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Abstract: There have been many empirical equations in studies on flow over Piano Key Weir (PKW), but these equations are only suitable for each research model. Thus, there cannot be a correct general equation for all cases. Additionally, machine learning is a research method that can gather all the data to build a training model. The more data, the more accurate the prediction. Moreover, it is also a solution that tends to suit current developments. Currently, there are many machine learning algorithms, each with advantages and disadvantages. In particular, the Support Vector Machine (SVM) algorithm is also a regression algorithm with high prediction performance. This study analyzed the factors affecting the flow over type-A PKW according to Pi theory. From there, it established the prediction objective function in machine learning and then applied the SVM algorithm for prediction. The results indicated that the Medium Gaussian SVM model has good predicting performance, comparing the predicted values and the measured values showed a very high correlation coefficient ($R^2 \approx 0.97$), other statistical indicators were very close to the ideal point (MSE = 0.001; RMSE = 0.033; MAE = 0.025). Furthermore, the largest percentage error was only 8.7%. This demonstrated that the SVM algorithm is suitable for studying and predicting flow characteristics over type-A PKW.

Keywords: PKW; Machine learning; SVM; discharge coefficient; Buckingham

1. Introduction

Increasing the ability to release water over dams with limited space and at the same time can well control the flow downstream. Therefore, the Piano Key Weir (PKW) was proposed, as it increases the discharge capacity compared to a conventional dam (the straight dam crest) from four to five times[1,2)].

Fig. 1: Dak Mi 3 hydropower plant - quang nam province, vietnam[3)].

The surveys showed that the Galours dam in France is a type-A PKW built in 2006[4)], it is one of the PKWs that it It was the earliest built in the world. Then, PKW proposals and improvements have expanded its application to many countries, such as India, Australia, and Switzerland, etc. In Vietnam, this type of dam is also widely applied (Fig. 1), such as the Van Phong dam (2015), Dak Mi 3 dam (2017), Phu Phong dam (2024) and so on. Currently, PKW is classified into four types (A, B, C, and D). The evaluation demonstrated that type-A PKW is widely applied in Vietnam (Fig. 1)[5)], so this research focuses on predicting discharge over type-A PKW.

According to studies on type-A PKW (Fig. 2 and 3), the dam structure has approximately 20 typical factors. It is identified through factors such as the total width of the PKW (W), total crest length (L), developed length of one PKW unit (L_u), width of outlet key (W_o), width of inlet key (W_i), width of a PKW unit (W_u), weir height (P), upstream-downstream length of PKW (B), overhang length of inlet key (B_i), overhang length of outlet key (B_o), weir base length (B_b), sidewall thickness (T_s), and slope of the inlet/outlet key apron (S_i) and characteristics that affect flow including PKW discharge (Q), specific discharge (q), total head (H_o), discharge coefficient (C_d)[6-8], etc.

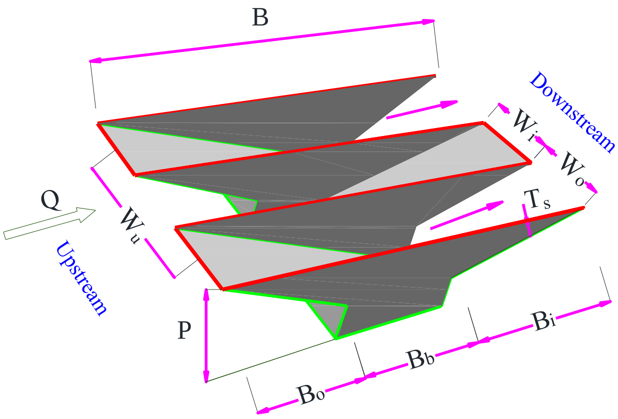

Fig. 2: Structure of the 3D model of type-A PKW[6].

The dimensions of the type-A PKW structure are shown as follows[5,6,9,10],

$$W_u = W_i + W_o + 2T_s \tag{1}$$
$$L_u = 2B + 2T_s + W_i + W_o \tag{2}$$
$$B = B_b + B_o + B_i \tag{3}$$

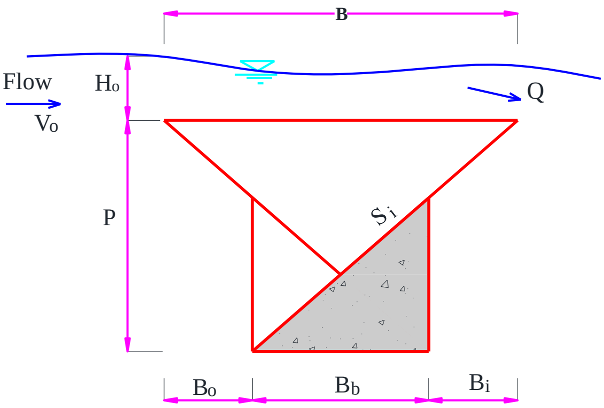

Fig. 3: Structure of the side view of the type-A PKW[5].

Studies on PKW primarily established empirical equations for determining discharge coefficient (C_d). Each study will have different influencing factors that affect the discharge coefficient, this is due to differences in research objectives and research data is often collected from only one or two physical models. The influencing factors considered are L/W, B/P, Bo/B, L/W, Wo/Wi and Pi/ Po, Ho/P, Ho/Lu, and Ho/Wu[6-8,11-14]. Each study has its meaning and scope of research; therefore, different equations will give different calculation results for the same research case. Hence, proposing a correct general equation for all research cases is challenging.

