A Simple Analysis for the Industrial Revolution: Based on the Lotka-Volterra Model

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A Simple Analysis for the Industrial Revolution: Based on the Lotka-Volterra Model*

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Abstract

The ascent of the modern production sector and the decline of its traditional counterpart stand as defining features of the Industrial Revolution. The disparity between these sectors lies in their approach to innovation. Traditional production relies on experiential knowledge for innovation, linking the innovation stock of this sector directly to population size. Conversely, the modern sector thrives on experimental innovations, resulting in a growth rate of innovation outpacing that of the population. The innovation stocks within each sector shape their potential output and competitive capacity. Applying a simple evolutionary industry competition model, this paper replicates the Industrial Revolution process, wherein the modern production sector supplanted the traditional, leading to a surge in per capita output.

Keywords: the Industrial Revolution, Lotka-Volterra Model, Industry Competition, the Great Divergence

JEL Codes: N1, O14

1 Introduction

Explaining the origin of the Industrial Revolution is one of the most intriguing problems in economic history and economic growth theory. The emergence of the modern industrial production sector and the sustained growth of output per capita began with the Industrial Revolution. The reasons for this economic development pattern transition occurring and first appearing in Western Europe but not other parts of the world have been widely discussed by economists, historians, and sociologists

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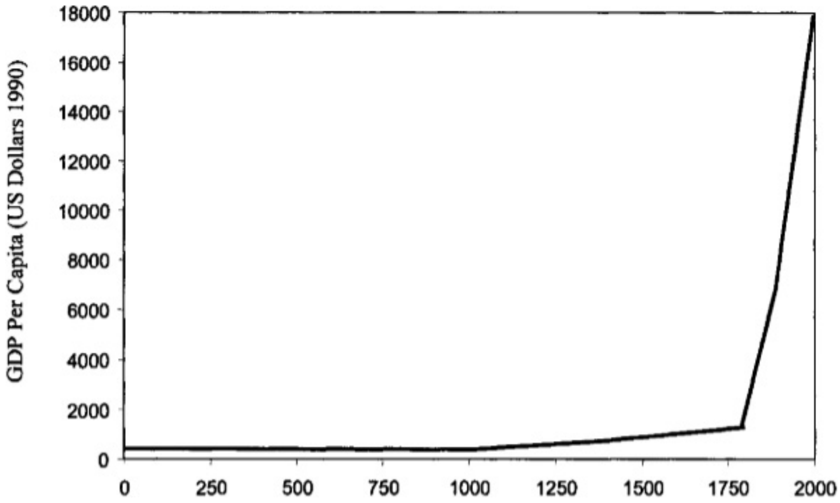


Figure 1: Output per Capita in Western Europe: 0-2000
Extract from Galor and Moav (2002), based on Maddison (2001)

(Needham (1974), Galor and Weil (2000), Pomeranz (2000), Allen (2009), Rosenthal and Wong (2011), Kelly et al. (2014)).

This paper tries to simulate the rise of the modern sector and the Industrial Revolution based on the biological evolution model. Applying the evolutionary method and Darwinian mechanisms to discuss long-term economic development has been widely seen in recent decades (Galor and Moav (2001), Galor and Moav (2002), Nunn (2021)). The Lotka-Volterra model is one of the biological evolution dynamic models used to describe the relationship between multiple species[1)], which has also been used in economic research for analyzing economic growth and industry competition (Goodwin (1967), Chen (2015)). In this paper, we apply the Lotka-Volterra competition model to simulate the competition between the traditional and modern production sectors and discuss the conditions for the emergence of the modern production sector. In our model, the two production sectors are analogous to the species in the original Lotka-Volterra model. They compete for production endowments, such as capital, labor, and land. Meanwhile, their competitive capacities depend on the ratio of their potential outputs. Moreover, these competitive capacities determine whether the two production sectors can coexist or if only one can survive. The model implies that only when the potential output of modern production is large enough, is the full-fledged transition to modern production feasible. Meanwhile, the larger the potential output of the traditional production sector, which weakens the competitive capacity of the modern production sector, the more it would thwart the Industrial Revolution. This result may shed light on the Great Divergence and the *Needham Puzzle*.

