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Influence of strategy adaptation speed on network reciprocity for evolutionary prisoner's dilemma games

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Abstract. Following our previous study (Tanimoto, submitted to *Physica A*) on how network reciprocity is affected when strategy adaptation speed is slower than gaming speed, we conducted a series of simulations to obtain a deeper insight. In the case of a spatial prisoner's dilemma on a scale-free network with a spatial distribution of the strategy updating time scale, we found that a negative correlation between degree and strategy updating speed brings an extremely large cooperation-enhancing effect. This might be because a cooperative hub agent who is insensitive to strategy adaptation can protect against defection invasion at the initial stage of a simulation episode to initiate a cooperative situation.

Keywords: Prisoner's Dilemma, Network reciprocity, Evolutionary game

1 Introduction

The puzzle of how cooperative behavior can emerge in the real world has attracted much attention. The classical metaphor for investigating this problem is the prisoner's dilemma (PD) game. Nowak (2006) identified five mechanisms that result in the evolution of cooperation (C) instead of defection (D) in PD games: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. All of these mechanisms can be somewhat related to the lessening of an opposing player's anonymity relative to the so-called well-mixed situation. Since 1992, when the first studies of the spatial PD (SPD) were conducted by Nowak and May (1992), hundreds of studies have reported the manner in which network reciprocity functions as a key protocol for evolving cooperation. In fact, SPD possibly offers a simple but meaningful metaphor for understanding network reciprocity. In SPD, there are only two strategies: C and D. Each player plays only with his or her immediate neighbors on the network. After the game, each player updates his or her strategy according to a specified update rule.

Most previous studies have investigated network reciprocity from the viewpoint of network topology. They have assumed that evolutionary games are played not only on regular graphs where respective vertices have identical degrees (e.g., a ring or square lattice

(Nowak & May 1992), which we refer to as homogeneous networks), but also on complex networks (e.g., Santos & Pacheco 2005, Santos et al. 2006a, Gómez-Gardeñes et al. 2007) where vertices have different degrees (which we call heterogeneous networks). In networks that are more "heterogeneous" in terms of degree distribution, such as scale-free networks, the stronger cooperation-enhancing effect can be observed because it allows for hub C agents, which compel cooperation among their neighbors and lead to strong and stable cooperation.

There have been several important precursors investigating the fundamental question of what constitutes the substantive issue of network reciprocity. Ohtsuki et al. (2006) deduced that, in general, a smaller average degree $<k>$ for the network implies more robust dynamics in solving the dilemma, regardless of network topology, although their deductive approach was premised on several restrictive assumptions. Yamauchi et al. (2011) found that "the effect of average degree, where a smaller degree network can be more robust for cooperation," is questionable and not universal. Through a series of simulations based on full-factorial design of experiments, they also clarified qualitatively that strategy update rules and dynamics are much more significant than network topology. Roca et al. (2009) and Tomassini et al. (2007) too reported from the same viewpoint.

Another important point in discussing the substantive aspects of network reciprocity has arisen in several studies dealing with co-evolutionary models that allow simultaneous evolution of networks and strategies. Pacheco et al. (2006) and Santos et al. (2006b) observed robust cooperation when the speed of network evolution exceeded the speed of strategy adaptation. This is understandable because network adaptation can serve to expel neighboring D agents effectively and quickly by severing links with them, thereby having the same function as a game exit option (e.g., Schuessler 1989). Immediately severing links with D agents is more effective than offering D against a D agent as retaliation in defending a C cluster from D agents' attacks. This leads to a sequitur that higher speed for network adaptation enhances C more effectively, as proved by their simulation studies. In other words, sustainable cooperation can be established more easily if a C agent is able to sever links with D neighbors before copying their D strategy.

In a relevant and novel study, Moyano and Sanchez (2009) considered a different type of co-evolution model. Their model permitted strategy plus strategy updating rules (not network adaptation) to evolve. An agent copies a strategy from his/her neighbors (including the focal player) based on a specified updating rule. At the same time, the agent copies an update rule itself from the selected neighbor whose strategy is copied. Moyano and Sanchez presume three variants of updating rules: Imitation Max (IM), wherein a focal player copies the strategy of the neighbor receiving the largest payoff in the current time step; Pairwise (PW), wherein a player compares his or her payoff with that of a randomly selected neighbor and copies the neighbor's strategy according to a certain function; and Pairwise comparison with all neighbors, which is analogous to Roulette selection (RS). There is a difference in how these three rules employ stochastic characteristics in the process of

selecting each opponent. Hence, IM is defined as deterministic in selecting opponents (there is no choice of copying from the best payoff neighbor), whereas PW is entirely stochastic. If we suppose a situation involving a spatial distribution of updating rules adopted by respective agents in a network, it can be said that strategy adaptation speed is also spatially distributed, because a stochastic updating rule means a lower speed of strategy adaptation vis-à-vis a deterministic updating rule. This particular spatial distribution in terms of strategy adaptation speed might help a C cluster grow, since a lower strategy adaptation speed avoids D diffusion among C agents (although it also prevents C dissemination among D agents).

