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https://doi.org/10.5109/7170853

出版情報：Reports of Research Institute for Applied Mechanics. 18 (61), pp.69-85, 1970. 九州大学応用力学研究所

Reports of Research Institute for Applied Mechanics
Vol. XVIII, No. 61, 1970

THEORETICAL STUDY ON THE VIBRATIONS OF RUDDER AND RUDDER-POST SYSTEM OF A HIGH SPEED CARGO LINER

By Jiro SUHARA* and Nobuyoshi FUKUCHI**

The possibility of arising the vibrations of the rudder and rudder-post system of a high speed cargo liner has been examined theoretically based on several assumptions.

Although it may not be predicted the vibration characteristics accurately owing to the complexities of the nature of the anticipated motions of the rudder and rudder-post system behind the propeller, the coincidence of the natural frequency of the fundamental mode of the system and the blade frequency of the propeller in service condition may be possible as a theoretical result based on the assumptions which are thought to be reasonable. However, even though such coincidence arises unfortunately, it is considered that the violent vibrations would not happen if the statically estimaed maximum deflections of the rudder-post due to pressure around the propeller blades are less than the order of the clearances between the rudder pintles and the gudgeons which cause the decrease of effective weight of rudder to the rudder-post i. e. the increase of the natural frequency of the system and check the resonance of the system.

1. Introduction

A cantilever type rudder-post attached with rudder of a high speed cargo liner was felt necessary to study on the possibility of occurrence of violent vibrations by the hydrodynamic disturbances by the propeller.

Theoretical estimations of the natural frequencies of the system have been made on the bases of assumptions to simplify the theoretical treatments. The fundamental data used in this theory are not yet sufficient to predict the exact value of the natural frequency of the system, because of the lack of example of measurement of an actual phenomena even for a similar case up to present. However the theoretical study would be useful for a rough prediction of the nature of the vibrations and the ranges of natural frequencies anticipated and for a design guidance of the scantlings of the system.

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2. Assumptions

Following assumptions are made as bases of the theory:

(1) The rudder is connected to the rudder-post through pintles which transmit the shearing forces only and sufficiently flexible not to transmit any moment of force about horizontal or vertical axis from the rudder to the rudder-post.

(2) The horizontal inertia forces of the rudder including the effects of added weight of surrounding water apply continuously to the rudder-post through infinite number of pintles assumed to be distributed over the vertical axis at the aft end of the rudder-post.

(3) Added weight of the rudder and rudder-post system for the motion of horizontal translation is estimated by the experimental formula obtained by Wereldsma of Netherlands Ship Model Basin [1] as follows:

$$\text{Added weight} = C_1 \cdot \frac{\pi}{4} \cdot \gamma \cdot B^2 dl$$

where

γ = weight per unit volume of the surrounding water.

B = chord length of the rudder and the rudder-post.

dl = infinitesimal length of the system in vertical direction.

For the value of the coefficient C_1, Wereldsma gave approximately 1/2 by the model experiment of the Mariner rudder with nearly similar profile of the system into consideration, however that value is thought to be used for the whole system including the three dimensional effect. The value of C_1 is to be variable along the length of the system when we estimate the local added weight per unit length at any section. In this report such effect has been ignored and the C_1 was taken as constant approximately.

The estimated added weights were divided proportionally to the both parts of the rudder and the rudder-posts by the ratio of respective chord length.

Added weight moment of inertia about the shear center of the rudder-post is estimated by Kumai's formula [2]:

$$\Delta I = C_2 \cdot \gamma \cdot \pi \cdot B_p^4$$

where

B_p = chord length of rudder-post

$C_2 \doteqdot$ coefficient determined by the location of the center of rotation

(4) Any inertia couple of the rudder around its vertical axis passed through the effective center of gravity of the rudder itself does not be transmitted to the rudder post by the action of pintles as it is assumed to be frictionless.

(5) The relative angle between the rudder and the rudder-post may varies in motion, consequently the effective position of the center of gravity of the

rudder including the effect of the added weight of the surrounding water for the horizontal motion of the rudder-post is assumed to be m times of the distance between the center of the pintle and the actual position of the center of gravity of the rudder including the effect of the added weight of the surrounding water. The value of the factor m is supposed to be in the range of 0 and 1, however it is thought to be difficult to determine this value exactly, we leave this value undetermined, and the natural frequencies are obtained in an range according to the variation of value m.

(6) The effect of the clearances between the gudgeons and the pintles is neglcted for the motion of the system in the theory at initial stage. However the discussions on this effect by the statical consideration are tried at the final stage of the study.

(7) The rudder post is connected rigidly to the hull.

(8) The stiffness of rudder stock is neglected, because it is supposed to be sufficiently flexible compared to the rudder post. However, its inertia forces including the added weight are taken into account by regarding this as a part of the rudder.

(9) The effect of the shearing deflection is neglected in the initial stage of the study, however it is esitmated approximately at the final stage.

(10) The effect of rotary inertia of the system about horizontal axis is neglected, because it is supposed to be neglesible order.

3. Fundamental Equations

The profile of the system is shown in Fig. 1. Any section of the rudder-

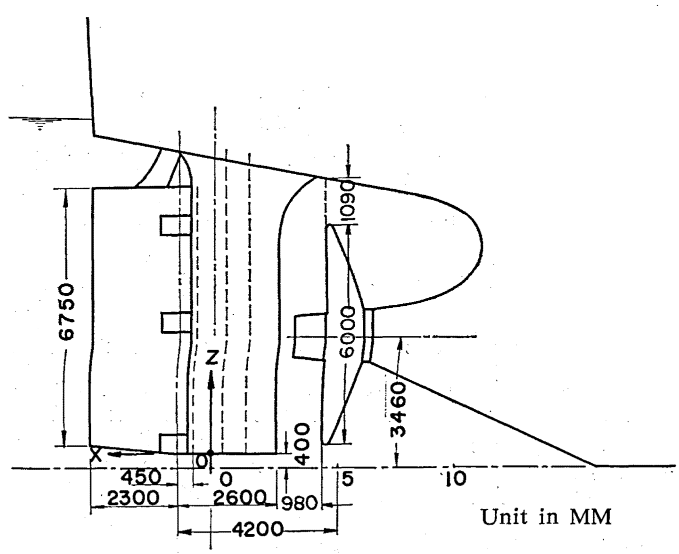

Fig. 1.

