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ON THE MOTION OF MULTIHULL SHIPS IN WAVES (II)

By Makoto OHKUSU* and Mikio TAKAKI**

This paper is the second report on the motion of multihull ships in waves and is especially concerned with the theoretical calculation of the seakeeping qualities of twin hull ships in head seas and beam seas. The theoretical results obtained by using the method which was developed in the first paper in order to compute added mass etc. of twin hull cylinders and by applying Strip method were found to be satisfactory in comparison with the experimental results.

The forces acting upon twin hull ship when it ran in waves were also calculated.

1. Introduction

Recently the merits and demerits of twin hull ship have been discussed from many points of view.[1)~3)] The twin hull ship does not seem to be able to get an economical advantage over the ordinary mono hull ship. The former, however, has several merits in its utility. Oceanographical research ships and floating stations for ocean development having twin hull have been recently built for such merits.

The research of twin hull ship has been carried out chiefly upon its resistance on calm sea and it is because the merit of the catamaran has been looked for from economical stand point. In the case of the research ships or the floating stations, however, the seakeeping qualities are more important. There are a few examples[4)~5)] of the studies upon the motion of catamaran in waves and these studies conducted so far are not necessarily based upon the modern ship motion theory which has been successfully applied to the motion of the ordinary ship and they neglect the hydrodynamic interaction between two hulls of twin hull ships.

In this paper we calculated the damping force, added mass of twin hull cylinders oscillating on the surface of a fluid and the wave exciting force upon these cylinders due to an incident wave by the method which was developed in the first paper[6)] on the motion of multihull ships in waves and which took the interaction effect into account as a matter of course, and solved the equations

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of the motions of catamaran in head seas and beam seas which could be formulated by using the damping force, added mass thus obtained of twin hull cylinders. In addition the solutions were compared with the measurements on model tests.

The forces exerted upon the twin hull ships moving in waves were also calculated.

2. The Motion of Twin Hull Ship in Head Sea

2.1. Comparison of theory and experiment

In the first paper[6] on the motions of multihull ships in waves one of the authors reported the method by which we can compute comparatively easily the hydrodynamic force exerted upon multiple 2-dimensional cylinders when they are oscillating on the surface of a fluid or they are in waves.

We can predict theoretically the motions of a catamaran in head seas by applying the method to the calculation of added mass and damping force etc. of its each cross section and with the aid of Strip method. It may be safely said that the accuracy of this prediction is not less satisfactory than it is in the motion of ordinary mono hull ships.

Two twin hull ship models were selected for our study (it will be called *TW1* and *TW2* hereafter). These two models are composed of two mono hulls with the form longitudinally symmetrical with respect to midship as shown in Fig. 1. The principal dimensions of these models and their component mono hull are given in Table 1. *TW1* and *TW2* are catamarans with $2p/T=3$ and 5 respectively where $2p$ is the distance between the longitudinal centers of their

Table Particulars of Models

Description	Single-hull	*TW 1*	*TW 2*
$2P/T$	—	3.00	5.00
Length between perpendiculars (L_{pp})	4.00 m	4.00 m	4.00 m
Breadth (B)	0.36 m	0.90 m	1.26 m
Draft (T)	0.18 m	0.18 m	0.18 m
Displacement (∇)	0.207m^3	0.414m^3	0.414m^3
Radius of longitudinal gyration (R_φ/L_{pp})	0.262	0.262	0.262
Radius of transverse gyration (R_θ/T)	—	1.772	2.514
Metacentric height (GM/T)	—	1.950	6.272
Height of center of gravity from keel (KG/T)	1.336	1.336	1.336

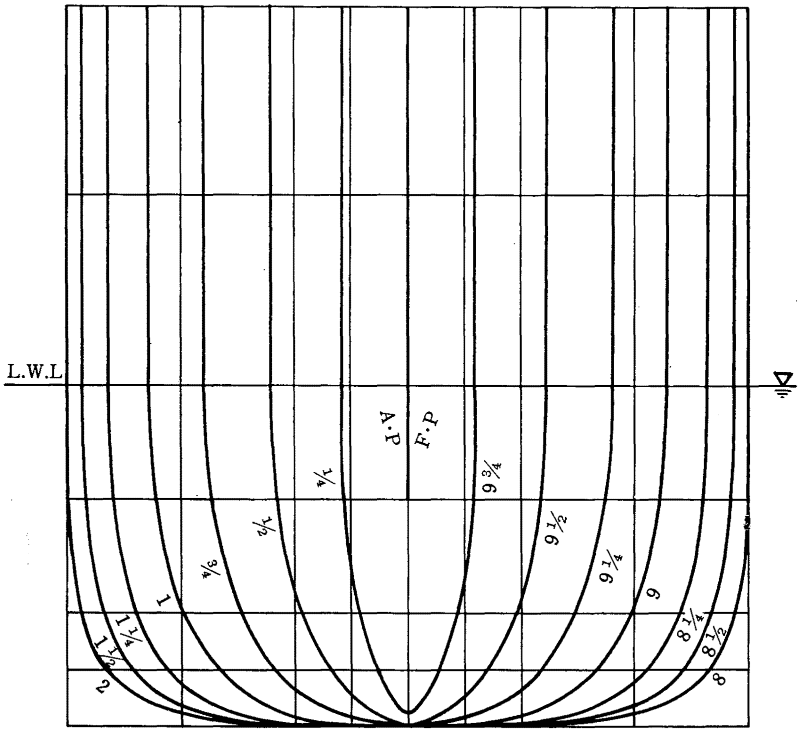

Fig. 1 Body plan

component mono hulls and T is the draft.

The heaving and pitching motions of twin hull ships *TW1* and *TW2* advancing in regular head waves were theoretically calculated along the afore-mentioned course, that is, to compute added mass and damping force of each cross section of the ships by our method and to apply the Strip method for connecting these two dimensional results to get the equations of the motions of the ships in head seas. In order to compute the added mass etc. of two-dimensional twin hull cylinder by the method described in the first report it is necessary to solve previously the so-called radiation problem of heaving, rolling and swaying motions of one cylinder which composes the twin hull cylinder. Then each cross section of the mono hull cylinder composing *TW1* and *TW2* was approximately expressed by a Lewis form and the radiation problem of this section was solved by Tasai method.[8]

The equations of the motions of twin hull ship in head seas can be formulated as follows[9] by the Strip method, where a coordinate system is taken as shown in Fig. 2.

