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https://doi.org/10.5109/7173428

出版情報：Reports of Research Institute for Applied Mechanics. 22 (70), pp.117-125, 1975. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. XXII, No. 70, 1975

PARAMETRIC INTERACTION OF LASER BEAM WITH INHOMOGENEOUS PLASMAS

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Parametric instability of electrostatic oscillations in a semi-infinite plasma subjected to electromagnetic radiation is studied. The possibility of the parametric decay of evanescent photon is pointed out. The threshold power of the incident radiation for this process is obtained by using the reductive perturbation method.

1. Introduction

In plasmas, electromagnetic wave with the frequency ω_0 and the wave-vector $\boldsymbol{k}_0$ can feed its energy into electron plasma wave with $(\omega_1, \boldsymbol{k}_1)$ and ion acoustic wave with $(\omega_2, \boldsymbol{k}_2)$ if the following conditions hold,

$$\omega_0=\omega_1+\omega_2, \boldsymbol{k}_0=\boldsymbol{k}_1+\boldsymbol{k}_2 \tag{1}$$

$$|\omega_0|>|\omega_1|, |\omega_2| \tag{2}$$

This phenomenon is wellkown as the parametric interaction of decay process of electromagnetic wave into electron plasma wave and ion acoustic wave and has been a subject of absorbing interest in recent years [1)-5)]. It has been already pointed out that electrostatic oscillations become unstable and grow in amplitude when they gain energy from electromagnetic radiations at a rate faster than they can dissipate it by their damping mechanism, collisional damping or Landau damping; that is to say, when the amplitude of the incident radiation exceeds a critical threshold, the parametric instability occurs.

In this paper we investigate the decay process of electromagnetic wave incident upon a semi-infinite plasma, which is in an overdense state ω_e(plasma frequency)$>\omega_0$. The electromagnetic wave cannot propagate through such a plasma and only penetrates it by a depth of c. $(\omega_e(\omega_e-\omega_0))^{-1/2}$, being called an evanescent mode. It will be shown that electrostatic oscillations can become unstable through the parametric interaction with the evanescent electromagnetic field though the condition (2) does not hold, that is, though $|\omega_0|<|\omega_1|$. This may be interpretted as follows: A finiteness of the life-time of evanescent mode yields a broadening in its frequency due to the uncertainty relations. In the present case, the frequency broadening, $|\Delta\omega_0|$,

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is estimated as $(\omega_e(\omega_e-\omega_0))^{1/2}$. The condition (2) is now made a little looser,

$$\omega_0+|\varDelta\omega_0|>\omega_1\sim\omega_e.$$

Thus, the parametric instability can be possible when $\omega_e/(\omega_e-\omega_0)>1$.

In Sec. 2, we derive the basic equations to be solved using the hydrodynamic description of the system. We assume the amplitude of the incident electromagnetic field and the frequency difference, $\omega_e-\omega_0$, to be small enough, so that the reductive perturbation method [6] can be applied. The equations so obtained are the ntreated in Sec. 3 as the eigenvalue problem by imposing appropriate boundary conditions. The threshold value is obtained for the parametric instability of the decay process of the evanescent electromagnetic wave into electrostatic oscillations. Section 4 is devoted to the discussions and concluding remarks.

2. Derivation of the Basic System of Equations

We shall apply the reductive perturbation method to derive the coupled equations for plasma waves interacting each other. The following hydrodynamic and Maxwell equations applies

$$\frac{\partial}{\partial t}n_s+\nabla(n_s\boldsymbol{u}_s)=0 \tag{3}$$

$$\frac{\partial}{\partial t}\boldsymbol{u}_e+(\boldsymbol{u}_s\cdot\nabla)\boldsymbol{u}_s=\frac{e_s}{m_s}(\boldsymbol{E}+\frac{1}{c}\boldsymbol{u}_s\times\boldsymbol{B})-\nu_s\boldsymbol{u}_s-\frac{\gamma_sT_s}{m_sn_s}\nabla n_s. \tag{4}$$

