FRICTIONAL EFFECTS ON THE RENORMALIZATION OF PROPAGATOR IN TURBULENT PLASMAS

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FRICTIONAL EFFECTS ON THE RENORMALIZATION OF PROPAGATOR IN TURBULENT PLASMAS

By M. KONO, H. SANUKI[+] and J. TODOROKI*

It is shown that there are two kinds of effects of fluctuations on the particle trajectries in a weakly turbulent plasma; diffusive effect and frictional one. The new propagator including the frictional processes is obtained, which is different from the Dupree type one, by summing up the most secular terms of iterative series in particle orbit. The propagator takes the following form;

$$G(\boldsymbol{k}, \omega; \boldsymbol{v}, \boldsymbol{v}')\sim\left[\omega-1/2\boldsymbol{k}\cdot(\boldsymbol{v}+\boldsymbol{v}')+i\nu\right]^{-1}\exp\left[-\frac{(\boldsymbol{v}-\boldsymbol{v}')^2}{2\Delta}\right].$$

The set of kinetic equations is also obtained.

1. Introduction

Recent advances in the theory of plasma turbulence have been based on the inclusion of diffusive effects on particle trajectories in phasespace [1,2]. In this regard several authors have studied the saturation mechanism of linearly unstable waves and anomalous resistivity and so on [3~8]. All of them paid their attention to the Dupree type effect of fluctuation on the particle trajectries. However there are two kinds of effects of fluctuation on particle trajectries in weakly turbulent plasmas. In order to see this situation, we shall consider the problem of the stochastic acceleration of particles in a prescribed random electric field. As is well-known, the motion of such particles is described by the equation;

$$\frac{d}{dt}\boldsymbol{v}(\mathrm{t})=\frac{e}{m}\boldsymbol{E}(t)-\boldsymbol{A}(\boldsymbol{v}, |E|^2), \tag{1}$$

where $\boldsymbol{A}$ is the dynamical friction, which is related to the diffusion coefficient in the following way [9]

$$\boldsymbol{A}(\boldsymbol{v})=\frac{\partial}{\partial \boldsymbol{v}}\cdot\boldsymbol{D}(\boldsymbol{v}). \tag{2}$$

If we put $\boldsymbol{v}(t)=\boldsymbol{v}_0+\Delta\boldsymbol{v}(t)$ and linearize eq. (2), provided the electric field is small, we have the equation of $\Delta\boldsymbol{v}(t)$;

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$$\left(\frac{d}{dt}+\frac{1}{\tau_s}\right)\Delta v(t)=\frac{e}{m}E(t), \tag{3}$$

where $\tau_s \equiv [(\partial/\partial v)^2 : D]^{-1}$.
From eq. (3), we have

$$\Delta r(t)=\tau_s\cdot\frac{e}{m}\int_0^t dt'\left(1-e^{-\frac{t-t'}{\tau_s}}\right)E(t'),$$

and the simultaneous correlation function

$$<\Delta r(t)\Delta r(t)>=\left(\frac{e}{m}\right)^2\tau_s^2\int_0^t dt_1\int_0^t dt_2\left(1-e^{-\frac{t-t_1}{\tau_s}}\right)\left(1-e^{-\frac{t-t_2}{\tau_s}}\right)<E(t_1)E(t_2)>. \tag{4}$$

If $<E(t_1)E(t_2)>$ is assumed to be a function of time difference $|t_1-t_2|$, the following two cases should be distinguished to estimate eq. (4).

For the case $\tau_s>\tau_c$, τ_c being the correlation time, we evaluate the correlation function as following

$$<\Delta r(t)\Delta r(t)>\simeq\frac{t^3}{3}\left(\frac{e}{m}\right)^2\int_{-\infty}^{\infty}d\tau<E(\tau)E(0)>, \tag{5}$$

which reduces to the propagator derived by Dupree first

$$\int_0^\infty dt<e^{-ik\cdot r(t)+i\omega t}>\simeq\int_0^\infty dt\ e^{i(\omega-k\cdot v)t-\frac{t^3}{3}kkD} \tag{6}$$

For the case $\tau_s<\tau_c$, the frictional process is important to determine the behavior of the system. In this case we have

$$<\Delta r(t)\Delta r(t)>\simeq t\tau_s^2\left(\frac{e}{m}\right)^2\int_{-\infty}^{\infty}d\tau<E(\tau)E(0)>, \tag{7}$$

and the propagator

$$\int_0^\infty dt<e^{-ik\cdot r(t)+i\omega t}>\sim\frac{i}{\omega-kv+i\tau_s^2 kk:D} \tag{8}$$

Furthermore in this case the propagator takes the Gaussian form in velocity.

The purpose of this paper is to clarify the above situation and to derive the new propagator. In the next section we give the outline of the method to obtain the propagator. In Sec. 3, the asymptotic solution of the equation for the particle orbit is obtained by summing up the most secular terms in the iterative series. The average propagator is evaluated in Sec. 4 with the aid of cumulant expansion. The set of kinetic equations is discussed on the basis of the new propagator in Sec. 5. Some remarks are made in the last section.

2. Basic Concept

The distribution function $F_a(\boldsymbol{r}, \boldsymbol{v}, t)$ in a prescribed electric field in plasmas satisfies the Klimontvich equation

$$\left\{\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{r}}+\frac{e_a}{m_a}\boldsymbol{E}(\boldsymbol{r}, t)\cdot\frac{\partial}{\partial \boldsymbol{v}}\right\}F_a(\boldsymbol{r}, \boldsymbol{v}, t)=0, \tag{9}$$

where a denotes species of particles. The electric field

$$\boldsymbol{E}(\boldsymbol{r}, t)=\int\frac{d\boldsymbol{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\boldsymbol{E}(\boldsymbol{k}, \omega)e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t} \tag{10}$$

is assumed to have a random nature and be specified statistically by $<\boldsymbol{E}(\boldsymbol{k}, \omega)>$, $<\boldsymbol{E}(\boldsymbol{k}, \omega)\boldsymbol{E}(\boldsymbol{k}', \omega')>$, etc., where $<\cdots>$ stands for the statistical average over the ensemble of the initial phases of the field. For simplicity of the following analysis we assume the homogeneity of the ensemble with respect to space and time especially

$$<\boldsymbol{E}(\boldsymbol{k}, \omega)>=0,$$
$$<\boldsymbol{E}(\boldsymbol{k}, \omega)\boldsymbol{E}(\boldsymbol{k}', \omega')>=<|\boldsymbol{E}|^2>_{k\omega}(2\pi)^4\delta(\boldsymbol{k}+\boldsymbol{k}')\delta(\omega+\omega'). \tag{11}$$

