

$$\chi = g(I) \equiv \left[\frac{A_K \Omega(I)}{I} - (n + \delta)\right] - \frac{1}{\theta}\left[\frac{A_K \Omega(I)}{1 - \tau} - \delta - \rho\right]. \tag{29}$$

If $A_K/(1 - \tau) - \delta - \rho > 0$, then the $\dot{\chi} = 0$ line will always lie below the $\dot{I} = 0$ line as shown in Fig. 4. I can confirm that a unique equilibrium path such that $(I(t), \chi(t))$ converges to point E in this case. On the equilibrium path, $I(t)$ increases toward its upper bound of one, while $\chi(t)$ decreases and converges to

$$\chi^* = A_K - (n + \delta) - \frac{1}{\theta}\left[\frac{A_K}{1 - \tau} - \delta - \rho\right]. \tag{30}$$

The convergence of $I(t)$ to one corresponds to a situation in which capital per worker can grow unboundedly. In fact, from Eq. (23), the growth rate of capital per worker converges to

$$g^* = \frac{1}{\theta}\left[\frac{A_K}{1 - \tau} - \delta - \rho\right]. \tag{31}$$

On a balanced growth path, the output and consumption per capita also grow at the same rate, as in Eq. (31).

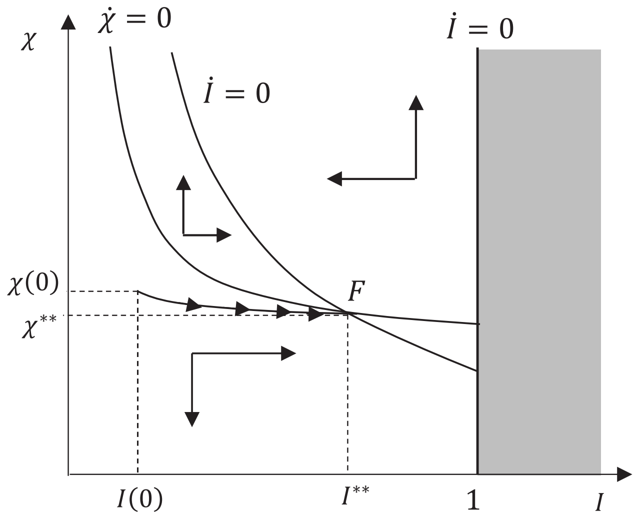

Figure 5: Phase Diagram on the (I, χ) Plane (A case of limited growth)

In contrast, if $A_K/(1-\tau)-\delta-\rho<0$, the $\dot{\chi}=0$ line intersects the $\dot{I}=0$ line once, as shown in the Fig. 5. Therefore, this intersection, point F, is the steady state of the model. In this case, there exists a unique equilibrium path such that $(I(t), \chi(t))$ converges at point F. On the equilibrium path, $I(t)$ increases toward I^{**} while $\chi(t)$ decreases and converges to χ^{**}. This implies that the capital per worker converges to k^{**} satisfying $I^{**}=\varphi(k^{**})$, so the growth rate of output per capita will eventually converge to zero. In other words, sustainable economic growth is not realized in this case. From the above discussion, I immediately obtain the following proposition.

Proposition 3: When the level of capital-augmenting technology (A_K) and the subsidy rate for automation (τ) are sufficiently high, this economy will achieve sustainable growth even without perpetual technological progress.

This proposition is one of the most important contributions of this study. It shows that if the productivity parameter associated with capital stock and the rate of automation subsidies are sufficiently high, capital accumulation will increase task automation and the economy will achieve sustainable growth. In typical endogenous growth theory, the production function becomes AK-shaped owing to various externalities, such as knowledge spillovers from capital stocks. However, this study shows that task automation weakens the effect of diminishing marginal productivity of capital, resulting in an (asymptotically) AK-type production function as in Eq. (17). This study shows that automation can be another path to long-term economic growth.

Moreover, the subsidy rate for automation is a determinant of the long-run growth rate. Increasing the subsidy rate for automation increases the demand for machines and the rental price of capital in equilibrium.

This is confirmed using Eq. (18). Therefore, a higher subsidy rate raises the interest rate, leading to faster capital accumulation, and consequently, a higher economic growth rate. Consequently, government subsidies for automation can stimulate economic growth.