$$\left.\begin{aligned}&\nabla\cdot\boldsymbol{E}=4\pi\sum_s n_se_s,\quad \nabla\cdot\boldsymbol{B}=0,\\&\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t}+\nabla\times\boldsymbol{E}=0,\quad \nabla\times\boldsymbol{B}-\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}=\frac{4\pi}{c}\sum_s n_se_s\boldsymbol{u}_s,\end{aligned}\right\} \tag{5}$$

where subscript s denotes particle species, s=i for ions and e for electrons, and e, m, ν, n and $\boldsymbol{u}$ are respectively the electric charge, mass, collision frequency, number density and flow velocity. The vector $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric field and magnetic flux density, respectively.

All the physical quantities are divided into three parts corresponding to electromagnetic oscillations which is called photon in what follows, electron plasma wave (plasmon) and ion acoustic wave (phonon):

$$n_s=n_0+\frac{1}{2}\{n_{s,t}e^{-i\omega_0t}+n_{s,p}e^{i(kx-\omega t)}+n_{s,a}e^{i(kx-\Omega t)}+c.c.\}, \tag{6}$$

$$\boldsymbol{u}_s=\frac{1}{2}\{\boldsymbol{u}_{s,t}e^{-i\omega_0t}+\boldsymbol{u}_{s,p}e^{i(kx-\omega t)}+\boldsymbol{u}_{s,a}e^{i(kx-\Omega t)}+c.c.\}, \tag{7}$$

$$\boldsymbol{E}=\frac{1}{2}\{\boldsymbol{E}_te^{-i\omega_0t}+\boldsymbol{E}_pe^{i(kx-\omega t)}+\boldsymbol{E}_ae^{i(kx-\Omega t)}+c.c.\}, \tag{8}$$

$$B=\frac{1}{2}\{B_t e^{-i\omega_0 t}+c.c.\},\tag{9}$$

where n_0 is unperturbed plasma density, subscripts t, p and a denote the photon, plasmon and phonon parts respectively and $c.c.$ is short for complex conjugate. The electric fields E_t, E_p and E_a satisfy the condition

$$\nabla\cdot E_t=0,\quad \nabla\times E_p=\nabla\times E_a=0.$$

The amplitudes with subscripts, t, p and a in eqs. (6)-(9) are the slowly varying functions of (t, x) which describe spatial variations created by the presence of evanescent photon and changes in frequencies due to the parametric interactions. In eqs. (6)-(9), ω_0, (ω, K) and (Ω, K) denote the frequency and wave-vector of photon, plasmon and phonon, respectively.

Introducing a small parameter ε, we write

$$\begin{bmatrix} n_{s,t} \\ u_{s,t} \\ E_t \\ B_t \end{bmatrix}=\sum_{m=1}^{\infty}\varepsilon^m\begin{bmatrix} n_{s,t}^{(m)} \\ u_{s,t}^{(m)} \\ E_t^{(m)} \\ B_t^{(m)} \end{bmatrix}\tag{10}$$

$$\begin{bmatrix} n_{s,p} \\ u_{s,p} \\ E_p \end{bmatrix}=\sum_{m=1}^{\infty}\varepsilon^{p+m}\begin{bmatrix} n_{s,p}^{(m)} \\ u_{s,p}^{(m)} \\ E_p^{(m)} \end{bmatrix}\tag{11}$$

$$\begin{bmatrix} n_{s,a} \\ u_{s,a} \\ E_a \end{bmatrix}=\sum_{m=1}^{\infty}\varepsilon^{a+m}\begin{bmatrix} n_{s,a}^{(m)} \\ u_{s,a}^{(m)} \\ E_a^{(m)} \end{bmatrix}\tag{12}$$

