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DISSIPATION IN A COLLISIONLESS SHOCK WAVE

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The dissipation, in a sub-critical collisionless shock wave propagating perpendicular to the magnetic field, is shown to be consistent with the model of current-driven ion-wave turbulence. The observed turbulence in the shock front $S(k)$ can be expressed by Kadomtsev's formula but reduced by a factor of 1/24. The observed effective collision frequency in the shock front ν^* can be expressed by Sagdeev's formula but reduced by a factor 1/12.

We present a model for the stochastic heating of the electrons which explains the observed dissipation in terms of the observed turbulence. Comparison with the Kadomtsev-Sagdeev model shows that if the theoretically predicted level of turbulence is reduced by a factor 1/24 then theory and experiment are in good agreement.

2. Introduction

We discuss the dissipation within a collisionless shock wave propagating perpendicular to the magnetic field at sub-critical Alfven Mach numbers $M_A<3$. The shocks are generated by the radial compression of an initially low β plasma within a linear z pinch-discharge [1]. The collisionless nature of the shock is demonstrated by the fact that the electron heating [2] within the shock requires a resistivity two orders of magnitude larger than the Spitzer value. Theoretical papers [3] have proposed a variety of possible origins for this anomalous resistivity. However, there are experimental evidences to suggest that current driven ion-wave turbulence is responsible; namely, i) the measured electron and calculated ion temperatures together with the electron drift velocity give plasma parameters within the shock which are unstable for the ion-waves [4], ii) the observed suprathermal density fluctuations within the shock have frequency spectra which are consistent with the dispersion relations for the ion-wave [5], iii) the wave-number spectra [5] of the turbulence fit that predicted by Kadomtsev [6] for a balance of linear growth of ion-wave instability against the scattering of waves by ions, and iv) the anisotropy of turbulence is consistent with the predictions of linear ion-wave instability theory and one-dimensional adiabatic compression of ions [7].

In the present paper, therefore, we assume that the observed anomalous

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resistivity is due to current driven ion-wave turbulence and show that the further consequences are consistent with observations. The effective collision frequency of electrons in this turbulence has been derived from Kadomtsev's theory and expressed by the Sagdeev formula [8] for anomalous resistivity. Substituting this resistivity into the electron energy equation for the shock gives the scaling of shock thickness against plasma parameters. This scaling is found in the experiments but the numerical factor 1/12 that given by Sagdeev.

Using this scaling, we can show that the spectrum of turbulence obtained by collective scattering of laser light for different plasma parameters scales as predicted by Kadomtsev in magnitude as well as in wave number. The measured absolute level of turbulence is lower than that predicted by a factor varying between 1/24 & 1/47 depending on the assumption for the anisotropy of the turbulence.

We present a model of the stochastic interaction of the electrons with the turbulence which relates the collision frequency to the level of turbulence. By assuming the distorted cone of turbulence described by Muraoka et al [7], this model shows good agreement between the collision frequency derived from the observed turbulence and that observed in the experiment.

By reducing the numerical constant in Kadomtsev's formula by a factor 1/24 to fit experiment, we obtain from the above stochastic model a collision frequency 1/11 that predicted by Sagdeev and in good agreement with observation.

2. Scaling of shock thickness

The electron energy equation for a resistive shock, in which the adiabatic compression term is neglected and the resistivity is assumed to heat electrons only, is expressed in the shock coordinates as follows [10]:

$$\bar{\eta}(F-1)^2\left(\frac{B_{z1}}{\mu_0 L_s}\right)^2=(n_{e1}V_s)1.5\kappa(T_{e2}-T_{e1}) \qquad (1)$$

The compression ratio F and electron temperature behind the shock $T_{e2}(\gg T_{i2})$ are obtained from the Rankine-Hugoniot relations for the value of β in front of the shock and the Alfven Mach number. From Eq. (1), we can derive an effective resistivity $\bar{\eta}$, and consequently an effective collision frequency $\bar{\nu}(=\bar{\eta}n_e e^2/m_e)$ from the observed shock thickness L_s, the shock velocity V_s, parameters of initial plasma n_{e1} and T_{e1}, and B_{z1}. For the standard shock condition (Table 1 *a*) $\bar{\nu}_{obs}=3$ *GHz*.

