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ON THE MOTION OF MULTIHULL SHIPS IN WAVES (I)

By Makoto OHKUSU*

In this paper are proposed an approximate method to calculate hydrodynamic force and moment working upon multiple cylinders with an arbitrary cross section which are forced to make a heaving, swaying and rolling oscillation about their mean position.

If we know the hydrodynamic coefficients (added mass and damping coefficient etc.) for a cylinder, we can comparatively easily calculate the coefficients for multiple cylinders composed of an arbitrary number of the cylinders by the approximate method. The accuracy of the coefficients obtained by the method was confirmed to be satisfactory by the comparison with the exact solutions for two circular cylinders.

This simple method will be effective for the theoretical treatment of the motions of multihull ships in waves.

1. Introduction

There have been reported many researches[1] upon the motion of catamaran due to waves, while there seem to be few examples that the motion is treated theoretically, for example, by the theory of ship motions which has been successfully applied to ordinary ships. The floating rig which has been recently used for the development of the ocean resources has common characteristic with catamaran ship that it is composed of several simple hulls. And theoretical researches may be also necessary for understanding its behaviour in the sea and designing its performances.

This paper is concerned with the most fundamental problem for the theoretical treatment of the motion of such multihull ships, that is, the theoretical calculation of the hydrodynamic forces, the amplitude and the phase of the waves diverging at infinity, when two or more infinitely long cylinders, submerged or floating in a fluid, are given a swaying, heaving and rolling forced oscillation about their mean position, under the assumption of inviscid fluid, no surface tension and the linearization of boundary conditions on the free surface of a fluid and the cylinders' surface. In the previous paper[10] the theoretical procedure to calculate the hydrodynamic pressure and the wave amplitude caused by the heaving motion of two circular cylinders on the surface of a fluid was developed and it was found that the results of numerical calculation agreed

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with the experimental results. Just the same method can be applied to the other motions, swaying and rolling, of two circular cylinders (Appendix 1). The calculation by this method, however, is complicated and not convenient to solve the more general problem for obtaining the hydrodynamic forces and the waves generated due to the oscillation of two or more cylinders with an arbitrary cross section. Then in this paper there is shown an approximate method suitable to treat such a general problem and its accuracy is examined through the comparison of the numerical results by this approximation with the exact solution for two circular cylinders by the procedure described in the previous paper.

If we know the hydrodynamic forces, the amplitude and the phase of waves outgoing at infinity when a body, whether it may be two dimensional or three dimensional, makes a heaving, swaying and rolling oscillation, the phase as well as the amplitude of wave forces and moment which act upon the body when it is held fixed on incoming plane waves can be easily completely derived by Haskind relation[9] extended by Bessho[8] (Appendix 2). Accordingly the equations of the motion of multiple cylinders due to waves can be mathematically formulated through only the solutions of the radiation problem for the multiple cylinders, which are the purpose of this paper, if the viscosity effect of a fluid is not taken into account.

2. The hydrodynamic force on multiple cylinders

Many analytical solutions are known for the radiation problem for one cylinder. The solutions are, for example, Ursell's one[2][3] for floating and submerged circular cylinder and the solution by Tasai[4][5] and others for a cylinder with a cross section derived from a circle by conformal mapping. In addition Faltinsen,[6] Maeda[7] have recently reported the results for cylinders with an arbitrary cross section which have been calculated through solving numerically integral equation with respect to singularity distribution on the cylinder surface so that the boundary consition may be satisfied.

In this section is shown an approximate method to calculate the hydrodynamic coefficients (added mass and moment of inertia, damping coefficients) and the diverging waves of N cylinders with an arbitrary cross section by making use of these quantities for one cylinder given in the above mentioned various solutions.

Let us consider the heaving motion $y=Re[e^{i\omega t}]$, swaying motion $x=Re[e^{i\omega t}]$ and the rolling motion about the origin O, $\theta=Re[e^{i\omega t}]$ of "a" cylinder composed of two cylinders as shown in Fig. 1, each of which need not have the same cross section while they must be symmetrical with respect to their own center line, where the period of the motions is $2\pi/\omega$.