Since the plasmon-part and the phonon-part are induced by the incident photon, their amplitude are assumed to be of the order of magnitude lower than that of photon, *i.e.*, $p, a<1$ in eqs. (11) and (12). The stretched variables are introduced,

$$\xi=\varepsilon x,\quad \tau=\varepsilon t\tag{13}$$

to describe the slow variation of amplitudes. Suppose that the photon frequency ω_0 is less than the plasma frequency ω_e, $\omega_s=(4\pi n_0 e_s^2/m_s)^{1/2}$, by the quantity of the order ε^2 and ν_s is of the order ε, that is,

$$\omega_0=\omega_e-\varepsilon^2\Sigma\quad\text{and}\quad \nu_s=\varepsilon\bar{\nu}_s,\tag{14}$$

Substituting eqs. (6)-(13) into eqs. (3), (4) and (5), taking into account the resonant condition,

$$\omega=\omega_0+\Omega, \tag{15}$$

and equating coefficients of like powers of ε for the same harmonics, we get the basic system of equations to be solved. The contribution of ions to the photon-part and the plasmon-part is neglected because of its large inertia; i.e.

$$n_{i,t}=n_{i,p}=0, \quad \boldsymbol{u}_{i,t}=\boldsymbol{u}_{i,p}=0 \tag{16}$$

In the lowest order, we then obtain

$$\left.\begin{aligned} &n_{e,t}^{(1)}=0, \quad \boldsymbol{u}_{e,t}^{(1)}=-i\frac{e}{m^e\omega^e}\boldsymbol{E}_t^{(1)}, \\ &\boldsymbol{B}_t^{(1)}=0, \end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned} &\omega^2=\omega_e^2(1+\boldsymbol{K}^2\lambda_0^2), \\ &\boldsymbol{u}_{e,p}^{(1)}=\boldsymbol{K}\omega/(n_0K^2)n_p^{(1)}, \quad \boldsymbol{E}_p^{(1)}=i4\pi\boldsymbol{K}/K^2\cdot n_p^{(1)} \end{aligned}\right\} \tag{18}$$

$$\left.\begin{aligned} &\Omega^2=K^2c_s^2/(1+K^2\lambda_D^2), \quad c_s^2=\gamma_e\, T_e/m_i, \\ &n_{i,a}^{(1)}=\omega^2/\omega_e^2n_a^{(1)}, \quad \boldsymbol{E}_a^{(1)}=-i4\pi e\boldsymbol{K}\lambda_D^2n_a^{(1)}, \\ &\boldsymbol{u}_{i,a}^{(1)}=\boldsymbol{K}\Omega\omega^2/(n_0K^2\omega_e^2)n_a^{(1)}, \quad \boldsymbol{u}_{e,a}^{(1)}=\boldsymbol{K}\Omega/(n_0K^2)n_a^{(1)}, \end{aligned}\right\} \tag{19}$$

where $n_p\equiv n_{e,p}^{(1)}$, $n_a^{(1)}\equiv n_{e,a}^{(1)}$, $\lambda_D=(\gamma_eT_e/4\pi n_0e^2)^{1/2}$=Debye length and ion is assumed to be cold, $T_i=0$.

In the next order, we have

$$\left.\begin{aligned} &\frac{\partial \boldsymbol{E}_t^{(1)}}{\partial t}=-\frac{\nu_e}{2}\boldsymbol{E}_t^{(1)}, \quad \frac{\partial \boldsymbol{E}_t^{(1)}}{\partial \xi}=0, \\ &n_{e,t}^{(2)}=0, \quad \boldsymbol{u}_{e,t}^{(2)}=\frac{1}{4\pi n_0e}\frac{\partial \boldsymbol{E}_t^{(1)}}{\partial t}, \\ &\boldsymbol{E}_t^{(2)}=0, \quad \boldsymbol{B}_t^{(2)}=-i(c/\omega_e)\frac{\partial}{\partial \xi}\times \boldsymbol{E}_t^{(1)}. \end{aligned}\right\} \tag{20}$$