post is symmetrical about x-axis in longitudinal direction and non-symmetrical about the axis y taken in transverse direction which passes through the shear center of the section of the rudder-post. (see Fig. 2)

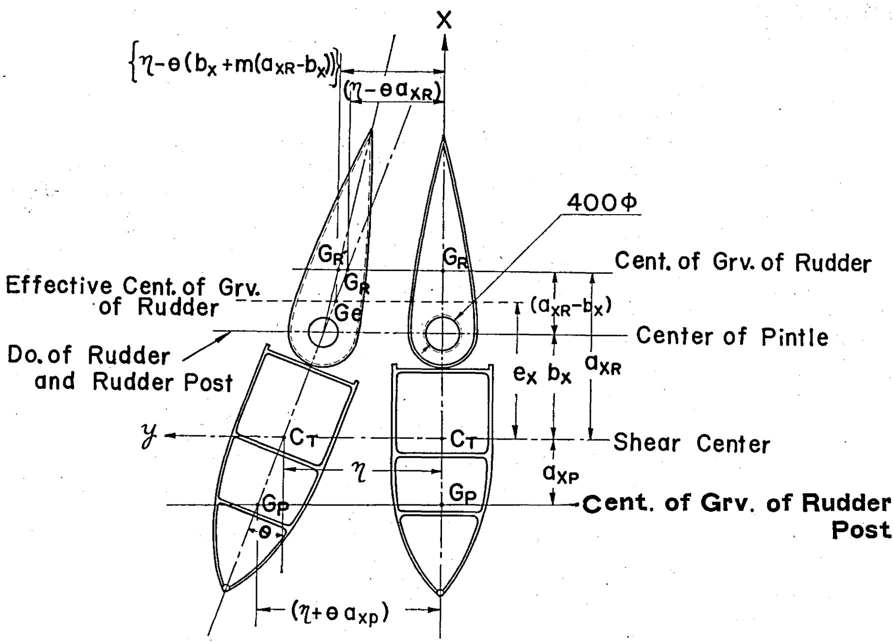

Fig. 2.

It is easily seen that the bending vibration in longitudinal direction is independent to the flexual vibrations in transverse direction and the torsional vibrations, owing to the symmetry of the sectional form and that the natural frequencies of the former ones are sufficiently high to meet the blade frequency of the propeller.

The flexual vibrations in transverse direction and torsional vibration about vertical axis passing through the shear center of the rudder-post may couple because the shear center of the rudder-post and the effective center of gravity of the system are located separately and it is thought that these are the most important modes of the vibrations which we are going to study.

We take y-axis in transverse direction passing through the shear center of the horizontal section at the lowest end of the rudder-post and taking z-axis vertically upward which has the same origin.

At initial stage, we neglect the effect of the shearing deflection to the natural frequencies of the system, the warping effect of the section of the rudder-post, the rotary inertia of the system about horizontal axis and the deviations of the locations of the shear centers in the horizontal sections of the rudder-post from a same straight line (z-axis).

However for the effect of the shearing deflections, the approximate estimations are discussed at the end of this report.

Then the equations of the coupled bending and torsional vibration of the system are

$$\left.\begin{aligned} &\frac{\partial^2}{\partial z^2}\left\{EI_x\frac{\partial^2\eta}{\partial z^2}\right\}=-\frac{\mu_y}{g}\frac{\partial^2\eta}{\partial t^2}+\frac{\mu a}{g}\frac{\partial^2\theta}{\partial t^2}\\ &\frac{\partial}{\partial z}\left\{GI_d\frac{\partial\theta}{\partial z}\right\}=\frac{\mu a}{g}\frac{\partial^2\eta}{\partial t^2}-\frac{\gamma' I_T}{g}\frac{\partial^2\theta}{\partial t^2}\end{aligned}\right\} \tag{1}$$

where following notation is used

η=deflection at shear center of rudder-post in y-direction
θ=rotational angle of cross section of rudder-post
E=modulus of elasticity
I_x=moment of inertia of section of rudder-post about x-axis
I_w=sectorial moment of inertia of rudder-post [3]
GI_d=torsional rigidity of rudder-post
μ_{yp}=weight of rudder-post including virtual weight per unit length (used later)
μ_{yR}=do. of rudder (do.)
$\gamma' I_p/g$=mass polar moment of inertia of rudder-post including added mass around rudder-post only
g=gravitational accelerlation
t=time
$\zeta=(l-z)/l$
l=effective length of rudder-post
e_x=distance between shear center of rudder-post and eflective center of gravity of system including added weight.
a_{xp}, a_{xR}=coordinates of the center of gravity of rudder-post and rudder including virtual weight respectively (used later)
b_x=distance between shear center of rudder-post and center of pintle.