The distances from the origin of the coordinate system to the center L of the cylinder at left (hereafter referred to cylinder L) and the center R of the cylinder at right (cylinder R) are denoted respectively by P_L, P_R. Let they have

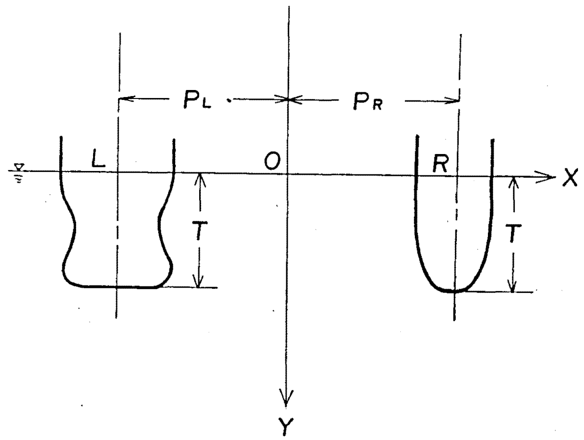

Fig. 1 Coordinate system.

the same draft T for simplicity. The volume per unit length displaced by the cylinder L and R are respectively defined to be $V_L=k_L\cdot T^2$ and $V_R=k_R\cdot T^2$.

Suppose that the added mass coefficients (the added mass moment of inertia coefficients in the case of rolling oscillation) and the diverging waves at $x\rightarrow+\infty$ for each of the two cylinders oscillating without the existence of the other cylinder (the center of rolling is the point L or R for the cylinder L or R) are known and expressed as follows.

Table 1.

Motion	Wave at $X=+\infty$	Added mass coefficient
$x=e^{i\omega t}$,	$\overline{A}_S^{L,R} e^{i\varepsilon_S^{L,R}} e^{i(\omega t-KX_{L,R})}$,	$m_S^{L,R}$
$y=e^{i\omega t}$,	$\overline{A}_H^{L,R} e^{i\varepsilon_H^{L,R}} e^{i(\omega t-KX_{L,R})}$,	$m_H^{L,R}$
$\theta=e^{i\omega t}$,	$T\cdot\overline{A}_R^{L,R} e^{i\varepsilon_R^{L,R}} e^{i(\omega t-KX_{L,R})}$,	$m_R^{L,R}$

Where K is the wave number ω^2/g(g is the gravity acceleration) and clockwise rolling motion around L or R is taken to be positive and superscript L and R denote the quantities for the cylinder L and R respectively. X_L means the X coordinate with respect to the origin L for the case of the oscillation of the left cylinder and X_R with respect to the origin R for the right cylinder. Added mass coefficients and added mass moment inertia coefficient are defined to be the component of the hydrodynamic force per unit length of the cylinder 180° out of phase with the acceleration of motion/($\rho V_{L,R}\times$ the acceleration) and the component of the hydrodynamic moment per unit length 180° out of phase with the acceleration of rolling motion/(the acceleration$\times\rho V_{L,R}\times T^2$), where ρ is the density of a fluid. In addition we introduce the following expression for the hydrodynamic cross coupling, that is the rolling moment exerted upon swaying cylinder, which is given by the summation of the components proportional to $-\dfrac{d^2x}{dt^2}$ and $-\dfrac{dx}{dt}$

$$\rho V_{L,R}\cdot T\cdot M^{L,R}\left(-\frac{dx^2}{dt^2}\right)+\frac{\rho g^2}{\omega^3}\cdot T\cdot N^{L,R}\left(-\frac{dx}{dt}\right). \tag{1}$$

It should be noted, here, that for the application of the procedure described below it is not necessarily required that the left or right cylinder must be one cylinder but that the hydrodynamic coefficients of the left and right cylinder are previously known. As a result if we know the hydrodynamic coefficients for one cylinder and $(N-1)$ cylinders with an arbitrary cross section we can calculate the hydrodynamic coefficients for N cylinders with that cross section through the use of the procedure, because if the left cylinder is considered to be a bundle of $(N-1)$ cylinders, the hydrodynamic characteristics of which are known, and the right cylinder to be one cylinder, then we can calculate the hydrodynamic force or moment for "two" cylinders made of the left and the right cylinder, that is, N cylinders. Summing up we can calculate the hydrodynamic coefficients for a bundle of cylinders composed of an arbitrary number of cylinders if we know those of one cylinder.

i). Heaving

At first consider the heaving motion of two cylinders.

The method adopted here to calculate the hydrodynamic coefficients of two cylinders is based upon an approximation that there exists the interference between two cylinders only with respect to the progressing waves generated by the oscillation of each of the cylinders, but the standing waves being mainly in the immediate vicinity of one cylinder have no influence upon the other cylinder. It is of course that the larger the distance between the cylinders is as compared with the wave length and the dimension of the cylinders, we can obtain the more exact solution under this approximation.

