VIRTUAL MASS ASSOCIATED WITH MARINE PROPELLER BLADE VIBRATION

KUMAI, Toyoji
Research Institute for Applied Mechanics, Kyushu University : Professor

https://doi.org/10.5109/7172230

出版情報：Reports of Research Institute for Applied Mechanics. 19 (65), pp.283-288, 1972. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. XIX, No. 65, 1972

NOTE

VIRTUAL MASS ASSOCIATED WITH MARINE PROPELLER BLADE VIBRATION

By Toyoji KUMAI

The virtual mass of water in the flexural vibration of a planetary ellipsoid is applied to that of the marine propeller blade vibration as an approach. The theoretical results of the reduction factors are compared with that of model propeller experiments.

1. Introduction

Since it is a hard task to calculate the three-dimensional virtual mass of water surrounding vibrating marine propeller blade, so the three-dimensional reduction factor of a planetary ellipsoid which vibrating with similar distribution of pressure as that of the propeller blade is calculated as an approach in the present note. There are some mathematical investigations into the virtual mass of the circular plate with symmetric or antisymmetric mode of bending vibration in water[1)2)] by using different method from that of the present note. The results of the present calculation of the factor J versus span-breadth ratio of the blade are compared with that obtained from pressure measurements of vibrating blade of model propellers.

2. Pressure Distribution

The co-ordinates of the planetary ellipsoid or oblate spheroid are given by

$$\begin{aligned} x &= a\mu\zeta, \\ y &= a(1-\mu^2)^{\frac{1}{2}}(1+\zeta^2)^{\frac{1}{2}}\sin\omega, \\ z &= a(1-\mu^2)^{\frac{1}{2}}(1+\zeta^2)^{\frac{1}{2}}\cos\omega. \end{aligned} \tag{1}$$

The equation of continuity, in which using the above co-ordinates becomes

$$\frac{\partial}{\partial\mu}\left\{(1-\mu^2)\frac{\partial\phi}{\partial\mu}\right\}+\frac{\partial}{\partial\zeta}\left\{(1+\zeta^2)\frac{\partial\phi}{\partial\zeta}\right\}+\frac{\mu^2+\zeta^2}{(1-\mu^2)(1+\zeta^2)}\frac{\partial^2\phi}{\partial\mu^2}=0. \tag{2}$$

Professor, Research Institute for Applied Mechanics, Kyushu University, Fukuoka, Japan.

The solutions of the equation in tesseral harmonics[3] are written by

$$\phi = P_n^s(\mu)\, q_n^s(\zeta) \left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} s\omega. \tag{3}$$

The motion of the planetary ellipsoid is assumed to be of flexural vibration and whose pressure distribution is similar to that of the propeller blade vibration, in which the tip of the blade is coinside with a point of the peripheral of given ellipsoid as shown in Figure 1. After some trial computations,

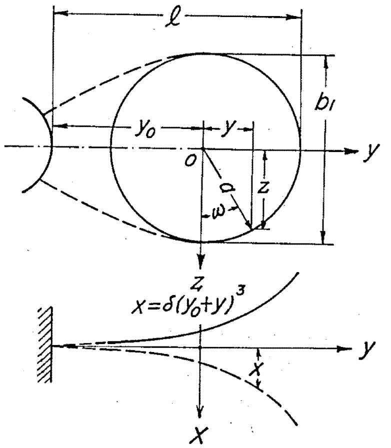

Fig. 1 Co-ordinate of planetary ellipsoid and propeller blade.

the flexural mode of vibration of the blade mentioned above is near to that assumed to be the cube of length co-ordinate that,

$$X = \delta(y_0 + y)^3. \tag{4}$$

Now, put $\eta = y/a$ and $\eta_0 = y_0/a$, where a is the radius of the ellipsoid, since the maximum breadth b_1 is approximately equals to $2a$, so the relation between η_0 and the span-breadth ratio l/b_1 is written by

$$\eta_0 = \frac{2l}{b_1} - 1. \tag{5}$$

The distribution of the velocity of fluid induced by the vibration of the ellipsoid is given by

$$U = \Omega(\eta_0 + \eta)^3 \frac{\partial x}{\partial \zeta} \cos \Omega t, \tag{6}$$

where, Ω is circular frequency. Since the velocity component in y-direction is considered to be small compared with normal velocity, the term of tangential flow is ignored as shown in the above equation.

