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Automation and Economic Growth in a Task-based Neoclassical Growth Model*

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Abstract

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Keywords: automation, task, neoclassical growth model, capital accumulation, AK model, sustained growth

JEL Classification: E23, E25, O33, O41

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Abstract

This paper incorporates a task-based approach into the Solow growth model to analyze the effects of automation on economic growth. We find that if task producers smoothly adopt automation technology along the capital accumulation path, sustained growth is possible even without technological progress. This result is brought about by the fact that task automation makes the aggregate production function linear. In addition, we demonstrate that both the rental price of capital and the wage are constant on the growth path. In sum, while the interaction between task automation and capital accumulation can be a pathway for sustained growth in output, it leads to the cease of wage growth in the long run.

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1. Introduction

Recently, digital innovations in areas such as artificial intelligence, industrial robots, and other intelligent devices have been progressing rapidly. This technological progress has raised concerns that machines deprive workers of their jobs as production processes become automated. Frey and Osborne (2017) report that roughly 47% of the U.S. workforce will be replaced by machines within 10 to 20 years. In addition, Brynjolfsson and McAfee (2014), Ford (2015), and Davenport and Kirby (2016) point out that, owing to the rapid development of digital technologies, machines are becoming advantageous even for tasks traditionally performed by human workers. The decline in the labor share of income observed in many developed countries is interpreted as evidence that workers find it increasingly difficult to compete with machines.

On the other hand, policymakers in many countries expect automation to be a driver of economic growth. For example, the Japanese government is aggressively pursuing the adoption of digital technologies and automation to avoid labor shortages and economic stagnation caused by its rapidly shrinking and aging population.[1] The European Commission (EU) has proposed the *2030 Digital Compass: the European way for the Digital Decade*, which outlines specific goals for successfully driving digital transformation in the EU.[2] One of the goals of this document is to achieve fair and competitive growth in the digital economy.

Several studies have already analyzed possible linkages between automation and economic growth. Prettner and Strulik (2020) find that automation increases income using an R&D-driven endogenous growth model. Acemoglu and Restrepo (2018) construct an elaborated dynamic general equilibrium model that integrates a task-based approach and the directed technical change. They show that technological progress represented as the creation of new tasks generates sustained economic growth. Prettner (2019) introduces a concept of automation capital into the analysis and concludes that the accumulation of automation capital leads to sustained growth even without technological progress in automation. Overall, these research findings suggest

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[1] The Japanese government's commitment to the digitization of society is summarized in the *6th Science, Technology, and Innovation Basic Plan*. For more information, see Cabinet Office (2021).

[2] For more details, see European Commission (2021).

that there are various channels through which automation can impact economic growth and that we have not yet fully identified how automation contributes to economic growth.

Automation enhances economic growth *quantitatively* by introducing new technologies through it. In addition, however, it has the potential to change economic growth *qualitatively* by affecting the nature of the underlying technology itself. To address this important issue, we employ a task-based approach in a neoclassical growth framework and propose an alternative channel through which automation can affect economic growth. The fundamental assumptions of our study are that all tasks are potentially automatable and that the number of automated tasks is immediately adjusted to clear the factor markets. Under these assumptions, we demonstrate that the availability of automation technology substantially alters the aggregate production technology and the nature of economic growth. To make the impact of automation on the production technology clear, we utilize an analytically tractable neoclassical growth model, the Solow growth model.

The contributions of this paper are threefold. First, we show that the aggregate production function becomes linear (or asymptotically linear in the extended model) if automation technologies are available for all task production. In standard neoclassical growth models, capital accumulation reduces the marginal productivity of capital. In our model, however, it raises the marginal productivity through promoting task automation. This effect weakens the degree of diminishing the marginal productivity of capital, and as a result, the aggregate production function becomes linear in capital. This clearly shows that the availability of automation technology leads to qualitative changes in the aggregate production technology.

Second, if task producers smoothly adopt automation technology on the capital accumulation path, sustained growth is possible even without technological progress. Similar to an AK model, the linearity of the production function with respect to capital leads the sustainable growth. In economic growth theory, knowledge spillover and externality are the main drivers of AK production technology. However, our research shows that task automation can be a generator of AK technology. Unlike the studies conducted by Prettner and Strulik (2020) and Acemoglu and Restrepo (2018), our model does not need technological progress for sustained growth. Additionally, in contrast to Prettner's (2019) research, our model does not need to include automation capital. Consequently, our findings demonstrate that automation can be a driver of economic growth without deviating considerably from the standard neoclassical growth model setting. This result is the most important contribution of this paper as it shows there exists an alternative pathway through which automation can impact economic growth.

Third, the rental price of capital and the wage become constant (or converge to constant values in the extended model). In particular, the effect of automation on the wage is important in our study. As the conventional neoclassical growth model suggests, capital accumulation raises the marginal productivity of labor and increases the demand for labor. In addition to this effect, our model indicates that it stimulates task automation and reduces labor demand. These two effects offset each other and the wage remains constant. In addition, this constancy in factor prices leads to a decline in the labor share of income along the path of economic growth. It potentially explains the recent declining trend in labor share observed in many developed countries.

The model presented in this paper can be considered as a fusion of the neoclassical growth model and the task-based approach. The task-based approach analyzes the structure of the labor market by considering work as a bundle of tasks. In this approach, tasks range from those requiring high skills to those that performed with low skills. Each task can be performed by human workers with different skills or machines, depending on its characteristics. There are two advantages to using the task-based approach in modeling automation. First, it divides the production process into many different tasks. Then, we can measure the degree of progress in automation by counting the number of automated tasks. Second, the task-based approach separates tasks from skills. This feature enables us to illustrate which factor of production is chosen by producers when they perform a specific task. These two advantages in the task-based approach allow us to analyze which tasks are performed by machines (automated).

The task-based approach has been developed in both growth theory and labor economics.[3] In the economic growth literature, Zeira (1998) embeds the task-based

[3] In labor economics, Autor, Levy, and Murnane (2003) are pioneers. They develop a framework that distinguishes between routine and non-routine tasks and show that progress in information technology automates routine tasks and increases inequality. In addition, paring the

approach in a framework of dynamic general equilibrium. Acemoglu and Zilibotti (2001) apply the task-based approach to a model of endogenous technological change and analyze productivity differences across countries. Nakamura and Zeira (2018) analyze technological unemployment due to automation using an endogenous growth model. Hémous and Olsen (2018) investigate the relationship between automation and income inequality by building a growth model that considers the differences between unskilled and skilled workers. Gomes (2019) also introduces task heterogeneity into a standard neoclassical model to develop discussions that automation technology may cause job polarization and economic anxieties.

Among the studies applying the task-based approach to growth theory, our study is closely related to Acemoglu and Restrepo's (2018). They provide a theoretical framework built on a task-based approach to comprehensively analyze task automation, capital accumulation, and technological progress. They examine the relationship between automation and economic growth under the assumption that R&D activities by technology monopolists determine the number of automated tasks. In their model, the availability of automation technology is restricted and only some task producers have access to it. In contrast, we assume a competitive economy and mainly focus on a situation in which all tasks are potentially automatable, and the number of automated tasks is immediately adjusted to clear the factor markets. Consequently, we find that automation can be another pathway to long-term economic growth.

