# Dynamic Coupling Studies of Cooperative Behavior and Synchronization on Complex Networks

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# Dynamic coupling studies of cooperative behavior and synchronization on complex networks

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**Abstract:** *Synchronous and cooperative behaviors are widely observed in economic, social and theoretical research. However, studies that integrate these two aspects are relatively rare. This paper introduces the concepts of cost and benefit into the classical Kuramoto model, based on an ER network. By comparing the traditional random selection method with the optimal selection method, we find that the optimal selection method significantly better promotes the maintenance of cooperative and synchronized behaviors.*

**Keywords:** Complex network; synchronization; Kuramoto model; cooperation; Optimal selection.

## 1. INTRODUCTION

Synchronous and cooperative behaviors are widespread in both natural and social systems, playing crucial roles in the emergence of collective behaviors. Synchronization refers to the process where individual components within a system coordinate their activities to operate in unison, often observed in both natural phenomena and engineered systems [1-3]. Examples include pendulum clocks hanging side by side swinging with the same frequency, schools of fish moving in the same direction, the consensus of opinions among individuals, and all generators in a power-grid system rotating at the same frequency. In the past decade, research into complex networks [4,5] has advanced significantly, introducing new structures such as scale-free and small-world networks, which have enhanced our understanding of synchronization in these networks. The pioneering work by Huygens in 1673 on the synchronization of pendulum clocks initiated the formal study of synchronization, which has since evolved to encompass various fields, including chaos theory, biology, and network science [6-8].

In addition to the research of synchronous behavior, research on cooperative behavior is also a popular topic in the study of collective behavior. With the deepening of evolutionary game theory research, many scholars have proposed various mechanisms to promote cooperation. Nowak summarized five mechanisms that facilitate the evolution of cooperation [9]: kin selection, network reciprocity, group selection, direct reciprocity, and indirect reciprocity. Among these, network reciprocity has become one of the most critical mechanisms in the study of cooperative evolution. Based on this foundation, mechanisms such as rewards, asymmetric information, punishment, reputation, and environmental influences have gradually become research topics, opening new directions for the study of the evolution of cooperation [10-16].

Previous research typically investigates synchronous and cooperative behavior separately. However, real-world interactions inherently involve cost and benefit, suggesting a dynamic coupling between cooperation and synchronization [17,18]. The Kuramoto model [19-24], a well-established framework for studying synchronization, and it has been extended to incorporate these interactions. This allows for a more realistic representation of how individual behavior influences and is influenced by collective dynamics. The research of Peroca and Carroll [6] has explored the control and synchronization of chaotic systems, while recent developments have integrated evolutionary game theory to understand the emergence of cooperation within synchronized networks. In the Kuramoto model, many oscillators with heterogeneous natural frequencies couple through the sine of their phase differences. When this coupling is strong enough, all oscillators eventually rotate at the same frequency. However, interactions are often costly. Due to the existence of costly interactions, some agents may choose not to interact with their neighbors at certain times, which impedes the synchronization of the entire system.

Understanding the emergence of synchronization with costly interactions is a challenging issue. Recently, Antonioni and Cardillo incorporated evolutionary game theory into the Kuramoto model. They assumed that agents could decide whether to interact with their neighbors. All agents receive benefits from local synchronization, but those who choose to interact with their neighbors must pay a cost. Each agent's benefit is thus given by the net gain of benefit minus cost. After each time step, agents are allowed to update their strategies by comparing benefits with their neighbors. Agents with higher benefits are more likely to have their strategies imitated. This model can be named the evolutionary Kuramoto game (EKG).

The EKG sheds light on the study of self-organized interactions. Agents in the EKG face a dilemma: interacting with neighbors helps enhance synchronization of the whole system, but the interaction cost reduces the willingness of agents to participate. So far, how to alleviate this dilemma and enhance the global level of synchronization in the EKG remains unclear. This paper finds that when the average degree of the underlying interaction network is moderate, the global synchronization level reaches the highest. We also examine the effect of oscillator's rationality in choosing strategies on synchronization.

## 2. MODEL

In this research, we consider an Erdős–Rényi (ER) [25] random network consisting of N nodes with an average degree $< k >$.