About the question of how strategy adaptation speed influences network reciprocity, other important studies (e.g., Guan et al. 2009, Chen et al. 2008, Droz et al. 2009) reported that a larger cooperation enhancement can be observed if an attenuation factor on the PW process is presumed, which makes "learning activity" (copying an opponent's strategy process) slower. Likewise, another group of studies (e.g., Szolnoki & Perc 2008, Perc et al. 2008, Szabo & Szolnoki 2009, Szolnoki & Perc 2009, Szolnoki et al. 2009) reported that a larger cooperation enhancement can be observed if an attenuation factor on PW processes for imposing the focal agent's strategy on the opponent is presumed, which makes the "teaching activity" (imposing a focal agent's strategy on the opponent) slower.

Szolnoki et al. (20008) clarified that high cooperation enhancement was found if strategy adaptation was limited for some agents. In other words, some agents had different strategy adaptation abilities from others.

In sum, all of these previous studies have highlighted that a strategy adaptation speed that is slower than gaming speed leads to high cooperation by network reciprocity.

Recently, Yamauchi et al. (2010) demonstrated simulation results in which effective network reciprocity can be established by varying spatial distribution of strategy updating, update time scale, and its time lag, all of which actually control strategy adaptation speed. In addition, he also validated that the central effect on network reciprocity limits the destruction of C clusters by D strategy expansion in the early stage of evolution. Cooperation can never be established unless C clusters survive D strategy invasion, which is crucial during initial evolutionary stages of the game. This particular effect is dependent on strategy adaptation speed.

This paper follows his work as a continuous report. We show that offering agents a stochastically distributed strategy adaptation speed enhances C strategies more effectively than imposing a single slower strategy adaptation speed. Moreover, the effect can be amplified if an appropriate correlation is established between agents' degree distribution and strategy adaptation speed.

2 Model for PD on time-constant networks

2.1 PD game

We consider a 2 × 2 game as an archetype. Each player can adopt either of strategies C or D. Players receive a reward (R) for mutual Cs and punishment (P) for mutual Ds. If one chooses C and the other chooses D, the player choosing D obtains a temptation (T) payoff, while the player choosing C is labeled a saint (S). According to Tanimoto and Sagara (2007), we define the dilemma from a stag hunt type of game as $D_r = P - S$ and from a chicken type game as $D_g = T - R$. We can assume that $R = 1$ and $P = 0$ without losing mathematical generality. In this case, the payoff matrix can be given as

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1+D_g & 0 \end{pmatrix} . \qquad (1)$$

In PD games, the elements are assumed to satisfy $T > R > P > S$ and $2R > T + S$. In this paper, we limit the PD game class by assuming that $0 \le D_g \le 1$ and $0 \le D_r \le 1$. At every time step, each agent plays 2 × 2 games with all other neighboring agents connected via links explained in the following section. The total payoff is evaluated by summing all games played by a specified agent at a specified time step.

2.2 Agents and strategy updating

We assumed $N = 1500$ agents in a scale-free network (SF) based on a Barabasi-Albert (BA) model (Barabasi & Albert 1999) with average degree $\langle k \rangle = 8$.

At every time step, agent i refreshes his strategy (C or D) with probability $p_{CorD}(i)$. We adopt the strategy update rule Imitation Max, in which the focal player i imitates the strategy having the maximum payoff among all strategies adopted by the focal player and his or her immediate neighbors. For update dynamics, we use synchronous updating, in which all players update their strategies simultaneously after playing all games.

2.3 Simulation setting

Each simulation runs as follows. Initially, an equal percentage of strategies is distributed randomly to players allocated on different vertices of the network. Then several generations are run until the frequency of cooperation arrives at a quasi-equilibrium. If the frequency of cooperation continues to fluctuate, we obtain the frequency of cooperation for the final 250 generations over a 3000-generation run. We conduct this procedure at 11 × 11

points of the PD area ($0 \le D_g \le 1$, $0 \le D_r \le 1$) 50 times to draw the respective 121 ensemble averages.