Meanwhile, machine learning is a tool with many advantages in analyzing statistical data, especially in research focused on predicting convergent factors. Machine learning algorithms perform well and have high accuracy in predicting. These algorithms include Artificial Neural Networks (ANN) and Adaptive Neuro Fuzzy Inference System (ANFIS) for the research and analysis of discharge[15-18]; Gene Expression Programming (GEP) and Extreme Gradient Boosting (XGBoost) algorithms for analyzing the energy of the flow over a type-C PKW[19]; GEP for predicting energy consumption of type-A PKWs[20]. The Tree and Random Forest algorithms are used to predict the discharge over PKW with rectangular and trapezoidal keys[21]. Besides, the SVM and the Firefly optimization-based Support Vector Regression (SVR-FA) have also been used to predict the discharge coefficient of the labyrinth dams[22]. In addition, the SVM regression algorithm has been applied to predict many hydraulic factors, such as predicting hydraulic characteristics of open flow in channels[23,24].

This shows that the SVM algorithm has not yet fully considered its applicability and suitability for studying hydraulic characteristics. Meanwhile, there are currently no complete studies on applying machine learning algorithms in flow forecasting for type-A PKW, which limits the connection of research data. Research has not been generalized to ensure effective flow forecasting through PKW in water resource control. Therefore, type-A PKW was introduced in this study with hydraulic characteristics observed on the physical model. Buckingham's Pi theory was used to establish the relationship between the discharge coefficient and the influencing factors. Then, the study's influencing variables and target variables were established by applying the SVM algorithm to predict the discharge coefficient (C_d), which serves the process of calculating the flow over the type-A PKW.

2. Study method

Applying machine learning algorithms to factor prediction research includes two basic stages: training and forecasting, each stage corresponding to a set of research data[25]. A suitable research algorithm is needed to identify the influencing factors in datasets. In studying the hydraulic problem, Buckingham's Pi theory has been applied to examine influencing factors[26]. Subsequently, the input data of the SVM algorithm were determined. Additionally, data from observations on physical models was collected. Finally, we developed steps to apply machine learning algorithms to research hydraulic characteristics.

2.1 Objective function

The objective function equation for determining the factors affecting the flow through the type A PKW is described in equation (4) as follows,

$$f(Q_u, W_u, B, L_u, P, S_i, \gamma, H_o) = 0 \tag{4}$$

In addition, the discharge of a free-flowing type-A PKW for a unit is determined according to a common equation[9],

$$Q_u = C_d W_u \sqrt{2g} H_o^{3/2} \tag{5}$$

So:

$$C_d = \frac{Q_u}{W_u \sqrt{2g} H_o^{3/2}} \tag{6}$$

The natural flow was studied so downstream factors do not affect the flow. Duplicate factors were eliminated: factors without effect on the predictive value to be studied or the interconnected factors. By applying Buckingham's Pi theory, the following was obtained,

$$f\left(C_d, \frac{H_o}{W_u}, \frac{H_o}{L_u}, \frac{H_o}{P}, Re\right) = 0 \tag{7}$$

The Reynolds number (Re) is not considered, because the flow in experimental models and flows in reality are alway turbulent. From this, it can conclude,

$$C_d = \Psi\left(\frac{H_o}{W_u}, \frac{H_o}{L_u}, \frac{H_o}{P}\right) \tag{8}$$

The discharge coefficient (C_d) is related to the data system of the Ho, Lu, Wu, and P factors. This is similar to previous studies[1,5,28-30].

In machine learning, the data fields were defined as follows,

+ Variable target: C_d

+ Variable influences: $\frac{H_o}{W_u}, \frac{H_o}{L_u}, \frac{H_o}{P}$

2.2 SVM algorithm

SVMs are effective machine learning models to solve classification, regression, and individual element detection problems. The SVM algorithm is a solution to find the optimal "Hyperplane" determined by the support vector and the margin (Fig. 4). The principle of SVM is to separate a dataset with n points in space, and each point belongs to a class denoted (+) or (-)[22-24,35]. Thus, the data were divided into two classes and an optimal "hyperplane" (H) was determined to separate two classes (+ and -).

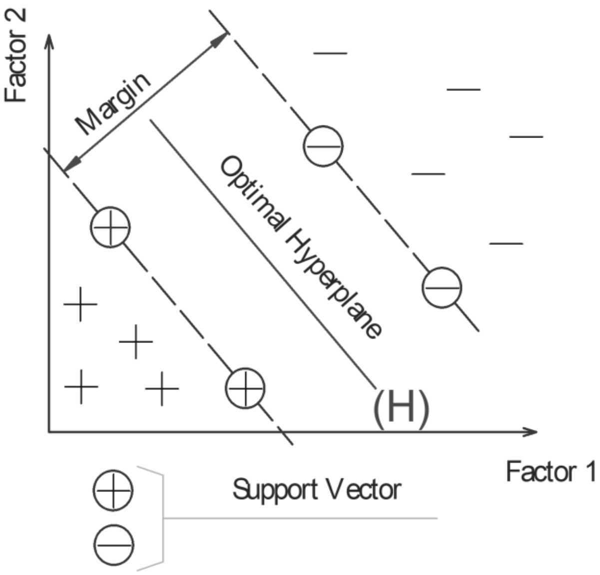

Fig. 4: Structure of SVM algorithm[31-33].

Determining the optimal "superplane" will be based on the characteristics of the algorithm structure. This study analyzed six different algorithms. Each algorithm had functions to determine different "hyperplanes" (Linear, Quadratic, Cubic, Fine Gaussain, Medium Gaussain, and Coarse Gaussain)[21,27,31-34]. In this study, SVM algorithms are studied in Matlab R2022b software.

