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SHIMIZU, Shigetoshi
Faculty of Engineering, Yamaguchi University : Professor

KAWANO, Shunichi
Faculty of Engineering, Yamaguchi University : Research Associate

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NOTE

A CONSIDERATION ON THE FINITE ELEMENT METHOD FOR CALCULATING STRESS INTENSITY FACTOR K_I USING THE J-INTEGRAL METHOD

By Shigetoshi SHIMIZU* and Shunichi KAWANO**
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Introduction

A number of studies on the stress intensity factors have been investigated by various investigators[1)-12)]. But it is so difficult to obtain analytically the stress intensity factors on the complicated boundary conditions that the finite element method is applied to the numerical analysis of ones. As Anderson et al[13)]. showed the numerical solutions of the direct displacement method were poor accuracy compared with the energy method barring fine mesh. But the energy method[14)] is very complicated process for calculation by using the finite element method.

Rice[15)] has shown a formula for what he called the J-integral,

$$J=\int_{\Gamma}\left(V dy-\boldsymbol{T}\frac{\partial \boldsymbol{u}}{\partial x}ds\right) \tag{1}$$

where Γ is an arbitrary contour surrounding the crack tip, V is the strain energy density, and $\boldsymbol{T}$ and $\boldsymbol{u}$ are traction and displacement vector.
He also showed the following relation as

$$\left.\begin{array}{ll} J=K_I^2/E & \text{(Plane stress)} \\ J=(1-\nu^2)K_I^2/E & \text{(Plane strain)} \end{array}\right\} \tag{2}$$

where K_I is the stress intensity factor for the mode I, E and ν is Young's modulus and Poisson's ratio, respectively. Levy, et al[16)]. and Leverenz[17)] have used the J-integral to the calculation of the stress intensity factor by finite element method.

On the other hand, Y. Yamamoto and et al.[18)-21)] suggested an effective technique called the method of superposition of analytical and finite element

* Professor, Faculty of Engineering, Yamaguchi University,
** Research Associate, Faculty of Engineering, Yamaguchi University.
*** Associate Professor, Research Institute for Applied Mechanics, Kyushu University.

solutions, and they obtained the agreement with the analytical results.

In this paper the authors make an application of the method of superposition to the J-integral for calculating the crack tip stress intensity factors.

The results agreed sufficiently with the analytical ones in spite of using extremely rough meshes in the vicinity of crak tip.

General Procedure

If the thin plate with a crack is subjected to the load $\boldsymbol{\sigma}_0{}^a$ along the boundary S, the stress vector $\boldsymbol{\sigma}^a$ in the vicinity of crack tip of the plate can be defined as

$$\boldsymbol{\sigma}^a = \boldsymbol{\sigma}^\infty + \boldsymbol{\sigma}^F \tag{3}$$

where $\boldsymbol{\sigma}^\infty$ is the stress vector in the infinite plate with a crack acted by a load $\boldsymbol{\sigma}_0$ at infinity, and the value of $\boldsymbol{\sigma}^\infty$ may be obtained by an analytical method.

While the stress vector $\boldsymbol{\sigma}^F$ in Eq. (3) is calculated by the finite element method for the plate which has the same boundary S and the crack, subjected to a load $\boldsymbol{\sigma}_0{}^F$ along the boundary.

The load $\boldsymbol{\sigma}_0{}^F$ is obtained by the following consideration; cutting out the plate, which boundary is same as S and including crack, from the previously considered infinite plate, we find the load distribution along the boundary S, then subtracting the load which should act along the boundary of the plate treated by the method of finte element.

Using the superscripts a, F and ∞ for the stress intensity factor K_I, coresponding to the stress vector $\boldsymbol{\sigma}^a$, $\boldsymbol{\sigma}^F$ and $\boldsymbol{\sigma}^\infty$, respectively, at a point (r, θ) which is a small distance from the crack tip, $\boldsymbol{\sigma}^a$ is generally expressed as

$$\left.\begin{aligned} &\boldsymbol{\sigma}^a = [K_I{}^\infty/(2\pi r^{1/2} \cdot F(a/W)]\boldsymbol{f}^a(r, \theta) \\ &F(a/W) = 1 + K_I{}^F/K_I{}^\infty \end{aligned}\right\} \tag{4}$$

where $F(a/W)$is the correction factor, W and a is the half breadth of rectangular plate and crack length, respectively.

