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NOTE

EFFECT OF EDGE THRUST ON THE NATURAL FREQUENCY OF FLEXURAL VIBRATION OF RECTANGULAR PLATE

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An investigation into the effect of edge compression or tention upon the natural frequencies of flexural vibration of the rectangular plate is carried out as a fundamental study on the local vibration of ship structure.

1. Introduction

As one of the vibration problems, it should be noted that the natural frequency of the vibration of a rectangular plate is different from the frequency under the static action of the edge compression or tension in its neutral plane. If the edge compression increases as high as the critical load of the rectangular plate, which is an extreme case, there is no fundamental natural frequency of the vibration of the plate. The investigation into the extreme case of the problem mentioned above was treated by Sezawa[1)] for obtaining buckling load and natural frequency of the clamped rectangular plate, though the relation between the natural frequencies and edge compression is not considered in his paper.

The present note shows the effect of edge compression or tension upon the natural frequencies of flexural vibration of the supported rectangular plate with a numerical example. The results thereby will be applicable to the estimation of the natural frequencies of the rectangular panel of the web plate of the longitudinal or transverse girder as a problem of the local vibration of ship hull structure.

2. Basic Equation and Solutions

The basic equation of the flexural vibration of the rectangular plate under the action of edge compressions in x and y direction and shear force, is written by

$$D\nabla^4 w + \rho h\frac{\partial^2 w}{\partial t^2} = -\left\{P_x\frac{\partial^2 w}{\partial x^2} + 2Q\frac{\partial^2 w}{\partial x \partial y} + P_y\frac{\partial^2 w}{\partial y^2}\right\}, \tag{1}$$

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where,

$$D=\frac{Eh^3}{12(1-\nu^2)}$$

E, ν	Young's modulus and Poisson's ratio respectively
ρh	mass of plate per unit area
w	deflection of the plate
P_x, P_y	edge compresions parallel to x and y-axis respectively
Q	shear force at four edges
x, y	cartesian co-ordinate

In the above equation, if there is no inertia force in the above plate, the equation shows that of buckling of the plate, or if there are no edge compressions and shear force, the equation shows that of flexural vibration of a rectangular plate.

Now we put $Q=0$, and assume the deflection, that

$$w=\sin\frac{m\pi x}{a}Y\cos\omega t,\quad m=1,\ 2,\ 3\ \ldots\ldots \tag{2}$$

Substitute (2) into (1), and the equation will become

$$Y^{IV}-\left(\frac{2m^2\pi^2}{a^2}-\frac{P_y}{D}\right)Y''-\left(\frac{\lambda^2\pi^4}{a^4}-\frac{m^4\pi^4}{a^4}+\frac{m^2\pi^2}{a^2}\frac{P_x}{D}\right)Y=0\ , \tag{3}$$

where, $$\lambda^2=\frac{\rho h\omega^2a^4}{D\pi^4}$$

The solutions of the above equation are written by

$$Y=A\cos qy+B\sin qy+C\operatorname{Cos}py+D\operatorname{Sin}py\ , \tag{4}$$

where, $$\left.\begin{matrix}p^2\\q^2\end{matrix}\right\}=\frac{\pi}{a}\left[\sqrt{\frac{\lambda^2\pi^2}{a^2}+\frac{m^2(P_x-P_y)}{D}+\frac{a^2}{4\pi^2}\frac{P_y^2}{D^2}}\begin{matrix}+\\-\end{matrix}\right\}\frac{m\pi}{a}\Bigg] \tag{5}$$

If the two oposite edges $y=0$ and $y=b$ are both assumed to be supported, the constants A, C, and D vanish and the solution becomes

$$Y=B\sin qy\ ,$$

where, $$q=\frac{n\pi}{b}\ ,\quad n=1,\ 2,\ 3\ \ldots\ldots \tag{6}$$

3. Relation between Eigenvalue and Edge Compression

From the above two equations (5) and (6), the following equation is obtained,

$$\left(\frac{n^2\pi^2}{b^2}+\frac{m^2\pi^2}{a^2}\right)^2=\frac{\pi^2}{a^2}\left\{\frac{\lambda^2\pi^2}{a^2}+\frac{m^2(P_x-P_y)}{D}+\frac{a^2}{4\pi^2}\frac{P_y^2}{D^2}\right\} \tag{7}$$

From the above equation, if we assume $P_x=P_y=0$, the eigen-value λ of the natural frequency of the rectangular plate are obtained. If we put $\lambda=0$, however, the buckling load P_x or P_y is determind, provided that the ratio P_y/P_x is given.