On the other hand, several studies investigate automation and economic growth without using a task-based approach. For example, Prettner and Strulik (2020) analyze the effect of automation on economic growth and inequality by combining a R&D-driven endogenous growth model with workers' skill acquisition. Berg et al. (2018), Dujava and Labáj (2019), and Prettner (2019) introduce the concept of automation capital that is highly substitutable for labor and analyze the relationship between automation and economic growth.[4] Above all, our study is related to Prettner's (2019).

U.S. representative data on task input with samples of employed workers, they find that progress in information technology has polarized the U.S. labor market by reducing labor demand for routine tasks. Acemoglu and Autor (2011) point out the problems with the skill premium model and explain some advantages of the task-based approach.

[4] To be precise, Berg et al. (2018) and Dujava and Labáj (2019) employ the concept of robotic

Incorporating two types of capital: traditional capital and automation capital into the analysis, Prettner (2019) demonstrates that sustained growth is possible even without perpetual technological progress.[5] In contrast, without introducing automation capital, we obtain the same result under the assumption that the automation technology is smoothly adopted. Consequently, our study highlights the importance of the availability of automation technology for sustainable growth.

The remainder of this paper is organized as follows. Section 2 presents the baseline model and clarifies the dynamics. In the baseline model, we assume that the technologies are the same across all production lines of the task. With these simplifying assumptions, the intuition behind our main results becomes clear and understandable. Section 3 extends the basic model to allow the productivity of labor to differ among tasks. The setting is similar to the original task-based approach in the literature. Section 4 concludes the paper and discusses future research.

2. Baseline Model

In this section, we describe the baseline model. Although the model is a simplified version of that by Acemoglu and Restrepo (2018), it is useful in analyzing the relationship between capital accumulation and task automation.

2.1 Production

In the model presented in this paper, the economy is assumed to produce a unique final good. The following production function is assumed in Acemoglu and Restrepo (2018):

$$Y = \tilde{B}\left(\int_{N-1}^{N} x(i)^{\frac{\sigma-1}{\sigma}}\, di\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where Y is the final output, $x(i)$ is the amount of task i, $\tilde{B} > 0$, and $\sigma \in (0, \infty)$ is an elasticity of substitution. N stands for the index of the most leading-edge (complex)

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capital, which is similar to automation capital, in their analysis.

[5] Sachs and Kotlikoff (2012), on the other hand, use a two-period OLG model to show that smart machines take the jobs of young unskilled workers, making the new generation potentially worse off than its predecessor.

task. Since the measure of tasks in production is always 1, an increase in N implies the upgrade in the tasks in production. In other words, the introduction of new technology in addition to the automation of existing tasks are considered simultaneously in their setting.

In our model, to focus on the relationship between automation and capital accumulation, we abstract upgrading tasks, and hence, N is assumed to be constant at 1. Also, to clarify the role of the automation of tasks, we assume that σ is 1. Assuming without the loss of generality that $\tilde{B} = 1$, if we takes the limit of $\sigma \to 1$ when $N = 1$ in Eq. (1), then we have the following production function:

$$Y = \exp\left(\int_0^1 \ln x(i)\, di\right). \tag{2}$$

By the assumption that $\sigma = 1$, Eq. (2) is regarded as a Cobb-Douglas production technology with a continuum of inputs.[6]

In this model, two technologies exist for producing tasks. One is a technology that uses only labor to perform tasks, which we refer to as a *non-automation technology*. This study assumes that the production function of task i with the non-automation technology is given as:

$$x(i) = A_L l(i), \tag{3}$$

where $l(i)$ is the amount of labor used for the production of task i, and A_L represents the productivity parameter for labor. When a task is produced by Eq. (3), it is called *not automated*.

The other is a technology that performs a task using only machines (capital) and is referred to as an *automation technology*. The production function of the automation technology is given as:

$$x(i) = A_K k(i), \tag{4}$$

where $k(i)$ is the amount of capital used for the production of task i, and A_K represents productivity parameter for the capital. When a task is produced by Eq. (4), it is called *automated*. Later in this paper, we show that the availability of automation technology has an impact on the results of this study.

In the baseline model, we assume that A_K and A_L are the same across all task

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[6] This type of production function is frequently used in the literature. See, for example, Acemoglu and Autor (2011), Acemoglu and Zilibotti (2001), Aghion et al. (2005).

production. This assumption may seem too restrictive because it ignores the productivity differences in task production. However, this assumption makes it possible to present the structure of the model more concisely. Finally, in this study, an increase in A_K is interpreted as technological progress in automation. Because we focus on the relationship between capital accumulation and automation, this parameter plays an important role in our analysis.

Here, we show how task automation affects the aggregate production technology of the final goods. As with Acemoglu and Restrepo's model (2018), I denotes the number of automated tasks. Because the total number of tasks is normalized to unity, the number of tasks that are not automated is $1 - I$. We temporarily assume that I is fixed at a constant value. This assumption implies that the range of tasks for which automation technology can be available is restricted. If the same amount of capital is used in the production of all automated tasks, we then have $k(i) = K/I$. Substituting this result into Eq. (4) yields the output of an automated task as $x(i) = A_K K/I$. Similarly, we derive the output of a non-automated task as $x(i) = A_L L/(1 - I)$. Substituting these results into Eq. (2) yields a different representation of the aggregate production function as:

$$Y = \frac{(A_K K)^I (A_L L)^{1-I}}{I^I (1 - I)^{1-I}}. \tag{5}$$

Equation (5) implies that the production technology of final goods becomes Cobb-Douglas with capital and labor when the use of automation technology is restricted (i.e., when I is exogenously given). In the next section, we make different assumptions about the availability of automation technology and examine how these changes influence the production of final goods.

2.2 Equilibrium

We characterize the equilibrium of this economy. To simplify our analysis, we assume that all markets in the economy are perfectly competitive and normalize the price of the final good to unity. In the previous section, we suppose that two types of technologies exist for producing tasks: automation and non-automation technologies. Regarding the availability of these technologies, we make the following assumption:

Assumption 1: *Automation and non-automation technologies are available for all task producers.*

The above assumption is important because it differs from that of Acemoglu and Restrepo (2018). Acemoglu and Restrepo (2018) posited that R&D activities by technology monopolists determine the number of automated tasks. However, our study assumes that all tasks are potentially automatable, and that the number of automated tasks is immediately adjusted to clear the factor markets. Under Assumption 1, each task producer uses only one of the two technologies to produce the task.[7] With these premises, we will show that the aggregate production function of the final goods largely differs from Cobb-Douglas technology.

First, let us describe the relationship between the factor prices. R denotes the rental price of capital and W denotes the wage. When a task is automated, it is produced using capital. Then, the price of the task is equal to the unit cost of producing the task, that is, $p(i) = R/A_K$. In contrast, when a task is not automated, $p(i) = W/A_L$ must hold. If $R/A_K < W/A_L$, then all tasks are produced with capital, and $I = 1$. However, this implies that labor is not used in production. This cannot occur in equilibrium. Similarly, $R/A_K > W/A_L$ also cannot occur in equilibrium. As a result, both unit costs must be equalized in equilibrium:

$$p(i) = \frac{R}{A_K} = \frac{W}{A_L}. \tag{6}$$

Equation (6) implies that, in equilibrium, the cost of task production is the same regardless of whether capital or labor is used in the task production. It assures the existence of the interior solution $I \in (0,1)$ in equilibrium.