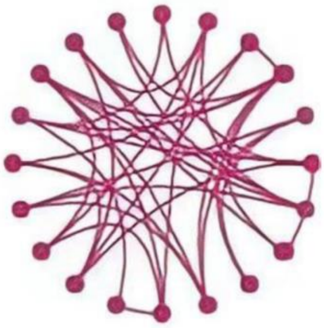

Fig. 1: Structure of the ER network.

Each node corresponds to a different Kuramoto oscillator, and each oscillator can decide whether to interact with its surrounding neighbors. When an oscillator chooses to interact with its surrounding neighbors, we refer to this oscillator as a cooperator. Conversely, when an oscillator chooses not to interact with its surrounding neighbors, we refer to this oscillator as a defector. Unlike the classical Kuramoto model, node $m$ changes its phase according to the following equation [26-28]:

$$\dot{\theta}_m = \omega_m + s_m \lambda \sum_{z=1}^{N} a_{mz} \sin(\theta_z - \theta_m) \tag{1}$$

Where $\dot{\theta}_m$ represents the instantaneous phase over time, $\omega_m$ denotes the natural frequency of the oscillator, and $s_m$ signifies the strategy selection parameter. When $s_m = 1$, the oscillator is a cooperator, whereas when $s_m$ =0, the oscillator is a defector. The parameter $\lambda$ represents the coupling strength of the system and $a_{mz}$ refers to the elements in the adjacency matrix. When $a_{mz} = 1$, there is a connection between nodes m and z, and when $a_{mz} = 0$, there is no connection between them. Initially, both $\theta$ and $\omega$ are uniformly distributed within the interval [-1, 1].

We consider the synchronization level of an oscillator with its surrounding neighbors, calling it the local synchronization rate, denoted as $r_m$, with a range of [0,1]. The closer the value of $r_m$ is to 1, the higher the local synchronization level, indicating a greater degree of phase synchronization between an oscillator and its surrounding neighbors. The local synchronization level $r_{mz}$ of a pair of oscillators is defined as follows [29]:

$$r_{mz} e^{\frac{i(\theta_m+\theta_z)}{2}} = \frac{e^{i\theta_m} + e^{i\theta_z}}{2}, r_{mz} \in [0,1] \tag{2}$$

Therefore, for oscillator $m$, the local synchronization rate $r_m$ can be expressed as [30]:

$$r_m = \frac{\sum_{z=1}^{N} a_{mz} r_{mz}}{\sum_{z=1}^{N} a_{mz}} \tag{3}$$

At the same time, we define the local synchronization rate $r_m$ as the oscillator 's benefit, which is also the basis for the oscillator to decide its strategy in the synchronization and game system. However, when an oscillator chooses to synchronize with other oscillators, there is also a "cost," meaning that cooperators must incur the cost of interaction. In this paper, we set the interaction cost as $c_m$, derived from the absolute value of angular acceleration, that is, the change in angular velocity of the node at each time step, defined as follows:

$$c_m = \left|\dot{\theta}_m(t) - \dot{\theta}_m(t-\varepsilon)\right| \tag{4}$$

Here, $\varepsilon$ is the time step of the formula calculated using the fourth-order Runge-Kutta method [31], with $\varepsilon = 0.01$ in this paper. The absolute value is used to ensure that the cost is non-negative. If an oscillator chooses to defect, they do not need to incur any cost and only need to wait for other oscillators to synchronize with them, acting as a free rider, in which case $c_m = 0$.

Therefore, we can calculate the net benefit $P_m$ of an oscillator, which can be expressed as follows:

$$P_m = r_m - \alpha \frac{c_m}{2\pi} \tag{5}$$

Here, the cost is divided by $2\pi$ to facilitate comparison with the benefits. Among them, $\alpha$ is an adjustable parameter representing the relative cost. After calculating the benefit of each oscillator, we can use the same method to calculate the benefits of the neighbors surrounding this oscillator.