Proceeding to the third order of ε, we finally obtain

$$\frac{\partial^2}{\partial \xi^2}\boldsymbol{E}_t^{(1)}-\frac{2\omega\sum}{c^2}\left(1-\frac{\nu_e^2}{8\omega_e\sum}\right)\boldsymbol{E}_t^{(1)}=i\varepsilon^{p+a-3}\,\boldsymbol{K}\frac{2\pi e\omega_e}{n_0c^2K^2}(\omega+\Omega)n_p^{(1)}n_a^{(1)*}, \tag{21}$$

$$\left(\frac{\partial}{\partial \tau}+\frac{d\omega}{d\boldsymbol{K}}\cdot\frac{\partial}{\partial \xi}\right)n_p^{(1)}+\frac{\nu_e}{2}n_p^{(1)}=-\frac{e}{4m_e\omega_e}\left(1+\frac{\Omega}{\omega}\right)(\boldsymbol{K}\cdot\boldsymbol{E}_t^{(1)})n_a^{(1)}, \tag{22}$$

$$\left(\frac{\partial}{\partial\tau}+\frac{d\Omega}{dK}\cdot\frac{\partial}{\partial\xi}\right)n_a^{(1)}+\frac{\bar{\nu}_i}{2}n_a^{(1)}=e\frac{4}{m_i\Omega}\left(\frac{\omega_e}{\omega}\right)^3\left(1+\frac{\Omega}{\omega}\right)(K\cdot E_t^{(1)*})n_p^{(1)} \quad (23)$$

In deriving above equations, the terms including the factor (m_e/m_i) are neglected compared with terms of the order unity and $(m_e/m_i)\nu_e \ll \nu_i$ is assumed. The quantities $\frac{d\omega}{dK}$ and $\frac{d\Omega}{dK}$ in eqs. (22) and (23) are the group velocity of plasmon and phonon, i.e.,

$$\frac{d\omega}{dK}=\frac{\gamma_e T_e}{m_e\omega}K,\quad \frac{d\Omega}{dK}=\frac{\gamma_e T_e}{m_i\Omega}\left(\frac{\omega_e}{\omega}\right)^4 K \quad (24)$$

Equations (21), (22) and (23) with (24) are the coupled equations which describe the parametric interaction of the decay process, photon → plasmon + phonon, and is solved in the next section.

3. Decay of Evanescent Photon into Plasmon and Phonon

We take the coordinate system that the x-axis is perpendicular to the boundary surface between semi-infinite plasma and vacuum, the region $x>0$ being filled by a uniform plasma and $x<0$ vacuum. We now assume that the amplitudes of induced plasmon and phonon are sufficiently small so that the r.h.s. of eq. (21) can be neglected; that is $p+a>3$. In this case, the spatial variation of the evanescent photon is given by

$$\frac{\partial^2}{\partial\xi^2}E_t^{(1)}-\frac{2\omega_e\sum}{c^2}\left(1-\frac{\bar{\nu}_e^2}{8\omega_e\sum}\right)E_t^{(1)}=0 \text{ for } \xi>0, \quad (25)$$

When $\bar{\nu}_e^2<8\omega_e\sum$, the solution vanishing at $\xi=\infty$ can exist,

$$E_t^{(1)}=E_0 e^{-\kappa\xi}, \quad (26)$$

$$\kappa=\sqrt{2\omega_e\sum(1-\bar{\nu}_e^2/(8\omega_e\sum))/c^2} \quad (27)$$

where κ is the attenuation coefficient of evanescent photon field.