2.3 Steps to apply the SVM algorithm

A study on applying machine learning algorithms to predict the discharge coefficient of the flow over PKW is conducted according to the following steps:

+ Step 1: Set up the objective function equation (this study uses equation 8).

+ Step 2: Screen input data (removing data with large mutations, the depth values less than 3 mm to avoid the effects of surface tension, etc.[5]).

+ Step 3: Identify the influencing variables and the target variables

+ Step 4: Set up training and testing datasets.

+ Step 5: Set up the training model according to SVM algorithms.

+ Step 6: Test the SVM model with test data and use statistical indicators for evaluation (R^2, MSE, RMSE, MAE, and MAPE[5,26,34]).

With the above six steps, a data control model will be built and applied to analyze and predict hydraulic characteristics for actual projects. A data control model will be built and applied to analyze and predict hydraulic characteristics for real projects. Interpretation of the research process is simulated according to the diagram in Fig. 5 as follows,

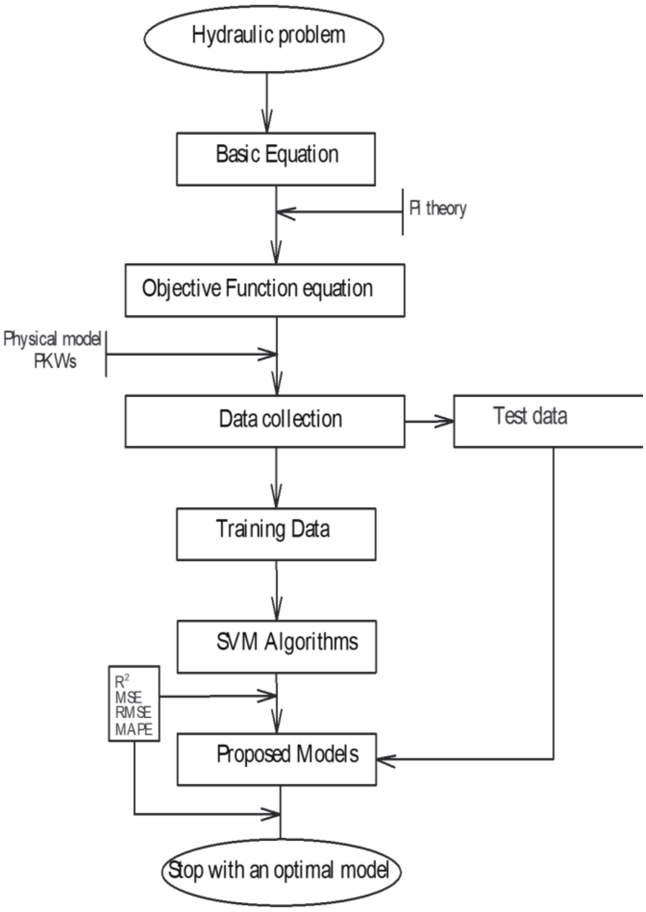

Fig. 5: Process chart for identifying machine learning models with high predictive performance.

3. Collecting experimental data

This study collected data from various sources on flow experiments over type-A PKW. The data was classified and the data structure in SVM models was established. The experimental model structures are shown in Table 1.

Table 1. Physical characteristics of type-A PKW models.

Ref.	L/W	P/W_u	W_i/W_o	B/P
Đ.T.M. Yen[29,30]	5.0	0.5÷ 1.1	1.30	1.87 ÷ 4.52
N. T. Hai et al.[36]	4.3÷ 8.2	0.3÷ 2.4	1.2	1.5 ÷ 4.5
O.Machiels et al.[37]	5.0	0.33÷ 2.0	1.5	1.0 ÷ 6.0
A.Noui et al.[10]	5.9	0.9	0.96÷ 1.53	2.73
A. Kabiri-Samani et al.[38]	6.0÷8.1	0.63÷ 1.79	0.33÷ 1.67	2.0

As it is displayed in Table 1, there are fire different type-A PKW models. These physical models were established to study the flow over the PKW.

The data collected from the physical models were processed and analyzed into dimensionless quantities. The hydraulic characteristics from the experiments are described in Table 2 as follows.

Table 2. Experimental data of physical models.

Ref.	q ($m^3/s/m$)	H_0/P	H_0/W_o	H_0/W_u	Type of PKW
Đ.T.M. Yen[29)-30)]	0.03 ÷ 0.32	0.17 ÷ 2.09	0.31 ÷ 2.08	0.136 ÷ 0.920	A
N. T. Hai et al.[36)]	0.03 ÷ 0.31	0.13 ÷ 2.15	0.23 ÷ 3.16	0.105 ÷ 1.355	A
O.Machiels et al.[37)]	0.04 ÷ 0.41	0.06 ÷ 2.68	0.26 ÷ 2.45	0.207 ÷ 0.810	A
A.Noui et al.[10)]	0.05 ÷ 0.17	0.15 ÷ 0.95	0.26 ÷ 2.05	0.105 ÷ 0.980	A
A. Kabiri-Samani et al.[38)]	0.05 ÷ 0.20	0.13 ÷ 0.56	0.16 ÷ 1.87	0.150 ÷ 1.00	A

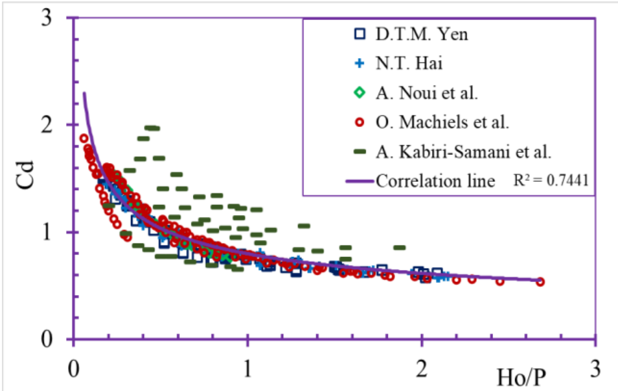

Fig. 6: Relationship between C_d and Ho/P according to all study data.