Putting

$$\left.\begin{aligned} &\eta=\psi_0 l\psi(\zeta)\sin k\tau, && \theta=\chi_0\chi(\zeta)\sin k\tau\\ &\tau=t/T, && k\tau=k\ t/T\\ &b'=\frac{\mu_{yTo}l^4}{EI_{xo}gT^2}, && e''=GI_{do}l^2/EI_{wo}\\ &g'=\frac{\mu_{yTo}a_{xpo}l^5}{EI_{wo}gT^2}, && r'=\frac{f'}{e''},\quad j'=\frac{g'}{e''}\\ &f'=\frac{\gamma' I_{po}l^4}{EI_{wo}gT^2}, && s=\frac{e_x}{a_{xp}}=\frac{b_x+m(a_{xR}-b_x)}{a_{xp}}\end{aligned}\right\} \tag{2$_1$}$$

$$\left.\begin{aligned} \mu_{yT} &\equiv \mu_{yR}+\mu_{yp} \\ \mu a &\equiv \mu_{yR}e_x+\mu_{yp}a_{xp} \\ \gamma' I_T &\equiv \gamma' I_p+\mu_{yR}e_x{}^2+\mu_{yp}a_{xp}^2 \end{aligned}\right\} \quad (2)_2$$

$$\left.\begin{aligned} C_{Ix}(\zeta) &\equiv I_x(z)/I_{xo} \quad (\text{for vertical member only}) \\ C_{Iw}(\zeta) &\equiv I_w(z)/I_{wo} \quad (\quad '' \quad) \\ C_{Id}(\zeta) &\equiv I_d(z)/I_{do} \quad (\quad '' \quad) \end{aligned}\right\} \quad (2)_3$$

$$\left.\begin{aligned} & C_{\mu yT}(\zeta) \equiv C_{\mu R}(\zeta)+C_{\mu p}(\zeta) \equiv \frac{\mu_{yT}(\zeta)}{\mu_{yTo}} \\ & C_{\mu R}(\zeta) \equiv \mu_{yR}(z)/\mu_{yTo}, \quad C_{\mu p}(\zeta) \equiv \mu_{yp}(z)/\mu_{yTo} \\ & C_{axp}(\zeta) \equiv a_{xp}(z)/a_{xpo} \\ & C_{\mu a}(\zeta) \equiv C_{axp}(\zeta)\{C_{\mu R}(\zeta)\cdot s+C_{\mu p}(\varsigma)\} \\ & C_{Ip}(\zeta) \equiv \frac{\gamma' I_T(z)}{\gamma' I_{po}} \\ & \qquad \equiv C_{Ipo}(\zeta)+\left(\frac{\mu_{yTo}a^2{}_{xpo}}{\gamma' I_{po}}\right) C^2{}_{axp}(\zeta)\{s^2 C_{\mu R}(\zeta)+C_{\mu p}(\zeta)\} \\ & C_{Ipo}(\zeta) \equiv \frac{\gamma' I_p(z)}{\gamma' I_{p0}} \end{aligned}\right\} \quad (2)_4$$

The illustrations of above-mentioned quantities are shown in Fig. 3 to Fig. 10, where the suffix 0 indicates the average value of the quantity through the length of the rudder-post.

T=unit time (1 sec.)

m=factor to specify the effective position of center of gravity of rudder including added weight in motion

Then we have fundamental equations in non-dimensional form as follows:

$$\left.\begin{aligned} & \frac{\partial^2}{\partial \zeta^2}\left\{C_{Ix}(\zeta)\frac{\partial^2 \psi}{\partial \zeta^2}\right\} = k^2\cdot b'\{C_{\mu yT}(\zeta)\psi+\left(\frac{a_{xpo}}{l}\right)C_{\mu a}(\zeta)\frac{\chi_0}{\psi_0}\chi\} \\ & \frac{\partial}{\partial \zeta}\left\{C_{Id}(\zeta)\frac{\partial \chi}{\partial \zeta}\right\} = -k^2\{j' C_{\mu a}(\varsigma)\psi+r' C_{Ip}(\varsigma)\frac{\chi_0}{\psi_0}\chi\} \end{aligned}\right\} \quad (3)$$

The end conditions are as follows:

at the free end ($\varsigma=1$):

$$\frac{\partial}{\partial \varsigma}\left\{C_{Ix}(\varsigma)\frac{\partial^2 \psi}{\partial \varsigma^2}\right\}=0, \quad C_{Ix}(\varsigma)\frac{\partial^2 \psi}{\partial \varsigma^2}=0, \quad C_{Id}(x)\frac{\partial \chi}{\partial \varsigma}=0$$

and at the clamped end ($\varsigma=0$): (4)

$$\frac{\partial \psi}{\partial \varsigma}=0, \quad \psi=0, \quad \chi=0$$

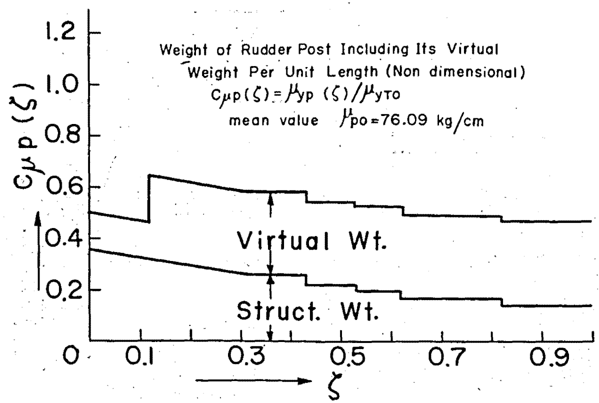

Fig. 3.

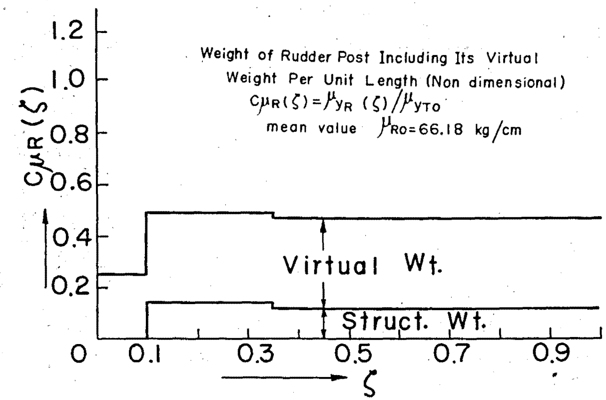

Fig. 4.