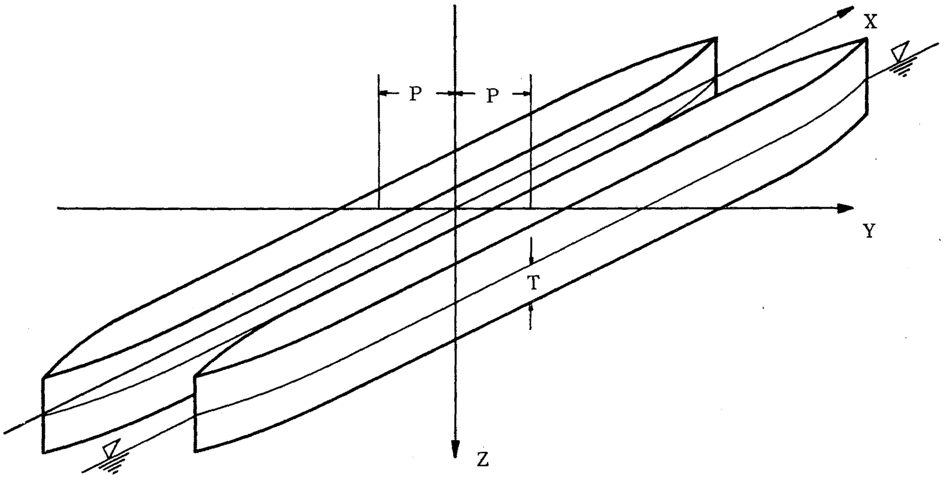

Fig. 2 Coordinate system

$$(m+a)\ddot{Z}_G + b\dot{Z}_G + cZ_G + d\ddot{\theta} + e\dot{\theta} + g_1\theta = Fe^{i\omega_e t}, \tag{1}$$

$$(J_\theta + A)\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{Z}_G + E\dot{Z}_G + G_1 Z_G = Me^{i\omega_e t}, \tag{2}$$

$$\left.\begin{aligned}
a &= \int_{L_a}^{L_f} \rho S m_H dx, \quad b = \int_{L_a}^{L_f} n_H dx, \quad c = \rho g A_W, \\
d &= \int_{L_a}^{L_f} \rho S x m_H dx, \quad e = \int_{L_a}^{L_f} (n_H x - V\rho S m_H) dx, \\
g_1 &= \int_{L_a}^{L_f} (\rho g \cdot 2bx - n_H V) dx, \quad A = \int_{L_a}^{L_f} x^2 \rho S m_H dx, \\
B &= \int_{L_a}^{L_f} \left(x^2 n_H + \left(\frac{V}{\omega_e}\right)^2 n_H \right) dx, \quad C = -\int_{L_a}^{L_f} V^2 \rho S m_H dx + \rho g I_W, \\
D &= \int_{L_a}^{L_f} x \rho S m_H dx, \quad E = \int_{L_a}^{L_f} (n_H x + V\rho S m_H) dx, \\
G_1 &= \int_{L_a}^{L_f} \rho g \cdot 2bx dx, \\
F &= -\zeta_a \int_{L_a}^{L_f} [(\rho g \cdot 2b - \omega \cdot \omega_e \rho S m_H) + i\omega n_H] e^{-KTm+iKx} \cdot dx, \\
M &= \zeta_a \int_{L_a}^{L_f} \Big[(\rho g \cdot x \cdot 2b - x\omega \cdot \omega_e \cdot \rho S m_H) - i(x\omega n_H - V\omega \rho S m_H) \Big] \\
&\quad \times e^{-KTm+iKx} dx.
\end{aligned}\right\} \tag{3}$$

where L_a and L_f are the x coordinate of the fore and aft end of the ship,
m: mass of the ship,
J_0: londitudinal mass moment of inertia of the ship,
A_W: water plane of area of the ship,
I_W: londitudinal moment of the water plane area about the center of floatation.
S is the area under the water and $2b$, T, and T_m are the breadth, the draft and half of the draft of a section at x of the ship. m_H and n_H are added mass and damping coefficient of this section. The ship runs in regular head waves of an elevation $\zeta_a e^{i(\omega t+Kx)}$ with advance speed V where K is ω^2/g and $\omega_e=\omega+KV$ and ρ, g are the density of a fluid and gravity acceleration. Z_G is the heavng displacement of the center of gravity and θ the pitching angle (bow down is taken to be positive). It is of course that the equations have the same form as those of ordinary ship, but m_H, n_H of each section at the coordinate x are calculated as the values of twin hull cylinder by our method of the first paper.

The amplitudes of heaving and pitching motions Z, Θ of *TW1* and *TW2* at the Froude number $F_n=0$ and 0.1 which were obtained by solving the equations (1), (2) and (3) are shown in Fig. 3, 4, 5 and 6 as theoretical values. In these figures the amplitudes Z, Θ are nondimensionalized by wave amplitude ζ_a and maximum wave slope $K\zeta_a$ respectively and λ, L are the wave length and the ship length. And the curves named as single hull in these figures give the amplitudes of the motions of the mono hull ship which is a constituent element of our twin hull ships. They were calculated by putting m_H, n_H etc. of this mono hull into the equations (1), (2), and (3). If we neglect the interaction effect between

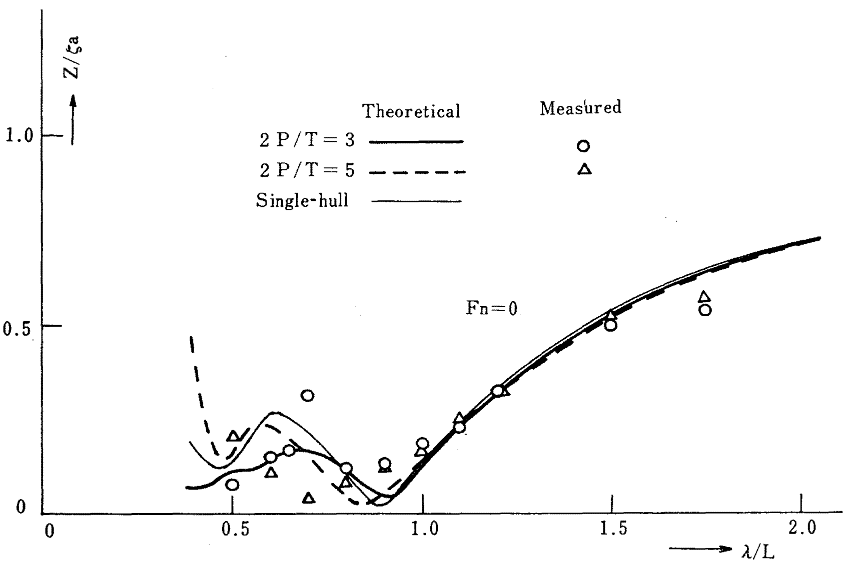

Fig. 3 Heaving motion of twin hull ships in head seas ($F_n=0$)

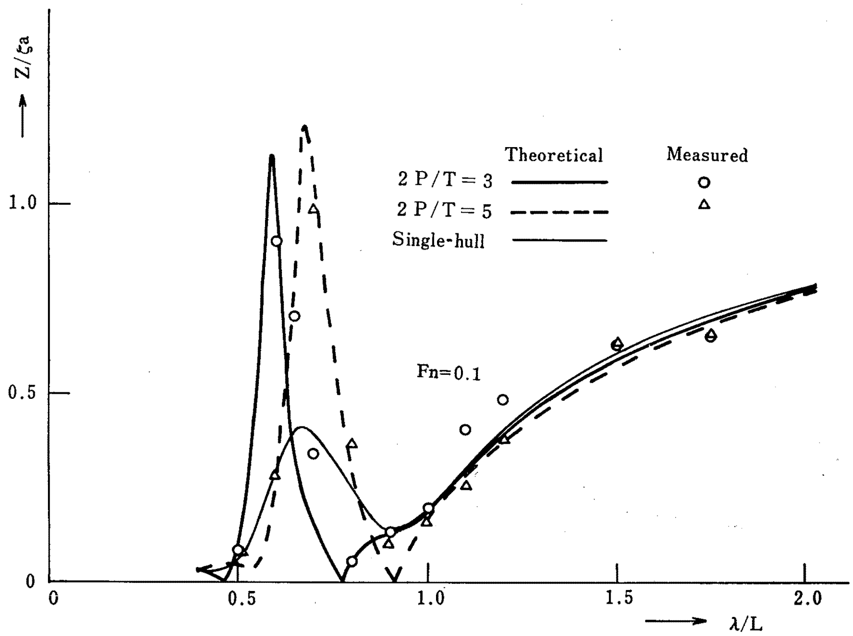

Fig. 4 Heaving motion of twin hull ships in head seas (F_n=0.1)