The research data were surveyed on six published studies on experimental research for the type-A PKW (Tables 1 and 2 and Fig. 6). A total of 320 datasets were identified and divided into training data and test data as follows,

+ Training data include 265 datasets (Table 3).

+ Test data include 55 datasets (equivalent to 20% of training data; this proportion of test data ensures objectivity in predicting and eliminates overfitting errors in predicting[32)]). The data characteristics are shown in Table 3.

Table 3. Training data of the machine learning model.

Values	H_o/P	H_o/W_u	H_o/L_u	C_d
Min	0.06	0.105	0.02	0.534
Max	2.68	1.355	0.22	1.975

The relationship between C_d and hydraulic factors is shown in Fig. 6. These relationships represent the physical law between hydraulic factors of the flow over the type-A PKW.

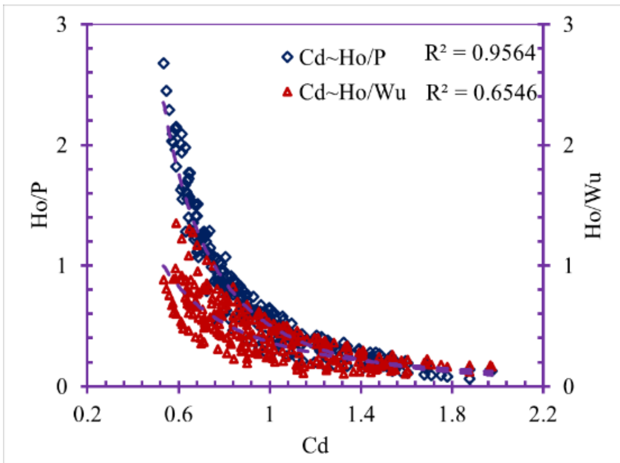

Fig. 7: Relationship between C_d and factors according to training data.

Table 3 and Figure 7 show that the relationship between C_d and Ho/P is tight (R^2 = 0.95). However, the correlation points are scattered around the average value, so if this relationship is used to predict C_d coefficients, there will be large errors. Therefore, it is necessary to consider many relationships to determine the C_d value accurately.

Table 4. Test data of the machine learning model.

Values	H_o/P	H_o/W_u	H_o/L_u	C_d
Min	0.24	0.106	0.03	0.558
Max	2.2	1.222	0.20	1.355

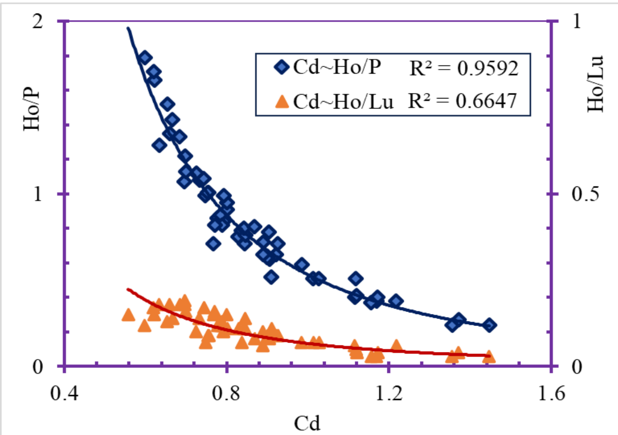

Fig. 8: Relationship between C_d and influencing factors according to test data.

The test data (Table 4) were used commonly for all machine learning models; the prediction results of the test data were employed to evaluate the suitability of the research according to each machine learning model.

From Figures 7 and 8, the relationship between C_d and influencing factors had correlation coefficients from 0.65 to 0.95, showing the ability to establish regression relationships.

4. Results and discussions

Study on applying SVM algorithm in machine learning for predicting the discharge coefficient, including six structural algorithms about the hyperplane (H) in Matlab software. The training data in Table 3 were used to implement training models according to SVM algorithms. The trained models were evaluated using statistical indicators to determine whether the prediction model performs well.

Evaluation of SVM algorithms is carried out based on statistical indicators between predicted values and measured values. These statistical indicators are analyzed directly in Matlab software and these indicators are shown in Table 5.

Table 5. Statistical indicators after training by svm models.

Models	RMSE	MSE	R^2	MAE
Linear SVM	0.164	0.027	0.742	0.121
Quadratic SVM	0.091	0.008	0.920	0.062
Cubic SVM	0.062	0.004	0.963	0.044
Fine Gaussain SVM	0.048	0.002	0.978	0.036
Medium Gaussain SVM	0.059	0.004	0.966	0.038
Coarse Gaussain SVM	0.107	0.011	0.891	0.072

From Table 5, the statistical indicators will be illustrated differently in Fig. 9.