The velocity potential ϕ which satisfies the boundary conditions on both the cylinder L and the cylinder R can be written as follows,

$$\phi=\phi_L^0+\phi_R^0+\phi_L^1+\phi_R^1+\phi_L^2+\phi_R^2+\cdots, \tag{2}$$

where ϕ_L^0, ϕ_R^0 show respectively the velocity potentials that represent a fluid motion as the cylinder L or the cylinder R oscillate without the other cylinder and ϕ_L^1 is determined as a diffraction potential of ϕ_R^0 due to the cylinder L which satisfies the condition $\frac{\partial}{\partial n}(\phi_R^0+\phi_L^1)=0$ on the surface of the cylinder L and ϕ_R^1 is a diffraction of ϕ_L^0 due to the cylinder R under the similar condition on this cylinder and so on.

According to the above mentioned approximation, near the cylinder L the velocity potentials ϕ_R^0, ϕ_R^1, ϕ_R^2,...... include only the progressing wave potentials and so the velocity potentials ϕ_L^0, ϕ_L^1, ϕ_L^2, ... near the cylinder R. It should be noted that on the cylinder L,

$$\frac{\partial}{\partial n}(\phi_L^1+\phi_L^2+\cdots+\phi_R^0+\phi_R^1+\phi_R^2+\cdots)=0. \tag{3}$$

It follows that since in the neighbourhood of the cylinder R the wave ζ_L^0 due to ϕ_L^0 is given by

$$\zeta_L^0 = a_L^0 e^{i(\omega t - KX_L)} = \overline{A}_H^L e^{i\varepsilon_H^L} e^{i(\omega t - KX_L)}, \tag{4}$$

the wave ζ_R^1 corresponding to ϕ_R^1 which is a reflection of ϕ_L^0 by the cylinder R becomes the progressing wave expressed by the equation (5) in the neighbourhood of the cylinder L.

$$\begin{aligned}\zeta_R^1 &= a_R^1 \cdot e^{i(\omega t + KX_L)} \\ &= iH_R^+(K) e^{-i2K(P_L+P_R)} \ a_L^0 e^{i(\omega t + KX_L)},\end{aligned} \tag{5}$$

where

$$H_R^+(K) = ie^{i\varepsilon_H^R}\cos\varepsilon_H^R - e^{i\varepsilon_S^R}\sin\varepsilon_S^R \tag{6}$$

(Appendix 2).
Similarly ζ_R^3 due to ϕ_R^3 is derived from the reflection by the cylinder R of the reflection of ζ_R^1 by the cylinder L.

Hence

$$\begin{aligned}\zeta_R^3 &= a_R^3 e^{i(\omega t + KX_L)} \\ &= iH_L^+(K) \cdot iH_R^+(K) e^{-i2K(P_L+P_R)} a_R^1 e^{i(\omega t + KX_L)}\end{aligned} \tag{7}$$

where

$$H_L^+(K) = ie^{i\varepsilon_H^L}\cos\varepsilon_H^L - e^{i\varepsilon_S^L}\sin\varepsilon_S^L \tag{8}$$

Therefore by repeating this procedure we can obtain the coming wave upon the cylinder L, which is due to the potential $\phi_R^1 + \phi_R^3 + \phi_R^5 + \cdots$, as follows,

$$\begin{aligned}& a_R^1(1+\alpha+\alpha^2+\alpha^3+\cdots)e^{i(\omega t + KX_L)} \\ &= iH_R^+(K) e^{-i2K(P_L+P_R)} \overline{A}_H^L e^{i\varepsilon_H^L}(1+\alpha+\alpha^2+\cdots)e^{i(\omega t + KX_L)}\end{aligned} \tag{9}$$

where

$$\alpha = iH_L^+(K) iH_R^+(K) e^{-i2K(P_L+P_R)}$$

Similarly the coming wave upon the cylinder L due to the velocity potential $\phi_R^0 + \phi_R^2 + \phi_R^4 + \cdots$ is expressed as

$$\overline{A}_H^R e^{i\varepsilon_H^R} e^{-iK(P_L+P_R)}(1+\alpha+\alpha^2+\cdots)e^{i(\omega t + KX_L)}. \tag{10}$$

The series (9) and (10) converge because $|H_{L,R}^+(K)|$ is necessarily smaller than the unity if the wave length is not so small in comparison with the depthwise dimension of the cylinders.