1）Pianka (2011)

The paper is structured as follows: Section 2 presents a simple Lotka-Volterra competition model to simulate the economic transition from traditional production to modern production. Section 3 demonstrates a numerical practice for this simple model. Section 4 concludes the paper and provides possible directions for future research.

2 Model

Let Y_t^T and Y_t^M represent the output of the traditional and modern sectors, respectively. Modifying the Lotka-Volterra competition model, we assume the following dynamics for $(Y_t^T,\ Y_t^M)\in[0,\ +\infty)^2$:

$$\begin{cases} \dfrac{dY_t^T}{dt}=Y_t^T\left(1-\dfrac{Y_t^T}{\exp(g^T t)K^T}-\dfrac{Y_t^M}{\exp(g^T t)K^T}\right) \\ \dfrac{dY_t^M}{dt}=Y_t^M\left(1-\dfrac{Y_t^M}{\exp(g^M t)K^M}-\dfrac{Y_t^T}{\exp(g^M t)K^M}\right) \end{cases} \tag{1}$$

$\exp(g^T t)K^T$ and $\exp(g^M t)K^M$ are the potential outputs for two production sectors at period t, g^T and g^M are the occurring rates of innovation in two production sectors. The growth of the potential output depends on the technology improvement and innovation. As Lin (1995) mentioned "*in premodern times, technological invention basically stems from experience, whereas in modern times, it mainly results from experiment cum science .... in a single production period an artisan or farmer can have only one trial, while an inventor can perform many trials by experiment*". Therefore, the stock of experience innovation is proportional to the size of the population. Meanwhile, before the Industrial Revolution, the tradition production sector dominated the society, and the output per capita was nearly stagnant[2]. Thus, we make the assumption that $0<g^T=g<g^M$, where g represents the population growth rate. The final term within the parentheses in Eq.(1) quantifies the level of competition originating from the opposing production sector. As the potential output of a production sector increases and the output of the opposing sector decreases, the level of competition faced by the production sector diminishes. Eq.(1) describes a non-autonomous dynamic system, contrasting with the standard Lotka-Volterra competition model, which represents an autonomous dynamic system. Historically, traditional production held sway over societal production prior to the Industrial Revolution. The subsequent assumption ensures the continued predominance of traditional production during the early stages of development.

Assumption 1:

$$\frac{K^T}{K^M}>(1+g)\exp[(g^M-g)t_0] \tag{A1}$$

2) Maddison (2001)

Additionally, we assume that in the initial period t_0, $Y_{t_0}^T>0$ while $Y_{t_0}^M=0$. Furthermore, in each period, a subset of pioneers will venture into modern sector production regardless of its sustainability, thereby generating a small output $Y^M=\epsilon$. Indeed, whether the ventures of the modern production pioneers could ignite the booming of modern production depends on the competitive capacities of the production sectors. The subsequent proposition gives the timing for the ascension of the modern production sector.

Proposition 1: (Timing of the Industrial Revolution) *Under Assumption 1,* $\exists\, t^i>t_0$*, such that*

- *When* $t\in[t_0,\ t^i]$*, only the traditional sector exists stably, the total output growth rate asymptotically converges to* g^T*, and the growth rate of the output per capita asymptotically converges to* $\frac{K^T}{N_{t_0}(1+g)}$*, which is a constant, and* N_{t_0} *is the population size at period* t_0*.*
- *When* $t\in(t^i,\ +\infty)$*, the modern sector begins to boom, the total output growth asymptotically converges to* g^M*, and the growth rate of the output per capita asymptotically converges to* g^M-g*.*

Proof: See details in Appendix

t^i in Proposition 1 represents the timing of the Industrial Revolution happened. The following lemma summarizes the comparative statics properties of t^i.