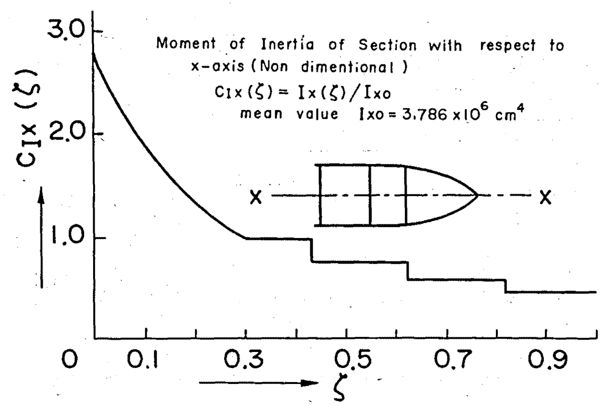

Fig. 5.

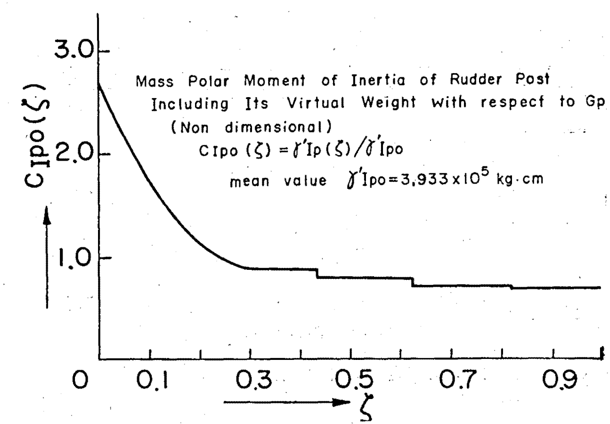

Fig. 6.

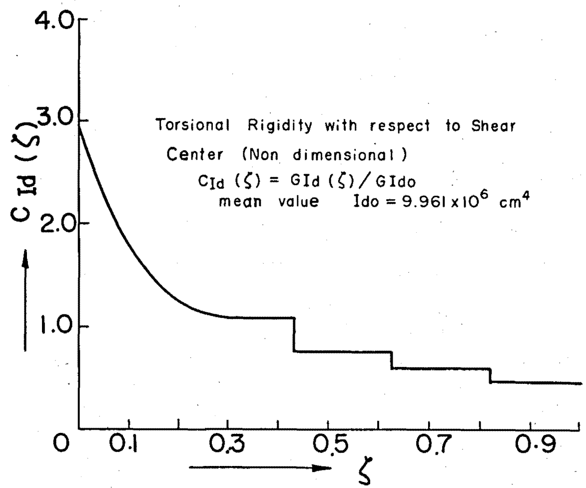

Fig. 7.

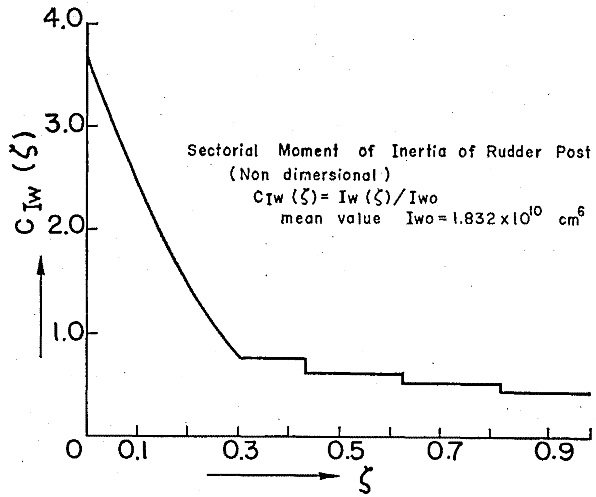

Fig. 8.

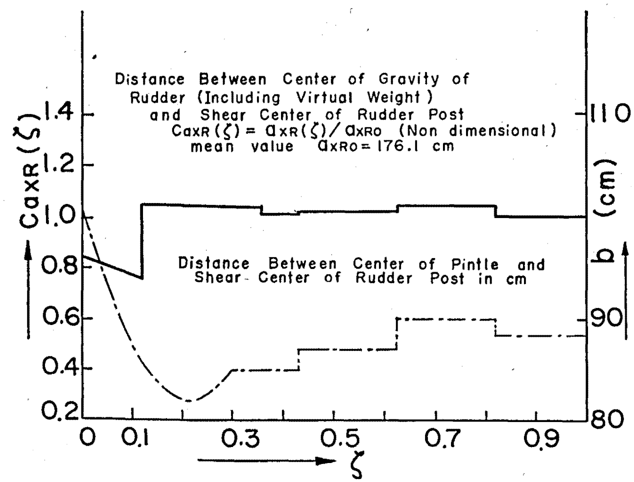

Fig. 9.

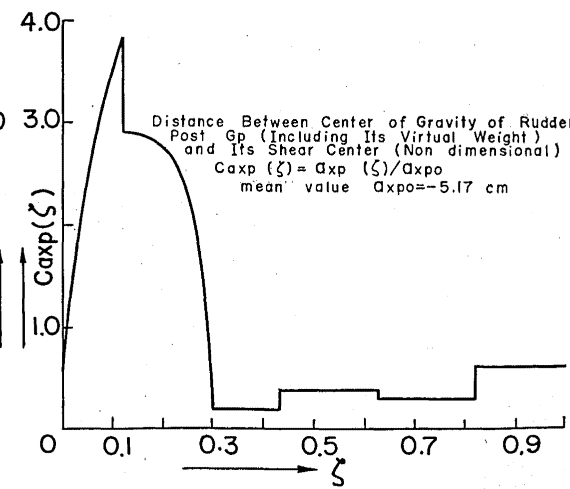

Fig. 10.

The equations (3) are simultaneous differential equations of the fourth and second order respectively with variable coefficients which are given numerically.

These equations are hard to solve analytically for complexities of variations of the coefficients.

In such case the numerical method of solution is applicable directly by use of electronic digital computer.