It may concluded that under our approximation the hydrodynamic force upon the cylinder L is given by the sum of the one due to ϕ_L^0 which is the velocity potential when the cylinder L makes a heaving oscillation independently of the cylinder R and the one due to the incident wave ζ_R which is the sum of the series (9) and (10) such as

$$\zeta_R = a_R \cdot e^{i(\omega t + KX_L)}$$
$$= \frac{iH_R^+(K)e^{-i2K(P_L+P_R)}\overline{A}_H^L e^{i\varepsilon_H^L} + \overline{A}_H^R \cdot e^{i\varepsilon_H^R} \cdot e^{-iK(P_L+P_R)}}{1 - iH_R^+(K) iH_L^+(K) e^{-i2K(P_L+P_R)}} e^{i(\omega t + KX_L)}. \tag{11}$$

The latter include, of course, the effect of the diffraction potential by the cylinder L of the wave ζ_R, that is, $\phi_L^1 + \phi_L^2 + \phi_L^3 + \cdots$ according to the equation (3). Referring to Appendix 2 the components of the force due to ζ_R in various directions are

Table 2.

direction	force (or moment)
x ,	$\frac{i\rho g}{K} a_R \cdot \overline{A}_S^L \cdot e^{i\varepsilon_S^L} e^{i\omega t}$
y ,	$\frac{i\rho g}{K} a_R \cdot \overline{A}_H^L e^{i\varepsilon_H^L} e^{i\omega t}$
Mt. about L ,	$\frac{i\rho g T}{K} a_R \cdot \overline{A}_R^L e^{i\varepsilon_R^L} e^{i\omega t}$

By just the same procedure we can calcualte the hydrodynamic force upon the cylinder R as the sum of the force by the potential ϕ_R^0 and the incident wave ζ_L upon the cylinder given by

$$\zeta^L = a_L \cdot e^{i(\omega t - KX_R)}$$
$$= \frac{iH_L^+(K)e^{-i2K(P_L+P_R)}\overline{A}_H^R e^{i\varepsilon_H^R} + \overline{A}_H^L \cdot e^{i\varepsilon_H^L} \cdot e^{-iK(P_L+P_R)}}{1 - iH_R^+(K) iH_L^+(K) e^{-i2K(P_L+P_R)}} e^{i(\omega t - KX_R)}. \tag{12}$$

The force due to the latter is also shown in the following table

Table 3.

direction	force (or moment)
x ,	$-\frac{i\rho g}{K} a_L \cdot \overline{A}_S^R e^{i\varepsilon_S^R} e^{i\omega t}$
y ,	$\frac{i\rho g}{K} a_L \cdot \overline{A}_H^R e^{i\varepsilon_H^R} e^{i\omega t}$
Mt. about R ,	$-\frac{i\rho g T}{K} a_L \cdot \overline{A}_R^R e^{i\varepsilon_R^R} e^{i\omega t}$

The hydrodynamic force working on the two cylinders can be computed by adding to the forces shown in these tables the hydrodynamic forces due to the potential ϕ_L^0 and ϕ_R^0 which are the forces acting upon the cylinder L and the cylinder R when they are making an independent heaving oscillation of each other, and given by the solution for one cylinder as shown in Table 1. Then the components of forces, F_x in X direction, F_y in Y direction and a moment M_z about the origin O, become

$$F_x=\frac{i\rho g}{K}(\overline{A}_S^L\cdot e^{i\varepsilon_S^L}\cdot a_R-\overline{A}_S^R\cdot e^{i\varepsilon_S^R}\cdot a_L)e^{i\omega t}, \tag{13}$$

$$\begin{aligned}F_y=&\frac{i\rho g}{K}(\overline{A}_H^L e^{i\varepsilon_H^L}\cdot a_R+\overline{A}_H^R\cdot e^{i\varepsilon_H^R}\cdot a_L)e^{i\omega t}\\&+\rho\left(-\frac{d^2y}{dt^2}\right)(V_L\cdot m_H^L+V_R\cdot m_H^R)+\frac{\rho g^2}{\omega^3}\cdot\left(-\frac{dy}{dt}\right)(\overline{A}_L^{H^2}+\overline{A}_H^{R^2}),\end{aligned} \tag{14}$$

$$\begin{aligned}M_z=&\frac{i\rho g}{K}(T\cdot\overline{A}_R^L e^{i\varepsilon_H^L}\cdot a_R-T\cdot\overline{A}_R^R\cdot e^{i\varepsilon_R^R}\cdot a_L)e^{i\omega t}\\&+\frac{i\rho g}{K}(-P_L\cdot\overline{A}_H^L e^{i\varepsilon_H^L}\cdot a_R+P_R\cdot\overline{A}_H^R\cdot e^{i\varepsilon_H^R}\cdot a_L)e^{i\omega t}\\&+\rho\left(-\frac{d^2y}{dt^2}\right)(-P_L\cdot V_L\cdot m_H^L+P_R\cdot V_R\cdot m_H^R)+\frac{\rho g^2}{\omega^3}\left(-\frac{dy}{dt}\right)\\&\times(-P_L\cdot\overline{A}_H^{L^2}+P_R\cdot\overline{A}_H^{R^2}),\end{aligned} \tag{15}$$