In the initial state of the system, defectors and cooperators are randomly distributed. Through iterations, each player accumulates their own gains and costs, then updates their strategy for the next time step. Here, we introduce the probability that a player chooses a neighbor $z$ as a learning target, defined as follows:

$$\pi_z = \sum_x \frac{e^{WP_z}}{e^{WP_x}} \tag{6}$$

Here, $W$ is a selected parameter, here called the optimal choice parameter [32]. $W$ represents the player's preference for the state when choosing surrounding neighbors as learning targets. When $W = 0$, the oscillator randomly selects neighbors for learning, a choice unrelated to benefits, indicating a casual state preference. When $W > 0$, the player selects based on benefits, with higher benefits from neighbors increasing the probability of being chosen as strategy contributors, indicating a preference for maximizing benefits. Conversely, when $W < 0$, it shows that the oscillator will choose neighbors with lower benefits, indicating an "unusual" state preference. This neighbor selection mechanism differs from the original random selection, better reflecting the different choices individuals might make under varying state preferences. Additionally, the introduction of this new mechanism makes our model more realistic. In real life, people's preferences and psychological states vary in different environments, leading to different choices. Most of the time, people tend to follow a stronger individual ($W > 0$); occasionally, they make random decisions in unpredictable environments ($W = 0$,); but sometimes, due to factors like morality or status, people are forced to follow less successful individuals in rare cases ( $W < 0$ ).

Subsequently, whether to update the strategy is based on the Fermi rule [33]. Specifically, the probability that oscillator $m$ adopts the strategy of the chosen neighbor $z$ is defined as follows:

$$W(s_m \leftarrow s_z) = \frac{1}{1+e^{(p_z-p_m)/K}} \quad (7)$$

Where $K$ represents the noise level, and its reciprocal $1/K$ is also referred to as the selection intensity. When the selection intensity is very low, players are highly likely to adopt the strategy of their chosen neighbor.

Next, we calculate the global order parameter $r_G$ of the system to measure the global synchronization level of the system, as shown below:

$$r_G e^{i\psi} = \frac{1}{N}\sum_{z=1}^{N} e^{i\theta_z} \quad (8)$$

Here, $i$ is the imaginary unit, and $\psi$ is the average phase of the system. The value of $r_G$ ranges from [0,1]. When $r_G = 1$, the system is in a state of complete synchronization, meaning all oscillators in the system are in the same phase. When $r_G = 0$, the system is in an incoherent state, with all oscillators rotating at their own intrinsic frequencies.

Without loss of generality, this paper studies the model in an ER network of size N = 2000 and sets the step size $\varepsilon = 0.01$ (it has been verified that the qualitative results remain unchanged over a wide range of $\varepsilon$ values). In all simulations, this paper first waits for 20,000 Monte Carlo time steps to allow the system to reach a steady state, and then runs for an additional 2,000 Monte Carlo time steps to calculate the average cooperation level and synchronization level.

## 3. RESULTS

In the interaction process, individuals incur a cost when they subjectively choose to synchronize with others, which can be understood as cooperative behavior. Conversely, free-riding, or defection, occurs when individuals avoid this cost and merely wait passively for cooperators to synchronize with their phase. Previous studies employed random selection for choosing neighbors to learn strategies, without considering the neighbors' benefit levels. In contrast, this research utilizes optimal selection when choosing neighbors as strategy contributors, with neighbors having higher benefits being more likely to be chosen as strategy contributors. Therefore, the following analysis focuses on random selection versus optimal selection.

To investigate the variation in cooperation rates under the methods of random selection and optimal selection, this research has plotted the cooperation phase diagrams under different relative costs and coupling strengths, as shown in Figure 2:

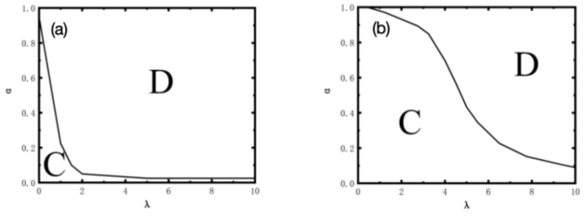


Fig. 2: The cooperation phase diagrams under different values of $\alpha$ and $\lambda$. (a) shows the cooperation phase diagram with random selection, while (b) shows the cooperation phase diagram with optimal selection. Here, $\lambda$ is the coupling strength, and $\alpha$ is the relative cost. The average degree $< k >= 12$, and the optimal selection parameter $W = 40$.

In the figures 2, C denotes cooperators, D denotes defectors, $\alpha$ represents the relative cost, and $\lambda$ signifies the coupling strength. The results indicate that cooperation is more easily sustained with optimal selection. In contrast, with random selection, cooperators are invaded by defectors at lower relative costs $\alpha$ and coupling strengths $\lambda$, ultimately leading to the complete extinction of cooperators within the population.