Suppose now the τ-dependence of $n_p^{(1)}$ and $n_a^{(1)}$ as

$$n_p^{(1)},\, n_a^{(1)}\sim\exp(\bar{\Gamma}\tau). \quad (28)$$

Substituting eqs. (26) and (28) into eqs. (22) and (23) and usingeq. (24) yields

$$\frac{dn_p^{(1)}}{d\xi}+\frac{m_e\omega}{\gamma_e T_e K_x}\left(\bar{\Gamma}+\frac{\bar{\nu}_e}{2}\right)n_p^{(1)}=-\frac{e\omega}{4\gamma_e T_e\omega_e K_x}\left(1+\frac{\Omega}{\omega}\right)(K\cdot E_0)e^{-\kappa\xi}n_a^{(1)}, \quad (29)$$

$$\frac{dn_a^{(1)}}{d\xi}+\frac{m_i\Omega}{\gamma_e T_e K_x}\left(\frac{\omega}{\omega_e}\right)^4\left(\bar{\Gamma}+\frac{\bar{\nu}_i}{2}\right)n_a^{(1)}=\frac{e\omega}{4\gamma_e T_e\omega_e K_x}\left(1+\frac{\Omega}{\omega}\right)(K\cdot E_0^*)e^{-\kappa\xi}n_0^{(1)} \quad (30)$$

where K_x is the x-component of K. We eliminate $n_a^{(1)}$ or $n_p^{(1)}$ from eqs. (29) and (30), to get

$$\frac{d^2 n_p^{(1)}}{dz^2}+(1-2\alpha)\frac{1}{z}\frac{dn_p^{(1)}}{dz}+\left(\beta^2+\frac{\alpha_2-\mu_+^2}{z^2}\right)n_p^{(1)}=0, \tag{31}$$

$$\frac{d^2 n_a^{(1)}}{dz^2}+(1-2\alpha)\frac{1}{z}\frac{dn_a^{(1)}}{dz}+\left(\beta^2+\frac{\alpha^2-\mu_-^2}{z^2}\right)n_a^{(1)}=0, \tag{32}$$

$$z=e^{-\kappa\xi} \tag{33-a}$$

$$\alpha=\frac{1}{2\kappa}\left\{\frac{m_e\omega}{\gamma_e T_e K_x}\left(\bar{\Gamma}+\frac{\bar{\nu}_e}{2}\right)+\frac{m_i\Omega}{\gamma_e T_e K_x}\left(\bar{\Gamma}+\frac{\bar{\nu}_i}{2}\right)+\kappa\right\}, \tag{33-b}$$

$$\beta^2=\left(\frac{e\omega}{4\kappa\gamma_e T_e\omega_e K_x}\right)^2\left(1+\frac{\Omega}{\omega}\right)^2|\boldsymbol{K}\cdot\boldsymbol{E}_0|^2, \tag{33-c}$$

$$\mu_\pm^2=\frac{1}{4\kappa^2}\left\{\frac{m_i\Omega}{\gamma_e T_e K_x}\left(\frac{\omega}{\omega_e}\right)^4\left(\bar{\Gamma}+\frac{\bar{\nu}_i}{2}\right)-\frac{m_e\omega}{\gamma_e T_e K_x}\left(\bar{\Gamma}+\frac{\bar{\nu}_e}{2}\right)\pm\kappa\right\}^2. \tag{33-d}$$

Imposing an appropriate boundary conditions to eqs. (31) and (32), we obtain the eigenvalue $\mu_\pm^2$ which determines the growth rate $\bar{\Gamma}$ of the paramitric excitation of the electrostatic oscillations. We will not, however, go into details of this problem beyond this point and only give the threshold value of photon amplitude to bring about the parametric instability.

We now estimate the order of magnitude of $\mu_\pm$. By applying eqs. (14), (18) and (19), the resonance condition (15) reduces to

$$\varepsilon^2\Sigma/\omega_p=(1+\sqrt{m_e/m_i}K\lambda_D)-[1+(K\lambda_D)^2]^{1/2}.$$