The added mass coefficient m_H^{L+R} of the heaving two cylinders (added mass $/\rho(V_L+V_R)$) and the damping coefficient N_H^{L+R} are easily derived from F_y and they are

$$\begin{aligned}m_H^{L+R}=&\frac{k_L}{k_L+k_R}m_H^L+\frac{k_R}{k_L+k_R}m_H^R\\&+\frac{1}{(KT)^2(k_L+k_R)}Re\,[i(\overline{A}_H^L e^{i\varepsilon_H^L}\cdot a_R+\overline{A}_H^R\cdot e^{i\varepsilon_H^R}\cdot a_L)],\end{aligned} \tag{16}$$

$$N_H^{L+R}=\frac{\rho g^2}{\omega^3}(\overline{A}_H^{L^2}+\overline{A}_H^{R^2})-\frac{\rho g^2}{\omega^3}Im\,[i(\overline{A}_H^L e^{i\varepsilon_H^L}\cdot a_R+A_H^R\cdot e^{i\varepsilon_H^R}\cdot a_L)]. \tag{17}$$

Nextly the progressing wave elevation ζ_∞ at $X=+\infty$ generated by the heaving motion of the two cylinders may be regarded as the sum of the wave caused by the independent motion of the cylinder R and the transmitted component of the incident wave ζ_L by the cylinder R and it is given by

$$\begin{aligned}\zeta_L&=\overline{A}_H e^{i\varepsilon_H}e^{i(\omega t-KX)}\\&=[\overline{A}_H^R e^{i\varepsilon_H^R}+(1+iH_R^-(K))\cdot a_L]\,e^{iKP_R}\cdot e^{i(\omega t-KX)},\end{aligned} \tag{18}$$

where

$$H_R^-(K)=ie^{i\varepsilon_H^R}\cos\varepsilon_H^R+e^{i\varepsilon_S^R}\cdot\sin\varepsilon_H^R \tag{19}$$

and since X coordinate is measured from the origin O, the term e^{iKP_R} is multiplied. And the wave at $X=-\infty$ can be obtained similarly.

ii). Swaying

Just the same procedure as in the heaving motion of two cylinders can be applied to the computation of the hydrodynamic force upon the swaying two cylinders. Then here are shown only the results derived from this application.

Using the identical notation as in i),

$$\zeta_R = a_R \cdot e^{i(\omega t + KX_L)}$$
$$= \frac{iH_R^+(K) e^{-i2K(P_L+P_R)} \overline{A}_S^L \cdot e^{i\varepsilon_S^L} - \overline{A}_S^R \cdot e^{i\varepsilon_S^R} e^{-iK(P_L+P_R)}}{1 + H_R^+(K) \cdot H_L^+(K) e^{-i2K(P_L+P_R)}} e^{i(\omega t + KX_L)}, \tag{20}$$

$$\zeta_L = a_L \cdot e^{i(\omega t - KX_R)}$$
$$= \frac{-iH_L^+(K) e^{-i2K(P_L+P_R)} \cdot \overline{A}_S^R e^{i\varepsilon_S^R} + \overline{A}_S^L e^{i\varepsilon_S^L} e^{-iK(P_L+P_R)}}{1 + H_R^+(K) \cdot H_L^+(K) e^{-i2K(P_L+P_R)}} e^{i(\omega t - KX_R)}, \tag{21}$$

The hydrodynamic force upon the two cylinders is given as the sum of the one which the wave $a_R \cdot e^{i(\omega t + KX_L)}$ acts upon the cylinder L and the wave $a_L \cdot e^{i(\omega t - KX_R)}$ upon the cylinder R, and the one to which they are respectively subjected when they are making a swaying oscillation independently of each other. We can calculate as well the moment upon the two cylinders as the force.

Then the force F_x in x-direction, F_y in y-direction and the moment M_z about the origin O are given by

$$F_x = \frac{i\rho g}{K} (\overline{A}_S^L e^{i\varepsilon_S^L} \cdot a_R - \overline{A}_S^R e^{i\varepsilon_S^R} a_L) e^{i\omega t}$$
$$+ \rho \left(-\frac{d^2x}{dt^2}\right) (V_L \cdot m_S^L + V_R \cdot m_S^R) + \frac{\rho g^2}{\omega^3} \cdot \left(-\frac{dx}{dt}\right) (\overline{A}_S^{L^2} + \overline{A}_S^{R^2}), \tag{22}$$