By introduction of the Green function $\hat{G}_a$

$$\hat{G}_a(\boldsymbol{r}, \boldsymbol{v}, t\,|\,\boldsymbol{r}', \boldsymbol{v}', t')=\delta[\boldsymbol{r}-\boldsymbol{R}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')]\delta[\boldsymbol{v}-\boldsymbol{V}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')], \tag{12}$$

we can write the solution of eq. (5) in the form

$$F_a(\boldsymbol{r}, \boldsymbol{v}, t)=\int d\boldsymbol{r}'\int d\boldsymbol{v}'\hat{G}_a(\boldsymbol{r}, \boldsymbol{v}, t\,|\,\boldsymbol{r}', \boldsymbol{v}', t')F_a(\boldsymbol{r}', \boldsymbol{v}', t')\equiv\hat{G}_a(t\,|\,t')F_a(t'), \tag{13}$$

where $\boldsymbol{R}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')$ and $\boldsymbol{V}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')$ satisfy the equations

$$\frac{\partial}{\partial t}\boldsymbol{R}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')=\boldsymbol{V}_a(t;\boldsymbol{r}', \boldsymbol{v}', t'),$$
$$\frac{\partial}{\partial t}\boldsymbol{V}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')=\frac{e_a}{m_a}\boldsymbol{E}[\boldsymbol{R}(t;\boldsymbol{r}', \boldsymbol{v}', t'), t], \tag{14}$$

with the initial condition

$$\boldsymbol{R}_a(t';\boldsymbol{r}', \boldsymbol{v}', t')=\boldsymbol{r}',$$
$$\boldsymbol{V}_a(t';\boldsymbol{r}', \boldsymbol{v}', t')=\boldsymbol{v}'. \tag{15}$$

If we put

$$\boldsymbol{R}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')=\boldsymbol{r}_0+\boldsymbol{v}_0t+\delta\boldsymbol{r}_a(t;\boldsymbol{r}_0, \boldsymbol{v}_0),$$
$$\boldsymbol{V}_a(t;\boldsymbol{r}', \boldsymbol{v}', t')=\boldsymbol{v}_0+\delta\boldsymbol{v}_a(t;\boldsymbol{r}_0, \boldsymbol{v}_0), \tag{16}$$

where $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ are constants, then $\delta\boldsymbol{r}_a(t)=\delta\boldsymbol{r}_a(t;\boldsymbol{r}_0, \boldsymbol{v}_0)$ and $\delta\boldsymbol{v}_a(t)=\delta\boldsymbol{v}_a(t;\boldsymbol{r}_0, \boldsymbol{v}_0)$

satisfy the following equations

$$\frac{\partial}{\partial t}\delta \boldsymbol{r}_a(t)=\delta \boldsymbol{v}_a(t), \tag{17-a}$$

$$\frac{\partial}{\partial t}\delta \boldsymbol{v}_a(t)=\frac{e_a}{m_a}\boldsymbol{E}(\boldsymbol{r}_0+\boldsymbol{v}_0 t+\delta \boldsymbol{r}_a(t), t)$$

$$=\frac{e_a}{m_a}\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2\pi}\boldsymbol{E}(\boldsymbol{k}, \omega)e^{i\boldsymbol{k}\cdot\boldsymbol{r}_0-i(\omega-\boldsymbol{k}\cdot\boldsymbol{v})t+i\boldsymbol{k}\cdot\delta\boldsymbol{r}(t)} \tag{17-b}$$

The equations (15) are the constraint condition to determine $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$;

$$\boldsymbol{r}'=\boldsymbol{r}_0+\boldsymbol{v}_0 t'+\delta \boldsymbol{r}(t';\boldsymbol{r}_0, \boldsymbol{v}_0),$$

$$\boldsymbol{v}'=\boldsymbol{v}_0+\delta \boldsymbol{v}(t';\boldsymbol{r}_0, \boldsymbol{v}_0) \tag{18}$$

Here we assume that $\delta \boldsymbol{r}$ and $\delta \boldsymbol{v}$ vanish at $t\to-\infty$ i.e.

$$\lim_{t\to-\infty}\delta \boldsymbol{r}(t;\boldsymbol{r}_0, \boldsymbol{v}_0)=0,$$

$$\lim_{t\to-\infty}\delta \boldsymbol{v}(t;\boldsymbol{r}_0, \boldsymbol{v}_0)=0. \tag{19}$$

Since the Jacobian $\partial(\boldsymbol{r}', \boldsymbol{v}')/\partial(\boldsymbol{r}_0, \boldsymbol{v}_0)=1$, we can write the Green function in the form

$$\hat{G}_a(\boldsymbol{r}, \boldsymbol{v}, t\,|\,\boldsymbol{r}', \boldsymbol{v}', t')$$

$$=\int\mathrm{d}\boldsymbol{v}_0\int\mathrm{d}\boldsymbol{r}_0\;\delta[\boldsymbol{r}-\boldsymbol{r}_0-\boldsymbol{v}_0 t-\delta\boldsymbol{r}(t)]\delta[\boldsymbol{v}-\boldsymbol{v}_0-\delta\boldsymbol{v}(t)]\delta[\boldsymbol{r}'-\boldsymbol{r}_0-\boldsymbol{v}_0 t'-\delta\boldsymbol{r}(t')]$$

$$\times\delta[\boldsymbol{v}'-\boldsymbol{v}_0-\delta\boldsymbol{v}(t')]$$

$$=\int\mathrm{d}\boldsymbol{v}_0\int\mathrm{d}\boldsymbol{r}_0\frac{1}{(2\pi)^{12}}\int\mathrm{d}\boldsymbol{k}\mathrm{d}\boldsymbol{k}'\mathrm{d}\boldsymbol{q}_1\mathrm{d}\boldsymbol{q}_2\exp\Big\{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}_0)-i\boldsymbol{k}'\cdot(\boldsymbol{r}-\boldsymbol{r}_0)$$

$$+i\boldsymbol{q}_1\cdot(\boldsymbol{v}-\boldsymbol{v}_0)-i\boldsymbol{q}_2\cdot(\boldsymbol{v}'-\boldsymbol{v}_0)-i\boldsymbol{k}\cdot\boldsymbol{v}_0 t+i\boldsymbol{k}'\cdot\boldsymbol{v}_0 t'$$

$$-i\boldsymbol{k}\cdot\delta\boldsymbol{r}(t)+i\boldsymbol{k}'\cdot\delta\boldsymbol{r}(t')-i\boldsymbol{q}_1\cdot\delta\boldsymbol{v}(t)+i\boldsymbol{q}_2\cdot\delta\boldsymbol{v}(t')\Big\} \tag{20}$$

Now we express $\hat{G}$ in terms of $<\hat{G}>$. Noting the Green function satisfies the equation

$$\left\{\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{r}}+\frac{e}{m}\boldsymbol{E}(\boldsymbol{x}, t)\cdot\frac{\partial}{\partial \boldsymbol{v}}\right\}\hat{G}(t\,|\,t')=0, \tag{21}$$

with $\hat{G}(t'\,|\,t')=0$, and putting $\hat{G}=<\hat{G}>+\delta\hat{G}$, we get the equation for $\delta\hat{G}$;

$$\left\{\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{r}}+\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial}{\partial \boldsymbol{v}}\right\}\delta\hat{G}(t\,|\,t')=-\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial}{\partial \boldsymbol{v}}<\hat{G}>+\frac{e}{m}\,\frac{\partial}{\partial \boldsymbol{v}}<\boldsymbol{E}\delta G>. \tag{22}$$