However, the transition paths of factor prices change as they satisfy Eqs. (18) and (19). In the case shown in Fig. 4, the wage rate increases but approaches its upper limit, $A_L\,\gamma(1)/(1-\nu)$. This wage rate behavior is attributed to the fact that an increase in the quantity of capital weakens the demand for labor through task automation. Therefore, wage growth is suppressed, and labor share declines during the growth process. Thus, while automation positively affects economic growth, it also makes income distribution unfavorable for workers.

In recent years, a decline in labor share has been observed in developed countries, and many researchers have discussed the underlying causes. Karabarbounis and Neiman (2014) point out that the lower cost of capital due to advances in IT reduced the labor share. However, their results are valid only when the elasticity of substitution between capital and labor is greater than one. In this study, on the other hand, the elasticity of substitution is sufficiently large because the production function is asymptotically linear. Consequently, increases in capital and advances in automation technology lead to decreased labor share.

4. Concluding Remarks

By incorporating a task-based approach into the Ramsey model, this study analyzes the impact of subsidy policies on automation and economic growth. The analysis emphasizes which tasks are automated because of firms' cost-minimization behavior. Therefore, if machines are cheaper than labor for performing a task, the task will be mechanized (automated). Consequently, the capital accumulation and productivity of machines affect the automation of a task through the rental price of capital.

The main consequences of this study are as follows. (i) The aggregate production function is asymptotically linear if firms' cost-minimizing behavior determines the task automation. (ii) If capital productivity and the subsidy rate for automation are sufficiently high, the economy will achieve sustainable growth, even without perpetual technological progress. (iii) The rental prices of capital and wages converge to constant values on the equilibrium path. In addition, while an increase in the subsidy rate for automation increases the long-term rental price of capital, it does not affect the long-term wage rate.

In ordinary neoclassical growth models, capital accumulation reduces marginal capital productivity. By contrast, this study shows that capital accumulation promotes task automation, which increases the demand for capital and decreases the demand for labor. This effect reduces the degree to which the marginal productivity of capital declines and the production function asymptotically becomes linear. Consequently, an economy can achieve sustained growth without permanent technological progress.

This study also finds that government subsidies play a crucial role in stimulating economic growth.

Specifically, increasing the subsidy rate for automation increases the demand for machines and the equilibrium rental price of capital. This leads to higher interest rates and faster capital accumulation, which ultimately promotes long-term economic growth. These results underscore the importance of government subsidies in driving economic growth.

Although this study provides essential results on automation and economic growth, some critical questions remain. First, this study assumes that the total number of tasks is set at one because it focuses on the relationship between task automation and capital accumulation. However, as Acemoglu and Restrepo (2018) point out, we often observe the introduction of new tasks and disappearance of old ones during the production process. Introducing task-related dynamics into the analysis would enrich my analysis.

On the policy side, this study shows that subsidies for automation effectively promote long-term economic growth. By contrast, employment subsidies are less important because they only raise wages. However, these results rely on the assumption that the task production function is Cobb-Douglas and labor supply is exogenously given. Therefore, to obtain realistic policy effects, it is important to use a more general class of production functions and endogenize labor supply.

Finally, this study demonstrates that capital productivity plays a vital role in sustainable economic growth. However, this study does not discuss the factors that determine this parameter. Therefore, analyzing the sources of capital productivity growth is essential for clarifying the macroeconomic impact of automation.

Appendix 1: Proof of Proposition 1

This appendix presents the proof for Proposition 1. This proof is essentially the same as that of Ikeshita et al. (2013). Since $\gamma(i)$ is continuously differentiable and increasing with i, the sign of the derivative of $\gamma(i)$ is positive. In addition, Eq. (20) implies that $I = \varphi(k) > 0$ for any $k > 0$. Therefore, $\gamma(I) > 0$ and $\gamma'(I) > 0$. Applying the implicit-function theorem to Eq. (20) yields

$$\frac{dI}{dk} = \varphi'(k) = \frac{(1-I)\frac{A_K}{A_L}}{\gamma(I) + I\gamma'(I) + \frac{A_K}{A_L}k} > 0. \tag{A1}$$

Moreover, differentiating $\Omega(I)$ with respect to I yields

$$\frac{d\Omega}{dI} = -(1-I)\frac{\gamma'(I)}{\gamma(I)}\Omega(I) < 0. \tag{A2}$$