In addition, Equations (1)-(4) show that the coupling strength $\lambda$ is positively correlated with the cost $c_m$. The increase of the coupling strength will lead to an increase in the cost and a decrease in the benefit-to-cost ratio, which will eventually lead to the disappearance of cooperators and the incoherence of the system. In addition, previous studies [18,21] have also found the relationship between relative cost $\alpha$, coupling strength $\lambda$, and average degree $< k >$. The cooperators can only survive when the above parameters meet the Equation (9)-(11).

$$\frac{b}{c} > k \quad (9)$$

$$\frac{\sqrt{2+2\sin(\varepsilon\lambda)}-\sqrt{2}}{\alpha\varepsilon\lambda}\pi > k \quad (10)$$

$$\frac{\sqrt{2+2\sin(\varepsilon\lambda)}-\sqrt{2}}{\varepsilon\lambda k}\pi > \alpha \quad (11)$$

As $\lambda$ and $\alpha$ increase, the left side of equation (11) decreases, while the right side increases. When the inequality no longer holds, cooperation will cease to occur. This phenomenon is also demonstrated in Figure 2. Therefore, while the increase in $\alpha$ and $\lambda$ typically promotes the evolution of cooperation, when $\alpha$ or $\lambda$ is too large and exceeds a certain threshold or does not satisfy the inequality in Equation (11), cooperation cannot be sustained. Overall, compared to random selection, the optimal selection allows cooperators to survive within a larger parameter space.

Figure 3 shows the synchronization phase diagrams with different relative costs and coupling strengths. It explores the difference in the synchronization rate under the two methods of random selection and optimal selection.

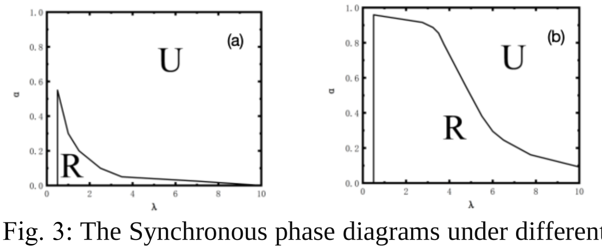


Fig. 3: The Synchronous phase diagrams under different values of α and λ. (a) shows the synchronized phase diagram with random selection, while (b) shows the synchronized phase diagram with optimal selection. Here $\lambda$ is the coupling strength, and $\alpha$ is the relative cost. The average degree $< k >= 12$, and the optimal selection parameter $W = 40$.

Where R represents the state of complete synchronization within the system, also known as coherence, while U represents the state of complete desynchronization, also known as incoherence. Figure 3 investigates the synchronization rates under random selection and optimal selection, demonstrating that the synchronization rate is higher with optimal selection. Additionally, it shows that synchronization does not occur when the coupling strength is low.

First, when an oscillator chooses to cooperate, it will pay the cost to synchronize with other oscillators, so when the cooperation rate is high, the synchronization rate usually increases. Second, Equation (1) shows that when the coupling strength is small, the oscillator tends to move at its own natural frequency, so synchronization does not occur when the coupling strength is small. Previous studies have also shown that there is a threshold for coupling strength when the system reaches a synchronous state or transitions to an incoherent state [18,25], defined as $\lambda_1$. When the coupling strength is lower than $\lambda_1$, the system will be in an incoherent state, and when λ is large, the synchronization rate will also decrease with the increase of λ because the cooperation is difficult to maintain.

Figure 4 shows the influence of network topology on cooperation. Here, we mainly study the influence of network average degree $< k >$.

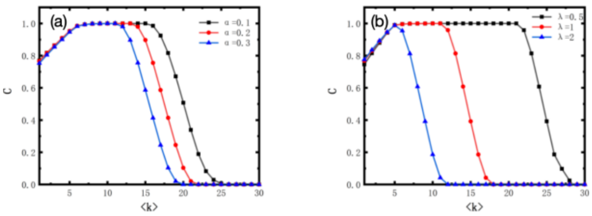

Fig. 4: The correlation between the average degree <k> and the cooperation rate C under different relative cost $\alpha$ and different coupling strength $\lambda$. The black, red, and blue lines in figure (a) correspond to $\alpha = 0.1, \alpha = 0.2, \alpha = 0.3$, and the coupling strength λ=1, respectively. The black, red, and blue lines in figure (b) correspond to $\lambda = 0.5, \lambda = 1, \lambda = 0.2$, and the relative cost $\alpha = 0.2$, respectively. The optimal selection parameter $W = 40$.