The formal solution for $\delta\hat{G}$ is easily obtained by means of eq. (21);

$$\delta\hat{G}(t|t') = -\frac{e}{m}\int dt_1 \hat{G}(t|t_1)\left\{\boldsymbol{E}(t_1)\cdot\frac{\partial}{\partial \boldsymbol{v}}<\hat{G}(t_1|t')> - <\boldsymbol{E}(t_1)\delta\hat{G}(t_1|t')>\right\}$$

$$= -\frac{e}{m}\int dt_1 \hat{G}(t|t_1)\boldsymbol{E}(t_1)\cdot\frac{\partial}{\partial \boldsymbol{v}}<\hat{G}(t_1|t')>$$

$$-\left(\frac{e}{m}\right)^2\int dt_1 dt_2 \hat{G}(t|t_1)\frac{\partial}{\partial \boldsymbol{v}}\cdot<\boldsymbol{E}(t_1)\hat{G}(t_1|t_2)\boldsymbol{E}(t_2)>\cdot$$

$$\frac{\partial}{\partial \boldsymbol{v}}<\hat{G}(t_2|t')>+\cdots.$$

Then, the nonlinear integral equation for $\hat{G}$ is following

$$\hat{G}(t|t') = <\hat{G}(t|t')> - \frac{e}{m}\int dt_1 \hat{G}(t|t_1)\boldsymbol{E}(t_1)\cdot\frac{\partial}{\partial \boldsymbol{v}}<\hat{G}(t_1|t)>$$

$$-\left(\frac{e}{m}\right)^2\int dt_1 dt_2 \hat{G}(t|t_1)\frac{\partial}{\partial \boldsymbol{v}}\cdot<\boldsymbol{E}(t_1)\hat{G}(t_1|t_2)\boldsymbol{E}(t_2)>\cdot\frac{\partial}{\partial \boldsymbol{v}}<\hat{G}(t_2|t')>+\cdots. \tag{23}$$

If this equation is iterated, we obtain the expression of $\hat{G}$ in powers of $\boldsymbol{E}$ and $<\hat{G}>$. As is pointed out by Dupree, above expansion converges rapidly, and it is sufficient to retain up to the second order with respect to $\boldsymbol{E}$;

$$\hat{G}(t|t') = G(t|t') - \frac{e}{m}\int d\tau\, \hat{G}(t|\tau)\boldsymbol{E}(\tau)\cdot\frac{\partial}{\partial \boldsymbol{v}}G(\tau|t'), \tag{24}$$

where $G(t|t') = <\hat{G}(t|t')>$. Hence we restrict our consideration to getting the average propagator on the present work.

3. Frictional Process due to the Wave-Particle Interaction

In this section we solve eq. (17) with the initial condition (18), using the perturbation expansion and summing up the most seclar terms.

If the magnitude of the fluctuating electric field is small, we can regard the quantities $\delta\boldsymbol{r}(t)$ and $\delta\boldsymbol{v}(t)$ as small and expand them with respect to the smallness of the fluctuation. Hence expanding the right hand side of eq. (17-a) with respect to the smallness of the fluctuation, we can solve eq. (17) formally by the successive approximation. Each term of the iterative series corresponds to taking into account the successive scattering of the particle by the fluctuation. In our case it is sufficient to seek the asymptotic solution of eq. (17) to evaluate the cumulative effect of the wave-particle interaction. As is well-known, it is performed by summing up the most secular terms so long as the sum converges.

Fourier transforming $\delta\boldsymbol{r}(t)$ and $\delta\boldsymbol{v}(t)$ as

$$\delta\boldsymbol{r}(t) = \int\frac{d\omega}{2\pi}\,\delta\boldsymbol{r}(\omega)e^{i\omega t},$$

$$\delta \boldsymbol{v}(t)=\int \frac{\mathrm{d}\omega}{2\pi}\ \delta \boldsymbol{v}(\omega) e^{-i\omega t}, \tag{25}$$

we obtain the following expression for $\delta \boldsymbol{r}(\omega)$

$$\begin{aligned}\delta \boldsymbol{r}(\omega)=&-\frac{1}{\omega^2}\ \frac{e}{m}\int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\boldsymbol{E}(\boldsymbol{k},\ \omega+\boldsymbol{k}\boldsymbol{v}_0)\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}_0}\\ &+\frac{1}{\omega^2}\left(\frac{e}{m}\right)^2\int \frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\int \frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}\int \frac{\mathrm{d}\omega_1}{2\pi}\boldsymbol{E}(\boldsymbol{k}_1,\ \omega_1+\boldsymbol{k}_1\boldsymbol{v}_0)e^{i(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{r}_0}\frac{1}{(\omega-\omega_1)^2}\\ &\times i\boldsymbol{k}_1\cdot\boldsymbol{E}(\boldsymbol{k}_2,\ \omega-\omega_1+\boldsymbol{k}_2\cdot\boldsymbol{v}_0)-\\ &-\frac{1}{2}\left(\frac{e}{m}\right)^3\frac{1}{\omega^2}\int \frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\int \frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}\int \frac{\mathrm{d}\boldsymbol{k}_3}{(2\pi)^3}\int \frac{\mathrm{d}\omega_1}{2\pi}\int \frac{\mathrm{d}\omega_1}{2\pi}\boldsymbol{E}(\boldsymbol{k}_1,\ \omega_1+\boldsymbol{k}_1\cdot\boldsymbol{v}_0)e^{i(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}_0}\\ &\times\frac{1}{\omega_1{}^2}i\boldsymbol{k}_1\cdot\boldsymbol{E}\ (\omega_2+\boldsymbol{k}_2\cdot\boldsymbol{v}_0)\frac{1}{(\omega-\omega_1-\omega_2)^2}i\boldsymbol{k}_2\cdot\boldsymbol{E}(\boldsymbol{k}_3,\ \omega-\omega_1-\omega_2+\boldsymbol{k}_3\cdot\boldsymbol{v}_0)+\cdots.\end{aligned} \tag{26}$$

We notice that the diagramatic representation introduced below has the definite advantage in analysing the higher order effects of the series. In Fig. 1, a thin line represents the propagator $1/\omega^2$ and a dotted line stands for the electric field $\boldsymbol{E}(\boldsymbol{k},\ \omega+\boldsymbol{k}\boldsymbol{v}_0)$. With the aid of these diagrams, the first four terms of the expansion, the analytic form of which are given in eq. (26), can be illustrated as shown in Fig. 2.