Lemma 1: (Comparative Statics of t^i)

$$\frac{\partial t^i}{\partial K^T}>0,\quad \frac{\partial t^i}{\partial K^M}<0$$

Proof: See details in Appendix

Lemma 1 suggests that a higher potential output of the traditional sector could postpone the onset of the Industrial Revolution, while a higher potential output of the modern sector might expedite it. The insights from Lemma 1 also shed light on the reason behind the West's industrialization preceding that of Asia, known as the Great Divergence. Throughout history, agriculture served as a prime example of the traditional production sector. Tamura (2002) referenced Mokyr (1992)'s argument, stating, "*the greater productivity of rice in Asia relative to wheat in Europe (where there are two and sometimes three rice crops per year in Asia versus one or sometimes two wheat crops per year in Europe).*" According to Tamura (2002)'s model, the high potential for agriculture production in China acted as a deterrent to industrialization. In our model, China possesses a larger K^T compared to Europe, leading to a delay in China's adoption of modern production methods. Moreover, China insisted on an agricultural-biased policy during the premodern era, bolstering the competitive strength of the traditional production sector while weakening that of the modern production sector. Additionally, scientific and technological developments in China predominantly relied on empirical knowledge. Conversely, during the ancient Greek era, rudimentary forms of modern scientific ideologies began to emerge in Europe, flourishing during the Renaissance and the Enlightenment. In our model, Europe's cultural heritage may result in a higher K^M compared to China, thus facilitating the Industrial Revolution. Consequently, during the period when the traditional sector held sway in both Europe and China, China exhibited superior economic performance compared to Europe, primarily owing to its larger K^T. However, this dominance of the traditional sector in China acted as a hindrance to society's long-term development by impeding the onset of the Industrial Revolution.

3 Numerical Simulation

In this part, we specify the parameter values of Eq.(1) and check the dynamics of two production sectors' outputs, the total output and the output per capita. Table.(1) lists the choice of the parameter values and the initial conditions.

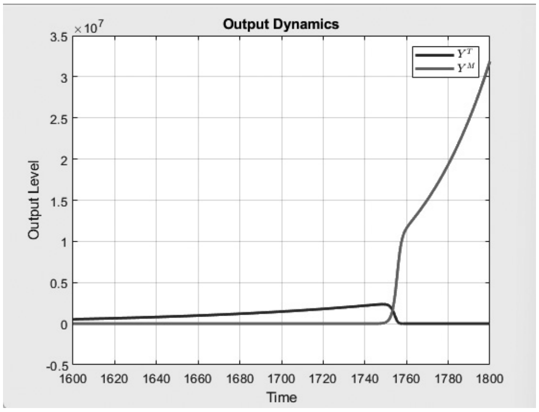
Fig.(2a) depicts the evolution of outputs in both the modern and traditional sectors. During the initial developmental phase, the output of modern production remained stagnant, while that of traditional production experienced gradual growth. Notably, in Fig.(2a), the emergence of a booming modern production sector occurred around 1750. Subsequently, post-1750, the output of the traditional production sector declined, while that of the modern sector surged significantly. Figs.(2b)-(2c) illustrate the trajectory of total output and output per capita, respectively. Post-1750, both total output and output per capita escalated rapidly compared to the preceding period.

4 Conclusion and Discussion

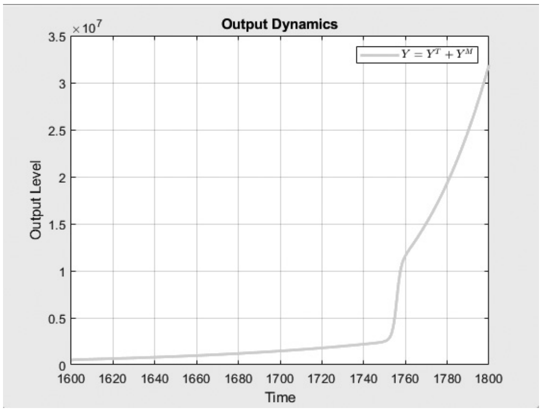
This paper presents a simple Lotka-Volterra model to illustrate the competition between the