From the second of Eqs. (4) $K_I{}^a$ is obtained as

$$K_I{}^a = K_I{}^\infty\,[1 + K_I{}^F/K_I{}^\infty]. \tag{5}$$

Substituting Eq. (5) into the first of Eqs. (2), the following equation for a plane stress condition is obtained.

$$J^a = J^\infty[1 + (J^F/J^\infty)^{1/2}]^2. \tag{6}$$

Likewise, the correction factor $F(a/W)$ is shown in

$$F(a/W) = 1 + (J^F/J^\infty)^{1/2} \tag{7}$$

Thus, calculating J^∞ and J^F for the analytical and finite element results, respectively, the correction factor $F(a/W)$ of the plate containing a straight crack with the boundary of arbitrary contour S is obtained from Eq. (7).

By using the Westergaard's stress function ψ[22], we can solve the problem of a stress free crack at $-a<x<a$, $y=0$ and with the boundary conditions of uniform biaxial stress σ_0 at infinity,

$$\psi=\sigma_0 z/(z^2-a^2)^{1/2}, \quad z=x+iy \tag{8}$$

where x and y are cartesian coordinates.

Defining the components of traction vector as σ_x^∞, σ_y^∞ and τ_{xy}^∞ at the imaginary boundary S, they are expressed as

$$\left.\begin{aligned} \sigma_x^\infty &= \sigma_0[x\ cos(\Theta/2)+y\ sin(\Theta/2)-ya^2\ sin(3\Theta/2)/R]/R^{1/2} \\ \sigma_y^\infty &= \sigma_0[x\ cos(\Theta/2)+y\ sin(\Theta/2)+ya^2\ sin(3\Theta/2)/R]R^{1/2} \\ \tau_{xy}^\infty &= \sigma_0 ya^2\ cos(3\Theta/2)/R^{3/2} \end{aligned}\right\} \tag{9}$$

in which $R=(X^2+Y^2)^{1/2}$, $\Theta=\tan^{-1}(Y/X)$, $X=x^2-y^2-a^2$ and $Y=2xy$.

Examples

We try to calculate the correction factors of rectangular sheet containing three kinds of situation of crack, that is, (i) a straight centre crack, (ii) a single edge crack and (iii) double-symmetric edge cracks, they are subjected to uniform tension along the edges parallel to the crack, and the aspect ratio L/W is adopted 1.4 for each case. The number of meshes used in this calculation are 35, 110 and 215 as shown in Fig. 1.

To examine the convergency of solution used the J-integral on the finite element method, the results of two kinds of calculation method, one is the direct application of J-method to the results of finite element method and the other is the application to the superposition method using Eq. (7), are compared with the analytical solution of centre crack ($a/W=0.4$) obtained from Isida[11].

By taking the ratio of the length of integral path Γ to minimum length of mesh is α, Fig. 2 shows the relation between the values of percent error to α.

We find that the convergency is bad for the former method, especially the value of α is smaller than 20 and the converged values are about 3% errors for each number of meshes.

On the contrary, we can expect the good convergency such as error less than 1% for each number of mesh by using the latter method.

The calculated values of the correction factor $F(a/W)$ by using Eq. (7) are plotted against a/W for the case of centre crack, single edge crack and double-symmetric edge cracks, in Fig. 3, 4 and Fig. 5, respectively.

Each results are compared with analytical ones of Isida's[11], Gross'[12] and Irwin's[3].

Using the present superposition method, the calculated results of the correction factors $F(a/W)$ are coinside with the analytical ones less than 1% in spite of the extremely rough mesh, say number of meshes is 35 for each crack situation.

It is difficult to expect to the good agreement with the analytical solution using the direct application of J-integral to the finite elment results with the extremely rough mesh, but we can improvement the error of correction factor by calculating the finite element method with not so fine mesh.