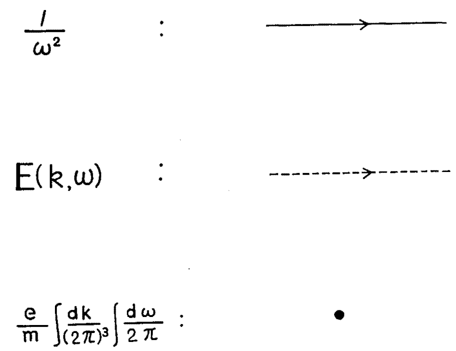

Fig. 1. Elements of diagramatic representation.

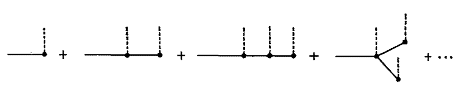

Fig. 2. First four terms of expansion, given in eq. (19)

Now let us consider the process in which the wave emitted by a particle at the i-th vertex is absorbed by the particle at the subsequent vertex. When this process takes place between the two neighboring vertexes, the third order process with respect to $\boldsymbol{E}$ is expressedas in Fig. 3.

Fig. 3. Third order processes with two identical propagation lines

In these diagrams the propagation lines on the two sides of a loop turn out to be identical (we shall call this a diagonal fragment), thus leading to the appearance of a four-fold pole with respect to ω. The self-energy part shown in Fig. 4, however, is proportional to ω. Hence sum of the third order diagrams (we shall call this a basic diagonal fragment) is characterized by a triple pole with respect to ω, which corresponds to secularity of t^2 in t-trepresentation.

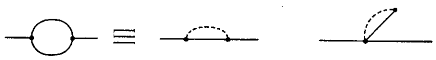

Fig. 4. Basic diagonal ftagment

There are thirty kinds of diagonal fragments in the fifth order which are not identical topologically. Among them four diagrams which are composed of combination of two kinds of basic diagonal fragments are summed up to give the time secularity proportional to t^3. The other diagrams contribute to the time secularity as proportional to t^2 at most. In the higher order processes, diagrams composed of repetition of the basic diagonal fragments give the most secular contribution, and the other diagonal fragments give less secular. Thus it is sufficient considering only the process of the repetition of the basic diagonal fragments to sum up all contribution of the higher order processes having the highest secularity in every order.

The operation to pick up the basic diagonal fragment is carried out, in our case, by taking the statistical average with respect to two neighbouring waves. Then we obtain the diagramatic expression for $\delta\boldsymbol{r}(\omega)$ as expressed in Fig. 5, which can be written explicitly as

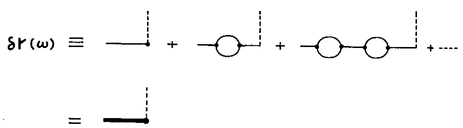

Fig. 5. Diagramatic expression for $\delta\mathbf{r}(\omega)$, composed of repetitions of the basic diagonal fragments

$$\omega U_a(\omega)\delta\boldsymbol{r}_a(\omega) = -\frac{e_a}{m_a}\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\boldsymbol{E}(\boldsymbol{k}, \omega+\boldsymbol{k}\boldsymbol{v}_0)e^{i\boldsymbol{k}\cdot\boldsymbol{r}_0}, \tag{27}$$

and

$$U_a(\omega)\delta v_a(\omega)=i\frac{e_a}{m_a}\int\frac{\mathrm{d}k}{(2\pi)^3}E(k,\omega+kr_0)e^{ik\cdot r_0}, \tag{28}$$

where

$$U_a(\omega)=\omega I+\left(\frac{e_a}{m_a}\right)^2\int\frac{\mathrm{d}k_1}{(2\pi)^3}\int\frac{\mathrm{d}\omega_1}{2\pi}<|E|^2>_{k_1,\,\omega_1+k_1\cdot v_0}\frac{\partial}{\partial\omega_1}\frac{1}{(\omega-\omega_1)\omega_1}k_1k_1$$

$$=\omega I+\Phi_a(\omega;v_0), \tag{29}$$

and I denotes the unit dyadic. Since we have summed up the partial series which dominates after long time interval to obtain eqs. (27) and (28), it may be justified to put $\omega=0$ in $\Phi(\omega;v_0)$. Then we have the expression

$$\Phi_a(v_0)=\frac{i}{\tau_{s,a}}=i\frac{\partial}{\partial v_0}\frac{\partial}{\partial v_0}D_a(v_0), \tag{30}$$

with

$$D_a(v_0)=\pi\left(\frac{e_a}{m_a}\right)^2\int\frac{\mathrm{d}k}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2\pi}<|E|^2>_{k,\omega}\delta(\omega-k\cdot v_0). \tag{31}$$

This agrees with the consideration given in Sec. 1. From eqs. (27) and (28) we have

$$\delta r_a(\omega)=\frac{i}{\omega}\delta v_a(\omega)$$

$$=-\frac{1}{\omega}U_a^{-1}(\omega)\cdot\left(\frac{e_a}{m_a}\right)\int\frac{\mathrm{d}k}{(2\pi)^3}E(k,\omega+kv_0)e^{ik\cdot r_0}, \tag{32}$$

where $U^{-1}(\omega)$ is the inverse dyadic of $U(\omega)$.

Here we should emphasize that the effect of the renormalization of wave-particle interactions plays an important role only when the behavior of $U^{-1}(\omega)$ near $\omega=0$ is examined, i.e. when the long time behavior of the system is investigated. On the contray, if we are interested in the short time evolution of the system, the effect of the renormalization of wave-particle interactions can be neglected and $U^{-1}(\omega)$ reduces to ω^{-1}, and $\delta r(\omega)$ has a double pole with respect to ω.

4. Average Propagator Taking Account of the Frictional Process

Now we can evaluate the average propagator G_a by using the expressions for $\delta r(t)$ and $\delta v(t)$ obtained in the preceding section. If we put $q_1=q+Q/2$, $q_2=q-Q/2$ and

$$\xi=k\cdot[\delta r(t)-\delta r(t')]+q\cdot[\delta v(t)-\delta v(t')]+\frac{Q}{2}[\delta v(t)+\delta v(t')],$$

then the average propagator takes the form

$$G_a(\boldsymbol{r}, \boldsymbol{v}, t \mid \boldsymbol{r}', \boldsymbol{v}', t')$$
$$=\frac{1}{(2\pi)^9}\int d\boldsymbol{v}_0\int d\boldsymbol{k}\int d\boldsymbol{q}\int d\boldsymbol{Q}\ \exp\Big\{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')+i\boldsymbol{q}\cdot(\boldsymbol{v}-\boldsymbol{v}')-i\boldsymbol{k}\cdot\boldsymbol{v}_0(t-t')$$
$$-i\boldsymbol{Q}\cdot(\boldsymbol{v}_0-\frac{\boldsymbol{v}+\boldsymbol{v}'}{2})\Big\}\times\langle\exp(-i\xi)\rangle. \tag{33}$$

Here we have carried out the integration with respect to $\boldsymbol{r}_0$ and $\boldsymbol{k}'$, using the fact that $\langle\exp(-i\xi)\rangle$ does not depend on $\boldsymbol{r}_0$.