Second, we turn to the factor demands. Since we have assumed that the production technology of the final output is Cobb-Douglas, and the price is unity, the demand for task i is given as:

$$x(i) = \frac{Y}{p(i)}. \tag{7}$$

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[7] To simplify the analysis, we exclude situations in which two technologies are used together to produce a single task. This assumption is justified, because using two different technologies to produce a task is likely to result in additional administrative costs for producers.

When a task is automated, the production function of the task is as Eq. (4). Substituting Eq. (7), and $p(i) = R/A_K$ into Eq. (4) yields the following demand for capital in the production of task i:

$$k(i) = \frac{Y}{R}. \tag{8}$$

Similarly, the demand for labor in the production of non-automated task is given as:

$$l(i) = \frac{Y}{W}. \tag{9}$$

Third, we consider the capital and labor markets. The demand for capital in each automated task is given by Eq. (8), and the number of automated tasks is I. This implies that the total demand for capital becomes IY/R. On the other hand, the supply for capital is given as K. Therefore, the capital market clearing condition is derived as:

$$K = I\frac{Y}{R}. \tag{10}$$

Similarly, the labor market clearing condition is:

$$L = (1 - I)\frac{Y}{W}. \tag{11}$$

Finally, we consider how the number of automated tasks, I, is determined in equilibrium. Dividing Eq. (10) by Eq. (11), and substituting Eq. (6) into the result, yield the following condition:

$$\frac{I}{1 - I} = \frac{A_K K}{A_L L}. \tag{12}$$

Figure 1 describes how the equilibrium number of automated tasks, I is determined from Eq. (12). While the LHS of Eq. (12) is a strictly increasing function of I, the RHS does not depend on I. Therefore, the equilibrium value of I exists uniquely.

[Insert Figure 1 around here]

Equation (12) is important because it shows how the relative effective factor endowment ($A_K K/A_L L$) determines the equilibrium number of automated tasks. In particular, an increase in K always shifts the right-hand side of Eq. (12) upward. This shift raises the equilibrium number of the automated tasks. Intuitively, an increase in capital stock seeks to reduce the rental price of capital. This promotes the replacement of labor by machines and raises the equilibrium number of automated tasks. In addition, the technological progress in automation, which is expressed as an increase in A_K, also raises the equilibrium number of automated tasks.

In the baseline model, we can solve Eq. (12) for the equilibrium number of the automated tasks, $I,$ as follows:

$$I = \frac{A_K k}{A_K k + A_L}, \tag{13}$$

where $k = K/L$ is the capital per unit of labor. Although the equilibrium output is given by Eq. (5), I is not exogenously given, but endogenously determined as Eq. (13). Therefore, substituting Eq. (13) into Eq. (5) yields the following production function of final output:

$$y = A_K k + A_L, \tag{14}$$

where $y = Y/L$ denotes the output per unit of labor. This result is summarized as the following proposition:

Proposition 1: *The production function per unit of labor becomes linear if automation and non-automation technologies are available for all task producers.*

This proposition implies that the marginal product of capital does not decrease when an increase in the capital-labor ratio leads to greater automation of task production. Intuitively, an increase in the capital-labor ratio produces two opposite effects: (i) it reduces the marginal productivity of capital as usual, and (ii) it increases the marginal productivity of capital because it raises the number of automated tasks as shown in Eq. (12) or Eq. (13). In our setting, the two effects are exactly cancelled out, so that the marginal product of capital becomes constant.

Proposition 1 shows that the aggregate production function of the final goods becomes linear if automation technologies are available for all task production. This result is simple but important because it reveals that smooth automation is necessary to linearize the aggregate production function. Some readers may think that this result is obtained only because the technologies of task production are also linear. However, we have already observed that the production function of the final goods becomes Cobb-Douglas, as in Eq. (5), when I is exogenously given.

[Insert Figure 2 around here]

Figure 2 illustrates the results of one numerical example. The figure shows graphs of per capita production functions in the case where I is determined by Eq. (12)

and those in the cases where I takes some fixed values. The blue line represents the production function, expressed by Eq. (14); the other three lines correspond to Eq. (5) for $I = 0.2, 0.3$, and 0.4. As shown in Fig. 2, the linear production function given by Eq. (14) is the envelope of the other three curves. For example, suppose that the capital-labor ratio is $k = 3.75$; then, we obtain the value of the equilibrium threshold as $I = 0.2$ from Eq. (12). In this case, the production function with $I = 0.2$ (corresponding to Eq. (5)) is tangent to the linear production function at point E. Intuitively, the two factors of production (K and L) are both used in the production of the final output under the conditions of Eq. (6). In addition, Eq. (6) is equivalent to the condition of cost minimization in task production. This implies that I is chosen to maximize the production of the final output, given the values of K and L.[8] As a result, the linear production function given by Eq. (14) becomes an envelope of other production functions with constant I.

Next, we consider the equilibrium factor prices. First, substituting Eq. (13) and Eq. (14) into Eq. (10) yields the following rental price of capital:

$$R = A_K. \tag{15}$$

Similarly, the equilibrium wage becomes

$$W = A_L. \tag{16}$$

Equation (15) and (16) indicate that the rental price of capital equals the productivity parameter for capital and the wage equals the productivity parameter for labor in equilibrium.

As in Proposition 1, the availability of automation technology is closely linked to the outcome that the two factor prices are independent of capital quantity. Below, we explain how the smooth adoption of automation technology leads to constant wage by considering the impact of automation on the labor market. An increase in capital has two opposite effects on the wage: (i) it raises the marginal productivity of labor and thus increases the demand for labor, as typically seen in the Cobb-Douglas production function, (ii) on the other hand, it also stimulates task automation as shown in Eq. (12), thereby reducing labor demand. These two effects cancel out each other, so the wage

[8] In fact, consider an optimization problem for determining I, so as to maximize the final output given by Eq. (5). The first-order condition for this optimization problem is then given by Eq. (12).

becomes constant and independent of the capital quantity. Similarly, we can prove that an increase in A_K does not affect the equilibrium wage.

The result that the wage becomes constant is closely related to Acemoglu and Restrepo's study (2018). They showed that, while an increase in I raises the demand for labor through higher productivity (called *the productivity effect*), it also promotes the automation of task production, which reduces the demand for labor (called *the displacement effect*). Their analysis indicates that, under reasonable parameter values, the displacement effect dominates, and automation can reduce the equilibrium wage. However, in our model, I is endogenously determined by Eq. (12). While an increase in K or A_K generates the same productivity and displacement effects as shown in Acemoglu and Restrepo (2018), the two effects completely cancel each other out so that the wage is independent of these variables.

The fact that the rental price of capital and the wage become constant has important implications for factor shares. From Eq. (10) and Eq. (11), we can confirm that the capital share and labor share are expressed as $RK/Y = I$ and $WL/Y = 1 - I$, respectively. These expressions imply that an increase in capital stock and technological progress in automation raise the capital share and lower the labor share. Intuitively, while an increase in capital promotes the automation of tasks and increases the output per unit of labor, it does not lead to a higher wage level. As a result, these changes reduce the labor share.