At the first stage we assume the approximate solutions which satisfy the end conditions (4) as follows:

$$\left.\begin{aligned} \psi(\varsigma)&\equiv\psi_1(\varsigma)\equiv\frac{1}{3}(6\varsigma^2-4\varsigma^3+\varsigma^4) \\ \chi(\varsigma)&\equiv\chi_1(\varsigma)=2\varsigma-\varsigma^2 \end{aligned}\right\} \tag{5}$$

Substituting (5) into the right members of the equation (3) and integrating with the end conditions (4), and putting $\chi_0=\psi_0=1$ we have the second approximate values of ψ (ς) and χ (ς) except the value of k^2.

Equating the maximum values (at $\varsigma=1$) of the first and second approximate values of ψ (ς) (the latter contains the unknown value of k^2), we get the first approximate value of k^2.

Using these approximate values of k^2 and the latter, we get the complete values of the second approximate solutions ψ_2 (ς) and χ_2 (ς).

Repeating this process until we get the solutions with the accuray wanted.

In this study, it has been taken following condition for the i th and $i+1$ th approximate values of k^2. $(k_{i+1}^2-k_i^2)/k_i^2<10^{-3}$

By this process, the values of ψ (ς) and χ (ς) for any value of ς tend to the respective limiting values with the convergence of k^2.

Such process of the numerical calculations have been performed for various values of m which specify the effective locations of rudder-post.

4. Solution by Rayleigh-Ritz Method

To check the above results, the Rayleigh-Ritz Method are also applied to this problem. As the additional advantage of this method, we can estimate the effects of warping of the rudder-post.

For the system which are undergoing coupled vibrations, the strain energy U and kinetic energy T are

$$\left.\begin{aligned} U&=\int_o^l\frac{EI_x}{2}\left(\frac{\partial^2\eta}{\partial z^2}\right)^2dz+\int_o^l\frac{EI_w}{2}\left(\frac{\partial^2\theta}{\partial z^2}\right)^2dz+\int_o^l\frac{GI_d}{2}\left(\frac{\partial\theta}{\partial z}\right)^2dz \\ T&=\int_o^l\frac{\mu_{yR}}{2g}\left\{\frac{\partial(\eta-\theta e)}{\partial t}\right\}^2dz+\int_o^l\frac{\mu_{yp}}{2g}\left\{\frac{\partial(\eta-\theta a_{xp})}{\partial t}\right\}^2dz+\int_o^l\frac{\gamma' I_p}{2g}\left(\frac{\partial\theta}{\partial t}\right)^2dz \\ &=\int_o^l\frac{\mu_{yT}}{2g}\left(\frac{\partial\eta}{\partial t}\right)^2dz-\int_o^l\frac{\mu a}{g}\cdot\frac{\partial\eta}{\partial t}\cdot\frac{\partial\theta}{\partial t}\cdot dz+\int_o^l\frac{\gamma' I_T}{2g}\left(\frac{\partial\theta}{\partial t}\right)^2dz \end{aligned}\right\} \tag{6}$$

Using the notations (2), the maximum values of the strain energy and kinetic energy are

$$\left.\begin{aligned}
\max U &= \frac{EI_{xo}\psi_o^2}{2l}\int_0^1 C_{Ix}(\varsigma)\left(\frac{\partial^2\psi}{\partial\varsigma^2}\right)^2 d\varsigma + \frac{EI_{wo}\chi_0^2}{2l^3}\int_0^1 C_{Iw}(\varsigma)\left(\frac{\partial^2\chi}{\partial\varsigma^2}\right)^2 d\varsigma \\
&\quad + \frac{GI_{do}}{2l}\chi_0^2\int_0^1 C_{Id}(\varsigma)\left(\frac{\partial\chi}{\partial\varsigma}\right)^2 d\varsigma \\
\max T &= k^2\Big[\frac{\mu_{yTo}l^3}{2gT^2}\psi_0^2\int_0^1\frac{1}{C_{\mu yT}(\varsigma)}\Big\{C_{\mu yT}(\varsigma)\psi\Big\}^2 d\varsigma \\
&\quad - \frac{\mu_{yT0}a_{xp0}l^2}{gT^2}\psi_0\chi_0\int_0^1 C_{\mu a}(\varsigma)\psi\chi d\varsigma + \frac{\gamma' I_{po}l}{2gT^2}\chi_0^2\int_0^1\frac{1}{C_{Ip}(\varsigma)}\{C_{Ip}(\varsigma)\chi\}^2 d\varsigma\Big]
\end{aligned}\right\} \tag{7}$$

Putting

$$\max U = \frac{1}{2}N \quad \text{and} \quad \max T = \frac{1}{2}Mk^2$$

where

$$\left.\begin{aligned}
N &= \left(\frac{EI_{x0}}{l}\right)A\psi_0^2 + \left(\frac{GI_{d0}}{l}\right)B\chi_0^2 \\
M &= \left(\frac{\mu_{yT0}l^3}{gT^2}\right)\Big\{C\psi_0^2 - 2\left(\frac{a_{x0}}{l}\right)D\psi_0\chi_0\Big\} + \left(\frac{\gamma' I_{p0}l}{gT^2}\right)Q\chi_0^2
\end{aligned}\right\} \tag{8}$$

Then we have $k^2 = N/M$ ....(9)

Equation (9) contains the amplitudes ψ_0 and χ_0, for which the optimum values are given by

$$\frac{\partial N}{\partial\psi_0} - k^2\frac{\partial M}{\partial\psi_0} = 0 \tag{10}$$