Based on the above results, I determine the property of $f(k)$. Differentiating $f(k)$ with respect to k yields

$$f'(k) = \left(A_K + A_L\gamma'(I)\frac{dI}{dk}\right)\Omega(I) + \left(A_K + A_L\gamma(I)\right)\frac{d\Omega}{dI}\frac{dI}{dk}. \tag{A3}$$

Substituting Eq. (A2) into Eq. (A3) and rearranging the result, the derivative of $f(k)$ can be expressed as

$$f'(k) = A_K\Omega(I) + \gamma(I)\Omega(I)\frac{dI}{dk}\left(A_L I - \frac{(1-I)\,A_K k}{\gamma(I)}\right). \tag{A4}$$

Since Eq. (14) implies $A_L I - (1-I)A_K k/\gamma(I) = 0$, the second term in Eq. (A4) becomes zero and I derive $f'(k) = A_K\Omega(\varphi(k)) > 0$. The second-order derivative of $f(k)$ is

$$f''(k) = -A_K(1-I)\frac{\gamma'(I)}{\gamma(I)}\Omega(I)\frac{dI}{dk}. \tag{A5}$$

As Eq. (A1) is positive, the sign of Eq. (A5) is negative. Finally, Eq. (14) implies that $I \to 1$ as $k \to \infty$. Given the definition of Ω and $\lim_{I\to 1}\Omega(I) = 1$, we obtain $\lim_{k\to\infty} A_K\Omega(\varphi(k)) = A_K$.

Appendix 2: Proof of Proposition 2

For the rental price of capital, I derive $R = f'(k)/(1-\tau)$ from Eq. (18) and Appendix 1, and the result of Proposition 2 holds. Therefore, this appendix presents only the wage rate results. Taking the logarithms of both sides of Eq. (19) and differentiating with respect to k yields

$$\frac{d\ln w}{dk} = \left(\frac{d\ln\gamma(I)}{dI} + \frac{d\ln\Omega(I)}{dI}\right)\frac{dI}{dk}. \tag{A6}$$

Substituting Eq. (A2) into Eq. (A6) yields

$$\frac{d\ln w}{dk} = I\frac{\gamma'(I)}{\gamma(I)}\frac{dI}{dk}. \tag{A7}$$

As Eq. (A1) is positive, Eq. (A7) is also positive. In addition, for the rental price of capital, $\lim_{k\to\infty} A_L\gamma(\varphi(k))\Omega(\varphi(k))/(1-\nu) = A_L\gamma(1)/(1-\nu)$ holds.

Appendix 3: Derivation of Eqs. (26) and (27)

First, I show the derivation of Eq. (26). Dividing both sides of Eq. (24), which represents the accumulation of capital stock, by k and using $I=\varphi(k)$, the motion of capital stock is

$$\frac{\dot{k}}{k} = \left[A_K + \frac{A_L\gamma(I)}{k}\right]\Omega(I) - \chi - (n+\delta), \tag{A8}$$

where $\chi \equiv c/k$. Transforming Eq. (14) yields $A_L\gamma(I)/k = (1-I)\,A_K/I$. Substituting this result into Eq. (A8) yields:

$$\frac{\dot{k}}{k} = \frac{A_K\Omega(I)}{I} - \chi - (n+\delta). \tag{A9}$$

On the other hand, from $I = \varphi(k)$, the growth rates of I and k have the following relationship:

$$\frac{\dot{I}}{I} = \frac{k\varphi'(k)}{I} \cdot \frac{\dot{k}}{k}. \tag{A10}$$

Here, using Eq. (14) and Eq. (A1), the following relationship is derived:

$$\frac{k\varphi'(k)}{I} = \frac{1 - I}{1 + (1 - I)I\frac{\gamma'(I)}{\gamma(I)}}. \tag{A11}$$

Substituting Eqs. (A19) and (A11) into (A10), Eq. (26) in the main text is obtained.

Next, I explain the derivation of Eq. (27). From the definition of χ, the growth rate of this variable is

$$\frac{\dot{\chi}}{\chi} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k}. \tag{A12}$$

Substituting Eqs. (23) and (A9) into Eq. (A12) immediately yields Eq. (27) in the main text.

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