This result has the potential to explain the recent declines in the labor share. Declines in the labor share have been observed in many countries in recent years, and some studies have been conducted on the causes.[9] In particular, Karabarbounis and Neiman (2014) point out that the decline in capital costs due to advances in information technology has reduced the labor share in many countries. However, this result is valid only when the elasticity of substitution between capital and labor is greater than one. On the other hand, in our study, the reduced form of the production function becomes

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[9] There are some influential hypotheses that can explain the reasons for the decline in the labor share. Elsby et al. (2013) and Böckerman and Maliranta (2012) present the hypothesis that the relocation of labor-intensive sectors abroad is the cause of this decline. Autor et al. (2020) use U.S. firm-level data to show that the emergence of firms with lower labor shares (the so-called "superstar firms") has led to a decline in the labor share at the aggregate level.

linear, and the elasticity of substitution is infinite. As a result, an increase in the amount of capital and advances in automation technology will lead to a decrease in the labor share.

2.3 Capital Accumulation and Economic Growth in the Baseline Model

The previous section confirms that the availability of automation technology qualitatively changes the aggregate production technology. In this section, we combine the baseline model with the neoclassical growth setting and analyze how automation affects economic growth. Since we mainly focus on the impact of automation on the aggregate production technology, we utilize the analytically tractable neoclassical growth model, the Solow growth model in our analysis. We assume that the labor supply remains constant over time. Let s be the savings rate ($0 < s < 1$), and $d > 0$ be the rate of capital depreciation. Under these assumptions, Eq. (14) implies that the growth rate of capital per unit of labor is expressed as follows:

$$\frac{\dot{k}}{k} = sA_K - \delta + \frac{sA_L}{k}. \tag{17}$$

[Insert Figure 3 around here]

Figure 3 shows the growth rate of capital per unit of labor, as described by Eq. (17). The right-hand side of Eq. (17) decreases for all k. Let us assume $sA_K > \delta$, as shown in Fig. 3. In this case, the growth rate of capital per unit of labor decreases and converges to a constant growth rate given by $sA_K - \delta$. Therefore, sustainable economic growth is achieved in the long run. However, if $sA_K < \delta$, the capital per unit of labor converges to a steady state satisfying $k^* = sA_L/(\delta - sA_K)$ and the growth ceases. This result is summarized in the following proposition:[10]

[10] Our dynamic model is similar to that of Jones and Manuelli (1990) because the marginal productivity of capital is asymptotically constant in both models (in our baseline model, it is always constant). However, their model differs in terms of wage behavior. In Jones and Manuelli's (1990) model, an increase in the capital-labor ratio raises the wage unboundedly. In contrast, our model shows that it becomes independent of the capital-labor ratio and remains constant. The extended model in Section 3 establishes the same feature of the wage.

Proposition 2: *Incorporate the baseline model into the Solow growth setting and suppose that Assumption 1 holds. Then, if the productivity parameter for capital is sufficiently high, the model generates perpetual growth even in the absence of technological progress.*

Proposition 2 implies that an economy can achieve sustained growth if task producers smoothly adopt automation technology along the capital accumulation path. In economic growth theory, knowledge spillover and externalities are the main causes of AK production technology. However, in our model, the AK-production function is generated by the smooth automation of task production. Consequently, our study shows that automation can be another pathway to long-term economic growth.

[Insert Figure 4 around here]

To clarify the model's implications, suppose that Assumption 1 is not satisfied. In this case, there is an upper bound on the number of automated tasks, and it is given by $J \in (0,1)$. Figure 4 describes the behavior of capital per unit of labor in this case. After the economy starts at $k(0)$, then capital per unit of labor will change according to Eq. (17), which corresponds to the solid red line. Along this transition path, the number of automated tasks increases as capital accumulates. Eventually, however, I will reach its upper bound, J, at some point in time. Simultaneously, k will reach $\tilde{k}$, as shown in Fig. 4.[11] Subsequently, the capital per unit of labor changes according to the following equation:

$$\frac{\dot{k}}{k} = s\left(\frac{A_K}{J}\right)^J \left(\frac{A_L}{1-J}\right)^{1-J} k^{J-1} - \delta. \tag{18}$$

Equation (18) corresponds to the solid blue line.[12] Since J is given as a constant, Eq. (18) represents a standard Solow model, and the capital per unit of labor converges to its steady-state value, k^*. This example shows that the smooth automation of tasks through capital accumulation is essential to generate sustainable growth.

In addition, Eq. (17) implies that an increase in the productivity parameter for capital, which corresponds to the technological progress in automation, raises the long-

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[11] We can derive $\tilde{k} = A_L J / A_K(1-J)$ by substituting J into Eq. (12).

[12] Since the linear production function shown in Eq. (14) is the envelope of the other production function with I constant, Eq. (17) is also the envelope of Eq. (18).

term rate of economic growth. In contrast, the productivity parameter for labor only affects transitional dynamics. This implies that automation technology plays a crucial role in achieving sustainable growth when the task model is combined with capital accumulation.

While Acemoglu and Restrepo (2018) did consider capital accumulation in their dynamic analysis, they mainly focused on its balanced growth path. In contrast, our analysis focuses on the transitional dynamics under the condition in which task producers smoothly adopt automation technology along the capital accumulation path. As a result, we find that task automation through capital accumulation can generate sustained growth. Moreover, Prettner (2019) incorporated automation capital into his analysis, such that it could be perfectly substitutable for labor and showed the same consequence as in Proposition 2. Our analysis shows that task automation weakens the diminishing marginal productivity of capital and achieves sustained growth. In this sense, our study can be regarded as an elaboration of Prettner's (2019) model using a task-based approach.

3. Extension of the Baseline Model: Task and Comparative Advantage

In the previous section, we assume that the productivity parameters for capital and labor are the same across all tasks. However, under this assumption, while the number of automated tasks is determined in the equilibrium, we cannot determine *which tasks are automated*. In this section, following Zeira (1998), Acemoglu and Autor (2011), and Acemoglu and Restrepo (2018), we extend the baseline model to allow different labor productivity for different tasks. This extension will enable us to specify which tasks are automated within the model.

3.1 Introducing Task-Specific Productivity

In the extended model, the production function of the automation technology is given by Eq. (4), while the production function of the non-automation technology is modified as follows:

$$x(i) = A_L \gamma(i) l(i), \tag{19}$$

where $\gamma(i)$ represents the task-specific productivity. As for $\gamma(i)$, we make the following assumption:

Assumption 2: *The function* $\gamma: [0,1] \to \mathbb{R}^{+}$ *is continuously differentiable and strictly increasing.*

Assumption 2 implies that labor has a comparative advantage in the production of higher-indexed tasks.

3.2 Equilibrium

Next, we consider the equilibrium of the extended model. Here, Eq. (6) does not hold, because the unit cost of production differs among tasks. Instead, the price of a task is given by:

$$p(i) = \min\left[\frac{R}{A_K}, \frac{W}{A_L \gamma(i)}\right]. \tag{20}$$

In Eq. (20), if the effective capital cost is smaller than the effective labor cost, the automation technology is used for production.

Since we have assumed that labor has a comparative advantage in the production of higher-indexed tasks, the equilibrium number of automated tasks, I, satisfies the following condition:

$$\frac{R}{A_K} = \frac{W}{A_L \gamma(I)}. \tag{21}$$

Figure 5 describes how the unique number of automated tasks, I, is determined. For all tasks $i < I$, the effective capital cost is lower than the effective labor cost, and these tasks are produced with capital. However, if $i > I$, the task is produced with labor because labor has a comparative advantage.