We defined three sub-classes of PD as the characteristic values used to evaluate cooperation. The first is a single algebraic average of those 121 ensemble averages covering all PD areas (hereafter AllPD). The second is another average of 11 points featured by $D_g = D_r$, the donor and recipient game (hereafter D&R). The last is another 11-point average collected from the region of $D_r = 0$, which consists of boundary games between the PD and chicken games featured without the stag hunt type dilemma (hereafter BPDC). Many previous studies prefer to postulate BPDC for representing PD because it can be characterized by the single dilemma parameter D_g.

3 Results and discussion

We first discuss the result if the strategy refreshing probability—i.e., the slowness of a specified strategy adaptation speed—is global and all agents have same $p_{CorD}(i)$. Figure 1 shows the relations between the frequency of cooperation and strategy refreshing probability classified by AllPD, D&R, and BPDC. Consistent with what Yamauchi et al. (2010) reported, the more robust network reciprocity obviously emerges with the slower strategy adaptation speed as compared with the speed of game processing. For example, in a boundary game of PD and chicken (BPDC), we are able to see a cooperative equilibrium over 0.9 in terms of cooperation frequency when the speed of strategy adaptation is defined as 1/2 of that of game processing ($p_{CorD}(i) = 0.5$).

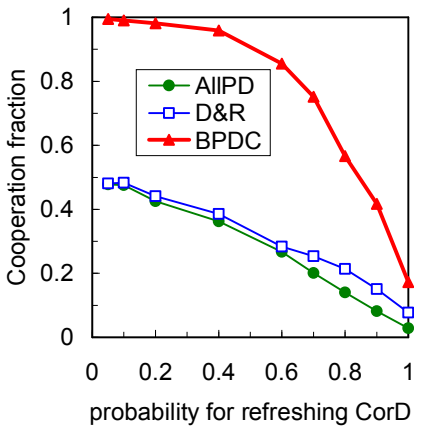

Fig.1 Relations between strategy refreshing probability—i.e., strategy adaptation speed—and cooperation fraction at equilibrium of AllPD (closed circle), D&R (open square), and BPDC (closed triangle).

Next, we discuss the result when $p_{CorD}(i)$ is defined to distribute stochastically among agents. Fig.2 shows the results when two uniform distributions, $p_{CorD} \in [0.6, 1.0]$ and $p_{CorD} \in [0.7, 0.9]$, are presumed. Three dotted horizontal lines imply the result when $p_{CorD}(i)$ = 0.8 from Fig.1, which is the central value of the two uniform distributions. On the left side of Fig.2, three solid horizontal lines show the results when $p_{CorD}(i)$ = 0.6. On the right side of the figure, three solid horizontal lines show the results when $p_{CorD}(i)$ = 0.7. Both sets of lines are the lower limits of the respective uniform distributions.

Fig.2 also displays three pairs of three plots connected by a solid vertical line. As indicated by the legend, each pair of three plots represents game situations AllPD, D&R, and BPDC. For each of the three plotted points, all distributed degrees and $p_{CorD}(i)$ are sorted by descending order, and the maximum degree and the minimum $p_{CorD}(i)$ are combined to define for an agent. The vertical distance spans three types of correlation between agents' degree and $p_{CorD}(i)$. The topmost plot indicates a negative correlation between agents' degree and $p_{CorD}(i)$. The bottom plot indicates a positive correlation, in which the inverse definition to the negative case is applied. The middle plot is the non-correlation case, where respective degree and $p_{CorD}(i)$ are randomly combined.