The following cumulant expansion holds exactly to all order in ξ;

$$\langle\exp(-i\xi)\rangle=\exp\Big\{-i\langle\xi\rangle_c-\frac{1}{2}\langle\xi^2\rangle_c+\sum_{n=3}^{\infty}\frac{(-i)^n}{n!}c_n(t, t')\Big\}, \tag{34}$$

where c_n is the n-th cumulant of ξ.

First, we estimate the second order cumulant $\langle\xi^2\rangle_c$. It is convenient to introduce the symmetric dyadic

$$A_{\alpha\beta}(t)=\frac{1}{2}\int\frac{d\omega}{2\pi}\ \frac{\langle\delta v_\alpha\delta v_\beta+\delta v_\beta\delta v_\alpha\rangle_\omega}{(\omega+io)^2}e^{-i\omega t}$$
$$=-\frac{1}{2}\Big(\frac{e_a}{m_a}\Big)^2\int\frac{d\omega}{2\pi}\int\frac{d\boldsymbol{k}}{(2\pi)^3}$$
$$\frac{(\mathrm{U}^{-1}(\omega)\cdot\boldsymbol{k})_\alpha{}^*(\mathrm{U}^{-1}(\omega)\cdot\boldsymbol{k})_\beta+(\mathrm{U}^{-1}(\omega)\cdot\boldsymbol{k})_\beta{}^*(\mathrm{U}^{-1}(\omega)\cdot\boldsymbol{k})_\alpha}{(\omega+io)^2k^2}\langle|E|^2\rangle_{\boldsymbol{k},\omega+\boldsymbol{k}\boldsymbol{v}_0}e^{-i\omega t}, \tag{35}$$

where suffices α, β denote the component. Then, we have

$$\langle\xi^2\rangle_c=\boldsymbol{kk}:[2\mathbf{A}(0)-\mathbf{A}(\tau)-\mathbf{A}(-\tau)]-\boldsymbol{qq}:[2\mathbf{A}_2(0)-\mathbf{A}_2(\tau)-\mathbf{A}_2(-\tau)]$$
$$-\frac{1}{4}\boldsymbol{QQ}:[2\mathbf{A}_2(0)+\mathbf{A}_2(\tau)+\mathbf{A}_2(-\tau)]-2\boldsymbol{kq}:[2\mathbf{A}_1(0)-\mathbf{A}_1(\tau)-\mathbf{A}_1(-\tau)]$$
$$-\boldsymbol{kQ}:[\mathbf{A}_1(\tau)-\mathbf{A}_1(-\tau)]-\boldsymbol{qQ}:[\mathbf{A}_2(\tau)-\mathbf{A}_2(-\tau)], \tag{36}$$

with $\tau=t-t'$ and $\mathbf{A}_l(\tau)=\Big(\frac{d}{d\tau}\Big)^l\mathbf{A}(\tau)$. With the aid of the relations which are easily examined;

$$A_{\alpha\beta}(\tau)-A_{\alpha\beta}(-\tau)=\frac{1}{2}\int\frac{d\omega}{2\pi}\langle\delta v_\alpha\delta v_\beta+\delta v_\beta\delta v_\alpha\rangle\Big[\frac{1}{(\omega+i0)^2}-\frac{1}{(\omega-i0)^2}\Big]e^{-i\omega\tau}$$
$$\simeq 2\tau\frac{dA_{\alpha\beta}(0)}{d\tau},$$
$$\frac{dA_{\alpha\beta}(0)}{d\tau}=-\frac{\pi}{2}\int\frac{d\omega}{2\pi}\langle\delta v_\alpha\delta v_\beta+\delta v_\beta\delta v_\alpha\rangle_\omega\delta(\omega), \tag{37}$$
$$\mathbf{A}_1(\tau)+\mathbf{A}_1(-\tau)=2\mathbf{A}_1(0),$$
$$\mathbf{A}_2(\tau)-\mathbf{A}_2(-\tau)=0,$$

eq. (36) can be reduced to

$$<\xi^2>_c=2\boldsymbol{kk}:[\mathbf{A}(0)-\mathbf{A}(\tau)+\tau\mathbf{A}_1(0)]-2\boldsymbol{qq}\cdot[\mathbf{A}_2(0)-\mathbf{A}_2(\tau)]$$

$$-\frac{1}{2}\boldsymbol{QQ}:[\mathbf{A}_2(0)+\mathbf{A}_2(\tau)]+2\boldsymbol{kQ}:[\mathbf{A}_1(0)-\mathbf{A}_1(\tau)]. \tag{38}$$

Since $\mathbf{A}(\tau)$ tends to zero for sufficiently large time, we can drop $\mathbf{A}_l(\tau)$ in eq. (38) to obtain the asymptotic form

$$<\xi^2>=2\boldsymbol{kk}:\mathbf{A}_1(0)\tau-2\boldsymbol{qq}:\mathbf{A}_2(0)-\frac{1}{2}\boldsymbol{QQ}:\mathbf{A}_2(0)+2\boldsymbol{kQ}:\mathbf{A}_1(10). \tag{39}$$

Here it should be noted that $\mathbf{A}_1(0)>0$ and $\mathbf{A}_2(0)\equiv-\mathbf{C}<0$.

We proceed to evaluate the first order cumulant. Since the expression (32) gives no contribution to $<\xi>_c$, we are forced to use the second order solution as shown in Fig. 6, and obtain

Fig. 6. Second order solution for $\delta\mathbf{r}(\omega)$

$$<\xi>_c\simeq-(\boldsymbol{k}\tau+\boldsymbol{Q})\cdot\boldsymbol{B},$$

$$\boldsymbol{B}=\left(\frac{e_a}{m_a}\right)^2\int\frac{d\omega}{2\pi}\int\frac{d\boldsymbol{k}}{(2\pi)^3}\frac{1}{\omega}\frac{\boldsymbol{k}\cdot\mathbf{U}^{-1}(0)\boldsymbol{kk}:\mathbf{U}^{-1}(\omega)^*}{k_2}<|E|^2>_{\boldsymbol{k},\omega+\boldsymbol{kv}_0}.$$

The higher order cumulants do not give any more contribution compared with first and second order ones. For example, third order contribution is proportional to t; at the same time it is forth order of smallness of the fluctuations. Thus, this term can be neglected. The similar discussions hold for the other higher cumulants.