[Insert Figure 5 around here]

Next, we consider capital and labor market clearing. Even in this case, the demand for capital and labor in task production is given by Eq. (8) and Eq. (9), respectively. Therefore, the equilibrium conditions of the capital and labor markets are the same as in Eq. (10) and Eq. (11), respectively. Dividing Eq. (10) by Eq. (11), and substituting Eq. (21) into the result yields the following condition:

$$\frac{I}{1-I}\gamma(I) = \frac{A_K K}{A_L L}. \tag{22}$$

Equation (22) determines the equilibrium number of the automated tasks in the extended model. Comparing this result with that in Eq. (12) in the previous section, the right-hand sides are the same, while the left-hand side of Eq. (22) includes $\gamma(I)$.

For tasks such that $i < I$, their production is automated, and the output is represented by $x(i) = A_K K/I$. However, for tasks such that $i > I$, their production is not automated, and the output of a non-automated task is $x(i) = A_L \gamma(i) L/(1-I)$. Substituting these results into Eq. (2) yields the following aggregate output:

$$Y = \frac{(A_K K)^I (A_L L)^{1-I}}{I^I (1-I)^{1-I}} \exp\left(\int_I^1 \ln \gamma(i) di\right). \tag{23}$$

While Eq. (23) is similar to Eq. (5), its exponential part, $\exp\left(\int_I^1 \ln \gamma(i) di\right)$, reflects the fact that labor productivity differs in the production of non-automated tasks. Combining Eq. (23) with Eq. (22) yields the following representation:[13]

$$y = \big(A_K k + A_L \gamma(I)\big)\Omega(I), \qquad \Omega(I) \equiv \exp\left(\int_I^1 \ln\left(\frac{\gamma(i)}{\gamma(I)}\right) di\right). \tag{24}$$

Again, we know that I is uniquely determined for each capital per unit of labor k from Eq. (22). This implies that I is represented as a function of k. This function is expressed as $I = \varphi(k)$. This function, $\varphi: \mathbb{R}^+ \to [0,1)$, is confirmed to be an increasing function of k.[14] Substituting $I = \varphi(k)$ into Eq. (24), we derive the per capita production function, given as:

$$y = \big(A_K k + A_L \gamma(\varphi(k))\big)\Omega\big(\varphi(k)\big). \tag{25}$$

From Eq. (25), we derive the following proposition:

Proposition 3: *Suppose that Assumption 2 holds and define a new function* $f: \mathbb{R}^+ \to \mathbb{R}^+$ *as* $f(k) \equiv \big(A_K k + A_L \gamma(\varphi(k))\big)\Omega\big(\varphi(k)\big)$. *Then, we derive* $f'(k) > 0$ *and*

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[13] Note that $\Omega : [0,1] \to [1,\infty)$ is strictly decreasing with I and $\Omega(1) = 1$. For details, see Appendix A1.

[14] We can obtain this result by applying the Implicit Function Theorem to Eq. (22). For details, see Appendix A1.

$f''(k) < 0$ *for any* $k > 0$. *Moreover,* $\lim_{k \to \infty} f'(k) = A_K$.

PROOF: See Appendix A1.

Proposition 3 shows that the production function described in Eq. (25) is no longer linear but a concave function. This result clearly illustrates the characteristics of the extended model. Even in the extended model, an increase in the capital-labor ratio produces two opposite effects: (i) it reduces the marginal productivity of capital, and (ii) it increases the marginal productivity of capital because it raises the number of automated tasks from Eq. (22). However, human labor has a comparative advantage in the production of higher-indexed tasks, and such tasks cannot be easily produced by machines. Therefore, an increase in the capital-labor ratio does not result in the smooth automation of tasks. As a result, the effect of task automation on the marginal productivity of capital is weakened, and the production function becomes concave.

However, according to Proposition 3, the marginal productivity of capital does not converge to zero but approaches A_K. In other words, the production function expressed in Eq. (25) is asymptotically linear. Therefore, Proposition 3 is similar to Proposition 1.

[Insert Figure 6 around here]

Figure 6 shows a numerical example of the extended model. The blue line corresponds to Eq. (25), and the other three lines correspond to Eq. (23) for $I = 0.2, 0.3$, and 0.4, respectively. In this numerical example, we specify $\gamma(i) = \alpha i^{\beta}$, and set $\alpha = 1$ and $\beta = 1$. In Fig. 6, the production function, represented by Eq. (25), is concave, as Proposition 3 indicates. Moreover, as shown in Fig. 2, the graph of this production function is the envelope of the other three lines. In the extended model, the equilibrium number of automated tasks is determined as a result of the cost minimization of task producers. This determination is described by Eq. (22). For society, Eq. (22) implies that the economy chooses I to maximize the production of the final good, given the values of K and L.[15] Therefore, we can more clearly understand that the graph of Eq.

[15] As in the case of the baseline model, Eq. (22) is a first-order condition in choosing I so as to maximize the final output given by Eq. (23).

(25) is the envelope of the production functions with a constant I.

Next, we derive the equilibrium factor prices. Substituting Eq. (22) and Eq. (24) into Eq. (10) yields the rental price of capital as:

$$R = A_K \Omega(I) = A_K \Omega(\varphi(k)). \tag{26}$$

Similarly, the wage is derived as:

$$W = A_L \gamma(I) \Omega(I) = A_L \gamma\big(\varphi(k)\big) \Omega(\varphi(k)). \tag{27}$$

By analyzing Eq. (26) and Eq. (27), we derive the following proposition:

Proposition 4: *While the rental price of capital is a decreasing function of the capital-labor ratio, the wage is an increasing function of the capital-labor ratio. Moreover, the rental price of capital converges to its lower bound,* A_K*, and the wage converges to its upper bound,* $A_L \gamma(1)$ *as the capital-labor ratio goes to infinity.*

PROOF: See Appendix A2.

The results of Proposition 4 are fundamentally brought about by the same mechanism as in Proposition 3. As in Proposition 3, human labor has a comparative advantage in the production of higher-indexed tasks. The production of these tasks cannot be smoothly automated, even if the capital-labor ratio increases. As a result, the rental price (or the wage) is a decreasing (or increasing) function of the capital-labor ratio, as observed in the usual neoclassical growth model. On the other hand, Proposition 4 also suggests that the rental price of capital converges to its lower bound, and the wage to its upper bound. In this respect, Proposition 4 is quite similar to the consequence that the two factor prices become constant in Section 2.

Next, we consider how the technological progress in automation, represented by an increase in A_K, affects factor prices. As in the case of Proposition 4, analyzing Eq. (26) and Eq. (27) yields the following proposition:

Proposition 5: *While the technological progress in automation represented by an increase in the productivity parameter for capital will raise both the rental price and the wage, the wage goes to the upper bound,* $A_L \gamma(1)$*, as the productivity parameter for capital goes to infinity.*

PROOF: See Appendix A3.