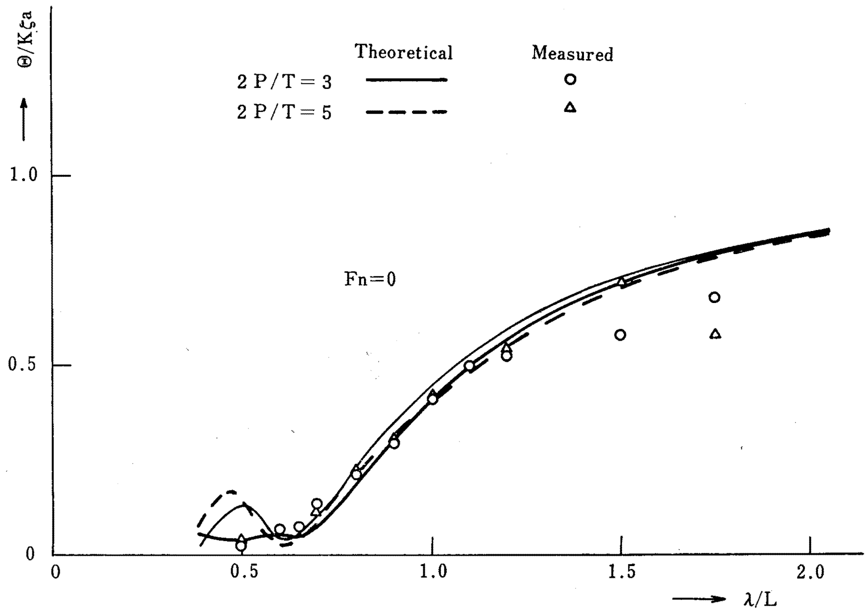

Fig. 5 Pitching motion of twin hull ships in head seas (F_n=0)

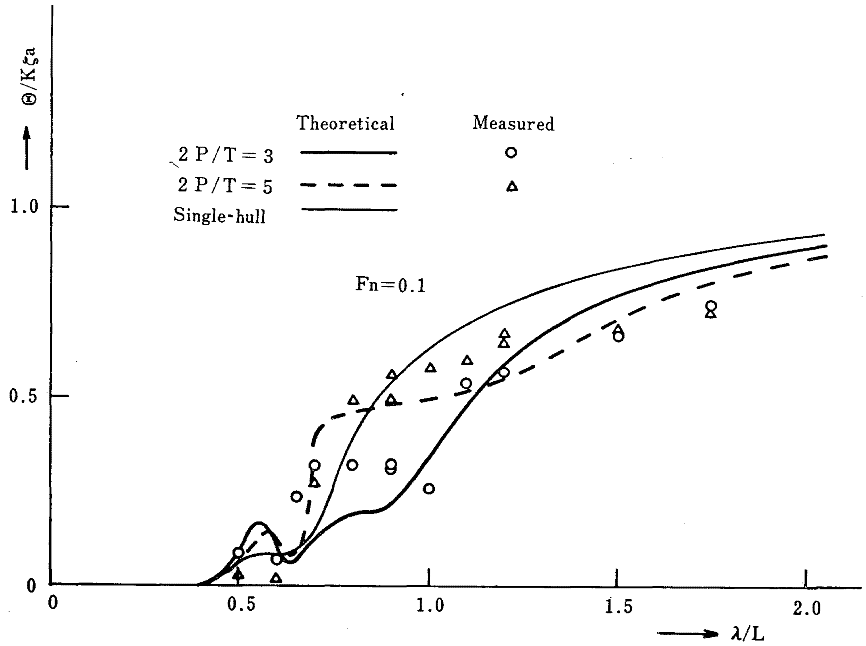

Fig. 6 Pitching motion of twin hull ships in head seas (F_n=0.1)

two mono hulls of *TW1* and *TW2*, we can consider the values of the single hull curves as the amplitude of the motions of *TW1* and *TW2*. From these comparison it is found that there is comparatively large difference between the motion amplitudes of catamarans with and without taking the interaction effect into account if F_n is not zero.

The experiments concerning the motions of our two catamaran models *TW1* and *TW2* in head waves were carried out at the large tank of Research Institute of Applied Mechanics, Kyushu University.

In our experiments the amplitudes of regular head waves generated by the wave maker at the end of the tank were made to be constant at any wave length, and they were measured at a fixed point in the tank before each experiment started. The apparatus used in our experiments are shown in Fig. 7, and heaving displacement and pitching angle are measured by potentio meters. The models are run by a propeller driven by a motor *M* and this propeller is not behind the after perpendicular of the models but between two hulls.

In our experiments some remarkable phenomena were observed. One is that the height of waves between two hulls caused by their motions and the incident waves is rather large even when their heaving motion is small. The others are that at a certain wave length the motion amplitudes make comparatively large change with a little variation of the wave length and that the measured amplitude of the motions does not become stationary but varies with

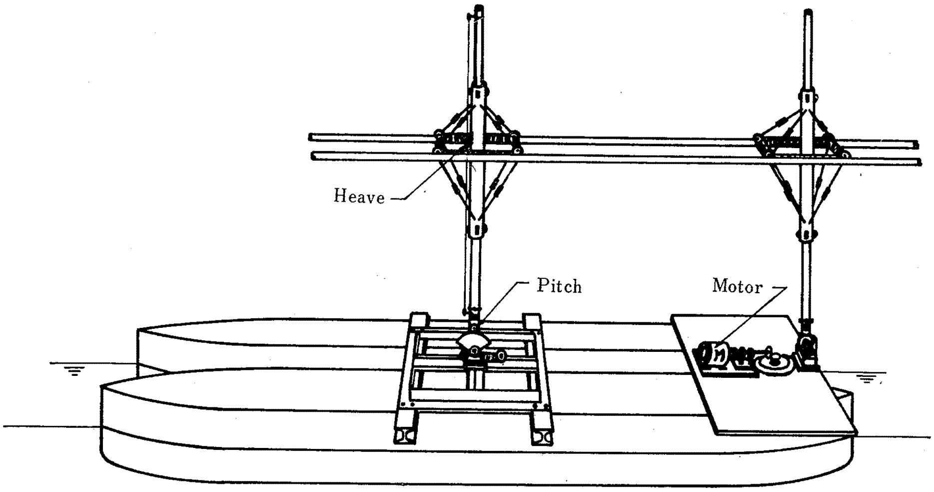

Fig. 7 Ship model and experimental apparatus

a time like a phenomenon of so-called "beat". The reason why this beat phenomenon occurs seems to be that the incident wave is not a sinusoidal wave with one component of frequency but includes other components of small amplitude or that a small drift speed of the models due to head waves causes a small variation in the frequency of encounter and as a result a change of motion amplitude because of the characteristic of catamarans, that is, the large variation of motion with the small change of wave frequency at some frequency. But it is not confirmed now.

It can be concluded from the comparison of theoretical and measured heaving motion in Fig. 3 and 4 that both are in good agreement. The heaving motion of twin hull ships *TW1* and *TW2* is very large compared with that of ordinary mono hull ships at resonance period and it is because the damping force of heaving motion of the twin hull ships become smaller than that of the mono hull ships at this period. When we neglect the interaction effect between two hulls, theory does not predict this large motion.