Now, we can write down the expression of the average propagator. Integrations with respect to $\boldsymbol{Q}$ and $\boldsymbol{q}$ are carried out in eq. (33) to give

$$G_a(\boldsymbol{r},\boldsymbol{v},t\,|\,\boldsymbol{r}',\boldsymbol{v}',t')=\frac{1}{(2\pi)^6}|\det\mathbf{C}|^{-3}\int d\boldsymbol{k}\int d\boldsymbol{v}_0\exp\Big\{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')-i\boldsymbol{k}\cdot\boldsymbol{v}_0\tau$$

$$-\boldsymbol{kk}:\mathbf{A}_1(0)\tau+i\boldsymbol{k}\cdot\boldsymbol{B}\tau-\frac{1}{4}\mathbf{C}^{-1}:(\boldsymbol{v}-\boldsymbol{v}')^2-\mathbf{C}^{-1}:[\boldsymbol{v}_0-\frac{\boldsymbol{v}+\boldsymbol{v}'}{2}+i\boldsymbol{k}\cdot\mathbf{A}_1(0)+\boldsymbol{B}]^2\Big\}. \tag{41}$$

As $\mathbf{A}$ is a function of $\boldsymbol{v}_0$, generally speaking, the integration with respect to $\boldsymbol{v}_0$ is complicated one. However, noting that the integrand has a peak about $\boldsymbol{v}_0\sim(\boldsymbol{v}+\boldsymbol{v}')/2$ with the half-value width $\frac{1}{2}\mathrm{T}_r\mathbf{C}$, we may approximate the above integral, in the case of low fluctuation level, in the following form,

$$G_a(\boldsymbol{r},\boldsymbol{v},t\,|\,\boldsymbol{r}',\boldsymbol{v}',t')$$
$$=\frac{\pi^{3/2}}{(2\pi)^6}|\det\mathbf{C}|^{-3/2}\int d\boldsymbol{k}\ \exp\Big\{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')-i(\boldsymbol{k}\cdot\frac{\boldsymbol{v}+\boldsymbol{v}'}{2}-i\boldsymbol{k}\boldsymbol{k}:\mathbf{A}_1(0)-\boldsymbol{k}\cdot\boldsymbol{B})\tau$$
$$-\frac{1}{4}\mathbf{C}^{-1}:(\boldsymbol{v}-\boldsymbol{v}')^2\Big\}, \tag{42}$$

where $\boldsymbol{v}_0$ in $\mathbf{A}$ and $\boldsymbol{B}$ should be replaced by $(\boldsymbol{v}+\boldsymbol{v}')/2$. As $|\det\mathbf{C}|^{1/2}$ measures the change of particle velocity in the interval τ_s, it is reasonable that the propagator has the Gaussian distribution in velocity space. This means that particles diffuse in velocity space due to the cumulative wave-particle scattering resulted in the increase of particle temperature. It is also worth noting that eq. (42) does not change the form when $\boldsymbol{v}$ and $\boldsymbol{v}'$ are interchanged.

Until now we restricted consideration to the case $\tau_c>\tau_s$. On the opposite case $\tau_c<\tau_s$, we can ignore the frictional term in eqs. (27) and (28), and find the propagator

$$G_a(\boldsymbol{r},\boldsymbol{v},t\,|\,\boldsymbol{r}',\boldsymbol{v}',t')$$
$$=\frac{1}{(2\pi)^3}\int d\boldsymbol{k}\ \exp\Big\{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')-i\boldsymbol{k}\cdot\boldsymbol{v}\tau-\frac{\tau^3}{3}\boldsymbol{k}\boldsymbol{k}:\mathbf{D}\Big\}\delta(\boldsymbol{v}-\boldsymbol{v}'), \tag{43}$$

where $\mathbf{D}=\left(\frac{e}{m}\right)^2\int\frac{d\boldsymbol{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}<|\mathrm{E}|^2>_{\boldsymbol{k},\omega}\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})$. We shall not get with this in detail here, since it has been investigated by several authors.

Before concluding the section, we consider the Fourier transformation of the propagator. Noting that the propagator depends on $\boldsymbol{r},\boldsymbol{r}$; t and t' only through differences $\boldsymbol{r}-\boldsymbol{r}'$ and $t-t'$, we introduce

$$G_a(\boldsymbol{k},\omega;\boldsymbol{v},\boldsymbol{v}')=\int_{-\infty}^{\infty}d\tau\ e^{i\omega\tau}\int d\boldsymbol{R}e^{-i\boldsymbol{k}\boldsymbol{R}}G_a(\boldsymbol{r},\boldsymbol{v},t\,|\,\boldsymbol{r}-\boldsymbol{R},\boldsymbol{v}',\ t-\tau), \tag{44-a}$$

$$G_a^{+}(\boldsymbol{k},\omega;\boldsymbol{v},\boldsymbol{v}')=\int_{0}^{\infty}d\tau\ e^{i\omega\tau}\int d\boldsymbol{R}e^{-i\boldsymbol{k},\boldsymbol{R}}G_a(\boldsymbol{r},\boldsymbol{v},t\,|\,\boldsymbol{r}-\boldsymbol{R},\boldsymbol{v}',t-\tau). \tag{44-b}$$

Then from eq. (42), we have

$$G_a^{+}(\boldsymbol{k},\omega;\boldsymbol{v},\boldsymbol{v}')=\frac{i}{\omega-\boldsymbol{k}\cdot\frac{\boldsymbol{v}+\boldsymbol{v}'}{2}+\delta\omega_a+i\eta_a}|4\pi\det\Delta_a|^{-3/2}\exp\Big[-\frac{1}{4}\Delta_a^{-1}:(\boldsymbol{v}-\boldsymbol{v}')^2\Big], \tag{45-a}$$

$$G_a(\boldsymbol{k},\omega;\boldsymbol{v},\boldsymbol{v}')=\frac{\eta_a}{\left(\omega-\boldsymbol{k}\cdot\frac{\boldsymbol{v}+\boldsymbol{v}'}{2}+\delta\omega_a\right)^2+\eta_a^2}|4\pi\det\Delta_a|^{-3/2}\exp\Big[-\frac{1}{4}\Delta_a^{-1}:(\boldsymbol{v}-\boldsymbol{v}')^2\Big], \tag{45-b}$$

where $\eta_a=\boldsymbol{k}\boldsymbol{k}:\Delta_1(0)+I_m\boldsymbol{k}\cdot\boldsymbol{B}$

$\delta\omega=R_e\boldsymbol{k}\cdot\boldsymbol{B}$

$\Delta_a=\mathbf{C}$

5. Kinetic Equations

In this section we derive the set of kinetice quations by using the propagator obtained in the preceding section.