$$F_y = \frac{i\rho g}{K} (\overline{A}_H^L \cdot e^{i\varepsilon_H^L} \cdot a_R + \overline{A}_H^R \cdot e^{i\varepsilon_H^R} \cdot a_L) \cdot e^{i\omega t}, \tag{23}$$

$$M_z = \frac{i\rho g T}{K} (\overline{A}_R^L e^{i\varepsilon_R^L} \cdot a_R - \overline{A}_R^R \cdot e^{i\varepsilon_R^R} \cdot a_L) e^{i\omega t}$$
$$+ \frac{i\rho g}{K} (-\overline{A}_H^L \cdot P_L \cdot e^{i\varepsilon_H^L} \cdot a_R + \overline{A}_H^R \cdot P_R \cdot e^{i\varepsilon_H^R} \cdot a_L) e^{i\omega t}$$
$$+ \rho T \left(-\frac{d^2x}{dt^2}\right) (V_L \cdot M^L + V_R \cdot M^R) + \frac{\rho g^2}{\omega^3} T \left(-\frac{dx}{dt}\right) (N^L + N^R). \tag{24}$$

Accordingly the added mass coefficient m_S^{L+R} of swaying motion, for example, is

$$m_S^{L+R} = \frac{k_L}{k_L + k_R} m_S^L + \frac{k_R}{k_L + k_R} m_S^R + \frac{1}{(KT)^2 (k_L + k_R)} \cdot \times$$
$$Re[i(\overline{A}_S^L e^{i\varepsilon_S^L} \cdot a_R - \overline{A}_S^R e^{i\varepsilon_S^R} \cdot a_L)], \tag{25}$$

and the damping coefficient N_S^{L+R} is

$$N_S^{L+R} = \frac{\rho g^2}{\omega^3} (\overline{A}_S^{L^2} + \overline{A}_S^{R^2}) - \frac{\rho g^2}{\omega^3} \cdot Im[i(\overline{A}_S^L \cdot e^{i\varepsilon_S^L} \cdot a_R - \overline{A}_S^R \cdot e^{i\varepsilon_S^R} \cdot a_L)]. \tag{26}$$

If we express the moment M_z as follows

$$\rho(V_L+V_R)\cdot T\cdot M^{L+R}\left(-\frac{d^2x}{dt^2}\right)+\frac{\rho g^2}{\omega^3}\cdot T\cdot N^{L+R}\left(-\frac{dx}{dt}\right), \tag{27}$$

then

$$M^{L+R}=\frac{k_L}{k_L+k_R}M^L+\frac{k_R}{k_L+k_R}M^R+\frac{1}{(KT)^2(k_L+k_R)}\times$$

$$Re[i(\overline{A}_R^L e^{i\varepsilon_R^L}\cdot a_R-\overline{A}_R^R e^{i\varepsilon_R^R}\cdot a_L-\frac{P_L}{T}\cdot\overline{A}_H^L e^{i\varepsilon_H^L}a_R+\frac{P_R}{T}\overline{A}_H^R\cdot e^{i\varepsilon_H^R}\cdot a_L)], \tag{28}$$

$$N^{L+R}=N^L+N^R-Im\,[i(\overline{A}_R^L\cdot e^{i\varepsilon_R^L}\cdot a_R-\overline{A}_R^R e^{i\varepsilon_R^R}\cdot a_L$$

$$-\frac{P_L}{T}\overline{A}_H^L\cdot e^{i\varepsilon_H^L}\cdot a_R+\frac{P_R}{T}\overline{A}_H^R e^{i\varepsilon_H^R}\cdot a_L)], \tag{29}$$

In addition the progressing wave ζ_∞ at $X=+\infty$ is

$$\zeta_\infty=\overline{A}_S e^{i\varepsilon_S}\cdot e^{i(\omega t-KX)}=[\overline{A}_S^R e^{i\varepsilon_S^R}+(1+iH_R^-(K))\cdot a_L]e^{iKP_R}\cdot e^{i(\omega t-KX)}. \tag{30}$$

iii). Rolling

The rolling motion of this two cylinders about the origin O can be regarded to be composed of two motions, the rolling motion $\theta=Re\,[e^{i\omega t}]$ of the cylinder L and R about their own centre L and R, and the heaving motion $y_L=-Re\,[P_L\cdot e^{i\omega t}]$ of the cylinder L and the motion $y_R=Re[P_R\cdot e^{i\omega t}]$ of the cylinder R, because the swaying of each cylinder caused by this rolling motion of the two cylinders is of secondary effect.