Since $\Sigma>0$ (overdense plasma), this yields

$$K\lambda_D<2(m_e/m_i)^{1/2},\ \varepsilon^2\Sigma/\omega_e<m_e/m_i. \tag{34}$$

Noting that $\nu_e\gg\nu_i$, $\nu_e\gtrsim\omega_i$, we obtain

$$\frac{m_i\Omega}{m_e\omega}\left(\frac{\omega}{\omega_e}\right)^4\frac{\bar{\nu}_i}{\bar{\nu}_e}\simeq\left(\frac{m_i}{m_e}\right)^{1/2}K\lambda_D\frac{\nu_i}{\nu_e}\ll 1,$$

$$\frac{\kappa\gamma_e T_e K_x}{m_e\omega\bar{\nu}_e}\simeq\cos\theta\frac{v_e}{c}K\lambda_D\frac{\omega_i}{\gamma_e}\sqrt{2\gamma_e\frac{m_i}{m_e}\frac{\varepsilon^2\Sigma}{\omega_e}}\ll 1.$$

where θ is an angle between $\boldsymbol{K}$ and the x-axis and $v_e=(T_e/m_e)^{1/2}$. We substitute this into eq. (33-d) to get

$$\mu_+^2\simeq\mu_-^2=\mu^2,$$

$$\mu^2=\left(\frac{c\omega^2/\omega_e^2}{2v_e K\lambda_D\cos\theta}\right)^2\frac{\omega_e}{2\gamma_e\Sigma}\left\{\frac{\bar{\nu}_e}{2\omega}+\left(1-\sqrt{m_i/m_e}\,K\lambda_D\right)\frac{\bar{\Gamma}}{\omega^2}\right\}. \tag{35}$$

Similar caluculation gives

$$\alpha=\frac{c\omega^2/\omega_e^2}{2v_eK\lambda_D\cos\theta}\sqrt{\frac{\omega_e}{2\gamma_e\sum}}\left\{\frac{\nu_e}{2\omega}+\left(1+\sqrt{m_i/m_e}K\lambda_D\right)\frac{\bar{\Gamma}}{\omega}\right\}. \tag{36}$$

The two independent solutions to eqs. (29) and (30) are readily obtained,

$$n_p^{(1)}, n_a^{(1)}\sim z^\alpha J_{|\mu|}(\beta z), z^\alpha N_{|\mu|}(\beta z). \tag{37}$$

Let $\bar{\Gamma}$ be positive. This is true for the present case that the parametric instability occurs. If so, $\alpha\pm|\mu|$ is positive for $\cos\theta>0$ and negative for $\cos\theta<0$. The solution (37) with positive $\bar{\Gamma}$, therefore, vanishes at $\xi=\infty(z=0)$ only when $\cos\theta>0$. The boundary condition, that $n_p^{(1)}$ and $n_a^{(1)}$ must be zero at the plasma boundary $\xi=0(z=1)$, is now imposed; i.e. $J_{|\mu|}(\beta)=0$, $N_{|\mu|}(\beta)=0$. The condition that at least one discrete eigenvalue exists gives the threshold value of the amplitude of incident photon for the parametric excitation of electrostatic oscillations. It follows from eq. (35) that for $K\lambda_D<\sqrt{m_e/m_i}$ the minimum value of $|\mu|$ is given by

$$|\mu_{min}|=\frac{c\omega^2/\omega_e^2}{8v_eK\lambda_D\cos\theta}\sqrt{\frac{\omega_e}{\gamma_e}}\frac{\nu_e}{\omega\sum}, \tag{38}$$

at $\bar{\Gamma}=0$. The parameter β must be larger than the first zero of the Bessel or Neumann function of the $|\mu_{min}|$-th order; that is to say,

$$\beta>j_{|\mu_{min}|} \text{ or } n_{|\mu_{min}|}.$$

Noting $|\mu_{min}|\gg1$ and using the asymptotic formula $j_\nu, n_\nu\sim\nu$ at large ν, we get

$$\left(\frac{|\boldsymbol{E}_0|^2}{4\pi n_eT_e}\right)^{1/2}\geq\frac{1}{2}\left(\frac{\gamma_e}{2}\right)^{1/2}\frac{\nu_e/\omega_e}{(1+\Omega/\omega)K\lambda_D|\cos\psi|}\equiv\bar{A}_1, \tag{39}$$

where ψ is the angle between $\boldsymbol{K}$ and $\boldsymbol{E}_0$. The growth rate is obtained from eqs. (35) and (39)

$$\bar{\Gamma}/\omega_e=\frac{1}{2}\frac{K\lambda_D|\cos\psi|}{1-\sqrt{m_i/m_e}K\lambda_D}\left(1+\frac{\Omega}{\omega}\right)\left\{\sqrt{\frac{|\boldsymbol{E}_0|^2}{4\pi n_0T_e}}-\bar{A}_1\right\}. \tag{40}$$