Conclusions

We have the following conclusions from the preceding numerical examples;

i) Appling directly the J-integral to the results obtained by the finite element method, the values of correction factor $F(a/W)$ are almost coincided with the analytical ones for each case except the extremely rough mesh, say 35.

ii) For the application of J-integral to present superposion method, the values were fairly well agreed with analytical ones for each case, in spite of the rough mesh.

iii) In general, the value of J, which is calculated by the finite element results, has a tendency to give an accurate result taking the integral path sufficiently remote from the crack tip.

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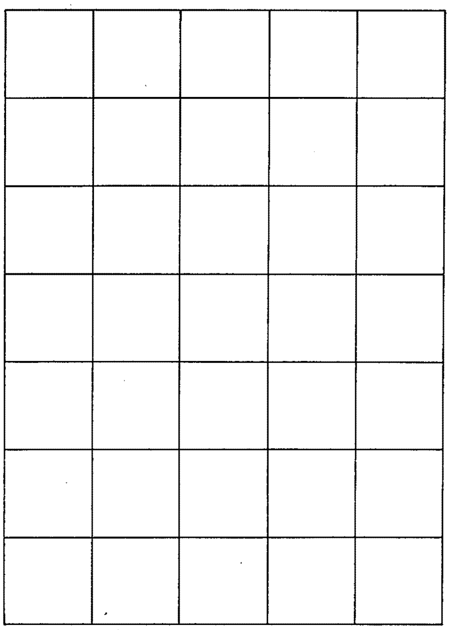
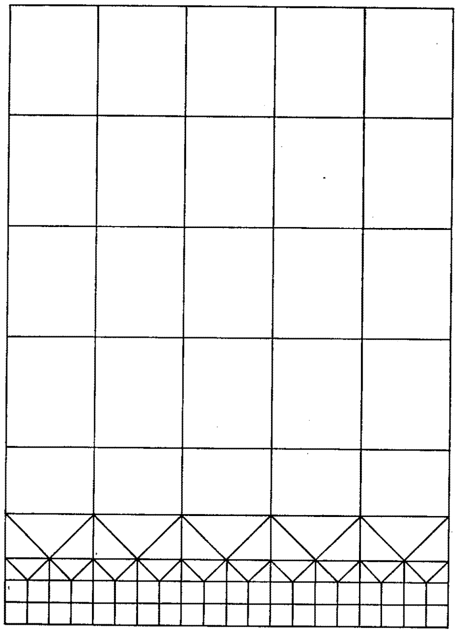


No. of mesh 35



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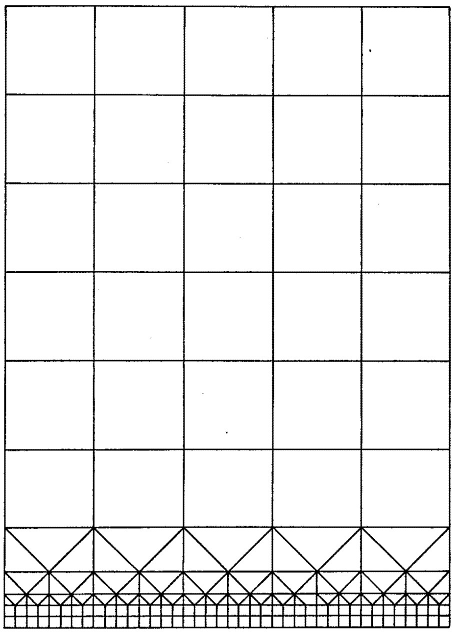

No. of mesh 215

Fig. 1 Finite element idealization for one-quater or half of plate.