If the Sagdeev collision frequency [8],

$$\nu_s=10^{-2}\frac{v_d}{v_e}\,\frac{T_e}{T_i}\omega_{pe} \qquad (2)$$

(v_d=electron drift velocity relative to ions, $v_e=\sqrt{\kappa T_e/m_e}$, and ω_{pe}=electron plasma frequency) represents this "measured" effective collision frequency of Eq. (1), we would get an expression for the shock thickness L_s by equating the collision frequencies of Eqs. (1) and (2). After proper approximation, this can be written as follows:

$$L_s=5.3\times10^7\frac{(F-1)^{3/2}}{(F+1)^{3/4}}\left(\frac{T_{em}}{T_{im}}\right)^{1/2}\frac{B_{z1}{}^{3/2}}{n_{e1}{}^{5/4}T_{e2}{}^{1/4}V_s{}^{1/2}}=5.3\times10^7\mathrm{f} \qquad (3)$$

where subscript m means the mean shock condition, L_s in m, T_e and T_i in ev, B_{z1} in *Tesla*, n_{e1} in m^{-3} and V_s in m/sec.

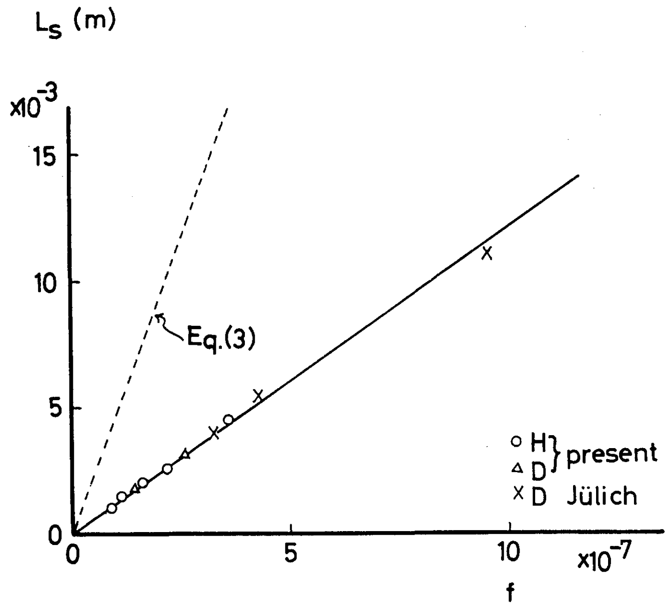

Fig. 1. Shock thickness L_S against f.

In Fig. 1, the observed L_s is plotted against f, in which the data by Dippel et al [11] are also included. It can be seen that L_s versus f gives a straight line as is consistent with the predicted scaling of Eq. (3), but the slope is different as is evident from the dotted line. The experimental values are well expressed by a collision frequency

$$\nu_E=8\times10^{-4}\frac{v_d}{v_e}\frac{T_e}{T_i}\omega_{pe}=\frac{1}{12}\nu_s$$

From the above arguments, we conclude that the collision frequency derived from the observed L_s is expressed by the Sagdeev formula but with an additional numerical factor of 1/12.