In the case of pitching motions of Fig. 5 and 6, there is some difference between theoretical and measured amplitude, though the theoretical values calculated by taking into account the ineraction effect are better. This difference seems to increase when the advance speed of the ships becomes larger.

2.2. The force exerted upon twin hull ship in head seas.

The wave bending moment at an arbitrary section of twin hull ship in head seas which is caused by the motion and the wave force can be also calculated[9] by applying our procedure to solve the radiation problem of twin hull cylinder.

Some examples of the moment at midship of *TW1* and *TW2* are compared in Fig. 8, where F_n is 0.1. The moment upon the twin hull ships when the hydrodynamic interaction between two hulls of the ships is not taken into account is also shown as single hull curve. It is, of course, twice of that which acts upon the midship of one element hull of *TW1* and *TW2* when the element hull runs in head seas as a mono hull ship.

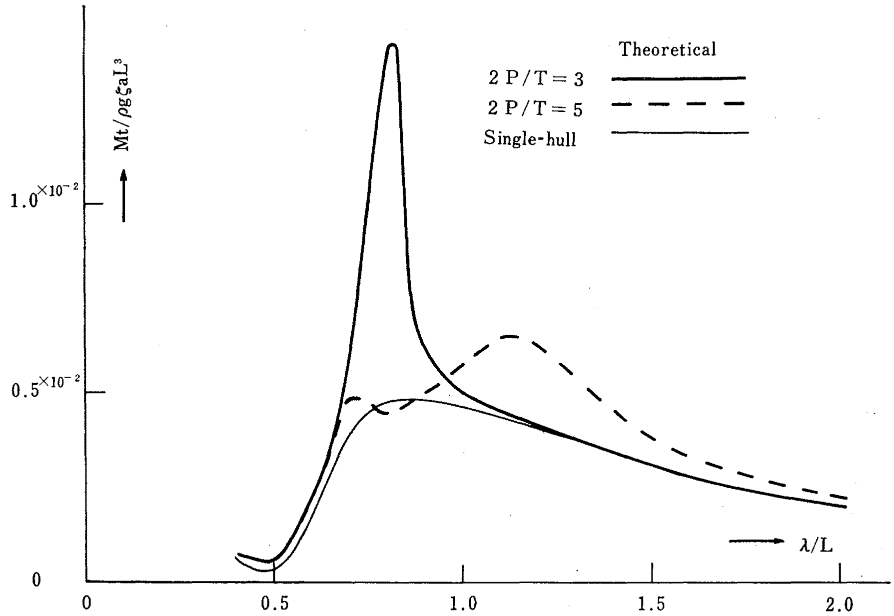

Fig. 8 Wave bending moment of twin hull ships in head seas (F_n=0.1)

It can be concluded from Fig. 8 that it is sometimes dangerous to consider the wave bending moment upon twin hull ship as only twice of those values of one mono hull ship which constitutes the twin hull ship.

As stated in the first paper when a twin hull cylinder as shown in Fig. 9 makes a heaving oscillation, a progressing wave ζ_R given approximately by the equaion (4) comes upon the left cylinder. And also a wave ζ_L with the same amplitude and the same phase as ζ_R but progressing rightward comes upon the right cylinder.

$$\zeta_R = \frac{\bar{A}_2 e^{i(\varepsilon_2+KP)}}{1-iH^+(KT)e^{-i2KP}} e^{i(\omega t+Kx)}, \tag{4}$$

$$\zeta_L = \frac{\bar{A}_2 e^{i(\varepsilon_2+KP)}}{1-iH^+(KT)e^{-i2KP}} e^{i(\omega t-Kx)}, \tag{5}$$

where

$$H^{\pm}(KT) = ie^{i\varepsilon_2}\cos\varepsilon_2 \mp e^{i\varepsilon_1}\sin\varepsilon_1. \tag{6}$$

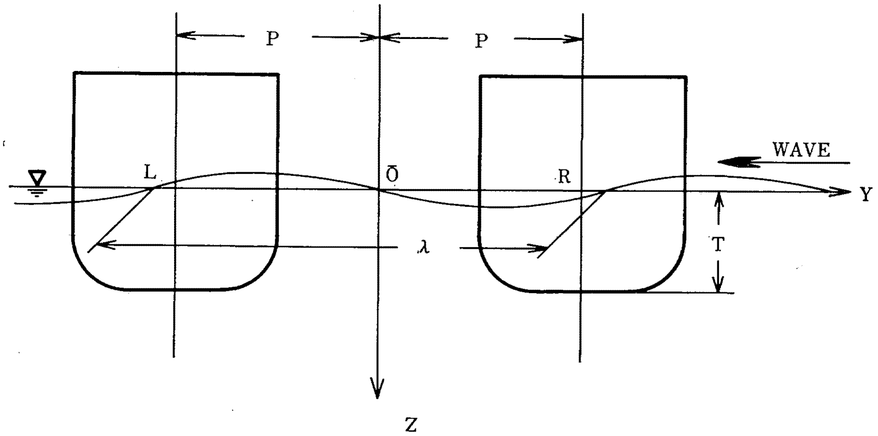

Fig. 9 Coordinate system

and $\overline{A}_j$ and ε_j are the amplitude and the phase of progressing wave at infinity when one cylinder of the twin cylinders makes j oscillation about the origin L or R ($j=1$ is swaying $y=e^{i\omega t}$ and $j=2$ is heaving $z=e^{i\omega t}$) without another cylinder.

Accordingly each cylinder receives a hydrodynamic force in Y direction and a moment due to these incident waves respectively. The forces and the moments upon the right and left cylinder have the same amplitude and the opposite direction with each other. The force of Y direction upon the right cylinder is given by

$$\frac{\rho S}{2} m_{HS}(-\ddot{z}) + \frac{\rho g^2}{\omega^3} n_{HS}(-\dot{z}) = -i\frac{\rho g}{K}\overline{A}_1 e^{i\varepsilon_1}\frac{\overline{A}_2 e^{i(\varepsilon_2+\omega t)}}{1-iH^+(KT)e^{-2KP}}. \tag{7}$$

It is to be supposed immediately that the similar force and moment act upon each mono hull of twin hull ships *TW1* and *TW2* when they run in head waves.

The formulation[9] to calculate vertical force on a cross section of a ship running in head waves which is, of course, based on Strip method can be also applied to the calculation of such transverse force or moment upon each mono hull of twin hull ship in head seas.

In this application the terms of inertia force and static water pressure force which appear in the equation of the vertical force must be removed, because these forces do not act in the transverse direction.