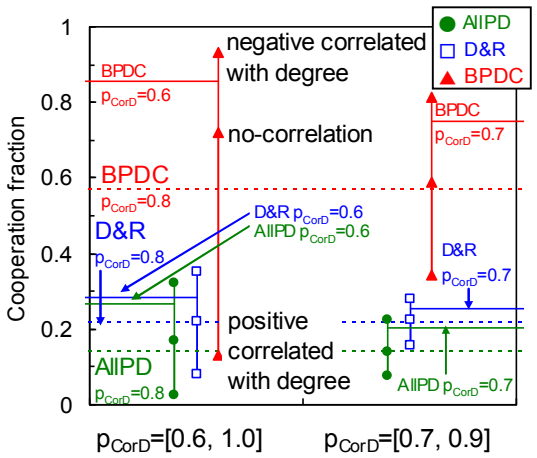

Fig.2 Cooperation fraction at equilibrium of AllPD (closed circle), D&R (open square), and BPDC (closed triangle) when the strategy refreshing probability of each agent obeys a uniform distribution, either [0.6, 1.0] or [0.7, 0.9]. Three plots placed in a vertical line indicate cases of different correlations between degree and strategy refreshing probability imposed on agents. Three dotted lines are results read from Fig.1 when p_{CorD} = 0.8, which is the central value of the two uniform distributions. Two sets of three bold lines are also results from Fig.1 when p_{CorD} = 0.6 and 0.7, which are the lower limits of the two uniform distributions, respectively.

Evidently, the least effective network reciprocity is observed when the correlation is positive as conformed above. Instances of non-correlation cases are better than the non-distribution result of $p_{CorD}(i)$ = 0.8, although the case of [0.7, 0.9] seems subtle. This fact

somewhat implies that the coherence-resonance feature (e.g., Perc 2006) is responsible for what we observe. This is consistent with the observed fact that the enhancement effect is significant when the range of stochastic distribution is wider.

The following reason might explain why the negative correlation case shows the best effective enhancement. Giving a negative correlation to agents makes the hub C agent insensitive to D agents' invasions, because he has less opportunity than other agents to refresh his strategy, meaning less chance to be overwritten by D strategy.

The fact that the non-correlation cases are better than the non-distribution result of $p_{CorD}(i) = 0.8$ suggests that a key factor is not only how slower adaptation speed is given to all agents on average but also which agent in the society is the slowest to adapt. Also the fact that the two negative correlation cases are better than the lower limit cases: $p_{CorD}(i) =$ 0.6 and 0.7 might imply that distribution of adaptation speed in agents plays an important role in encouraging such a high enhancement effect. To the end, a simple question might come up, say, whether the same cooperation-enhancing effect could be observed if only the maximum hub agent, who has the largest degree regardless C or D, is allowed to have a slower speed of strategy adaptation than other agents. Figure 3 provides one answers. It compares payoffs on the D_g vs D_r domain (as explained in 2.3) of the default case and the case where only the maximum hub agent is $p_{CorD}(i) = 0.5$ while all others $p_{CorD}(i) = 1.0$. The effect of making the maximum hub agent insensitive to strategy adaptation seems significant, especially for game structures with larger D_r, which is close to the boundary between PD and stag-hunt. This fact suggests a marvelous possibility: a highly cooperative society emerges if, among thousands agents, only one—the largest hub agent—could be persuaded to change strategies more slowly. He need not become a blind agent, but merely less sensitive (i.e., slower) in updating strategy.

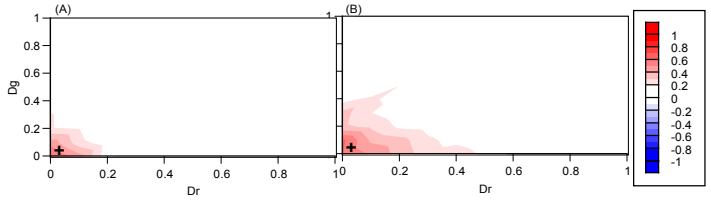

Fig.3 Payoff obtained within the limits $0 \le D_r \le 1$ and $0 \le D_g \le 1$. (A) is when $p_{CorD}(i) = 1.0$ for all agents, which is the default case with general setting, whereas (B) is when the case where only the maximum hub agent is $p_{CorD}(i) = 0.5$ while others $p_{CorD}(i) = 1.0$.

4 Conclusions

Following our previous report, we have presented simulation results to examine what substantially explains network reciprocity. Our simulation model involves typical Spatial PD games parameterized by strengths of both chicken type and stag hunt type dilemmas on

a time-constant, scale-free network. We vary agents' strategy adaptation speed by controlling strategy refreshing probability.

We confirmed that network reciprocity enhances cooperation significantly when slower strategy adaptation is imposed on all agents. And by presuming that adaptation speed is distributed stochastically, we find that enhancement is more effective than in the non-distribution case.

Moreover, by assuming a negative correlation between agents' degree and strategy adaptation speed, extremely significant enhancement can be realized. This is because hub agents who offer C can be insensitive to D agents' invasion, as they have less chance to be overwritten by D. Interestingly, desensitizing only the maximum hub agent's strategy adaptation speed has a non-negligible effect on enhancing cooperation through network reciprocity.

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