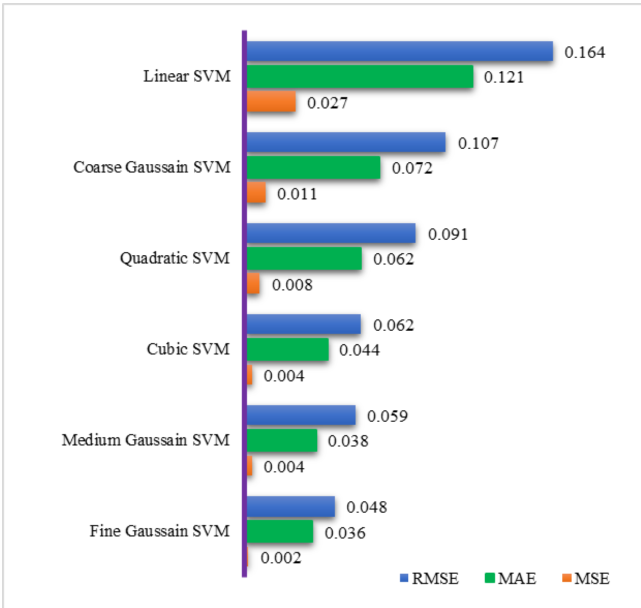

Fig. 9: Characteristics of statistical indicators according to training models.

Table 5 and Figure 9 describe the prediction effectiveness of the SVM model by statistical indicators. It was found that the Cubic SVM, Fine Gaussian SVM, and Medium Gaussian SVM models have the best training efficiency shown at $R^2 > 0.96$ and other statistical indicators (RMSE, MSE, and MAE) are also close to the ideal point (zero). The study selected three models with good statistical indicators to evaluate and test with the test data in Table 4.

After setting up the training model according to SVM algorithms, the models were tested against the test data to evaluate the prediction effectiveness of SVM models.

Table 6. Statistical indicators of the proposed svm models according to the test data.

Models	RMSE	MSE	R^2	MAE
Cubic SVM	0.044	0.002	0.951	0.036
Fine Gaussain SVM	0.036	0.001	0.968	0.029
Medium Gaussain SVM	0.033	0.001	0.973	0.025

Table 6 shows that the Medium Gaussian SVM model has the best performance, demonstrated in indices such as $R^2 = 0.97$ and other statistical indicators close to zero.

Table 7. Prediction results of C_d according to the proposed SVM models.

Values	Models			
	Fine Gaussain SVM		Medium Gaussian SVM	
	C_d	ε (%)	C_d	ε (%)
Min	0.617	0.0	0.593	0.1
Max	1.459	12.9	1.457	8.7

Tables 6 and 7 reveal that the Medium Gaussian SVM model performs better with the test data, in which the statistical indicators are close to the ideal point. Conversely, the largest error is only 8.7%, which is smaller than the Fine Gaussian SVM model (12.9%).

By studying the Medium Gaussian SVM model, the comparison results between the measured values and predicted values were analyzed and the forecast effectiveness was examined as follows.

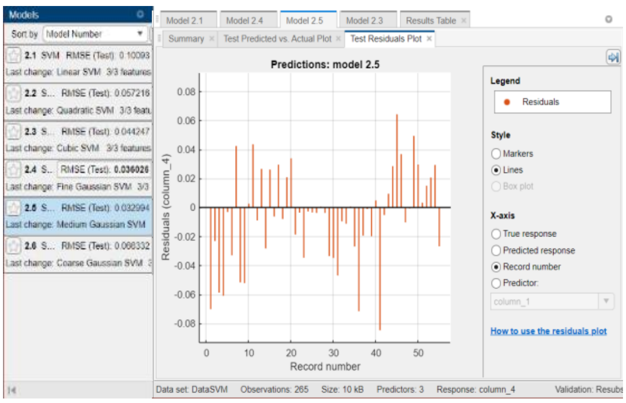

Fig. 10: Residuals of the measured values compared with the predicted values according to the medium gaussian SVM model.

From Fig. 10, the forecast results according to the test data of the Medium Gaussian SVM model have very convergent real measured and predicted values and the largest residual error is 0.085 (error 6.8%).

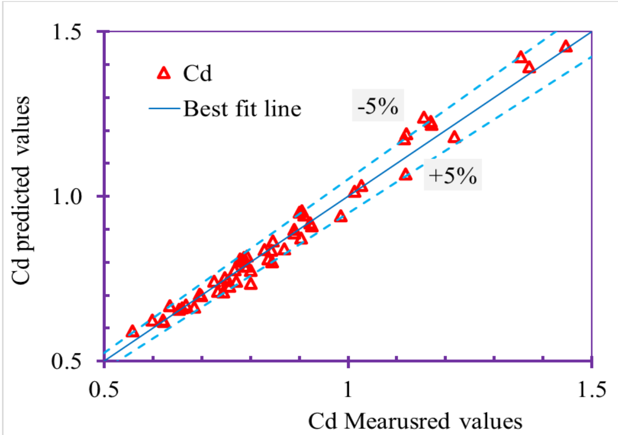

Fig. 11: Comparison between measured and predicted values according to test data.

Figure 11 shows the evaluation of the measured and predicted data using the Medium Gaussian SVM model demonstrated that the predicted values have ± 5% difference with the measured value. The percentage errors have an error greater than 5%, which accounts for 16.3% of the data (equivalent to a ratio of 9/55 datasets).

Comparing with other traditional methods, the study analyzed the test data according to the empirical equations of N.M. Ngoc et al.[5],

+ If $H_0/W_o < 0.5$

$$C_d = 1.856 - 1.729\frac{H_0}{P} - 0.92\frac{H_0}{L_u} \tag{9}$$

+ If $H_0/W_o \geq 0.5$

$$C_d = 0.694\frac{P^{0.294}.W_u^{0.148}}{H_o^{0.442}} \tag{10}$$

Equations 9 and 10 have been studied based on equivalent experimental data, the evaluations of these equations have been carried out in detail by N.M. Ngoc et al.[5]. In this study, it is used to compare with the predicted values by the SVM algorithm in Machine Learning. Statistical indicators analyzed according to the equations of N.M. Ngoc et al. as follows.