3. Spectrum of turbulence

The intensity of light scattered from a plasma yields the Fourier transform of the electron density fluctuations in the form

$$S(\omega, \vec{k}) \equiv \frac{\iint <\delta n_e(t,\vec{r})\delta n_e(t+\tau, \vec{r}+\vec{\xi})> e^{-i(\omega\tau+\vec{k}\cdot\vec{\xi})} d\tau d\vec{\xi}}{n_e} \quad (4)$$

In the experiment [5], ruby-laser light is used and $\vec{k}$ is taken collinear with the azimuthal current in the shock $|\vec{k}|=k$. A spectrally integrated signal is detected by a photomultiplier so that

$$S(k) = \int S(\omega, k) d\omega \quad (5)$$

Measurements have been performed for the following conditions;

Table 1

	$n_{e1}(m^{-3})$	M_A	$T_{e2}(ev)$	$n_{em}(m^{-3})$	$T_{em}(ev)$	$\lambda_{Dm}(m)$
a	6.4×10^{20}	2.4	42	11.0×10^{20}	22	1.05×10^{-6}
b	10.5×10^{20}	2.5	35	18.4×10^{20}	18	0.74×10^{-6}

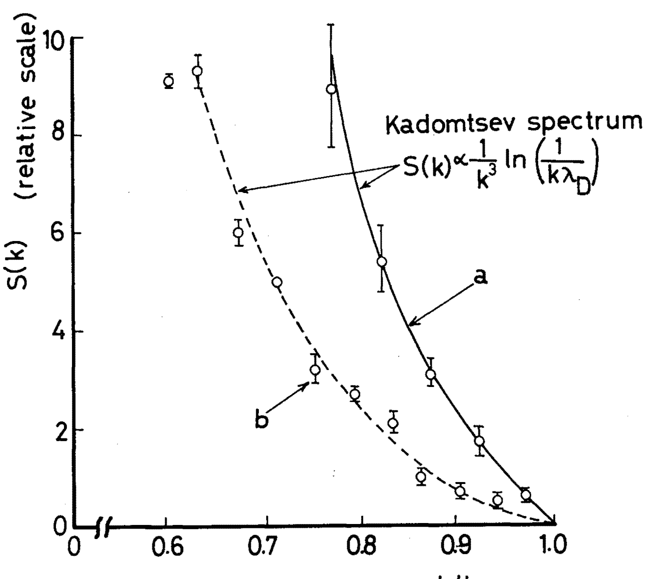

Fig. 2. Intensity of turbulence normalized by a cutoff wave number.

(λ_D=Debye length) with $B_{z1}=0.115$ *Tesla* for hydrogen plasma. The observed spectra show a clear cut-off (k_{co}) at high k and the results plotted in Fig. 2 are normalized in k to this cut-off.

The Kadomtsev spectrum [6] has been combined with the linear dispersion relations for ion-waves to yield [12]

$$S_{KAD}(k)=\frac{2\pi^2}{7}\frac{v_d}{v_e}\frac{T_e}{T_i}\frac{1}{\theta_c^2}n_e\frac{1}{k^3}ln\frac{1}{k\lambda_D} \quad (6)$$

where θ_c is the cone angle of the turbulence in k-space.

In the following, the measured turbulence for the two cases of Table 1 is shown to be consistent with the Kadomtsev specrum of Eq. (6), if we use the result of the previous section.

In order to see the wave-number dependence of the observed turbulence explicitly, the data of Fig. 2 are rearranged in Fig. 3($K=k\lambda_D$). From this figure, we can see the $1/k^3$ dependence of $S(k)$ as well as cut-off with $ln(1/K)$ which is evident in Fig. 2.