The force F of Y direction upon x section of the right hull of a twin hull ship running in head waves with advance speed V is expressed as

$$F=\left[\rho g^2\frac{V}{\omega_e^5}n_{HS}+x\rho\frac{S}{2}m_{HS}\right]\ddot{\theta}+\left[x\frac{\rho g}{\omega_e^3}n_{HS}-V\frac{\rho S}{2}m_{HS}-V\frac{\rho}{2}\cdot\frac{\partial}{\partial x}(xSm_{HS})\right]\dot{\theta}$$

$$+V^2\frac{\rho}{2}\frac{\partial}{\partial x}(S\cdot m_{HS})\theta+\frac{\rho S}{2}m_{HS}\ddot{z}_G+\left[\frac{\rho g^2}{\omega_e^3}n_{HS}-V\frac{\rho}{2}\frac{\partial}{\partial x}(S\cdot m_{HS})\right]\dot{z}_G$$

$$-\left[\left(\frac{\rho g^2}{\omega_e^3} n_{HS} - V\frac{\rho}{2}\frac{\partial}{\partial x}(S\cdot m_{HS})\right) i\omega - \frac{\rho S}{2} m_{HS}\omega\omega_e\right] \zeta_a e^{KTm+i(Kx+\omega et)} \tag{8}$$

Fig. 10 shows the total of such forces acting upon the right hull of *TW2* at $F_n=0.1$ in the non-dimensionalized form, where ζ_a is the amplitude of head waves. Such force or moment will have important meaning when it is necessary to know the stress at the bridge structure of a twin hull ship.

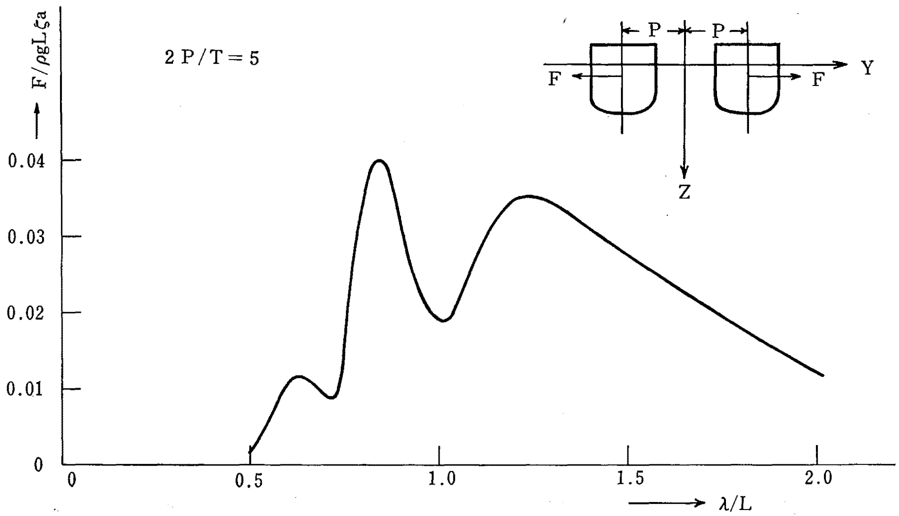

Fig. 10 Transverse force of twin hull ships in head seas (F_n=0.1)

3. The Motion of Twin Hull Ship in Beam Seas

3.1. The equations of motions

The added mass and damping coefficient of twin hull cylinders calculated by our method can be also used for formulating the equations of motions of twin hull ship in beam seas. The equations are formulated by integrating with respect to x the hydrodynamic forces (added mass and wave exciting force etc.) upon cross sections of the ship which are considered to be 2-dimensional.

Added mass (moment) coefficients m_j (j=1, 2, 3) and M, N of each section are calculated, where m_1 and m_2 are added mass coefficients of swaying and heaving motion of twin hull cylinder shown in Fig. 9, and are defined to be added mass divided by ρS (S is sectional area under the water of the twin hull cylinder) and m_3 is a coefficient of rolling motion given by a moment around the origin O divided by ρST. M and N are also defined by the following equation

$$f=\rho SM(-\ddot{y})+\frac{\rho g^2}{\omega^3}N(-\dot{y}), \tag{9}$$

where f is a moment about O due to hydrodynamic force when the cylinder makes a swaying motion $y=e^{i\omega t}$.

In addition we can calculate twin hull cylinder's $\overline{A}_j$ and ε_j by which the wave at $y=+\infty$ is expressed as $\overline{A}_j e^{i(\omega t-Kx+\varepsilon_j)}$ when the cylinder makes the oscillation j ($j=1$, 2 mean the swaying and the heaving motion, and $j=3$ is the rolling motion about the origin O). Accordingly the wave exciting force when a plane wave with the elevation $e^{i(\omega t+Ky)}$ comes upon the twin hull cylinder from rightward as shown in Fig. 9 can be exactly derived by Haskind-Bessho relation.[7] These wave forces (moment) are expressed by

$$i\frac{\rho g}{K}\overline{A}_j e^{i\varepsilon_j} e^{i\omega t}, \tag{10}$$

where $j=1$ and 2 mean the forces in x and y direction, and $j=3$ the moment around the origin O.

Then the equations of motions of the center of gravity G in beam seas $e^{i(\omega t+Ky)}$ are given by

$$\rho\ddot{z}\int_{L_a}^{L_f}(1+m_2)\,Sdx+\dot{z}\frac{\rho g^2}{\omega^3}\int_{L_a}^{L_f}\overline{A}_2{}^2dx+z\cdot\rho g\int_{L_a}^{L_f}2bdx$$

$$=\frac{i\rho g}{K}e^{i\omega t}\int_{L_a}^{L_f}\overline{A}_2 e^{i\varepsilon_2}dx,, \tag{10}$$

$$\rho\ddot{y}\int_{L_a}^{L_f}(1+m_1)\,Sdx+\dot{y}\frac{\rho g^2}{\omega^3}\int_{L_a}^{L_f}\overline{A}_1{}^2dx+\rho\,\ddot{\theta}\int_{L_a}^{L_f}ST\left(M+\frac{l_G}{T}m_1\right)dx$$

$$+\dot{\theta}\frac{\rho g^2}{\omega^3}\int_{L_a}^{L_f}T\left(N+\frac{l_G}{T}\overline{A}_1{}^2\right)dx=\frac{i\rho g}{K}e^{i\omega t}\int_{L_a}^{L_f}\overline{A}_1 e^{i\varepsilon_1}dx, \tag{11}$$

$$\ddot{\theta}\left\{\rho r_0{}^2\int_{L_a}^{L_f}Sdx+\rho\int_{L_a}^{L_f}ST^2\left[m_3+\left(\frac{l_G}{T}\right)^2 m_1+2\left(\frac{l_G}{T}\right)M\right]dx\right.$$

$$+\dot{\theta}\frac{\rho g^2}{\omega^3}\int_{L_a}^{L_f}T^2\left[\left(\frac{\overline{A}_3}{T}\right)^2+\left(\frac{l_G}{T}\right)^2\overline{A}_1{}^2+2\left(\frac{l_G}{T}\right)N\right]dx+\theta\cdot\rho g\cdot GM\int_{L_a}^{L_f}Sdx$$

$$+\ddot{y}\cdot\rho\int_{L_a}^{L_f}ST\left(M+\frac{l_G}{T}m_1\right)dx+\dot{y}\frac{\rho g^2}{\omega^3}\int_{L_a}^{L_f}T\left(N+\frac{l_G}{T}\overline{A}_1{}^2\right)dx$$

$$=\frac{i\rho g}{K}e^{i\omega t}\int_{L_a}^{L_f}T\left(\frac{\overline{A}_3}{T}e^{i\varepsilon_3}+\frac{l_G}{T}\overline{A}_1 e^{i\varepsilon_1}\right)dx, \tag{12}$$

where y and z are a swaying and a heaving displacement of the point G and θ rolling angle. And r_0 is the radius of gyration, GM the transverse metacentric height and l_G the z coordinate of the center of gravity of each cross section of the ship. Another symbols have the same meaning as in the section 2.