Figure 4(a) shows the variation in cooperation rate with respect to the average degree $< k >$ of the network under different relative costs. Firstly, a smaller $\alpha$ allows cooperation to be maintained over a wider range of $< k >$. Secondly, when the average degree $< k >$ is low, the cooperation rate increases with$< k >$, but stabilizes at a high level once $< k >$ reaches 6. Finally, depending on $\alpha$, cooperators begin to gradually disappear from the population at different average degrees, eventually being completely replaced by defectors.

Figure 4(b) shows the variation in cooperation rate with respect to the average degree $< k >$ of the network under different coupling strengths. Smaller λ allows cooperation to be maintained over a wider range of$< k >$. The evolutionary trend in Figure 4(b) is similar to that in Figure 4(a).

Figure 5 studies the effect of average degree on synchronization at different relative costs and coupling strengths.

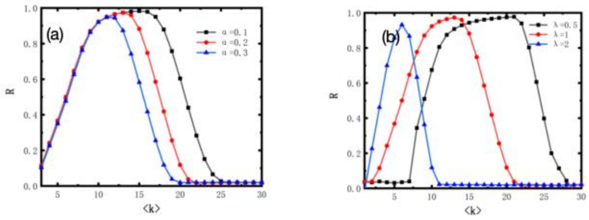

Fig. 5: The correlation between the average degree <k> and the synchronization rate R under different relative cost $\alpha$ and different coupling strength $\lambda$. The black, red, and blue lines in figure (a) correspond to $\alpha = 0.1, \alpha = 0.2, \alpha = 0.3$, and the coupling strength λ=1, respectively. The black, red, and blue lines in figure (b) correspond to $\lambda = 0.5, \lambda = 1, \lambda = 0.2$, and the relative cost $\alpha = 0.2$, respectively. The optimal selection parameter $W = 40$.

In the classical Kuramoto model, the increase of the average degree has a good effect on synchronization. However, in this research, the increase of the average degree does not monotonically increase the synchronization rate. Figure 5 shows that different relative costs and coupling strengths correspond to an optimal solution, that is, the synchronization rate can reach the highest value. In Fig. 5(a), when $\alpha = 0.1, 0.2, 0.3$, the optimal value of the average degree $< k >$ is 15, 13, 11, and when $\lambda = 0.5, 1, 2$, the optimal value of the average degree $< k >$ is 21, 13, and 6, respectively. With the increase of the values of $\alpha$ and $\lambda$, the synchronization rate reaches its optimal value earlier and begins to decline more quickly. Eventually, the system will transform into an incoherent state.

Fig. 6 shows the relationship between the optimal selection parameter $W$ and the synchronization rate $R$ and the cooperation rate $C$ under different relative cost $\alpha$ and coupling strength $\lambda$.

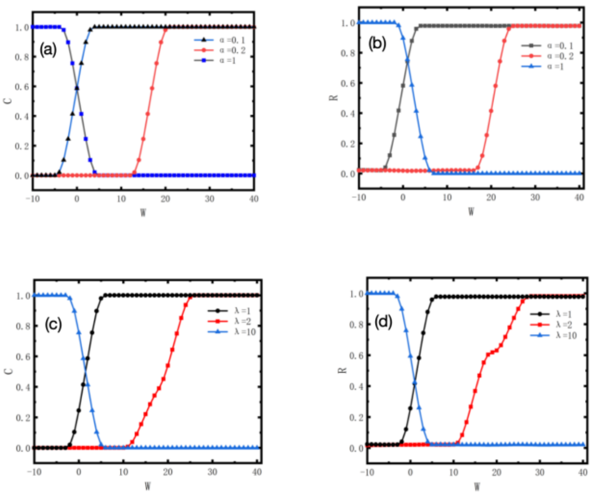

Fig. 6: The fraction of the synchronization rate $R$ and the cooperation rate $C$ under different optimal selection parameters $W$. The black, red, and blue lines in Figure (a) and (b) correspond to $\alpha = 0.1, \alpha = 0.2, \alpha = 1$, and the coupling strength $\lambda = 2$, respectively. The black, red, and blue lines in Figure (c) and (d) correspond to $\lambda = 1, \lambda = 2, \lambda = 10$ , and the relative cost $\alpha = 0.2$ , respectively. The average degree $< k >= 12$.