If we put $f_a(\boldsymbol{v}, t) = \langle F_a(\boldsymbol{r}, \boldsymbol{v}, t)\rangle$ and

$$F_a(\boldsymbol{r}, \boldsymbol{v}, t) = f_a(\boldsymbol{v}, t) + \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3} F_a(\boldsymbol{k}, \boldsymbol{v}, t) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{46}$$

from eqs. (13) and (24) we have

$$\begin{aligned} F_a(\boldsymbol{k}, \boldsymbol{v}, t) = &\int \frac{\mathrm{d}\omega}{2\pi} \int \mathrm{d}\boldsymbol{v}' e^{-i\omega(t-t')} G_a(\boldsymbol{k}, \omega : \boldsymbol{v}, \boldsymbol{v}') F_a(\boldsymbol{k}, \boldsymbol{v}', t') \\ &- \frac{e_a}{m_a} \int \frac{\mathrm{d}\omega}{2\pi} \frac{\mathrm{d}\omega'}{2\pi} \frac{\mathrm{d}\omega''}{2\pi} \frac{1}{i(\omega - \omega' - \omega'')} e^{-i\omega' t} \left\{ e^{-i\omega''(t-t')} - e^{-i(\omega-\omega')(t-t')} \right\} \\ &\times \int \mathrm{d}\boldsymbol{v} \int \mathrm{d}\boldsymbol{v}' \; G_a(\boldsymbol{k}, \omega; \boldsymbol{v}, \boldsymbol{v}') \boldsymbol{E}(\boldsymbol{k}, \omega') \cdot \frac{\partial}{\partial \boldsymbol{v}'} G_a(0, \omega''; \boldsymbol{v}', \boldsymbol{v}'') f_a(\boldsymbol{v}'', t). \end{aligned} \tag{47}$$

Here we have retained the first two terms in the expression of F, becauce the expansion of $\hat{G}$ converges rapidly. Using the relation

$$\int \mathrm{d}\boldsymbol{v}'' \; G_a(0, \omega''; \boldsymbol{v}', \boldsymbol{v}'') f_a(\boldsymbol{v}'', t') \simeq 2\pi\delta(\omega'') f_a(\boldsymbol{v}, t), \tag{48}$$

and noting that

$$G_a{}^{+}(\boldsymbol{k}, \omega; \boldsymbol{v}, \boldsymbol{v}') = \lim_{t-t'\to\infty} \int \frac{\mathrm{d}\omega'}{2\pi} \frac{1 - e^{-i(\omega-\omega')(t-t')}}{i(\omega' - \omega)} G_a(\boldsymbol{k}, \omega'; \boldsymbol{v}, \boldsymbol{v}'), \tag{49}$$

we find that

$$\begin{aligned} F_a(\boldsymbol{k}, \boldsymbol{v}, t) = &\int \frac{\mathrm{d}\omega}{2\pi} \int \mathrm{d}\boldsymbol{v}' e^{-i\omega(t-t')} G_a(\boldsymbol{k}, \omega; \boldsymbol{v}, \boldsymbol{v}') F_a(\boldsymbol{k}, \boldsymbol{v}', t') \\ &- \frac{e_a}{m_a} \int \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} \int \mathrm{d}\boldsymbol{v}' G_a{}^{+}(\boldsymbol{k}, \omega; \boldsymbol{v}, \boldsymbol{v}') \boldsymbol{E}(\boldsymbol{k}, \omega) \cdot \frac{\partial}{\partial \boldsymbol{v}'} f_a(\boldsymbol{v}', t). \end{aligned} \tag{50}$$

Substituting eq. (50) into Poisson's equation, we have

$$\varepsilon(\boldsymbol{k}, \omega) \boldsymbol{E}(\boldsymbol{k}, \omega) = -i \sum_a \frac{4\pi e_a}{k^2} \boldsymbol{k} \int dv \mathrm{d}\boldsymbol{v}' e^{i\omega t'} G_a(\boldsymbol{k}, \omega; \boldsymbol{v}, \boldsymbol{v}') F_a(\boldsymbol{k}, \boldsymbol{v}', t'), \tag{51}$$

where

$$\varepsilon(\boldsymbol{k}, \omega) = 1 - i \sum_a \frac{4\pi e_a{}^2}{m_a k^2} \int \mathrm{d}\boldsymbol{v} \, \mathrm{d}\boldsymbol{v}' G_a{}^{+}(\boldsymbol{k}, \omega; \boldsymbol{v}, \boldsymbol{v}') \boldsymbol{k} \cdot \frac{\partial}{\partial \boldsymbol{v}} f_a(\boldsymbol{v}', t). \tag{52}$$

From eq. (51), we can derive the wave kinetic equation in a similar fashion as is shown in ref. 10. In order to do so, we must relax the restrictions eq. (11) and allow the weak inhomogeneity of the ensemble. Then we multiply the both sides of eq. (51) by $E(k'\omega')\ e^{-i(\omega+\omega')t}$ and take the ensemble average. Then, the wave kinetic equation becomes

$$\varepsilon(k,\omega)\langle|E|^2\rangle_{k,\omega}+i\frac{1}{2}\frac{\partial}{\partial t}\langle|E|^2\rangle_{k,\omega}\frac{\partial\varepsilon(k,\omega)}{\partial\omega}$$

$$=\sum_a\frac{(4\pi e_a)^2}{k^2}\int\mathrm{d}v\int\mathrm{d}v'\frac{1}{\varepsilon^*(k,\omega)}G_a(k,\omega;v,v')f_a(v',t). \tag{53}$$

Here we have assumed that the particle correlation is zero at $t=t_1$ and used the relations

$$\langle F_a(k,v,t_1)F_b(k',v',t_1)\rangle=(2\pi)^3f_a(v,t_1)\delta_{ab}\delta(k+k')\delta(v-v'), \tag{54}$$

and

$$G_a(-k,\omega;v,v')G_a(v,\omega';v'',v')\simeq G_a(0,\omega+\omega';v'',v')G_a^*(v,\omega';v,v''). \tag{55}$$

Next we derive the particle kinetic equation. Averaging eq. (9) over the statistical ensemble, we have

$$\frac{\partial}{\partial t}f_a(v,t)$$

$$=-\frac{e_a}{m_a}\frac{\partial}{\partial v}\int\frac{\mathrm{d}k}{(2\pi)^3}\int\frac{\mathrm{d}k'}{(2\pi)^3}\int\frac{\mathrm{d}\omega'}{2\pi}\langle E^*(k',\omega')F_a(k,v,t)\rangle e^{-i\omega't}. \tag{56}$$

After substituting eq. (50) into the right hand side of eq. (56) we follow the same procedure as derivation of eq. (53). Then we obtain

$$\frac{\partial}{\partial t}f_a(v,t)=\frac{e_a}{m_a}\frac{\partial}{\partial v}\int\frac{\mathrm{d}k}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2\pi}\int\mathrm{d}v'\Big\{i\frac{4\pi e_a k}{k^2\varepsilon^*(k,\omega)}G_a(k,\omega;v,v')f_a(v',t)$$

$$-\frac{e_a}{m_a}\frac{k}{k^2}\langle|E|^2\rangle_{k,\omega}G_a^+(k,\omega;v,v')k\cdot\frac{\partial}{\partial v},\ f_a(v',t)\Big\}. \tag{57}$$