Table 1: Setting of Parameters

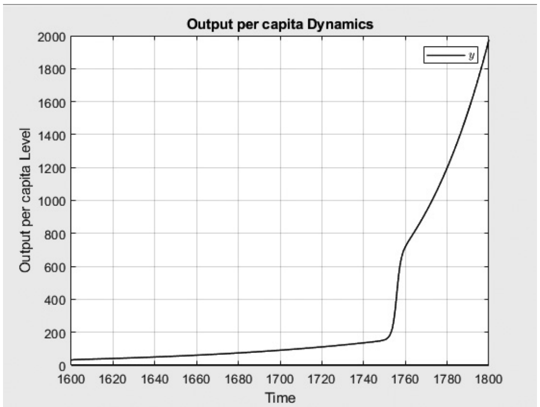
Parameter	Value
g	0.01
g^M	0.025
K^T	10000
K^M	10
N_{t_0}	40
Initial Period	$t_0=1200$
Initial Values	$Y^T_{t_0}=500000$, $Y^M_{t_0}=5$



(a)　Y^T and Y^M



(b)　Total Output



(c)　Output per Capita

Figure 2: Numerical Simulation

traditional and modern production sectors. The model replicates the emergence of the modern production sector and the subsequent increase in per capita output, both hallmark features of the Industrial Revolution. The findings suggest that a stronghold in the traditional sector might impede the adoption of the more efficient modern production methods by society. Despite China's advancements in science, mathematics, and engineering during the 14th century, surpassing those of any other region in the world[3)], these achievements were primarily the result of experience-based innovations. Moreover, China's higher agricultural productivity and sustained political favoritism towards agriculture further bolstered traditional production. Consequently, China possessed a greater potential output in the traditional sector. This larger output potential weakened the

3）Elvin (1972)

competitive edge of the modern production sector, consequently delaying China' s Industrial Revolution and economic transition.

Notice that the potential outputs in the two production sectors are exogenously given in our simple model. Meanwhile, the dynamics model of the two production sectors lacks the micro-foundation. Therefore, including the micro-foundation and endogenizing the potential output's evolution are two future research directions.

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References

[1] Allen, R. C., *The British industrial revolution in global perspective*, 2009.
[2] Chen, P., "Metabolic growth theory: market-share competition, learning uncertainty, and technology wavelets" *The Evolution of Economic and Innovation Systems*, pp. 41-68, 2015.
[3] Elvin, M., "The High-level Equilibrium Trap: The Cause of the Decline of Invention in the Traditional Chinese Textile Industries", *Economic organization in Chinese society* (eds.) Willmott, W. E., pp. 137-145, 1972.
[4] Galor, O. Moav, O., "Evolution and growth" *European economic review*, vol.45, no.4-6, pp. 718-729, 2001.
[5] Galor, O. Moav, O., "Natural selection and the origin of economic growth" *The Quarterly Journal of Economics*, vol. 117, no.4, pp. 1133-1191, 2002.
[6] Galor, O. Weil, D. N., "Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond" *American economic review*, vol.90, no.4, pp. 806-828, 2000.
[7] Goodwin, R. M., "A Growth Cycle: Socialism, Capitalism and Economic Growth, 1967, ED. CH Feinstein", *Essays in economic dynamics*, pp. 165-170, 1967.
[8] Kelly, M. Mokyr, J. Gráda, C. Ó., "Precocious Albion: a new interpretation of the British industrial revolution" *Annu. Rev. Econ.* vol.6, no.1, pp. 363-389, 2014.
[9] Lin, J. Y., "The Needham puzzle: why the industrial revolution did not originate in China" *Economic development and cultural change*, vol.43, no.2, pp. 269-292, 1995.
[10] Maddison, A., *A millennial perspective*, 2001.
[11] Mokyr, J., *The lever of riches: Technological creativity and economic progress*, 1992.
[12] Needham, J., *Science and civilisation in China*, 1974.
[13] Nunn, N., "History as evolution", *The handbook of historical economics*, pp. 41-91, 2021.
[14] Pianka, E. R., *Evolutionary ecology*, 2011.
[15] Pomeranz, K., *The great divergence: China, Europe, and the making of the modern world economy*, 2000.
[16] Rosenthal, J.-L. Wong, R. B., *Before and beyond divergence: the politics of economic change in China and Europe*, 2011.
[17] Tamura, R., "Human capital and the switch from agriculture to industry" *Journal of economic Dynamics and Control*, vol.27, no.2, pp. 207-242, 2002.