The ratio of cut-off wave numbers for the two cases is

$$\frac{(k_{co})_a}{(k_{co})_b}=0.78 \quad (7)$$

which agrees with the ratio of a reciprocal of the Debye lengths

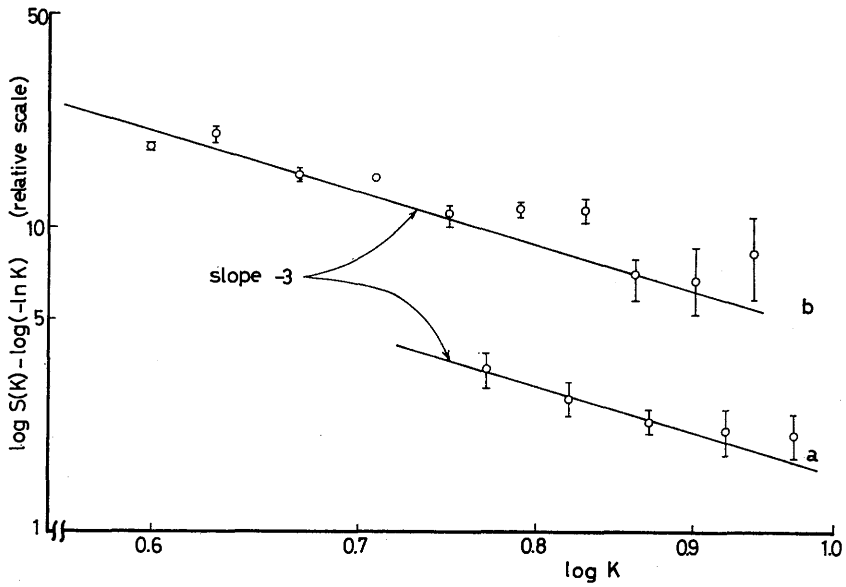

Fig. 3.

$$\frac{(\lambda_{Dm})_b}{(\lambda_{Dm})_a}=0.71 \tag{8}$$

within the error of 10 %.

The observed ratio of intensities of turbulence for the two cases for given k/k_{co} and fixed Δk is from Fig. 2,

$$\left(\frac{S_a}{S_b}\right)_{\Delta k}=2.4 \tag{9}$$

To compare with theory, we convert to the variable $K=k/k_{co}$

$$\left(\frac{S_a}{S_b}\right)_{\Delta K}=\left(\frac{S_a}{S_b}\right)_{\Delta k}\frac{(k_{co})_a}{(k_{co})_b}=1.8 \tag{10}$$

We expect from Eq. (6) for given K and fixed ΔK, the Kadomtsev spectrum will scale

$$(S_{KAD})_{\Delta K}\propto\frac{v_d}{v_e}\,\frac{T_e}{T_i}\,\frac{1}{\theta_c^2}n_e\lambda_D^2 \tag{11}$$

We now use the fact that

$$\left.\begin{aligned} v_d &\propto\frac{1}{n_{em}L_s}\\ v_e &\propto\sqrt{T_{em}}\\ \lambda_{Dm}^2&\propto\frac{T_{em}}{n_{em}}\end{aligned}\right\} \tag{12}$$

and assume

$$T_{ia}\sim T_{ib},\quad (\theta_c)_a\sim(\theta_c)_b$$

to get

$$(S_{KAD})_{\Delta K}\propto\frac{T_{em}^{3/2}}{n_{em}L_s} \tag{13}$$

From the results of section 2 on the scaling of L_s, we get

$$\left(\frac{S_{KAD,a}}{S_{KAD,b}}\right)_{\Delta K}=1.61 \tag{14}$$

which is in reasonable agreement with the experimental value of 1.8 from Eq. (10).

4. Level of turbulence

The measured level of turbulence for condition a above at $k_m=7.1\times10^5$ m^{-1} is given by [13]

$$S_E(k_m)=230 \tag{15}$$

and the measured cut-off angle in the plane containing the shock current J_θ and V_s is given by [7]

$$\varphi_c=50° \tag{16}$$

We can compare this measured level of turbulence with that predicted by Kadomtsev in Eq. (6) by substituting the experimental conditions including a cone angle $\theta_c=\varphi_c=50°$. Eq. (6) yields

$$S_{KAD}(k_m)=10,800=47S_E(k_m) \tag{17}$$

However, Muraoka et al [7] have pointed out that in this collisionless shock the ion temperature should be anisotropic and consequently the cone of instability for ion-waves should be distorted. They predict cut-off angles $\varphi_c=55°$ in the plane containing the shock current J_θ and V_s and $\psi_c=84°$ in the plane perpendicular to J_θ and to V_s. If we use the average of these $\theta_c=\varphi'_c=70°$ in Eq. (6), we obtain