The normal velocity at the surface of the ellipsoid given by (6) is represented by Legendre's P-functions that,

$$U=a\Omega\Big[(\eta_0^3+\frac{3}{5}\eta_0)P_1(\mu)-\frac{3}{5}\eta_0P_3(\mu)$$
$$+\Big\{(\eta_0^2+\frac{1}{7})P_2^1(\mu)-\frac{3}{70}P_4^1(\mu)\Big\}\sin\omega$$
$$-\frac{1}{10}\eta_0P_3^2(\mu)\cos 2\omega-\frac{1}{420}P_4^3(\mu)\sin 3\omega\Big]\cos\Omega t, \tag{7}$$

provided that the following relations are used in the coefficients of $\sin^2\omega$ and $\sin^3\omega$ respectively, for all the terms are to be the solutions of the equation of continuity, that

$$\mu(1-\mu^2)=\frac{2}{5}P_1(\mu)-\frac{2}{5}P_3(\mu),$$
$$\mu(1-\mu^2)^{\frac{3}{2}}=\frac{4}{21}P_2^1(\mu)-\frac{2}{35}P_4^1(\mu). \tag{8}$$

Since the thickness-breadth ratio of the blade considered is so small compared with uity, so the ellipsoid may be replaced by a circular disc, that is presented by $\zeta=\zeta_0\to 0$ in the above equation.

As the boundary condition at the surface $\zeta=\zeta_0$, the velocity U should be given by

$$U=\left(\frac{\partial\phi}{\partial\zeta}\right)_{\zeta=\zeta_0}, \tag{9}$$

so that, the velocity-potential satisfied the above condition is written by

$$\phi=a\Omega\Big[(\eta_0^3+\frac{3}{5}\eta_0)P_1(\mu)\frac{q_1(\zeta)}{[q_1(\zeta_0)]'}-\frac{3}{5}\eta_0P_3(\mu)\frac{q_3(\zeta)}{[q_3(\zeta_0)]'}$$
$$+\Big\{(\eta_0^2+\frac{1}{7})P_2^1(\mu)\frac{q_2^1(\zeta)}{[q_2^1(\zeta_0)]'}-\frac{3}{70}P_4^1(\mu)\frac{q_4^1(\zeta)}{[q_4^1(\zeta_0)]'}\Big\}\sin\omega$$
$$-\frac{1}{10}\eta_0P_3^2(\mu)\frac{q_3^2(\zeta)}{[q_3^2(\zeta_0)]'}\cos 2\omega-\frac{1}{420}P_4^3(\mu)\frac{q_4^3(\zeta)}{[q_4^3(\zeta_0)]'}\sin 3\omega\Big]\cos\Omega t. \tag{10}$$

where, dash denote the derivative with respect to ζ, and q-functions are calculated as follows[3)]

$$q_1(\zeta)=1-\zeta\cot^{-1}\zeta,$$
$$q_2(\zeta)=\frac{1}{2}(3\zeta^2+1)\cot^{-1}\zeta-\frac{3}{2}\zeta,$$
$$q_3(\zeta)=-\Big\{\frac{1}{2}(5\zeta^3+3\zeta)\cot^{-1}\zeta-\frac{1}{2}(5\zeta^2+3)\Big\},$$
$$q_4(\zeta)=\frac{1}{4}\Big(\frac{35}{2}\zeta^4+15\zeta^2+\frac{3}{2}\Big)\cot^{-1}\zeta-\frac{5}{4}\Big(\frac{7}{2}\zeta^3+3\zeta\Big),$$
$$q_n^s(\zeta)=(1+\zeta^2)^{\frac{s}{2}}\frac{d^s}{d\zeta^s}q_n(\zeta). \tag{11}$$

The pressure distribution is obtained from the above velocity-potential, that

$$p=\rho\left(\frac{\partial\phi}{\partial t}\right)_{\zeta=\zeta_0\to 0}. \tag{12}$$

The spanwise distribution of the pressure is shown provided that $\omega=\pm\pi/2$ and the chordwise that $\omega=0$ and π in equation (10). The spanwise distribution in the case $l/b_1=1.25$ is shown in Figure 2 comparing with that of measurements as an example.