In eqs. (53) and (57), we separate the contribution of the collective mode from that of thermal fluctuaton. We put

$$\langle|E|^2\rangle_{k\omega}=\langle|E|^2\rangle^0_k\delta(\omega-\omega_k)+\langle|E|^2\rangle'_{k\omega}, \tag{58}$$

where $\mathrm{Re}\varepsilon(k,\omega_k)=0$. Defining $\gamma_k=-\mathrm{Im}\varepsilon(k,\omega_k)/\left.\frac{\partial\mathrm{Re}\varepsilon(k,\omega)}{\partial\omega}\right|_{\omega=\omega_k}$, we obtain from eq. (53)

$$\frac{\partial}{\partial t}\langle|E|^2\rangle^0_k-2\gamma_k\langle|E|^2\rangle^0_k$$

$$=\sum_a\frac{(4\pi e_a)^2}{k^2\left[\left.\frac{\partial\mathrm{Re}\varepsilon(k,\omega)}{\partial\omega}\right|_{\omega=\omega_k}\right]^2}\int\mathrm{d}v\int\mathrm{d}v'G_a(k,\omega_k;v,v')f_a(v',t), \tag{59}$$

and from eq. (57)

$$\frac{\partial}{\partial t}f_a(\boldsymbol{v},t)=\frac{e_a}{m_a}\frac{\partial}{\partial \boldsymbol{v}}\cdot\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\int\mathrm{d}\boldsymbol{v}'\left\{\frac{4\pi^2 e_a}{2}\frac{\boldsymbol{k}}{k^2}\frac{1}{\left.\frac{\partial\mathrm{Re}\varepsilon^*(\boldsymbol{k},\omega)}{\partial\omega}\right|_{\omega=\omega_k}}\right.$$

$$G_a^+(\boldsymbol{k},\omega_k;\boldsymbol{v},\boldsymbol{v}')f_a(\boldsymbol{v}',t)-\frac{1}{2}\frac{e_a}{\mathrm{m}_a}\frac{\boldsymbol{k}}{k^2}<|E|^2>^0{}_k G_a^+(\boldsymbol{k},\omega_k;\boldsymbol{v},\boldsymbol{v}')\boldsymbol{k}\cdot\left.\frac{\partial}{\partial\boldsymbol{v}}f_a(\boldsymbol{v}',t)\right\}$$

$$+\frac{e_a}{m_a}\frac{\partial}{\partial\boldsymbol{v}}\cdot\int\mathrm{d}\boldsymbol{k}\int\mathrm{d}\omega\int\mathrm{d}\boldsymbol{v}'\left\{i\frac{4\pi e_a}{(2\pi)^4}\frac{\boldsymbol{k}}{k^2}\frac{1}{\varepsilon^*(\boldsymbol{k},\omega)}G_a(\boldsymbol{k},\omega;\boldsymbol{v},\boldsymbol{v}')f_a(\boldsymbol{v}',t)\right.$$

$$-\frac{e_a}{m_a}\frac{\boldsymbol{k}}{k^2}\sum_b\frac{(4\pi e_b)^2}{(2\pi)^4}\int\mathrm{d}\boldsymbol{v}''\mathrm{d}\boldsymbol{v}'''\frac{1}{|\varepsilon(\boldsymbol{k},\omega)|^2}G_b(\boldsymbol{k},\omega;\boldsymbol{v}'',\boldsymbol{v}''')f_b(\boldsymbol{v}''',t)$$

$$\left.\times G_a^+(\boldsymbol{k},\omega;\boldsymbol{v},\boldsymbol{v}')\frac{\boldsymbol{k}}{k^2}\cdot\frac{\partial}{\partial\boldsymbol{v}}f_a(\boldsymbol{v}',t)\right\}, \tag{60}$$

where the prime on the integral sign indicates to remove the pole of $\varepsilon(\boldsymbol{k},\omega)$ at $\omega=\omega_k$.

The set of eqs. (59) and (60) obtained above is the renormalized quasi-linear equation taking account of the particle discreteness. If we put $G_a(\boldsymbol{k},\omega;\boldsymbol{v},\boldsymbol{v}')=\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})\delta(\boldsymbol{v}-\boldsymbol{v}')$, they become to familiar ones.

6. Dicussions

In this paper we have investigated the problem of the stochastic acceleration of particles in a prescribed random electric field and shown that there are two kinds of effects of fluctuations on particle trajectries in weakly turbulent plasmas. One of these is the diffusive process, which is pointed out by Dupree first and later examined by several authors. It characterizes the behavior of the system which satisfies the inequality relation $\tau_s>\tau_c$, that is, in rather low fluctuation level. The other is the effect of frictional process, which is the subject of the present paper. This new process plays an important role in the case $\tau_s<\tau_c$, i.e. in rather large fluctuation level.

There appear two characteristic times in our theory. One is the slowing down time which is time for friction to become dominant in particle motions;

$$\tau_s^{-1}=\left(\frac{e}{m}\right)^2\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2\pi}\boldsymbol{kk}<|E|^2>_{k,\omega}\frac{\partial^2}{\partial\omega^2}\delta(\omega-\boldsymbol{kv}). \tag{61}$$

The other is the damping time of the propagator;

$$\tau_d^{-1}=\frac{1}{2}\left(\frac{e}{m}\right)^2\int\frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2\pi}<|E|^2>_{k'\omega}\delta(\omega-\boldsymbol{k}'\boldsymbol{v})\frac{(\boldsymbol{k}\cdot\boldsymbol{\tau}_s\cdot\boldsymbol{k}')^2}{k'^2}. \tag{62}$$

Eqs. (61) and (62) show that the small scale fluctuations play an essential role in scattering.

The new propagator leads to not only the damping of particle orbit but also the diffusion in velocity space. These facts suggest that the scattering of particle due to the fluctuations gives important contributions to the anomalous transport phenomena. Application to the case of ion sound turbulenc will be made in a forthcoming paper.

It is not easy to discuss the relation between the method used in this article and the conventional perturbation method [11]. However, in order to formulate the plasma turbulence theory, it is important to interpret our method in terms of the conventional perturbation theory. In this regard, the recent work of Misguich and Balescu [12], which has shown the derivation of a similar propagator as ours by means of the operator algebra, is also not reponsible for the requirement that the plasma turbulence theory should give the total grasp of microscopic processes, i.e. resonance, scattering and so on, as well as the macroscopic phenomena such as anomalous transport.

One of the characteristic features of the plasma wave phenomena is a formation of wave packets due to the dispersion. In this case trapped particles become important to determine the behavior of the system. Our method may be extended to treat the trapped particles using the diagramatic equation shown in Fig. 7. The construction of the kinetic equations in this stage is a interesting subject for the future.

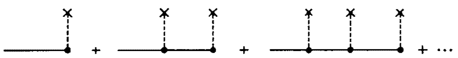

Fig. 7. Diagramatic expression for $\delta\mathbf{r}(\omega)$ taking acount of trapped particles denotes the resonant condition of the wave and particles

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