Table 8. Statistical indicators according to the equations of N.M. Ngoc et al.

Indicators	Unit	Values
MSE	-	0.002
RMSE	-	0.043
MAE	-	0.033
R^2	-	0.955
Maximum error	%	11.34
Minimum error	%	0.01

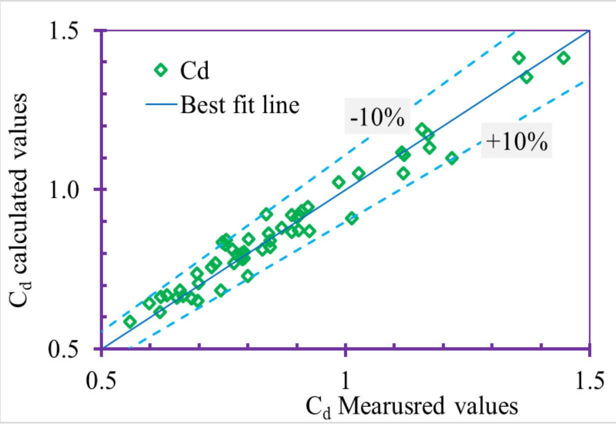

Fig. 12: Comparison between the measure test data and calculated values by N.M. Ngoc et al. (2023).

As shown in Table 8 and Fig. 12, it can be seen that the error calculated according to the equation of N.M. Ngoc et al. has a larger error (the largest error is 11.34% and the value with error greater than 5% accounts for 23.6% of the test data-set) than the prediction according to the Medium Gaussian SVM model (the largest error is 8.7%). The statistical indicators of the SVM model are better than those calculated by the empirical formula (Comparing Tables 6 and 8). This is due to the inflexibility of the equations. According to the machine learning model (using many different hyperplane models, can be seen in Table 5), the predicting process is flexible and determined according to the trends of data groups. So the error is smaller and it is possible to adjust the model to suit the input data system.

In general, the Medium Gaussian SVM model has shown very good results in predicting the C_d coefficient, ensuring small errors between actual measurements and predictions. This shows that the Medium Gaussian algorithm of the SVM model is very suitable for predicting structural, hydraulic characteristics.

5. Conclusion

Study on predicting with regression analysis in Machine Learning is not only a solution with many advantages and high prediction performance, but also eliminates research limitations. This has confirmed the superiority of Machine Learning algorithms compared to empirical equations. In the study on determining the dischagre coefficient of the flow over type-A PKWs, the study applied Buckingham's Pi theory to establish the objective function, thereby determining the data system of dimensionless quantities, these quantities have a direct impact on the research objectives. From there, the target variable and influencing variables in the SVM regression algorithm have been established. The study applied different kernels in the SVM algorithm to predict the discharge coefficient (C_d) in studying on the PKWs. To evaluate the effectiveness of SVM models, the study is based on statistical indicators of the training model and the test data field.

The result of applying the SVM algorithm to predict flow characteristics over type-A PKW, some assertions are made as follows,

+ The SVM regression algorithm in machine learning is suitable for predicting the hydraulic characteristics of flow over type-A PKW.

+ The prediction process of the machine learning algorithm for hydraulic characteristics includes the following: (1) applying Buckingham's Pi theory to build a predicting equation (equation 8); (2) collecting data to build training and testing datasets; (3) using machine learning algorithms to establish predicting models; (4) using prediction models to predict the results of the test data, evaluating the predicted data with the measured data.

+ The study demonstrates that according to the Fine Gaussian SVM algorithm, the training model gives the best prediction efficiency for the discharge coefficient (C_d). However, in some cases, the prediction efficiency for test data is not the best. Besides, the training model by the Medium Gaussian SVM algorithm does not have good training efficiency, but analysis with the test data gives better efficiency in terms of statistical indicators (R^2, MSE, and RMSE). The error is stable and also smaller than other models and the empiric equation.

Applying Machine Learning algorithms to predict the coefficient of the dischagre over the type-A PKW has many practical meanings. This makes the process of controlling the flow over the spillway more convenient and ensuring stability in the flow regulation process. At the same time, it helps integrate real-time data in short-term predicting. This is a new study and explores the application of the SVM algorithms for predicting the dischagre over the type-A PKW.

The study reveals that the Medium Gaussian SVM model is the best predicting model for flow over PKW. However, depending on each different case, there will be a more suitable predicting model. If applying machine learning models in regression predicting, it is necessary to consider many different models to get the best-predicted values.

Nomenclature

H_0	Total upstream head (m)
Q	Discharge (m^3/s)
q	Specific discharge ($m^3/s.m$)
L	Total crest length (m)
P	Height of the weir (m)
W	Width of PKW (m)
W_u	PKW-unit width (m)
W_i	Inlet keys' widths (m)
W_o	Outlet keys' widths (m)
C_d	Discharge coefficient
B	Key length (m)
Bo	Upstream overhangs length (m)

Bi	Downstream overhangs length (m)
Ts	Wall thickness (m)
L_u	Developed crest length (m)
R^2	R-Squared

$$R^2 = 1 - \frac{\sum (y_i - x_i)^2}{\sum (y_i - \overline{x_i})^2}$$

MAE	Mean absolute error

$$MAE = \frac{1}{n}\sum_{i=1}^{n}(y_i - x_i)^2$$

RMSE	Root Mean Square Error

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - x_i)^2}$$

MAPE	Mean Absolute Percentage Error (%)

$$MAPE = \frac{100}{n}\sum \frac{|x_i - y_i|}{x_i}$$

y and x	Calculated values and the observed values, respectively.
$\bar{x}$	Average observed value
n	Number of observations
ε	Percentage error (%)

$$\varepsilon = \frac{|X_{\text{predicted value}} - X_{\text{measured value}}|}{X_{\text{predicted value}}}.100\ (\%)$$