3.3 Capital Accumulation and Economic Growth in the Extended Model

Finally, in this section, we analyze how automation affects economic growth using an extended model. As in Section 2, the production structure of the extended model is combined with the Solow growth setting. Considering Eq. (25), the growth rate of capital per unit of labor can be expressed as follows:

$$\frac{\dot{k}}{k} = \frac{sf(k)}{k} - \delta = sA_K \Omega\big(\varphi(k)\big) - \delta + \frac{sA_L \gamma\big(\varphi(k)\big)\Omega\big(\varphi(k)\big)}{k}. \tag{28}$$

From Proposition 3, the average productivity of capital, $sf(k)/k$, decreases in k. In addition, it is shown that $\lim_{k\to\infty} A_K \Omega(\varphi(k)) = A_K$ and $\lim_{k\to\infty} A_L \gamma\big(\varphi(k)\big)\Omega(\varphi(k)) = A_L \gamma(1)$ in Appendices A1 and A2. Therefore, if $sA_K > \delta$, the growth rate of capital per unit of labor decreases and converges to a constant growth rate given by $sA_K - \delta$. In contrast, if $sA_K < \delta$, the capital per unit of labor converges to a positive steady-state value that satisfies $sf(k) = \delta k$, and economic growth ceases. This result is summarized in the following proposition:

Proposition 6: *Incorporate the extended model with the Solow growth setting and suppose that Assumption 2 holds. Then, if the productivity parameter for capital is sufficiently high, the model generates perpetual growth even in the absence of technological progress.*

This proposition is also important because it shows that Proposition 2 is not a special case. Proposition 6 implies that automation can be a driver of sustained growth, even if we consider differences in labor productivity across tasks, as assumed in Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018). Note that we have assumed that $\gamma(i)$ is continuously differentiable and strictly increasing. Given these weak assumptions, it is more generally accepted that task automation mitigates the diminishing marginal productivity of capital and leads to sustained growth.

4. Concluding Remarks

By incorporating the task-based approach into the neoclassical growth model, this paper investigated the impact of task automation on economic growth. In the baseline model, we assumed that the factor-augmenting technologies are the same across all task productions and derived some important results. In the extended model, we have modified the baseline model to allow tasks to differ in labor productivity and demonstrated how this extension changes the results obtained in the baseline model.

The main implications in this paper are as follows: (i) the aggregate production function is linear (or asymptotically linear) if automation technologies are available for all task productions; (ii) if task producers smoothly adopt automation technology along the capital accumulation path, sustained growth is possible even without technological progress; and (iii) the rental price of capital and the wage become constant (or converge to constant values) in the process of economic growth. In the standard neoclassical growth model, the accumulation of capital stock diminishes the marginal productivity of capital. In contrast, we show that capital accumulation promotes task automation and increases its demand for capital, while it decreases the demand for labor. This effect weakens the degree of the diminishing marginal productivity of capital and makes the production function linear. Consequently, the economy can realize sustained growth without perpetual technological progress. This result constitutes a significant contribution to the literature in the sense that task automation itself is a pathway to sustainable growth.

Although this paper has derived vital results on automation and economic growth, it also has restricting assumptions. Among them, first, the type and number of tasks required for production do not change over time. Under the assumption, we cannot analyze task dynamics, such as the emergence of new tasks and disappearance of old ones, as discussed in Acemoglu and Restrepo (2018). Therefore, incorporating the process of task creation and extinction into the model would make our analysis of automation and growth more realistic. Second, the productivity parameter for capital is treated as exogenous. The assumption makes it impossible to analyze what determines the productivity of automation technology. For example, although Acemoglu and Restrepo (2022) pointed out that the aging of the population is associated with innovations in automation technology, our model cannot provide any implications for this relationship. Relaxing these assumptions surely deserves future research.

Appendix A1: Proof of Proposition 3

Assumption 2 means that the derivative of $\gamma(i)$ must be positive. In addition, Eq. (22) implies that $I = \varphi(k) > 0$ for $k > 0$. Therefore, we obtain $\gamma(I) > 0$ and $\gamma'(I) > 0$. By applying the Implicit Function Theorem to Eq. (22), we derive:

$$\frac{dI}{dk} = \varphi'(k) = \frac{(1-I)\frac{A_K}{A_L}}{\gamma(I) + I\gamma'(I) + \frac{A_K}{A_L}k} > 0. \tag{A1}$$

In addition, differentiating $\Omega(I)$ with respect to I yields:

$$\frac{d\Omega}{dI} = -(1-I)\frac{\gamma'(I)}{\gamma(I)}\Omega(I) < 0. \tag{A2}$$

Next, differentiating $f(k)$ with respect to k leads to:

$$f'(k) = \left(A_K + A_L\gamma'(I)\frac{dI}{dk}\right)\Omega(I) + \left(A_K k + A_L\gamma(I)\right)\frac{d\Omega}{dI}\frac{dI}{dk}. \tag{A3}$$

Substituting Eq. (A2) into Eq. (A3), we obtain the following equation:

$$f'(k) = A_K\Omega(I) + \gamma'(I)\Omega(I)\frac{dI}{dk}\left(A_L I - \frac{(1-I)A_K k}{\gamma(I)}\right). \tag{A4}$$

Since Eq. (22) implies $A_L I - (1-I)A_K k/\gamma(I) = 0$, the second term of (A4) becomes zero, and we derive $f'(k) = A_K\Omega(\varphi(k)) > 0$. Furthermore, we derive the second derivative of $f(k)$ as:

$$f''(k) = -A_K(1-I)\frac{\gamma'(I)}{\gamma(I)}\Omega(I)\frac{dI}{dk}. \tag{A5}$$

Since Eq. (A1) is positive, $f''(k) < 0$. Finally, Eq. (22) implies that $I \to 1$ as $k \to \infty$. From the definition of Ω in Eq. (24), and $\lim_{I\to 1}\Omega(I) = 1$, we derive $\lim_{k\to\infty} A_K\Omega(\varphi(k)) = A_K$.

Appendix A2: Proof of Proposition 4

The results for the rental price of capital are shown in Appendix A1. Therefore, we focus on the wage. Differentiating the logarithm of Eq. (27) with respect to k yields:

$$\frac{d\ln W}{dk} = \left(\frac{d\ln\gamma(I)}{dI} + \frac{d\ln\Omega(I)}{dI}\right)\frac{dI}{dk}. \tag{A6}$$

Substituting Eq. (A2) into the above leads to:

$$\frac{d\ln W}{dk} = I\frac{\gamma'(I)}{\gamma(I)}\frac{dI}{dk}. \tag{A7}$$

Since Eq. (A1) is positive, Eq. (A7) is also positive. Moreover, as in the case of the rental price of capital, we derive $\lim_{k\to\infty} A_L\gamma\big(\varphi(k)\big)\Omega(\varphi(k)) = A_L\gamma(1)$.