Substituting $z=Ze^{i\omega t}$, $y=Ye^{i\omega t}$ and $\theta=\Theta\,e^{i\omega t}$ into the equations (11), (12), and (13) we can obtain the amplitudes and the phases of the motions.

3.2. Comparison of theoretical and experimental ship motions

The heaving, swaying and rolling motions in beam seas of *TW1* and *TW2* were measured at zero advance speed. The experimental apparatus and regular waves used in the measurement are the same one as used for the experiment in head seas.

The measured amplitudes of swaying, heaving and rolling oscillations of *TW1* and *TW2* are shown and compared with the theoretical ones obtained by solving the equations (11), (12) and (13) in Fig. 11, 12 and 13. The transverse radius of gyration r_0 of *TW1* and *TW2* used in the equations was determined experimentally from free rolling test.

It should be noted that the amplitude of swaying motion shown in Fig. 12 is not of gravity *G* but of the point 35.3 cm above the point *G*. Ship models drift due to the wave force in beam seas and accordingly the abscissa of these figures is $\frac{\omega_e^2}{g}T$, where ω_e is the circular frequency of encounter of incident waves.

Theoretical and measured values of the motions are generally in good agreement. Since damping force given rise to by the generation of progressing waves is larger, the theoretical and the measured values are also in comparatively good agreement even in the case of rolling in spite of the neglect of the force due to viscosity of a fluid.

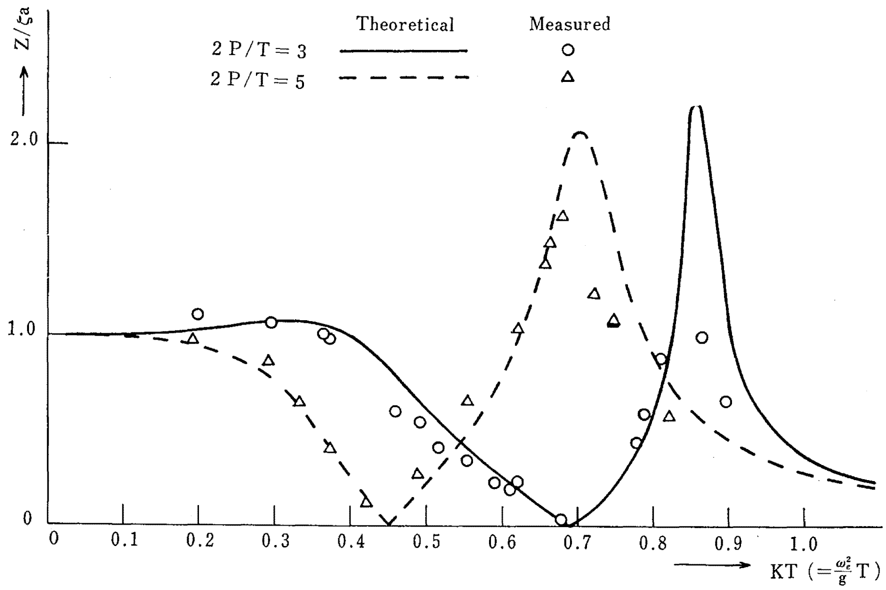

Fig. 11 Heaving motion of twin hull ships in beam seas

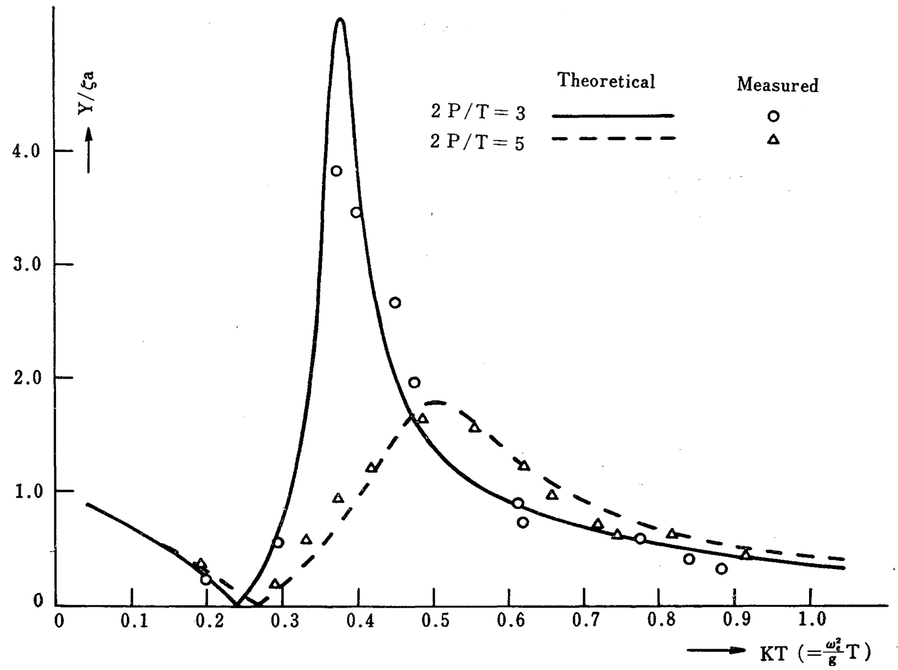

Fig. 12 Swaying motion of twin hull ships in beam seas

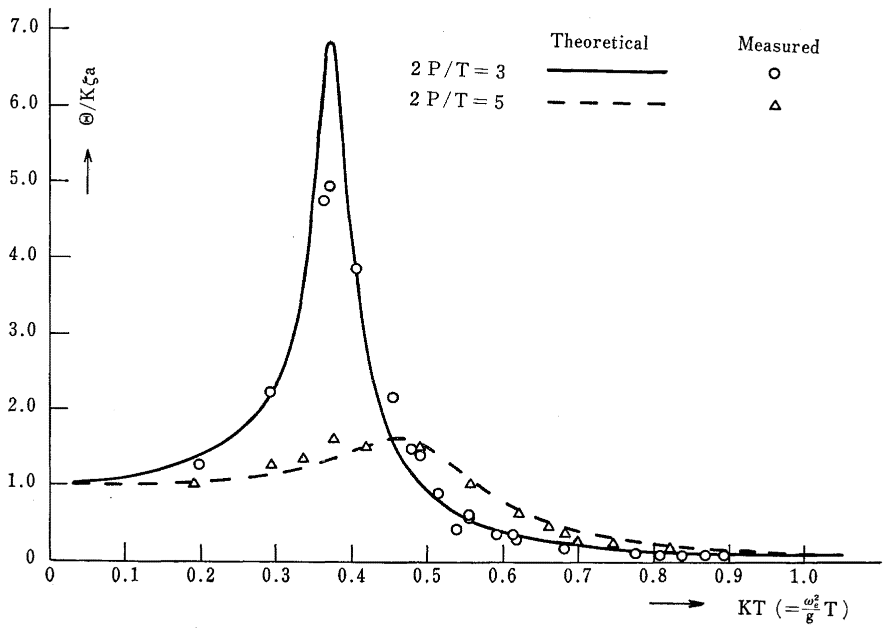

Fig. 13 Rolling motion of twin hull ships in beam seas

In the experiments it was observed that the waves between two hulls began to progress in the lengthwise direction of the ships especially at the frequency when the waves became very high and the heaving motion was larger. This fact means that three dimensional effect is strong at this frequency of motion. And at such frequency the measured amplitude of the heaving motion was not stationary but varied with a time like a beat. It will be due to these facts that at the resonant frequency the measured amplitude of the heaving motion does not reach to the level of the amplitude predicted by theory.