References

1) R. K. Bhukya, M. Pandey, M. Valyrakis and P. Michalis, "Discharge Estimation over Piano Key Weirs: A Review of Recent Developments," *Water*, 14 (19) 3029 (2022). doi: 10.3390/w14193029.
2) R. M. Anderson and B. P. Tullis, "Piano Key Weir: Reservoir versus Channel Application," *Journal of Irrigation and Drainage Engineering*, 138 (8) 773–776 (2012). doi:10.1061/(ASCE)IR.1943-4774.0000464.
3) "Dak Mi 3 and Đak Mi 4 Hydropower Plant", https://pecc2.com/en/dak-mi-3-and-dak-mi-4-hydropower-plant.html (accessed August 15, 2017).
4) F. Laugier, "Design and construction of the first Piano Key Weir spillway at Goulours dam," *Int. J. Hydropower Dams*, 14 (5) 94-100 (2007).
5) N. M. Ngoc, L. V. Nghi and D. T. M. Yen, "Experimental Study on the Discharge Coefficient of Type A Piano Key Weir for Water Resource Sustainable Development," *EVERGREEN*, 10 (4) 2299-2307 (2023). doi:10.5109/7160907.
6) J. Pralong, J. Vermeulen, B. Blancher, F. Laugier, S. Erpicum, O. Machiel, M. Pirotton, J. L. Boillat, M. L. Ribeiro, and A. J. Schleiss, "A naming convention for the Piano Key Weirs geometrical parameters," *Proceedings of the International Conference Labyrinth and Piano Key Weirs – PKW 2011*, Liège, Belgium. CRC Press, 271-278 (2011). https://infoscience.epfl.ch/handle/20.500.14299/69366 (Accessted May 11,2024).
7) A. Abhash and K. K. Pandey, "A review of Piano Key Weir as a superior alternative for dam rehabilitation," *ISH Journal of Hydraulic Engineering*, 28 (1) 541-551 (2020). doi:10.1080/09715010.2020.1767516.
8) D. Singh, and M. Kumar, "Hydraulic Design and Analysis of Piano Key Weirs: A Review," *Arab J. Sci. Eng*, 47 5093–5107 (2022). doi:10.1007/s13369-021-06370-4.
9) X. Guo, Z. Liu, T. Wang, H. Fu, J. Li, Q. Xia, Y. Guo, "Discharge capacity evaluation and hydraulic design of a piano key weir," *Water Supply*, 19 (3) 871–878 (2019). doi:10.2166/ws.2018.134.
10) A. Noui, and A. Ouamane, "Study of optimization of the Piano Key Weir," *International Conference on Labyrinth and Piano Key Weirs - PKW 2011*, Liège, Belgium, 175–182 (2011). https://lab.univ-biskra.dz/lahe/images/pdf/study%20of%20optimizing%20of%20pk-weir.pdf (Accessted March 10, 2024)
11) G. M. Cicero and J. R. Delisle, "Discharge characteristics of Piano Key weirs under submerged flow," *Labyrinth Piano Key Weirs II–CRC Press*, (2013). https://books.google.com.vn/books?id=zd7KBQAAQBAJ&lpg (Accessted June 09, 2024).
12) M. B. N. Al-Baghdadi and S. I. Khassaf, "Evaluation of crest length effect on piano key weir discharge coefficient," *Int. J. Energy Environ.*, 9 (5) 473–480 (2018). https://www.ijee.ieefoundation.org/vol9/issue5/IJEE_04_v9n5.pdf (Accessted June 12, 2024)
13) S. Li, G. Li, D. Jiang, and J. Ning, "Influence of auxiliary geometric parameters on discharge capacity of piano key weirs," *Flow Meas. Instrum.*, 72, 101719 (2020). doi:10.1016/j.flowmeasinst.2020.101719.
14) G. D. Singhal, N. Sharma, and C.S.P. Ojha, "Experimental study of hydraulically efficient piano key weir configuration," *ISH J. Hydraul. Eng.*, 17 (1) 18–33 (2012). doi:10.1080/09715010.2011.10515029.
15) N. I. Khattab, A. N. Altalib, and M. Y. Mohammed, "Estimating Discharge Coefficient for Piano Key Weir using ANN Technique," *AIP Conf. Proc.* 2651 (1) 020039. doi:10.1063/5.0132654.
16) O. F. Dursun, N. Kaya, and M. Firat, "Estimating discharge coefficient of semi-elliptical side weir using ANFIS," *Journal of Hydrology*, 426 55-62 (2012). doi:10.1016/j.jhydrol.2012.01.010.
17) A. H. Haghiabi, A. Parsaie and S. Ememgholizadeh, "Prediction of discharge coefficient of triangular labyrinth weirs using Adaptive Neuro Fuzzy Inference System," *Alexandria Engineering Journal*, 57 (3) 1773-1782 (2018). doi:10.1016/j.aej.2017.05.005.