Appendix

Heuristic Proof of Proposition 1

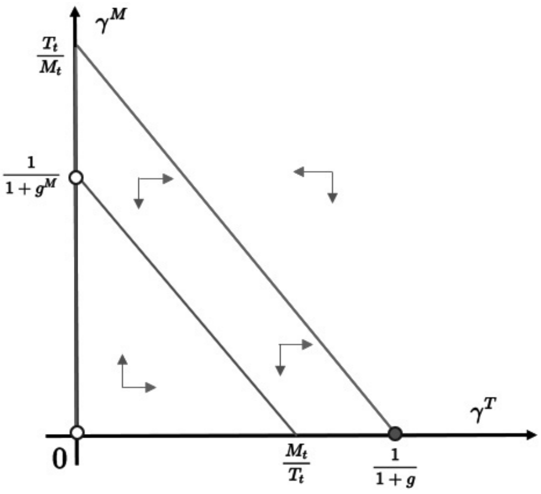
Let $T_t=\exp(g^T t)K^T$, $M_t=\exp(g^M t)K^M$, $\gamma_t^T=\frac{Y_t^T}{T_t}$ and $\gamma_t^M=\frac{Y_t^M}{M_t}$, then Eq.(1) could be rewritten as

$$\begin{cases} \frac{d\gamma_t^T}{dt}=\gamma_t^T\left[1-(1+g^T)\gamma_t^T-\frac{M_t}{T_t}\gamma_t^M\right] \\ \frac{d\gamma_t^M}{dt}=\gamma_t^M\left[1-(1+g^M)\gamma_t^M-\frac{T_t}{M_t}\gamma_t^T\right] \end{cases} \tag{2}$$

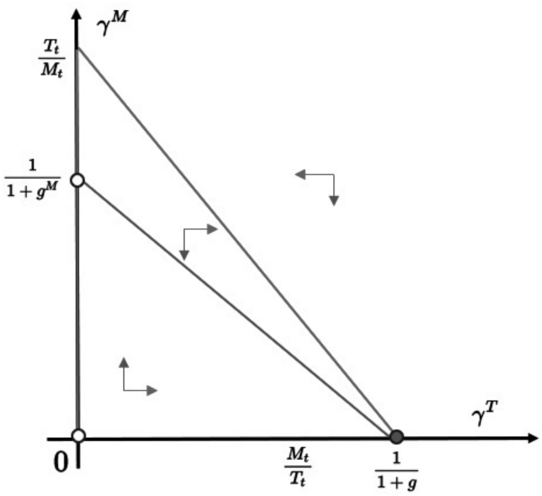
Under Assumption 1, we could have $\frac{M_{t_0}}{T_{t_0}}<\frac{1}{1+g}$, and $\frac{T_{t_0}}{M_{t_0}}>1+g>1>\frac{1}{1+g^M}$.