$$S'_{KAD}(k_m)=5,510=24S_E(k_m) \tag{18}$$

5. Relation of $S(k)$ and ν^*

We now wish to relate the observed level and spectrum of turbulence to the observed effective collision frequency. In particular, we wish to see whether a Kadomtsev form of spectrum but with 1/24 or 1/47th the theoretical level is consistent with an effective collision frequency 1/12 that predicted by Sagdeev from the Kadomtsev spectrum. To this end, we now present an extension of our earlier model [10),12)] of the stochastic interaction of electrons with the fluctuating electric field of the ion-wave turbulence. This model which relates ν^* directly to $S(k)$ depends on two main assumptions.

(1) 'STATIONARY WAVE' APPROXIMATION: We assume that all electron velocities c_e are sufficiently greater than the ion-wave velocity ($c_e \gg c_s$) that the waves can be regarded as stationary. As $v_e=10c_s$ most electrons satisfy this condition.

(2) STOCHASTIC ASSUMPTION: We assume that the dominant process in the interaction is small angle deflection of the electrons by the electric field of the waves and that these deflections result in an effective 90° collision rate given by the Fokker-Planck equation [14]

$$\nu^*(\vec{v})=\frac{1}{v^2}D_\perp(\vec{v}) \tag{19}$$

where

$$D_{\perp}(\vec{v})=\frac{1}{(2\pi)^3}\left(\frac{e}{m_e}\right)^2\iint_{\vec{k}\,\omega}\left[1-\frac{\vec{k}\cdot\vec{v}}{k^2v^2}\right]<|E(\omega,\vec{k})|^2>\delta(\omega+\vec{k}\cdot\vec{v})\,d^3k\,d\omega$$

$$=\frac{1}{(2\pi)^3}\left(\frac{e}{m_e}\right)^2\int_{\vec{k}}<|E(\vec{k})|^2>\delta(\vec{k}\cdot\vec{v})\,d^3k$$

The stationary wave assumption allows the integration over ω.

The validity of the stochastic assumption depends on the absence of large angle deflections (sometimes called trapping). Neglecting the effect of magnetic fields, large angle deflections are not important provided the electric potential φ satisfies the condition

$$e\varphi/\kappa T_e \ll 1 \tag{20}$$

The mean electric potential of the turbulence can be derived from $S(k)$ by using the known relation of $\varphi(k)$ and $\delta n_e(k)$[15]

$$\int<\varphi(\vec{r})\varphi(\vec{r}+\vec{\xi})>e^{-i\vec{k}\cdot\vec{\xi}}d\vec{\xi}=<|\varphi(k)|^2>=\left(\frac{m_e}{e}\right)^2\frac{v_e^4}{n_e}S(\vec{k}) \tag{21}$$

From this, the mean potential φ is given by

$$\overline{\varphi^2}=\frac{1}{(2\pi)^6}\left(\frac{m_e}{e}\right)^2\frac{v_e^4}{n_e}\int S(\vec{k})\,d^3k \tag{22}$$

For a Kadomtsev form of spectrum with level $S(k_0)$ at k_0

$$\overline{\varphi^2}=\frac{1}{(2\pi)^5}(1-\cos\theta_c)\left(\frac{m_e}{e}\right)^2\frac{v_e^4}{n_e}\left[\frac{S(k_0)k_0^3}{ln(1/k_0\lambda_D)}\right]\int\frac{ln(1/k\lambda_D)}{k}dk \tag{23}$$

where the integral has the value $\frac{1}{2}\left[\left\{ln(1/k_{min}\lambda_D)\right\}^2-\left\{ln(1/k_{max}\lambda_D)\right\}^2\right]$. Performing the integration from $k=1/L_s$ to $1/\lambda_D$ and substituting experimental values for $\theta_c=\varphi_c=50°$ yields

$$\frac{e\sqrt{\overline{\varphi^2}}}{\kappa T_e}=0.015\ll 1 \tag{24}$$

The possible effect of the magnetic field on large angle deflection is beyond the scope of this paper.