Appendix A3: Proof of Proposition 5

Applying the Implicit Function Theorem to Eq. (22) yields:

$$\frac{dI}{dA_K} = \frac{(1-I)\frac{1}{A_L}k}{\gamma(I) + I\gamma'(I) + \frac{A_K}{A_L}k} > 0. \tag{A8}$$

In addition, differentiating the logarithm of Eq. (26) with respect to A_K and substituting Eq. (A2) into the result yields:

$$\frac{d\ln R}{dA_K} = \frac{1}{A_K} + \frac{d\ln\Omega(I)}{dI}\frac{dI}{dA_K} = \frac{1}{A_K} - (1-I)\frac{\gamma'(I)}{\gamma(I)}\frac{dI}{dA_K}. \tag{A9}$$

By plugging Eq. (22) and Eq. (A8) into Eq. (A9), the following expression is obtained:

$$\frac{d\ln R}{dA_K} = \frac{1}{A_K}\left[1 - \frac{I(1-I)\gamma'(I)}{\frac{\gamma(I)}{1-I} + I\gamma'(I)}\right]. \tag{A10}$$

Equation (A10) is positive since $I(1-I)\gamma'(I) < I\gamma'(I)$ (remember that I is less than one). This implies that an increase in the productivity parameter for capital raises the rental price of capital. Similarly, differentiating the logarithm of Eq. (27) with respect to A_K yields:

$$\frac{d\ln W}{dA_K} = \left(\frac{d\ln\gamma(I)}{dI} + \frac{d\ln\Omega(I)}{dI}\right)\frac{dI}{dA_K} = I\frac{\gamma'(I)}{\gamma(I)}\frac{dI}{dA_K}. \tag{A11}$$

Equation (A11) is also positive, which implies that wage increases with A_K. Finally, Eq. (22) shows that $I \to 1$ as $A_K \to \infty$. Therefore, we derive $\lim_{A_K\to\infty} A_L\gamma(I)\Omega(I) = A_L\gamma(1)$.

References

Acemoglu, D., & Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings, in: Card, D., Ashenfelter, O. (Eds.), *Handbook of Labor Economics*, Vol.4, Part B, Elsevier, Amsterdam, 1043–1171. doi: 10.1016/S0169-7218(11)02410-5

Acemoglu, D., & Restrepo, P. (2018). The race between man and machine: Implications of technology for growth, factor shares, and employment, *American Economic Review*, 108, 1488–1542. doi: 10.1257/aer.20160696

Acemoglu, D., & Restrepo, P. (2022). Demographics and automation. *Review of Economic Studies*, 89, 1–44. doi: 10.1093/restud/rdab031

Acemoglu, D., & Zilibotti, F. (2001). Productivity differences. *The Quarterly Journal of Economics*, 116, 563–606. doi: 10.1162/00335530151144104

Aghion, P., Bloom, N., Blundel, R., Griffith, R., & Howitt, P. (2005). Competition and innovation: An inverted-U relationship. *The Quarterly Journal of Economics*, 120, 701–728. doi: 10.1093/qje/120.2.701

Autor, D., Dorn, D., Katz, L.F., Patterson, C., & Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, 135, 645–709. doi: 10.1093/qje/qjaa004

Autor, D., Levy, F., & Murnane, R.J. (2003). The skill content of recent technological change: An empirical exploration. *The Quarterly Journal of Economics*, 118, 1279–1333. doi: 10.1162/003355303322552801

Berg, A., Buffie, E.F., & Zanna, L.-F. (2018). Should we fear the robot revolution? (The correct answer is yes). *Journal of Monetary Economics*, 97, 117–148. doi: 10.1016/j.jmoneco.2018.05.014

Böckerman, P., & Maliranta, M. (2012). Globalization, creative destruction, and labour share change: Evidence on the determinants and mechanisms from longitudinal plant-level data. *Oxford Economic Papers*, 64, 259–280. doi: 10.1093/oep/gpr031

Brynjolfsson, E., & McAfee, A. (2014). *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies*. W.W. Norton & Company, New York.

Cabinet Office (2021). *6th Science, Technology, and Innovation Basic Plan*, URL: https://www8.cao.go.jp/cstp/english/sti_basic_plan.pdf

Davenport, T.H., & Kirby, J. (2016). *Only Humans Need Apply: Winners and Losers in the Age of Smart Machines*. Harper Business, New York.

Dujava, D., & Labáj, M. (2019). Economic growth and convergence during the transition to production using automation capital. Department of Economic Policy Working Paper 17. University of Economics in Bratislava.

Elsby, M.W.L., Hobijn, B., & Şahin, A. (2013). The decline of the US labor share. *Brookings Papers on Economic Activity*, 44, 1–63.

European Commission (2021). *2030 Digital Compass: the European way for the Digital Decade*, URL: https://commission.europa.eu/document/e2aa7a8e-f3d1-45d6-a115-bab5177f2dc2_en

Ford, M. (2015). *Rise of the Robots: Technology and the Threat of a Jobless Future.* Basic Books, New York.

Frey, C.B., & Osborne, M.A. (2017). The future of employment: How susceptible are jobs to computerisation? *Technological Forecasting and Social Change*, 114, 254–280. doi: 10.1016/j.techfore.2016.08.019

Gomes, O. (2019). Growth in the age of automation: Foundations of a theoretical framework. *Metroeconomica*, 70, 77–97. doi: 10.1111/meca.12229

Hémous, D., & Olsen, M. (2018). The rise of the machines: Automation, horizontal innovation and income inequality, Available at SSRN: http://dx.doi.org/10.2139/ssrn.2328774.

Jones, L.E., & Manuelli, R.E. (1990). A convex model of equilibrium growth: Theory and policy implications. *Journal of Political Economy*, 98, 1008–1038. doi: 10.1086/261717

Karabarbounis, L., & Neiman, B. (2014). The global decline of the labor share. *The Quarterly Journal of Economics*, 129, 61–103. doi: 10.1093/qje/qjt032

Nakamura, H., & Zeira, J. (2018). Automation and unemployment: Help is on the way. CEPR Discussion Papers 12974.

Prettner, K. (2019). A note on the implications of automation for economic growth and the labor share. *Macroeconomic Dynamics*, 23, 1294–1301. doi: 10.1017/S1365100517000098

Prettner, K. & Strulik, H. (2020). Innovation, automation, and inequality: Policy challenges in the race against the machine. *Journal of Monetary Economics*, 116, 249–265. doi: 10.1016/j.jmoneco.2019.10.012

Sachs, J.D., & Kotlikoff, L.J. (2012). Smart machines and long-term misery. NBER Working Papers 18629.

Zeira, J. (1998). Workers, machines, and economic growth. *The Quarterly Journal of Economics*, 113, 1091–1117. doi: 10.1162/003355398555847

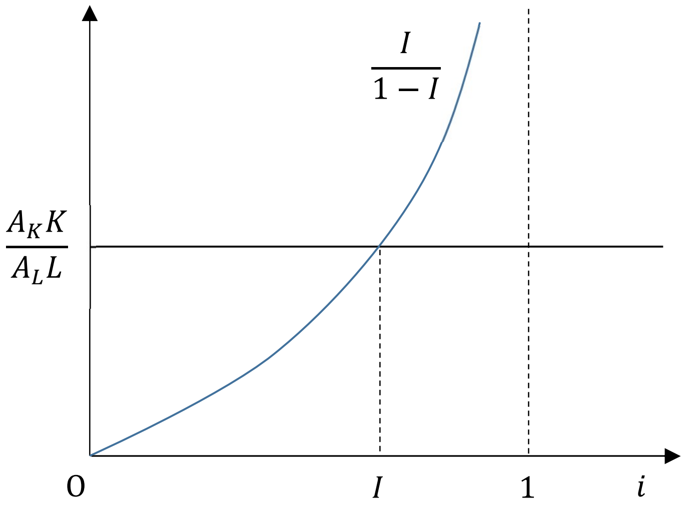

Figure 1: Relative effective factor endowment and equilibrium number of automated tasks