For $K\lambda_D>\sqrt{m_i/m_e}$, $|\mu|$ can vanish at

$$\bar{\Gamma}/\omega_e=\frac{1}{2}\frac{\nu_e}{\omega_e}/(\sqrt{m_i/m_e}\,K\lambda_D-1). \tag{41}$$

In this case, thethreshold power is determined by the first zero $n_{0.1}$ of $N_0(x)$, i.e.,

$$\left(\frac{|\boldsymbol{E}_0|^2}{4\pi n_0T_e}\right)^{1/2}\geq4|\tan\psi|\frac{v_e}{c}\sqrt{\frac{2\gamma_e\sum}{\omega_e}}n_{0.1}\equiv\bar{A}_2. \tag{42}$$

In deriving eq. (42), we assume the normal incidence of photons upon the plasma boundary to use the relation $|\cos\theta|=|\sin\psi|$, In this case the growth rate is geven by eq. (41).

4. Discussions

In this paper, we have shown the possibility that the interaction of the electromagnetic radiation with a semiinfinite, overdense plasma gives rise to the parametric instability if the difference between frequency of incident photon and plasma frequency is sufficiently small. Finitemess of the life time of evanescent photon in a semi-infinite plasma brings forth broadening in frequency of photons and makes the decay process of photon into electrostatic oscillations feasible. We mention that the possibility of the parametric instability accompanied with an evanescent photon has not been yet pointed out.

The results involve the two cases, a) $K\lambda_D<(m_e/m_i)^{1/2}$ and b) $K\lambda_D>(m_e/m_i)^{1/2}$, where K is the wavenumber of induced plasmon and phonon. For case a), plasmons have a group velocity smaller than that of phonons and are found in the vicinity of the plasma surface more than phonons. The incident photons feed their energy into plasmons near the plasma surface. Therefore the threshold powers for the instability depends mainly on the damping rate of plasmons. This circumstance is seen in eq. (39). The results show the threshold power is

$$\bar{\Lambda}_1{}^2\sim\frac{m_i}{m_e}\left(\frac{\nu_e}{\omega_e}\right)^2.$$

For case b), the group velocity of phonons becomes smaller than that of plasmons and, therefore, motion of phonons determines the threshold of instability: phonons escape from the interactin region near the plasma surface after the period $(\kappa c_s)^{-1}$. Therefore we may replace the collision frequency ν_e in the above expression by (κc_s), to get

$$\bar{\Lambda}_2{}^2\sim\left(\frac{v_e}{c}\right)^2\frac{m_e}{m_i},$$

which is equivalent to eq. (42). It should be noted that $\bar{\Lambda}_2$ is sufficiently small compared with $\bar{\Lambda}_1$.

It follows from eq. (34) that the decay of the evanescent photons through the parmetric interaction is possible only for the small frequency range, $\omega_e>\omega_0>(1-m_e/m_i)\omega_e$. Therefore, in real plasmas with a smooth boundary the parametric interaction of evanscent photon can only occur very close to the cut-off point $(\omega_0=\omega_e)$. Nevertheless, this process may become of importance because the threshold power is very small. The detailed calculation is needed by adopting a more realistic profile of plasma density than the present case.

At last we point out that for metal plasmas which are always overdense for the electromagetic wave to propagate, a new type of the parametric instability discussed in this paper may play a role.

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(Received November 30, 1974)