Next let us study the details of Fig. 11. Theory predicts that there are three resonance point of heaving motion for *TW1* in the range of *KT* shown in this figure, where a resonance point of heaving motion means a frequency at which the sum of inertia force and restoring force upon a ship making a heaving motion vanishes. One of these three points is a frequency where the amplitude of the heaving motion of *TW1* becomes the largest as shown in Fig. 11 and the other two points are $KT \fallingdotseq 0.4$ and 0.55. At the latter two points, the amplitude does not show a peak because $\overline{A}_2$, that is, damping force is large there (vertical wave exciting force becomes larger with $\overline{A}_2$ but damping force is much larger).

Although the resonance frequency of heaving motion of a mono hull ship which constitudes *TW1* or *TW2* is known to be at $KT \fallingdotseq 0.7$, the resonance point of *TW1* and *TW2* differs from the mono hull's one as a result of an interaction between two hulls. When $2p/T$ is from 3 to 10, it can be said that the larger the distance between two hulls is, the smaller the difference becomes. By the theory developed in the first paper, added mass, damping force of heaving motion and vertical wave exciting force of twin hull are known to be functions of *KT* and e^{-i2KP} as far as *KT* is not so small. Accordingly if a twin hull ship composed of some two hulls is at resonance of heaving motion with $KT=\alpha$ and $2KP=\beta$, then another twin hull ship which is composed of the same two hulls but has $2KP=\beta+2\pi$ at $KT=\alpha$ makes just the same heaving motion at this frequency. The heaving motion of *TW1*, for example, is in resonance at $KT \fallingdotseq 0.9$ and that of a twin hull ship with widened breadth as $2P/T=10$ is also in resonance at $KT \fallingdotseq 0.9$.

It is shown in the first paper that there are some frequencies at which $\overline{A}_2$ of twin hull cylinder is zero and then vertical wave exciting force upon the cylinder is also zero. In the heaving motion of *TW1* and *TW2* there is the frequency at which the wave exciting force become almost zero and as a result the ship does not make a heaving motion as shown in Fig. 11 at this frequency.

3.3. Miscellaneous

Let us compare the motion amplitudes of twin hull ships in beam seas calculated approximately without taking the interaction effect between two hulls into consideration with the exact ones obtained in 3.2.

When the interaction is neglected the added mass, damping force and the wave exciting force of a twin hull ship in heaving and swaying motion are considered to be the sum of the ones for the mono hulls which are elements of the twin hull ship. In rolling, added mass moment and damping coefficient are derived from the sum of the hydrodynamic forces acting upon each mono hull when it makes a heaving motion and a rolling motion about its center. The wave exciting forces of Y and Z directions ($j=1$ is Y direction and $j=2$ Z direction), for example, are expressed as follows

$$\frac{i\rho g}{K}\int_{L_\alpha}^{L_f}\left[\overline{A}_j e^{i\varepsilon_j} e^{i(\omega t-KP)}+\overline{A}_j\cdot e^{i\varepsilon_j} e^{i(\omega t+KP)}\right]dx, \tag{14}$$

and the wave exciting rolling moment about the origin is

$$\frac{i\rho g}{K}\int_{L_a}^{L_f}\left[-P\cdot\overline{A}_2 e^{i\varepsilon_2} e^{i(\omega t-KP)}+P\cdot\overline{A}_2 e^{i\varepsilon_2} e^{i(\omega t+KP)}+\overline{A}_3 e^{i\varepsilon_3} e^{i(\omega t-KP)}+\overline{A}_3 e^{i\varepsilon_3} e^{i(\omega t+KP)}\right]dx, \tag{15}$$

where $\overline{A}_j$, ε_j are the values of the mono hull cylinders.

The added mass and the wave exciting force thus calculated are substituted into the equations (11), (12) and (13), then we can obtain the motion amplitudes of twin hull ship when the interaction is neglected.

In Fig. 14 and 15, there are shown the heaving and the rolling motion amplitude of *TW1* with and without the interaction, where approximate denotes the values without the interaction. As seen in Fig. 14, if the interaction is not taken into account, the amplitude of the heaving motion is not only smaller than the exact one, but also the amplitude is rather large when the exact calculations show the zero amplitude. Both the exact and the approximate calculations give almost the same resonant frequency and the wave exciting moment in the rolling, but the damping force without the interaction is larger and the approximate

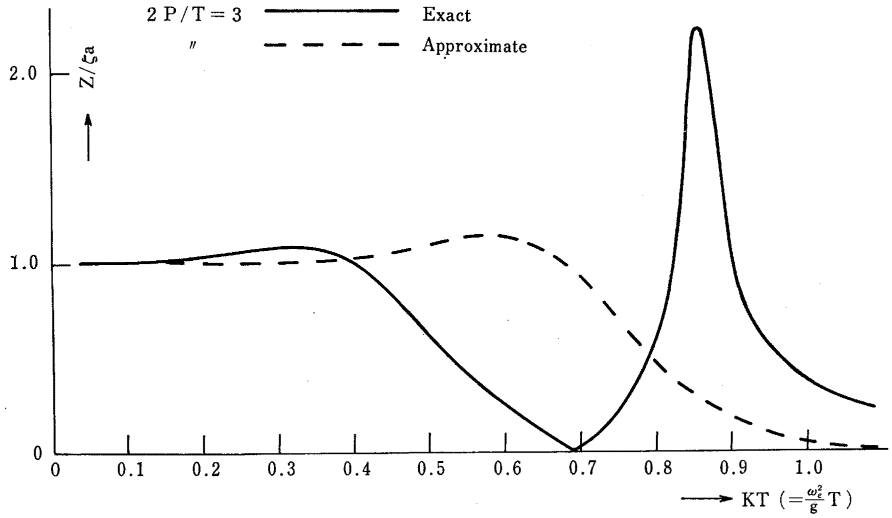

Fig. 14 Comparison of approximate and exact solutions (heaving)

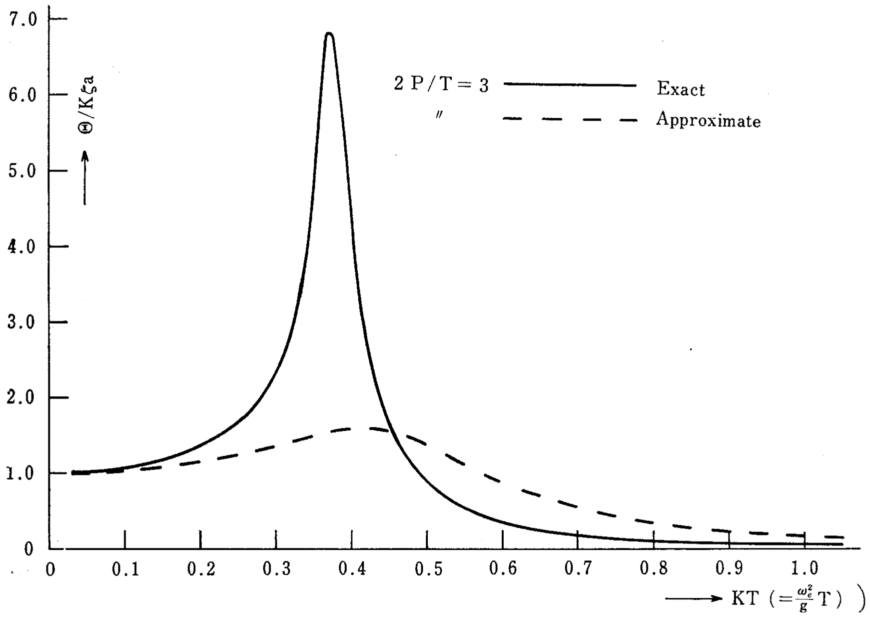

Fig. 15 Comparison of approximate and exact solution (rolling)

amplitude is much smaller the real value.