Since $g^M>g^T=g$, we have $\lim_{t\to\infty}\frac{M_t}{T_t}=\lim_{t\to\infty}\frac{K^M}{K^T}\exp[(g^M-g)t]=+\infty>1$. By the intermediate value theorem, $\exists\, t^i>t_0$, such that $\frac{M_{t^i}}{T_{t^i}}=1$. $\frac{M_t}{T_t}$ is an increasing function of t guarantees the uniqueness of t^i. $\forall t\in[t_0, t^i)$, $\frac{M_t}{T_t}<1$; $\forall t\in(t^i, +\infty)$, $\frac{M_t}{T_t}>1$, and $t^i=\frac{\ln(K_T/K_M)-\ln(1+g)}{g^M-g}$

● When $t\in[t_0, t^i)$, we could get the following "phase diagram"[4] of (γ^T, γ^M)



(a) $t\in[t_0, t^i)$



(b) $t=t^i$

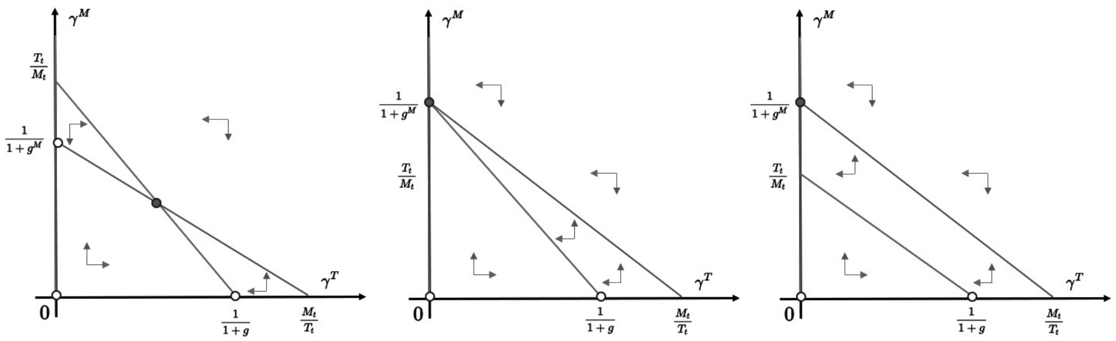
Figure 3: "Phase diagram" of (γ^T, γ^M): $t\in[t^0, t^i]$

4) Precisely, the phase diagram method could not be applied in the non-autonomous dynamics. Here, we still use the notation "phase diagram" to represent the slice of $(\gamma_t^T, \gamma_t^M, t)$ and the potential moving direction of every point at period t. Notice that at the next moment, the potential moving direction would change, but it will not influence the qualitative results that will be introduced in the following.

In this development stage, only the pure traditional production sector could stably exist for any disturbance in γ^M from $\left(\frac{1}{1+g},\ 0\right)$.

● When $t \in [t_0,\ t^i)$, $\exists\, t^{ii} = \dfrac{\ln(K_T/K_M) + \ln(1+g^M)}{g^M - g} > t^i$, such that $\dfrac{T_{t^{ii}}}{M_{t^{ii}}} = \dfrac{1}{1+g^M}$.

We could get the following "phase diagram"



(a) $t \in (t^i,\ t^{ii})$ (b) $t = t^{ii}$ (c) $t \in (t^{ii},\ +\infty)$

Figure 4: "Phase diagram" of $(\gamma^T,\ \gamma^M)$: $t \in (t^i,\ \infty)$

In this case, any positive disturbance of γ^M from $\left(\frac{1}{1+g},\ 0\right)$ could induce the emergence of the modern production sector. Finally, $(\gamma^T,\ \gamma^M)$ will asymptotically converge toward $\left(0,\ \frac{1}{1+g^M}\right)$

Proof of Lemma 1

From the proof of Proposition 1, we have $t^i = \dfrac{\ln(K_T/K_M) - \ln(1+g)}{g^M - g}$. Therefore,

$$\frac{\partial t^i}{\partial K^M} = -\frac{1}{(g^M - g)K^M} < 0 \text{ and } \frac{\partial t^i}{\partial K^T} = -\frac{1}{(g^M - g)K^T} > 0$$