Putting

$$A \equiv \int_0^1\frac{1}{C_{Ix}(\varsigma)}\Big\{C_{Ix}(\varsigma)\frac{\partial^2\psi}{\partial\varsigma^2}\Big\}^2 d\varsigma$$

$$B \equiv \Big[\lambda\Big(\frac{EI_{w0}}{GI_{ao}l^2}\Big)\Big]\int_0^1 C_{Iw}(\varsigma)\left(\frac{\partial^2\chi}{\partial\varsigma^2}\right)^2 d\varsigma + \int_0^1\frac{1}{C_{Id}(\varsigma)}\Big\{C_{Id}(\varsigma)\Big(\frac{\partial\chi}{\partial\varsigma}\Big)\Big\}^2 d\varsigma$$

where $\lambda = 1$ means warping effect included, and $\lambda=0$ means warping effect neglected. And

$$C \equiv \int_0^1\frac{1}{C_{\mu yT}(\varsigma)}\{C_{\mu yT}(\varsigma)\psi\}^2 d\varsigma \equiv \int_0^1 C_{\mu yT}(\varsigma)\psi^2 d\varsigma$$

$$D \equiv \int_0^1 C_{\mu a}(\varsigma)\psi\chi d\varsigma \quad , \quad K_1 \equiv \frac{\mu_{yTo}a^2_{xo}}{\gamma' I_{po}}\left(\frac{D^2}{CQ}\right) \tag{11}$$

$$Q \equiv \int_0^1\frac{1}{C_{Ip}(\varsigma)}\{C_{Ip}(\varsigma)\chi\}^2 d\varsigma \equiv \int_0^1 C_{Ip}(\varsigma)\chi^2 d\varsigma$$

$$\left.\begin{aligned}
(k_b)^2 &= \frac{EI_{x0}gT^2}{\mu_{Ty0}l^4}\left(\frac{A}{C}\right) , \quad (k_t)^2 = \frac{GI_{d0}gT^2}{\gamma' I_{p0}l^2}\left(\frac{B}{Q}\right) \\
R^2 &= \left(\frac{k_t}{k_b}\right)^2 \\
X &= \left(\frac{k}{k_b}\right)^2
\end{aligned}\right\}$$

we get

$$\left.\begin{aligned}
X &= \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \frac{1}{2(1-K_1)}\left\{(1+R^2)\pm\sqrt{(1+R^2)^2-4R^2(1-K_1)}\right\} \\
\frac{\chi_0}{\psi_0} &= \frac{\{k_i^2-(k_b)^2\}}{k_i\left(\frac{a_{x0}}{l}\right)\left(\frac{D}{C}\right)} = \frac{k_i^2\left(\frac{\mu_{yT0}l^2}{\gamma' I_{p0}}\right)\left(\frac{a_{x0}}{l}\right)\left(\frac{D}{Q}\right)}{\{k_i^2-(k_t)^2\}} , \quad (i=1 \text{ or } 2)
\end{aligned}\right\} \tag{12}$$

Then we have two values of k which correspond to the circular frequencies of the fundamental and second modes of vibrations.

By the way, the Lewis' formula [4] for estimating the natural frequency of the uncoupled flexual vibration of the cantilever beam is

$$f=\frac{60}{2\pi l^2}\left[\frac{Eg}{\left(\int_0^1\frac{(1-\varsigma)^2d\varsigma}{I}\right)\left(\int_1^0 \mu y^2\, d\varsigma\right)}\right]^{1/2} \tag{13}$$

The result from this formula is given in Table II for comparison.

Results are shown in Table II in which the values for $\lambda=1$ show the results taking accounted of the warping effect and $\lambda=0$ means values neglected it which correspond to the results obtained by successive approximations from equations (3).

The functions $\psi(\varsigma)$ and $\chi(\varsigma)$ should be chosen so as to satisfy the end condition (4). In this case the final results obtained by successive approximations have been used for them respectively.

Thus obtained values of k are numerically coincided to the corresponding values of k obtained by the successive approximations. These results verify that both processes of calculations are correct and practically usable. Flow chart of this prosess is shown in Table I.

5. The Effects of Shearing Deflections

The effects of shearing deflections are approximated by the formula [5].

$$N_s=N/\sqrt{1+r_s} \tag{14}$$

where N_s is the corrected frequency and N is the calculated one from the cou-

Table I

Flow Chart

Start

↓

$\zeta^2(6-4\zeta+\zeta^2)/3 \rightarrow \psi_1(\zeta)$
$2\zeta(1-\zeta^2) \longrightarrow \chi_1(\zeta)$

↓

Calculatc k_b, k_t, k_1, k_2, χ_0/ψ_0 by energy method using $\psi_1(\zeta)$ and $\chi_1(\zeta)$.

↓

$\chi_1(\zeta)\{\chi_0/\psi_0\} \rightarrow \chi_1(\zeta)$
$k_1 \rightarrow k_i\ (i=1)$

↓

$i=2$

↓

$$b\left\{C_{\mu yT}(\zeta)\psi_{i-1}(\zeta)+\left(\frac{a_{xo}}{l}\right)C_{\mu a}(\zeta)\chi_{i-1}(\zeta)\right\} \rightarrow \frac{1}{k_i^2}\frac{\partial^2}{\partial\zeta^2}\left\{C_{Ix}(\zeta)\frac{\partial^2\psi_i(\zeta)}{\partial\zeta^2}\right\}$$

↓

Calculate $\frac{1}{k_i^2}\ \psi_i(\zeta)$ by Simpson's rule.

↓

$$\psi_{i-1}(1) \Big/ \left\{\frac{1}{k_i^2}\psi_i(1)\right\} \longrightarrow k_i^2$$

↓

$$\left\{\frac{1}{k_i^2}\psi_i(\zeta)\right\} \times k_i^2 \longrightarrow \psi_i(\zeta)$$

↓

$$-k_i^2\left\{j'C_{\mu a}(\zeta)\psi_i(\zeta)+r'C_{Ip}(\zeta)\chi_{i-1}(\zeta)\right\} \longrightarrow \frac{\partial}{\partial\zeta}\left\{C_{Id}(\zeta)\frac{\partial\chi_i(\zeta)}{\partial\zeta}\right\}$$

↓

Calculate $\chi_i(\zeta)$ by Simpson's rule.

↓

$|(k_i^2-k_{i-1}^2)/k_i^2| \leqq \delta k^2$ .... ?

No → $i+1 \rightarrow i$ → (return to the step after $i=2$)

↓ Yes

Calculate k_b,k_t,k_1,k_2 by energy method using exact solution $\psi(\zeta)$ and $\chi(\zeta)$.