Applying the same procedure as in i), ii) to each of these two motions we get the wave ζ_R coming upon the cylinder L and ζ_L upon the cylinder R as follows,

$$\zeta_R=a_R e^{i(\omega t+KX_L)}$$

$$=\frac{1}{1+H_L^+(K)H_R^+(K)e^{-i2K(P_L+P_R)}}\times[-\overline{A}_H^L e^{i\varepsilon_H^L}P_L\cdot iH_R^+(K)e^{-i2K(P_L+P_R)}$$

$$+\overline{A}_H^R e^{i\varepsilon_H^R}P_R\cdot e^{-iK(P_L+P_R)}+\overline{A}_R^L Te^{i\varepsilon_R^L}iH_R^+(K)e^{-i2K(P_L+R_R)}-\overline{A}_R^R Te^{i\varepsilon_R^R}e^{-iK(P_L+P_R)}]$$

$$\times e^{i(\omega t+KX_L)}, \tag{31}$$

$$\zeta_L=a_L\cdot e^{i(\omega t-KX_R)}$$

$$=\frac{1}{1+H_L^+(K)H_R^+(K)e^{-i2K(P_L+P_R)}}\times[\overline{A}_H^R e^{i\varepsilon_H^R}P_R\cdot iH_L^+(K)e^{-i2K(P_L+P_R)}$$

$$-\overline{A}_H^L e^{i\varepsilon_H^L}P_L e^{-iK(PL+PR)}+\overline{A}_R^L\cdot T\cdot e^{i\varepsilon_R^L}e^{-iK(P_L+P_R)}-\overline{A}_R^R\cdot T\cdot e^{i\varepsilon_R^R}iH_L^+(K)e^{-i2K(P_L+P_R)}]$$

$$\times e^{(i\omega t-KX_R)} \tag{32}$$

Similarly as in the case of i) or ii) the hydrodynamic force or moment on the two cylinders is considered to be the total of the ones which ζ_R acts on the cylinder L and ζ_L on the cylinder R, and the ones due to the cylinder L's and R's individual heaving motion and rolling motion about the point L and R, and accordingly the total force F_x, F_y and moment M_z on the two cylinders become

$$F_x = \frac{i\rho g}{K}[\overline{A}_S^L e^{i\varepsilon_S^L} a_R - \overline{A}_S^R e^{i\varepsilon_S^R} a_L] e^{i\omega t} + \rho T\left(-\frac{d^2\theta}{dt^2}\right)(V_L M^L + V_R M^R)$$

$$+\frac{\rho g^2}{\omega^3} T\left(-\frac{d\theta}{dt}\right)(N^L + N^R), \tag{33}$$

$$F_y = \rho\left(-\frac{d^2 y_L}{dt^2}\right) V_L m_H^L + \rho\left(-\frac{d^2 y_R}{dt^2}\right) V_R m_H^R + \frac{\rho g^2}{\omega^3}\left(-\frac{dy_L}{dt}\right)\overline{A}_H^{L^2}$$

$$+\frac{\rho g^2}{\omega^3}\left(-\frac{dy_R}{dt}\right)\overline{A}_H^{R^2} + \frac{i\rho g}{K}[\overline{A}_H^L e^{i\varepsilon_H^L} a_R + \overline{A}_H^R e^{i\varepsilon_H^R} a_L] e^{i\omega t}, \tag{34}$$

$$M_z = \rho T^2\left(-\frac{d^2\theta}{dt^2}\right)\left((V_L\cdot m_R^L + V_R\cdot m_R^R) + \frac{\rho g^2}{\omega^3} T^2\left(-\frac{d\theta}{dt}\right)(\overline{A}_R^{L^2} + \overline{A}_R^{R^2})\right.$$

$$-\rho\left(-\frac{d^2 y_L}{dt^2}\right)\cdot P_L\cdot V_L\cdot m_H^L + \rho\left(-\frac{d^2 y_R}{dt^2}\right)\cdot P_R\cdot V_R\cdot m_H^R$$

$$-\frac{\rho g^2}{\omega^3}\left(-\frac{dy_L}{dt}\right)\cdot P_L \overline{A}_H^{L^2} + \frac{\rho g^2}{\omega^3}\left(-\frac{dy_R}{dt}\right)\cdot P_R\cdot\overline{A}_H^{R^2}$$

$$+\frac{i\rho g}{K}[T\cdot\overline{A}_R^L e^{i\varepsilon_R^L}\cdot a_R - T\cdot\overline{A}_R^R e^{i\varepsilon_R^R}\cdot a_L - P_L\cdot\overline{A}_H^L e^{i\varepsilon_H^L}\cdot a_R + P_R\cdot\overline{A}_H^R\cdot e^{i\varepsilon_H^R}\cdot a_L] e^{i\omega t}, \tag{35}$$