Each element hull of twin hull ship making oscillation in beam seas receives also a hydrodynamic force with Y direction and a hydrodynamic moment about its center line. This force and the moment upon right and left hull have the same magnitude but the opposite direction with each other. Such a force upon a twin hull ship moving in beam seas is expressed by

$$(-\ddot{z})\rho\int_{L_a}^{L_f}\frac{S}{2}m_{HS}dx+(-\dot{z})\frac{\rho g^2}{\omega^3}\int_{L_a}^{L_f}n_{HS}dx+\int_{L_a}^{L_f}\left(\overline{W}_1{}^R-\frac{\overline{W}_1{}^R+\overline{W}_1{}^L}{2}\right)dx, \tag{16}$$

where m_{HS} and n_{HS} of each section are calculated from the equation (7). $\overline{W}_1{}^R$ and $\overline{W}_1{}^L$ are wave exciting forces of Y direction upon the sections of right and left hull respectively and are given as follows.

When the wave $e^{i(\omega t+Ky)}$ comes upon the twin hull cylinder shown in Fig. 9, the exchange of diffraction waves between two cylinders described in the first paper produces the incident wave upon the right cylinder given by

$$E_1(KT)e^{i(\omega t-Ky_R)}=\frac{(1+iH^-(KT))\,iH^+(KT)}{1+H^+(KT)^2e^{-i4KP}}e^{-i3KP}\cdot e^{i(\omega t-Ky_R)}, \tag{17}$$

and the incident wave upon the left cylinder given by

$$E_2(KT)e^{i(\omega t+Ky_L)}=\frac{(1+iH^-(KT))}{1+H^+(KT)^2e^{-i4KP}}e^{-iKP}e^{i(\omega t+Ky_L)}, \tag{18}$$

where $H^{\pm}(KT)$ is given by the equation (6) and y_R and y_L are the Y coordinate measured from the center of the right and left cylinders. The wave exciting forces, $\overline{W}_1{}^L$ and $\overline{W}_1{}^R$ upon the right and the left cylinder are calculated as the exciting force due to these incident waves by

$$\left.\begin{aligned} \overline{W}_1{}^L &= \frac{i\rho g}{K}\overline{A}_1 e^{i\varepsilon_1} E_2(KT) e^{i\omega t}, \\ \overline{W}_1{}^R &= \frac{i\rho g}{K}\overline{A}_1 e^{i\varepsilon_1}[e^{iKP}+E_1(KT)]\, e^{i\omega t} \end{aligned}\right\} \tag{19}$$

Hydrodynamic moment upon each hull can be calculated by the same procedure.

Fig. 16 and 17 give the amplitudes of such force and moment of *TW1* and *TW2*. At small heaving amplitude the force or moment becomes rather large in some cases.

4. Conclusion

The method to solve the radiation problem of twin hull cylinder developed in the first report was applied to the theoretical calculations of the various motions of twin hull ships by Strip method and it was found that the theoretical predictions gave satisfactory results in comparison with the measurements on model tests.

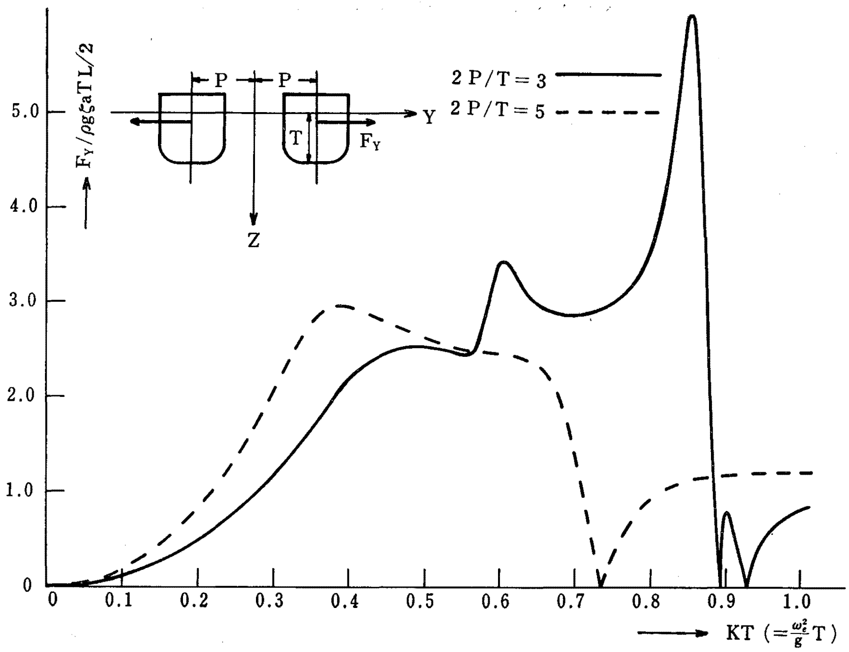

Fig. 16 Transverse force of twin hull ships in beam seas

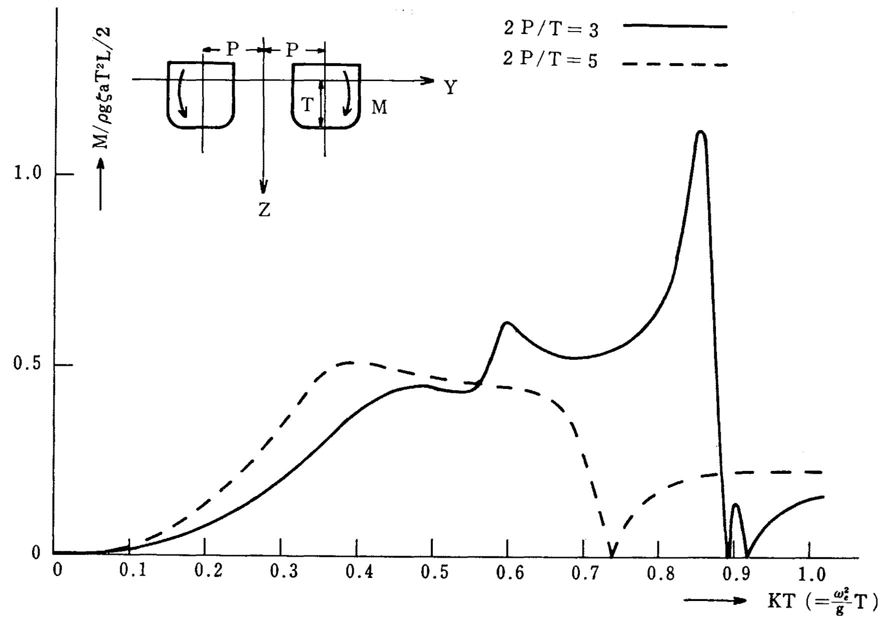

Fig. 17 Transverse moment of twin hull ships in beam seas

The motions of twin hull ship are some times much different from those of ordinary mono hull ship and this singular response characteristics of twin hull ship can not be predicted if the hydrodynamic interaction effect between its two hulls is neglected.

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