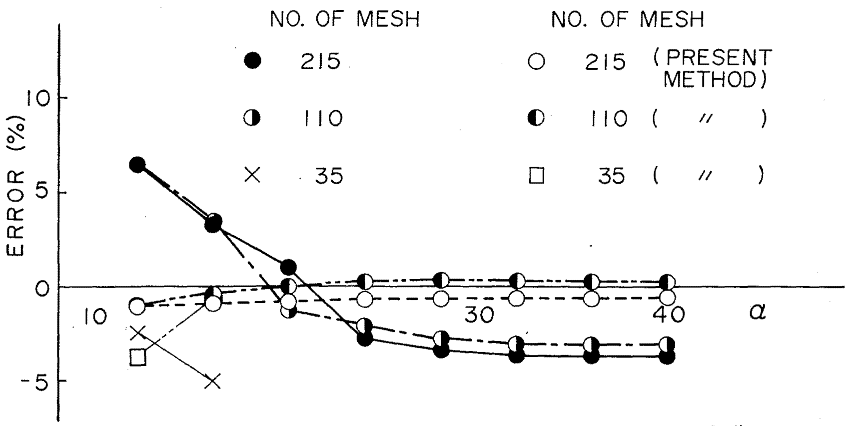

Fig. 2 Convergency of correction factor $F(a/W)$ $(a/W=0.4)$.
α is the ratio of the length of integral path to minimum length of mesh.

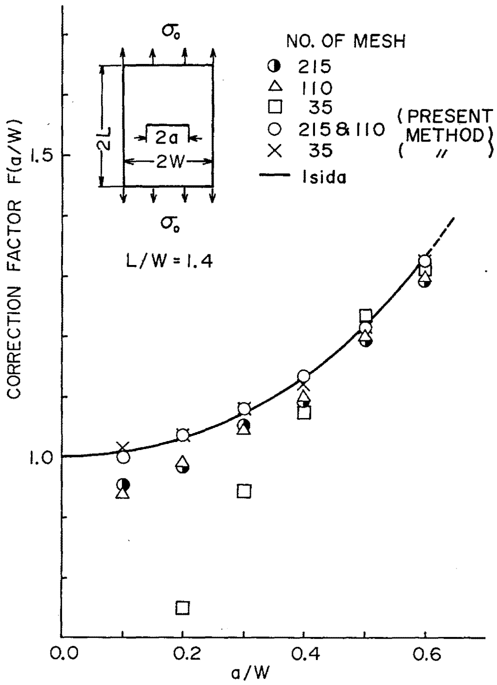

Fig. 3 Correction factor $F(a/W)$ for a straight centre crack.

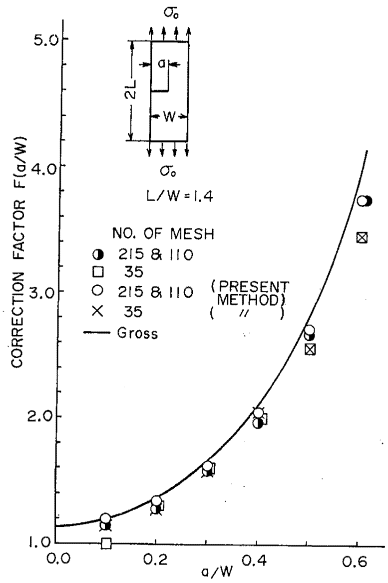

Fig. 4 Correction factor $F\ (a/W)$ for a single edge crack.

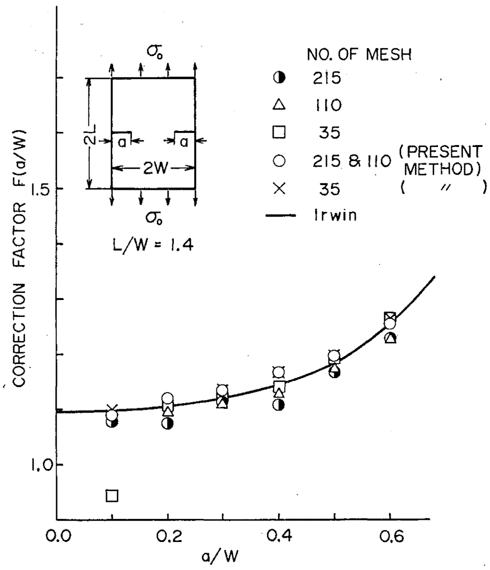

Fig. 5 Correction factor $F\ (a/W)$ for double -symmetric edge crack.