Having a measure of justification for our assumptions, we can now use Eq. (19) to obtain an average ν^* for a Maxwellian distribution of electron velocities

$$\overline{\nu^*}=\int\nu^*(\vec{v})f(\vec{v})\,d^3v$$

$$=\frac{1}{(2\pi)^3}\left(\frac{e}{m_e}\right)^2\left(\frac{s}{\pi}\right)^{3/2}\iint<|E(\vec{k})|^2>\delta(\vec{k}\cdot\vec{v})\exp(-sv^2)\frac{d^3v}{v^2}d^3k \tag{25}$$

where $s=m_e/2\kappa T_e$.

In Eq. (25), the integration over $\vec{v}$ and $\vec{k}$ are coupled through the geometrical term $\delta(\vec{k}\cdot\vec{v})$. If we consider the k-spectrum of ion-wave turbulence, i.e. a cone about J_θ, the coupling implies interaction with electrons within an inverse cone in v-space. However, we know from experiment that $\omega_{ce}>\nu^*$ so that gyration will make $f(v)$ symmetrical about B. As B is perpendicular to J_θ, this effect of the magnetic field will remove the anisotropy of $f(v)$ and allow the decoupling of the v and k integrations.

By substituting the identity $\delta(\vec{k}\cdot\vec{v})=\frac{1}{|k|}\delta(v_{/\!/})$, where $v_{/\!/}$ means parallel to k, we obtain from Eq. (25)

$$\overline{\nu^*}=\frac{1}{(2\pi)^3}\left(\frac{e}{m_e}\right)^2\left(\frac{s}{\pi}\right)^{3/2}I_k I_v \tag{26}$$

where

$$I_k=\int<|E(\vec{k})|^2>d^3k/|k|$$

$$I_v=\int\delta(v_{/\!/})\exp(-sv^2)d^3v/v^2$$

For a uniform cone of turbulence with half angle θ_c

$$I_k=2\pi(1-\cos\theta_c)\int<|E(k)|^2>kdk \tag{27}$$

The $\delta(v_{/\!/})$ has no significance because $f(v)$ is isotropic. The integral I_k diverges logarithmically as $v\rightarrow 0$ and consequently a lower cut-off velocity, v_{min}, is required. Then

$$I_v=\pi E_1(\varepsilon);\ \varepsilon=sv_{min}{}^2 \tag{28}$$

where E_1 is the exponential integral function.

The lower cut-off in velocity is analogous to the lower cut-off of impact parameter in the classical derivation of collision frequency. Large angle deflection is not important for velocities above v_{min} provided

$$\varepsilon=e\varphi/\kappa T_e$$

For the mean value of φ obtained from the measured $S(k)$ using Eq. (23), we find $\varepsilon=0.015$ and $E_1(\varepsilon)=3.6$.

Bringing together Eqs. (26), (27), and (28), we can relate this improved calculation of ν_2^* to that derived previously[10),12)] ν_1^*,

$$\nu_2^*=1.8(2\pi)^{-1/2}(1-\cos\theta_c)\nu_1^* \tag{29}$$

where

$$\nu_1^* = \frac{1}{(2\pi)^2}\left(\frac{e}{m_e}\right)^2 \frac{1}{v_e^3}\int <|E(k)|^2> kdk = \frac{1}{(2\pi)^2}\,\frac{v_e}{n_e}\int S(k)k^3 dk$$