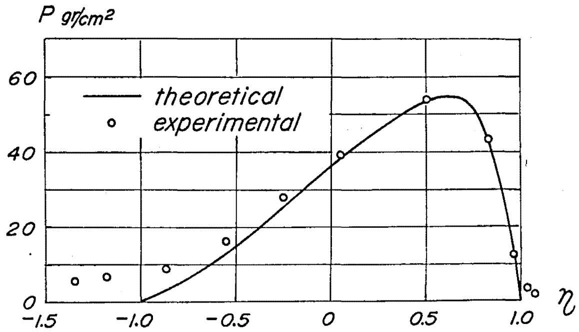

Fig. 2 Spanwise distribution of pressure along centre line of the blade.

3. Reduction Factor

The three-dimensional kinetic energy of the water is calculated by the use of the velocity-potential mentioned above, that

$$2T=-\rho a\int_{-1}^{1}\int_{0}^{2\pi}\left(\phi\frac{\partial\phi}{\partial\zeta}\right)_{\zeta=\zeta_0}d\omega d\mu. \tag{13}$$

in which,

$$\int_{-1}^{1}[P_n^s(\mu)]^2 d\mu=\frac{2}{2n+1}\frac{(n+s)!}{(n-s)!},$$

and $$\int_{-1}^{1}P_m^s(\mu)P_n^s(\mu)d\mu=0, \qquad m\neq n.$$

On the other hand, the two-dimensional energy of the water in the vibration of the circular disk with the same mode as mentioned above is obtained from the following equation;

$$2T_{\text{II}}=\rho\pi\Omega^2 a\int_{-1}^{1}z^2(\eta_0+\eta)^6 d\eta, \tag{14}$$

where, $$z^2=a^2(1-\eta^2).$$

From the above equations (13) and (14), the three-dimensional reduction factor J under consideration is thus obtained, that

$$J=\frac{T}{T_{\text{II}}}=\frac{2}{\pi}\cdot\frac{\frac{1}{3}\eta_0^6+\frac{4}{5}\eta_0^4+\frac{46}{175}\eta_0^2+\frac{43}{1225}}{\frac{1}{3}\eta_0^6+\eta_0^4+\frac{3}{7}\eta_0^2+\frac{1}{63}}. \tag{15}$$

In the above expression, since η_0 is replaced by span-breadth ratio l/b_1 from equation (5), so J-values are obtained for various values of l/b_1. The experiment results obtained from five model propellers with different l/b_1 are compared with that calculated by the present theoretical approach. Both results are shown fairly good agreement as seen in Figure 3.

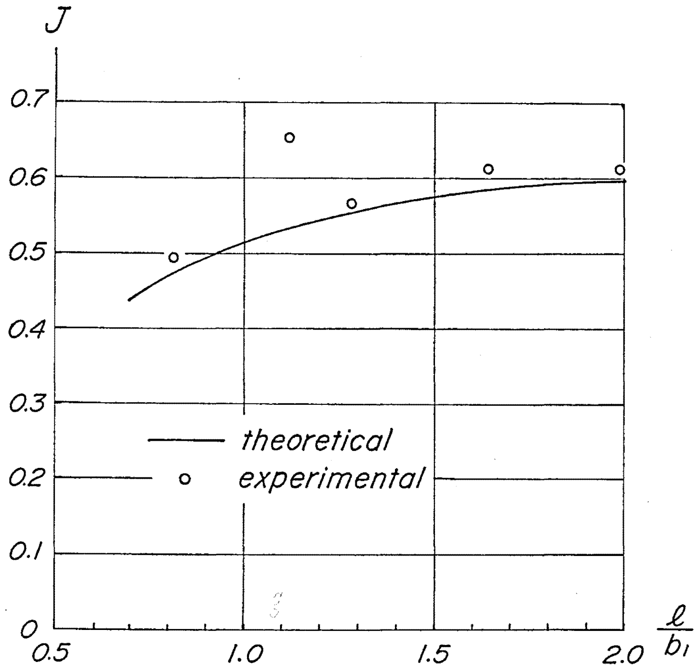

Fig. 3 Three-dimensional reduction factor of the propeller blade vibration.