18) M. Zounemat-Kermani and A. Mahdavi-Meymand, “Hybrid meta-heuristics artificial intelligence models in simulating discharge passing the piano key weirs”, *Journal of Hydrology*, 569 12-21 (2019). doi:10.1016/j.jhydrol.2018.11.052.
19) N. Bansal, D. Singh, and M. Kumar, “Computation of energy across the type-C piano key weir using gene expression programming and extreme gradient boosting (XGBoost) algorithm,” *Energy Reports*, 9 (4) 310-321 (2023). doi:10.1016/j.egyr.2023.04.003.
20) D. Singh and M. Kumar, “Computation of energy dissipation across the type-A piano key weir by using gene expression programming technique”, *Water Supply*, 22 (8) 6715–6727 (2022). doi:10.2166/ws.2022.255.
21) M. Kumar, P. Sihag, N. K. Tiwari, and S. Ranjan, “Experimental study and modelling discharge coefficient of trapezoidal and rectangular piano key weirs,” *Applied Water Science*, 10 (43) 1-9 (2020). doi:10.1007/s13201-019-1104-8.
22) A. H. Zaji, H. Bonakdari, S. R. Khodashenas and S. Shamshirband, “Firefly optimization algorithm effect on support vector regression prediction improvement of a modified labyrinth side weir's discharge coefficient”, *Applied Mathematics and Computation*, 274 14-19 (2016). doi:10.1016/j.amc.2015.10.070.
23) M. Dasineh, A. Ghaderi, M. Bagherzadeh, M. Ahmadi, and A. Kuriqi, “Prediction of Hydraulic Jumps on a Triangular Bed Roughness Using Numerical Modeling and Soft Computing Methods,” *Mathematics*, 9 (23) 3135 (2021). doi:10.3390/math9233135.
24) S. M. Saghebian, “Predicting the relative energy dissipation of hydraulic jump in rough and smooth bed compound channels using SVM,” *Water Supply*, 19 (4) 1110-1119 (2019). doi:10.2166/ws.2018.162.
25) I. H. Sarker, “Machine Learning: Algorithms, Real-World Applications and Research Directions,” *SN COMPUT. SCI.*, 2 160 (2021). doi:10.1007/s42979-021-00592-x.
26) N. M. Ngoc and T. N. Thang, “A Study of an Empirical Sequent Depths Equation of the Hydraulic Jump in a Horizontal Trapezoidal Channel,” Eng. Technol. Appl. Sci. Res., 13 (6) 12307–12313 (2023). doi:10.48084/etasr.6431.
27) A. Parsaie and A. H. Haghiabi, “Support Vector Machine to predict the discharge coefficient of Sharp crested w-planform weirs,” *AUT Journal of Civil Engineering*, 1 (2) 195-204 (2017). doi:10.22060/CEEJ.2017.13005.5309.
28) V. Rezaei, S. H. Musavi-Jahromi, A. Khosrowjerdi and B. Beheshti, “Experimental and Simulation Studies on Water Discharge Coefficients of Rectangular Piano Key Weirs,” *International Journal of Technology*, 13 (4) 695-705 (2022). doi:10.14716/ijtech.v13i4.5377.
29) L.V. Nghi and D. T. M. Yen, “Build a discharge coefficients of the flow over the Piano Key Weir,” *Journal Science and Technology Water Resources*, 54 141-149 (2019). https://vjol.info.vn/index.php/vawr/article/view/48440 (Accessted Jan 15, 2025)
30) D. T. M. Yen and L. V. Nghi, “Characterize the shape and sequence of flow over the piano key weir,” *Journal of Water resources sciences & Technology*, 42 1-10 (2018). https://vjol.info.vn/index.php/vawr/article/view/42150 (Accessted Jan. 15, 2025)
31) Mahesh K. Singh, D. Kishore, R. Anil Kumar, “Accent Recognition of Speech Signal Using MFCC-SVM and k-NN Technique,” *EVERGREEN*, 11 (02) 1305-1312 (2024). doi:10.5109/7183440
32) A. Parsaie, A. H. Haghiabi, S. Emamgholizadeh and H. M. Azamathulla, “Prediction of discharge coefficient of combined weir-gate using ANN, ANFIS and SVM,” *International Journal of Hydrology Science and Technology*, 9 (4) 412-430 (2019). doi:10.1504/IJHST.2019.102422
33) “What Is a Support Vector Machine?” MATLAB for Artificial Intelligence. https://www.mathworks.com/discovery/support-vector-machine.html (Accessed 1994-2004).
34) G. S. Fesghandis, A. Pooya, M. Kazemi and Z. N. Azimi, “Comparison of Multilayer Perceptron and Radial Basis Function Neural Networks in Predicting the Success of New Product Development”, *Eng. Technol. Appl. Sci. Res.*, 7 (1) 1425-1428 (2017). doi: 10.48084/etasr.936.
35) P. Ilius, M. Almuhaini, M. Javaid, and M. Abido, “A Machine Learning–Based Approach for Fault Detection in Power Systems”, Eng. Technol. Appl. Sci. Res., 13 (4) 11216–11221 (2023). doi:10.48084/etasr.5995.
36) N. T. Hai, “Report: Product report of scientific and technological project: Study on the application of piano key weir to build hydraulic works in Vietnam,” Southern institute of water resources research, HoChiMinh City; Vietnam (hardcopy, accessted Jan. 10, 2024).
37) O. Machiels, M. Pirotton, P. Archambeau, B. Dewals and S. Erpicum, “Experimental parametric study and design of Piano Key Weirs,” *Journal of Hydraulic Research*, 52 (3) 326-325 (2014). doi:10.1080/00221686.2013.875070.
38) A. Kabiri-Samani, and A. Javaheri, “Discharge coefficients for free and submerged flow over Piano Key weirs,” *Journal of Hydraulic Research*, 50 (1) 114–120 (2012). doi:10.1080/00221686.2011.647888.