↓

Stop

Note : Suffix i attached to ψ's, χ's etc. indicates the order of the approximation.

Table II.

m	ax (cm)	Natural frquencies (rpm)															
		Variable section							Uniform section								
		Succ. App.	Energy method					Lewis	Succ. App.	Energy method				Exact		Lewis
		N	N	N	N	N_B	N_T	N_B	N	N	N	N_T	N_T	N_T	N_B	N_B
		ψ, χ	ψ_0, χ_0	ψ, χ	ψ, χ	ψ	χ		ψ, χ	ψ, χ	ψ, χ	χ	χ			ψ
		$\lambda=0$	$\lambda=1$	$\lambda=1$	$\lambda=0$		$\lambda=1$		$\lambda=0$	$\lambda=1$	$\lambda=0$	$\lambda=1$	$\lambda=0$			
0	49.07	482.1	541.7 (2361.2)	483.7 (2309.2)		489.7	(1849.6)		404.4	405.6 (2138.8)	404.4 (1807.7)	1817.7	(1531.8)	(1553.3)	408.4	
0.25	56.33	481.0	539.4 (2172.1)	481.4 (2081.5)		489.7	(1629.3)							(1413.8)	408.4	
0.50	63.59	477.0	536.7 (2007.3)	478.6 (1889.1)		489.7	(1449.4)		401.7	403.0 (1746.7)	401.7 (1572.3)	1430.5	(1283.4)	(1292.2)	408.4	
0.75	70.85	473.6	533.5 (1864.0)	475.5 (1734.8)		489.7	(1307.5)							(1186.8)	408.4	
1.00	78.11	470.1	530.1 (1739.4)	472.1 (1606.3)	470.1 (1532.7)	489.7	(1190.9)		398.3	399.7 (1484.9)	398.3 (1384.2)	1180.6	(1096.5)	(1095.4)	408.4	
	$a_x \to 0$	489.7	549.1 (3785.1)	489.7		489.7		478.8	408.4		408.4		(2725.4)	(2726.7)	408.4	402.4

Note : Blade frequecies : Trial cond. 117×4=468 rpm and normal cond. 115×4=460 rpm.
( ) shows frquency of 2nd mode or torsional mode.

pled bending and torsional vibrations and γ_s is the ratio of the shear deflection to the coupled bending and torsional deflection. The value of γ_s is given as follows

$$\gamma_s \cong k\left(\frac{E}{G}\right)\left(\frac{I_{x0}}{A_0 l^2}\right)\int_0^1 \frac{1}{C_A(\varsigma)}\frac{\partial}{\partial\varsigma}\left\{C_{Ix}(\varsigma)\frac{\partial^2\psi}{\partial\varsigma^2}\right\} d\varsigma \tag{15}$$

where k is the coefficient which is known as lying its value between 1 and 1.5.

k = 1.0~1.5, coefficient of shearing deflection
G = Shear modulus, $(E/G)=8/3$
A = effective sectional area of rudder-post for shearing deflection
$C_A(\varsigma)$ = $A(z)/A_0$,
A_0 = mean value of A through length of rudder-post
$\psi(\varsigma)$ = approximately assumed solution of coupled bending and torsional vibration

Results are shown in Table III together with the effects of virtual mass factor β which means the ratio to be multiplied to the estimated added weight used above of translational motion for the reference, and used for introducing the effect of the clearance of the rudder pintle and gudgeon.

We can see that the both effects are fairly influential to the values of natural frequencies for the fundamental mode of the system and the obtained ones may meet the blade frequency of the propeller which is 460 rpm.

Table III. Natural frequencies (rpm) (Effect of shear and virtual weight factor)

β	k	N_s		
		m=1.0	m=0.5	m=0.0
1.0	S.E.N.	470.1	476.8	482.1
	1.0	454.0	460.5	465.6
	1.2	451.0	457.4	462.5
	1.5	446.6	452.9	458.0
0.75	S.E.N.	519.0	526.3	532.1
	1.0	501.2	508.3	513.9
	1.2	497.9	504.9	510.4
	1.5	492.8	500.0	505.4
0.5	S.E.N.	587.1	595.3	601.7
	1.0	567.0	574.9	581.1
	1.2	563.2	571.1	577.2
	1.5	557.7	565.5	571.6

B : Virtual weight coefficient.
k : Shear effect coefficicnt.
S.E.N. : Shear effect neglected.

6. Estimations of Statical Deflections

The pressure raise on the surface of the shell at the nearly equal distance of the tips of the propeller blade may be order of 0.1 kg/cm^2 as maximum in full load condition estimated from direct measurements for a actual and similar ship with a propeller of nealy equal diameter and blade frequency in service condition, though it may vary with the power of the engine. Assuming the magnitude of pressure applied to the system near the lower end of the rudder-post and its vertical distributions are taken variously as shown in Fig. 12. The statical bending deflection is given by

$$\delta = \frac{p_0 B l^4}{E I_0} \int_0^1 \frac{g(\varsigma)(1-\varsigma)}{C_{Ix}(\varsigma)} d\varsigma \tag{16}$$

where

$p(z)$ = pressure at z applied to the system
p_0 = maximum pressure at the lowest end of rudder-post assumed to be 0.1 (kg/cm^2)
B = chord length of rudder and rudder post approximately taken constant (490 cm)
$g(\varsigma)$ = $p(z)/p_0$

The calculated values of maximum deflection are less than 1.6 mm (see Table IV), which are same order or less than the clearances between the pintles and the gudeons. This means that the effectiveness of the inertia of the rudder and its surrounding water to the rudder-post should be decreased, i. e. virtual weight factor β become lower than unity. It is supposed that complicate nonlinear vibration may arise actually because the clearance effect is remarkable.