The conversion of $<|E(k)|^2>$ to $S(k)$ comes from the properties of ion waves [15]. Gary and Paul [9] have given a different development of this model which yielded

$$\nu_3^* = (2\pi)^{-1/2}(1-\cos\theta_c)(1-c_s/v_e)\nu_1^* \tag{30}$$

6. Self-consistent description of dissipation

Paul et al [13] reported previously that the experimental $S(k_m)=230$ and $\theta_c=\varphi_c=50°$ yielded from Eq. (29)

$$\nu_2^* = 1.3\ GHz \sim \frac{1}{2}\bar{\nu}_{obs} \tag{31}$$

In the light of the paper by Muraoka et al [7], we should now consider $\theta_c=\varphi'_c=70°$ which gives better agreement

$$\nu_2^* = 2.5\ GHz \sim \bar{\nu}_{obs} \tag{32}$$

Thus the observed level and spectrum of turbulence can, on a stochastic basis, explain the observed effective collision frequency and hence the dissipation.

We now wish to reconcile this self-consistent description with the prediction of Kadomtsev [6] and Sagdeev [8]. If the Kadomtsev prediction of Eq. (6) is substituted into Eqs. (29) and (30) together with Sagdeev's assumption $\theta_c=\pi/2$ and $v_d \gg c_s$

$$\nu_2^* = 2.0\nu_s,\quad \nu_3^* = 1.1\nu_s \tag{33}$$

in reasonable agreement. This disposes of the problem of the unexplained factor 10^{-2} in the usual Sagdeev formula.

We now have to consider the two alternatives for θ_c

(a) With $\theta_c=\varphi_c=\psi_c=50°$ in both ν_2^* (Eq. (30)) and $S_{KAD}(k)$ (Eq. (6)) we obtain

$$\nu_2^* = 2.4\nu_s \tag{34}$$

But if we use the corresponding observed level of turbulence $S(k)=\frac{1}{47}\times S_{KAD}(k)$, ν^* would be reduced in proportion

$$\nu_2^* = 2.4\times\frac{1}{47}\nu_s = \frac{1}{20}\nu_s \tag{35}$$

(b) With $\theta_c=\varphi_c'=70°$ in both ν_2^* and $S_{KAD}(k)$, we obtain

$$\nu_2{}^* = 2.2\nu_s \tag{36}$$

But the change of θ_c changes $S_{KAD}(k)$ so that the observed $S(k)=\frac{1}{24}\times S_{KAD}(k)$ and consequently

$$\nu_2{}^* = 2.2\times\frac{1}{24}\nu_s = \frac{1}{11}\nu_s \tag{37}$$

This case is consistent with the observed $\bar{\nu}_{obs}=\frac{1}{12}\nu_s$.

We have to remark that, because of the neglect of ion-ion collision (turbulence sink) in deriving the Kadomtsev spectrum, actual $S(k)$ is expected to depart from that given by Eq. (6) and eventually be cut-off at low k. If cut-off occurs at $k\lambda_D \sim 0.2$ the effective collision frequency is reduced to half the value given above.

7. Conclusions

The very close agreement between the observed collision frequency and that derived from the observed turbulence is, considering the chain of argument and averaging within the shock, fortuitous. In fact the use of $\nu_3{}^*$ instead of $\nu_2{}^*$ would produce a difference of a factor of about two. Also there is as yet no experimental evidence for the theoretical prediction that the cut-off angle $\psi_c=84° \neq \varphi_c=50°$. Summarizing the comparison with theory we observe:

(1) Collision frequency scales as predicted by Sagdeev but $\nu^*=\nu_s/12$.
(2) Turbulence spectrum scales as predicted by Kadomtsev but $S(k)<S_{KAD}(k)$.
(3) Turbulence has cone angle $\varphi_c=50°$.

We assume

(1) Stochastic heating of the electrons by 'stationary' ($c_s \ll v_e$) ion-waves with a Kadomtsev spectrum.
(2) A distorted cone of turbulence as predicted by Muraoka et al [7] with $\varphi_c=55°$, $\psi_c=84°$ giving a mean $\theta'_c=70°$.

We have shown that, if the formula for $S_{KAD}(k)$ in Eq. (6), derived from Kadomtsev, is reduced by a factor 1/24, then the observed and predicted values of $S(k)$ and ν^* are in agreement.

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