Putting $P_y=0$ in equation (7) for the sake of simplicity, the eigen-value will become,

$$\lambda^2=\left(m^2+n^2\frac{a^2}{b^2}\right)^2-\frac{m^2a^2}{b^2}\frac{P_xb^2}{D\pi^2} \tag{8}$$

The buckling load is obtained, provided that $\lambda=0$ in the above equation,

$$\frac{P_xb^2}{\pi^2D}=\left(m\frac{b}{a}+\frac{n^2}{m}\frac{a}{b}\right)^2 \tag{9}$$

It is a well known fact that the buckling load of the rectangular plate supported at all edges gives a minimum value when m takes equal value to a/b[2]. Hence the buckling load is written by

$$\frac{P_xb^2}{\pi^2D}=\left(\frac{b}{a}+\frac{a}{b}\right)^2 \tag{10}$$

Since the edge compression will change from zero to critical load in the present problem, the coefficent c will be multiplied by P_x in equation (10), in which $c=P_x/P_{cr}$ that changes from zero to unity. Then the eigen-value presented by equation (8) becomes

$$\lambda=\sqrt{\left(m^2+n^2\frac{a^2}{b^2}\right)-cm^2\left(1+\frac{a^2}{b^2}\right)^2} \tag{11}$$

In the case of the fundamental mode, the eigenvalue is written by

$$\lambda_{\substack{m=1\\n=1}}=\sqrt{\left(1+\frac{a^2}{b^2}\right)^2(1-c)} \tag{a}$$

When c becomes zero, λ shows eigen-value of the vibration of rectangular plate. If c takes unity or $P_x=P_{cr}$, λ becomes zero. If the plate vibrates under the tension in its neutral plane, p_x will become negative, or c will take negative, and the eigen-value increases, as seen in equation (a). Figure 1 shows change of eigenvalue for various values of edge compression and tension in some range of c in the case of the square plate.

Since there are higher harmonics in the vibration of the rectangular plate, for instance, the mode $m=2$, $n=1$, the eigenvalue in this case is written by

$$\lambda_{\substack{m=2\\n=1}}=\sqrt{\left(4+\frac{a^2}{b^2}\right)^2-4c\left(1+\frac{a^2}{b^2}\right)} \tag{b}$$

If we put $P_x=P_{cr}$ or $c=1$ in the above equation the eigen-value of the second mode becomes 60 % of that in the case $c=0$.

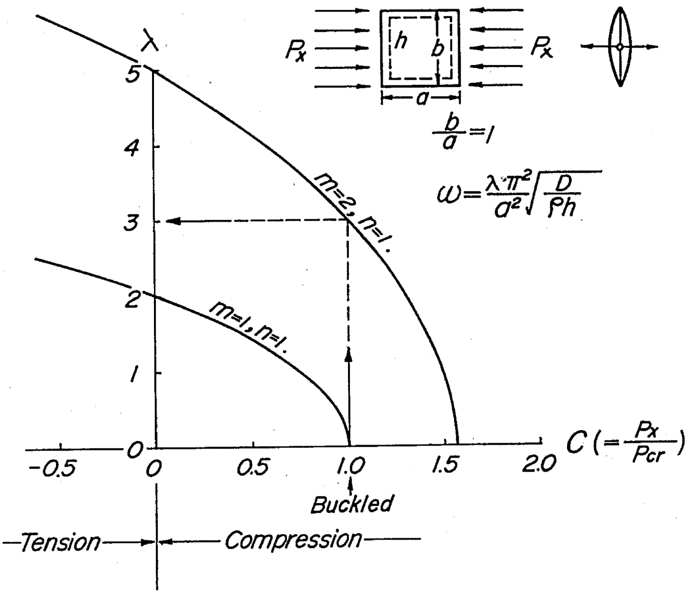

Fig. 1. Eigen-value versus edge compression and tension by the ratio of buckling load of square plate supported four edges.

4. Conclusions

The effect of edge compression or tension upon the natural frequences of the rectangular plate supported all edges was investigated. It is to be noted that the range of variation of the eigen-value for that of the edge compression or tension is considerably wide. The results will be applied for estimating natural frequency of web plate vibration in a local structure of ship hull. There is still much to be studied in the present problem for different types of external force, for instance, under the static action of bending moment and shear force in its neutral plane.

References

1) Sezawa, K., On the Lateral Vibration of a Rectangular Plate Clamped Four Edges and its Stability, Jour. Soc. N. A. Japan, Vol. 49, 1934, pp. 87-93.
2) Timoshenko, S., *Theory of Elastic Stability*, McGraw-Hill, 1936, p. 330.

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