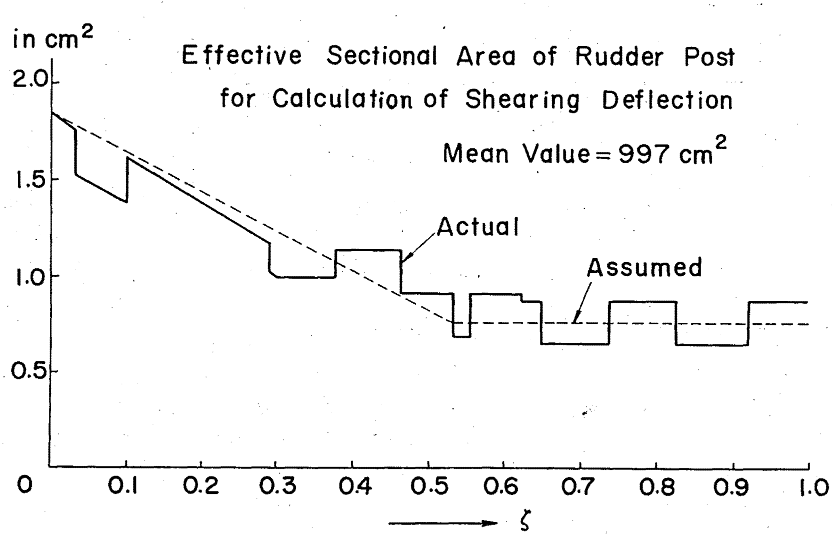

Fig. 11.

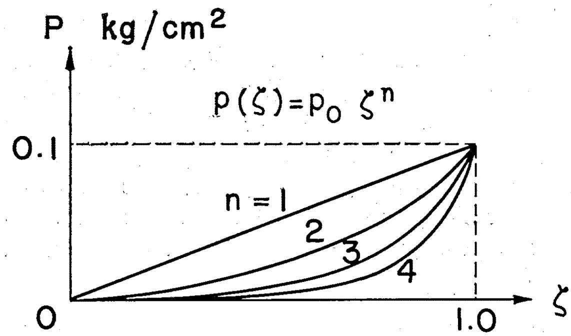

Fig. 12.

Table IV.

$g(\zeta)=p(\zeta)/p_0$		
No.	$g(\zeta)$	max. def.
1	ζ	0.160
2	ζ^2	0.128
3	ζ^3	0.107
4	ζ^4	0.092

max. def. : maximum deflection in cm

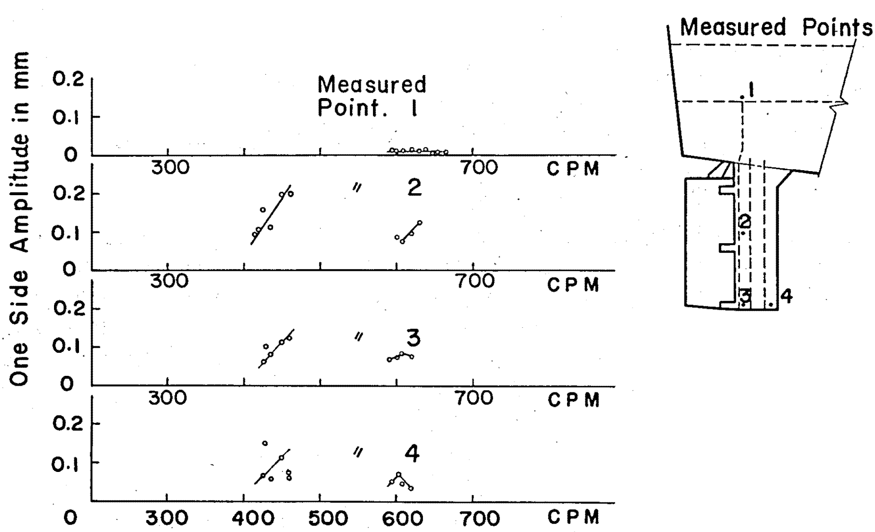

Fig. 13.

Therefore the violent resonance phenomenon is supposed to be avoided in actual condition because of the reduction of vitual weight, the nature of supposed nonlinear vibration and structural and hydrodynamical damping effect.

7. Comparison with Measured Trial Data and Discussion

Examples of measured data of the system are shown in Fig. 13. The actual amplitudes were far less than statically estimated ones obtained by above mentioned way and the estimated fundamental frequency (460 cpm) was clearly recognized from them which are throught to prove correctness of the assumptions in this theory within small amplitude of vibration. It is considered that the sufficient stiffness of the structure of rudder-post and the effect of friction between pintles and gudgeons made realized the vibration mode given by the linear theory in this case and the effect of clearance mentioned above and miscellaneous damping effects prevented violent vibrations.

However it may not be guaranteed for the case of the rudder-post with insufficient rigidity.

8. Conclusion

Methods of estimating the natural frequencies of rudder and rudder-post system of a ship of Mariner type have been developed.

If the possibility of arising resonance is anticipated by the linear theory, it is recommended to design the structure of rudder post so as to have sufficient rigidity of which order of magnitude should be determined by checking the rate of estimated statical deflection by induced pressure of propeller tips and the clearances of the gudgeons and pintles.

Acknowledgement

Authors wish to thank Miss J. Fukuda, Reserarch Assistant of Kyushu University who assisted us greatly in carrying out this study.

References

[1] Ir. R. Wereldsma. Model tests for determining critical vibrations of the rudder-ppost of a "Mariner" Rudder. International Shipbuilding Progress. Vol. 6, No. 57, May 1959.
[2] T. Kumai. Added mass moment of inertia induced by tosional vibration of ships. European Shipbuilding, No. 6, 1958.
[3] V. Z. Vlasov. Thin-walled elastic beams. 2nd edition published for the National Science Foundation, Washington, D. C. and the Department of Commerce, U. S. A. by the Israel Program for Scientific Translations, Jerusalem 1961, pp. 263-272.
[4] F. M. Lewis. A modification of Rayleigh's principle for calculating beam frequencies. Jour. of App. Mech. Sept. 1959, pp. 452.
[5] F. H. Todd. Ship hull vibration. Edward Arnold (Publishers) Ltd. London 1961, pp. 117-122.

(Received July 15, 1970)