$W$ is the optimal selection parameter. When $W$ is not equal to 0, the oscillator will choose based on the benefit. When $W > 0$, the probability that the neighbor with high benefit is selected as the strategy contributor is high, which indicates the state preference of pursuing the maximization of benefit. When $W < 0$, the oscillator tends to choose the neighbor with low benefit, indicating an "unusual" state preference.

With the increase of $\alpha$ and λ, the cost $c_m$ and the benefit $r_m$ will increase, but the increase rate of $r_m$ is slower than the increase of $c_m$, that is, the increased cost is higher than the excess benefit. Therefore, although in a system with more cooperators, the benefit of oscillator choice of defect is lower than that of cooperators, but the defector does not need to pay the cost and avoids the high cost of interaction. Therefore, at a larger α or λ, oscillators still tend to choose defect. The optimal choice is to choose the neighbor with higher benefits as the strategy contributor, and in this case, defect becomes the "optimal" strategy instead of cooperation.

When the relative cost $\alpha$ and coupling strength λ are small, the increase in the local synchronization rate caused by cooperation, that is, the increase in benefit will be greater than the cost. Therefore, cooperation is the option with a higher net benefit. With the increase of relative cost $\alpha$ and coupling strength $\lambda$, the benefit difference between cooperators and defectors decreases, when $W$ is positive and small, the optimal choice has little impact on the system, and the probability difference between choosing cooperation and defect is very small, and some oscillators will tend to choose the defect strategy. Eventually, the defector will invade the cooperator group, resulting in the demise of the cooperator and the incoherence of the system. When $W$ is large, the optimal choice has a greater impact on the system, and a small benefit difference will also reflect a large selection probability difference, so the system will strictly select the neighbor with higher benefit as the strategy contributor.

## 4. CONCLUSION

The evolution of synchronization and cooperation has been a popular research direction. This paper combines the two by focusing on the dynamic coupling of synchronous and cooperative behaviors on complex networks. By employing the optimal selection method, we investigate the dynamic changes in synchronization and cooperation.

This research employs Monte Carlo simulations and stipulates that oscillators incur a cost if they actively synchronize with other oscillators, which is defined as cooperation. Conversely, if they passively wait for others to synchronize with them, no cost is incurred, which is defined as defection. The local synchronization rate of an oscillator represents the oscillator's benefit, while the change in angular velocity at each time step represents the cost of cooperation. Each oscillator's net benefit is thus calculated as the benefit minus the cost. Subsequently, the optimal selection method is used to choose neighbors, with neighbors having higher net benefits being more likely to be selected as strategy contributors.

In this work, we compare the outcomes of optimal selection and random selection methods. It is evident that optimal selection significantly promotes the evolution of cooperation and synchronization. As the relative cost and coupling strength increase, in the random selection method, the group of cooperators is invaded by defectors, leading to the extinction of cooperators and the system being in a completely incoherent state. However, under the same parameters in the optimal selection method, cooperation remains at a high level, and the system is in a completely coherent state.

This paper also examines how optimal selection parameters affect cooperation and synchronization. Firstly, the trend of cooperation and synchronization rate is the same. Secondly, when the coupling strength and relative cost are small, the optimal selection parameter is negative, and the defect strategy with lower benefit will be chosen. If the optimal selection parameter is positive, a cooperative strategy with higher benefit will be chosen. On the contrary, when the coupling strength and relative cost are large, a negative optimal selection parameter indicates a preference for a cooperative strategy with lower benefits, while a positive optimal selection parameter indicates a preference for a defect strategy with higher benefits.

Finally, we hope that this research contributes to a deeper understanding of the dynamic coupling of synchronous and cooperative behaviors in complex networks, providing a theoretical foundation for addressing many real-world problems. Additionally, we aim to enrich the research on cooperation and synchronization in complex networks and encourage more scholars to engage in the study of the Evolutionary Kuramoto Game (EKG).

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