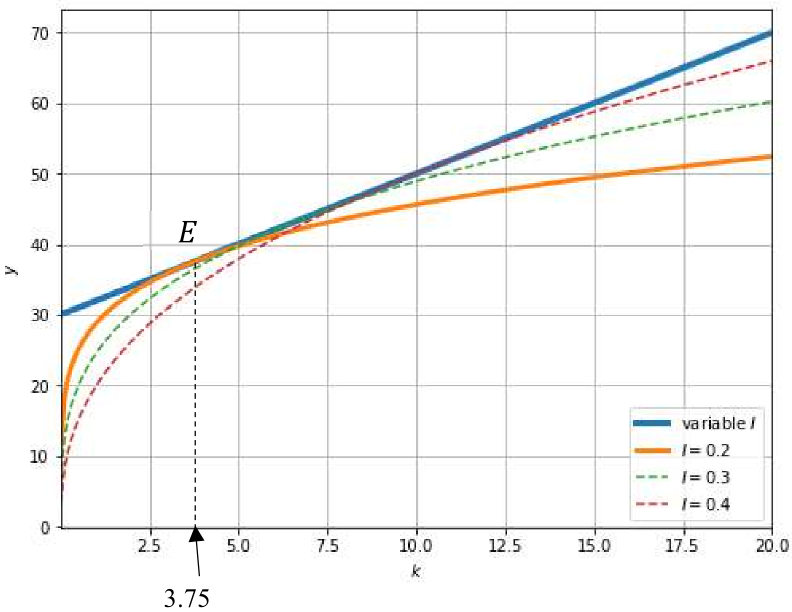

Figure 2: Per capita production functions in the baseline model ($A_K = 2$, $A_L = 30$)

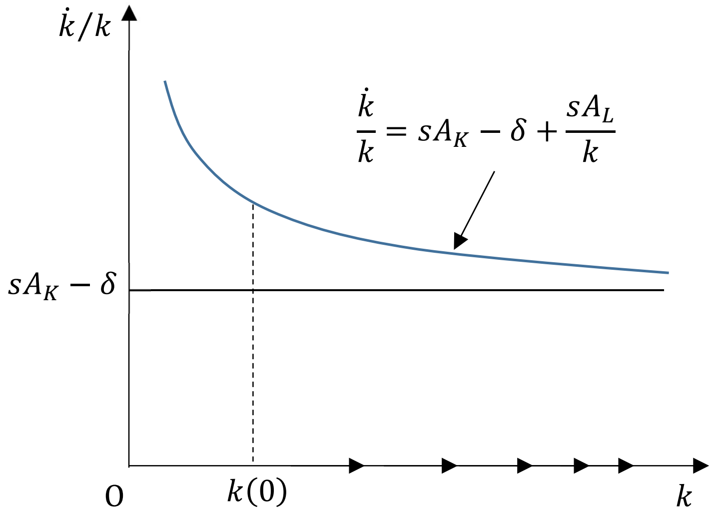

Figure 3: Transitional dynamics of the task-based Solow model

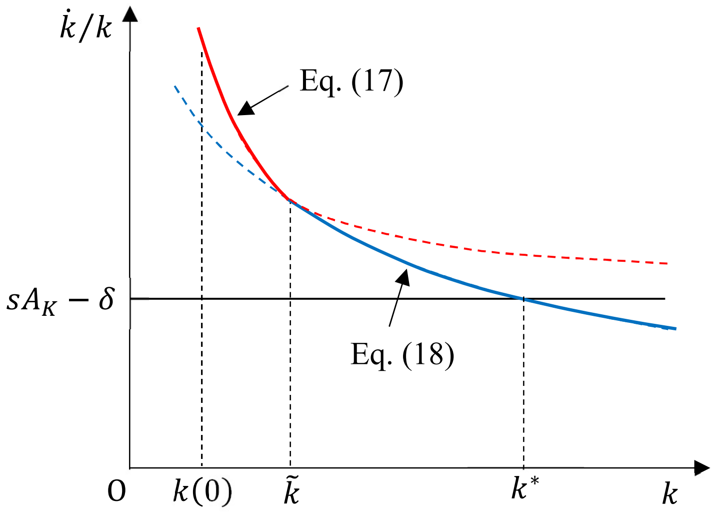

Figure 4: Transitional dynamics: a case in which there is an upper bound on the number of automated tasks

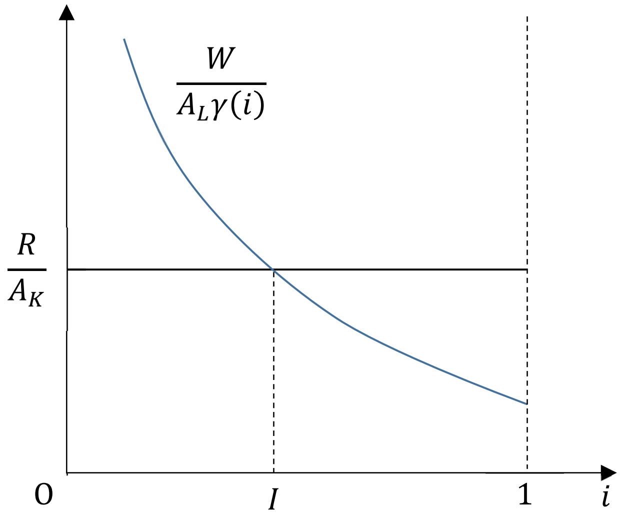

Figure 5: Determination of equilibrium number of automated tasks

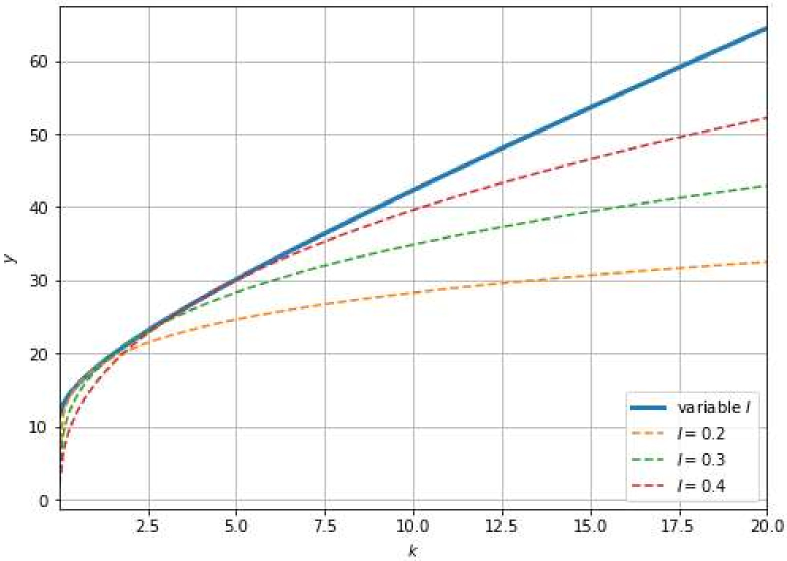

Figure 6: Per capita production functions in the extended model ($A_K = 2, A_L = 30, \gamma(i) = i$)