4. Ratio of Natural Frequencies of Blade Vibration in Water and in Air

The natural frequency of the propeller blade in water f_w may be appraised from that in air f_a using the ratio of the energy of the blade vibration in water to that in air ε, that

$$\frac{f_w}{f_a}=\frac{1}{(1+\varepsilon)^{\frac{1}{2}}}, \tag{16}$$

provided that

$$\varepsilon=\frac{J\int_0^1 m_e\xi^2 d\eta_1}{\int_0^1 m_1\xi^2 d\eta_1}=\frac{J\pi\rho b_0}{4c_a\rho t_0}\gamma ,$$

$$\gamma=\frac{\int_0^1\left(\frac{b}{b_0}\right)^2\xi^2 d\eta_1}{\int_0^1\left(\frac{bt}{b_0t_0}\right)\xi^2 d\eta_1},$$

where, m_e and m_1 denote two-dimensional virtual mass and mass of blade both in unit length of the blade respectively, γ is evaluated value proportional to

b_1/b_0 that $\gamma=2.61 b_1/b_0$ for the range $1\leq b_1/b_0\leq 2$ in general type of existing marine propeller[6], η_1 is the length co-ordinate with the origine at the root of the blade, ξ is the mode of vibration of the blade, b_0 and t_0 are breadth and the maximum thickness of the blade at $\eta_1=0$ respectively, and c_a is the area coefficient of the cross section of the blade.

The numerical values except ρ/ρ_1 in the above terms are assumed to be almost constants or that to be estimated for given value of b_1/b_0 and l/b_1 including J-value. Now, we put $l/b_1=1.25$, $b_1/b_0=8/5$, $b_0/t_0=5$, $c_a=0.75$ and $\rho/\rho_1=1/7.5$ as an example of the ideal propeller blade of a cargo ship, J takes 0.55 from the present theory and the ratio of energy and of frequencies are obtained, that

$$\varepsilon=1.64 \quad \text{and} \quad \frac{f_w}{f_a}=0.60 \quad . \tag{17}$$

The frequency ratios in actual ship's propellers measured by some researchers[4),5)] are shown almost nearly to the above value.

The empirical expression of the theoretical results of J-value is shown by

$$J=1-0.485\left(\frac{b_1}{l}\right)^{\frac{1}{3}}. \tag{18}$$

The approach will be conveniently used in practice.

5. Conclusions

The three-dimensional reduction factor, J-value of the virtual mass associated with vibrating planetary ellipsoid considered a circular disk is calculated and the results are applied to that of the marine propeller blade. The theoretical values are confirmed by the results of measurements of pressure of the vibrating blade of model propellers. The reduction factors under consideration show $J=0.5$ to 0.6 for span-breadth ratio 1.0 to 2.0 of the propeller blade respectively. The thoretical results of J-values are also confirmed by the experiments. It is to be noted that the J-value is considerably small, however the effect of span-breadth ratio of the marine propeller blade upon J-value is not so conspicuous.

References

1) Lamb, H., On the Vibration of an Elastic Plate in Contact with Water, Proc. Roy. Soc. London, Series A, Vol. XCVIII, March 1921, pp. 205-216.
2) Mclachlan, N. W., The Accession to Inertia of Flexible Discs Vibrating in a Fluid, Proc. Phys. Soc. Vol. 44, Sept. 1932, pp. 546-555.
3) Lamb, H., *Hydrodynamics*, Cambridge, 1932, pp. 142-146.
4) Schwanecke, H., Untersuchung einiger aufgetretener Propellerschäden. Schiff u Hafen, Heft 3, 1970.
5) Castanié, B. et Osouf, J., Vibrations des pales d'hélic Résultats expérimentaux. advance copy, Assoc. Tech. Mar. et Aero. 1971.
6) Kumai, T., On the Natural Frequencies of Propeller Blade Vibration. under preparation to publish.

(Received January 10, 1972)