From the equation (35) we obtain, for example, the added mass moment of inertia coefficient m_R^{L+R} of the two cylinders as follows (the added mass moment of inertia$/\rho(V_L+V_R)T^2$)

$$m_R^{L+R} = \frac{k_L}{k_L+k_R}\left(m_R^L + \left(\frac{P_L}{T}\right)^2\cdot m_H^L\right) + \frac{k_R}{k_L+k_R}\left(m_R^R + \left(\frac{P_R}{T}\right)^2\cdot m_H^R\right)$$

$$+\frac{1}{(KT)^2\cdot(k_L+k_R)} Re\left[i\left(\overline{A}_R^L\cdot e^{i\varepsilon_R^L}\cdot\frac{a_R}{T} - \overline{A}_R^R\cdot e^{i\varepsilon_R^R}\cdot\frac{a_L}{T}\right.\right.$$

$$\left.\left.-\overline{A}_H^L\cdot\frac{P_L}{T}\cdot e^{i\varepsilon_H^L}\cdot\frac{a_R}{T} + \overline{A}_H^R\cdot\frac{P_R}{T}\cdot e^{i\varepsilon_H^R}\cdot\frac{a_L}{T}\right)\right], \tag{36}$$

and the damping coefficient N_R^{L+R} is also

$$N_R^{L+R} = \frac{\rho g^2}{\omega^3} T^2(\overline{A}_R^{L^2} + \overline{A}_R^{R^2}) + \frac{\rho g^2}{\omega^3}(P_L^2\cdot\overline{A}_H^{L^2} + P_R^2\cdot\overline{A}_H^{R^2})$$

$$-\frac{\rho g^2}{\omega^3} Im[i(T\overline{A}_R^L \cdot e^{i\varepsilon_R^L} \cdot a_R - T \cdot \overline{A}_R^R e^{i\varepsilon_R^R} a_L - P_L \cdot \overline{A}_H^L e^{i\varepsilon_H^L} \cdot a_R + P_R \cdot \overline{A}_H^R e^{i\varepsilon_H^R} \cdot a_L)] \; , \quad (37)$$

The wave ζ_∞ diverging from the cylinders at $x=+\infty$ is given by

$$\zeta_\infty = T \cdot \overline{A}_R e^{i\varepsilon_R} e^{i(\omega t - KX)} = [T \cdot \overline{A}_R^R e^{i\varepsilon_R^R} + P_R \cdot \overline{A}_H^R e^{i\varepsilon_H^R} + (1 + iH_R^-(K)) a_L] e^{iKP_R} \cdot e^{i(\omega t - KX)}, \quad (38)$$

As for cross coupling of sway and rolling the same result as in ii) are obtained.

3. Numerical examples and discussion

Here are shown numerical examples of the hydrodynamic coefficients for some multiple cylinders which are obtained by our approximate method.

i). Two circular cylinders

The hydrodynamic coefficients, added mass and damping coefficient etc., for two circular cylinders of radius a when they are forced to make a heaving, a swaying and a rolling oscillation with their axes on the surface of a fluid were computed by the approximate method shown in the previous section. The results for the heaving and the swaying motion are compared in Fig. 2, 3, 4, 5 and 6 with the exact solutions obtained with the procedure reported in the previous paper[10] and Appendix 1.

From these figures we can find immediately that the results by our approximate method agree almost completely with the exact solution at least for $2P/a \geqq 3$ unless Ka is very small. Then we can infer that for any multiple cylinders our approximate method without complicated computation can give accurate solution enough to be used in the equation of motions and in the result the fairly correct wave exciting force will be derived from this solution by the relation shown in Appendix 2. Consequently the theoretical treatment of the motion of multiple cylinders with an arbitrary section is very easy.

Now substitute into the equations (18) and (30) the following relations in the case of the two circular cylinders $\overline{A}_H^L = \overline{A}_H^R = \overline{A}_H^O$, $\overline{A}_S^L = \overline{A}_S^R = \overline{A}_S^O$, $\varepsilon_H^L = \varepsilon_H^R = \varepsilon_H^O$, $\varepsilon_S^L = \varepsilon_S^R = \varepsilon_S^O$, $V_L = V_R = \frac{\pi}{2} a^2$, and $P_L = P_R = P$. and using the relations (6), (8) and (19) reduce them into more simple form, then for the heaving

$$\zeta_\infty = \frac{2(1+e^{i(2\varepsilon_S^O - 2KP)})}{2+(e^{i(2\varepsilon_H^O - 2KP)} + e^{(2i\varepsilon_S^O - 2KP)})} \overline{A}_H^O e^{i\varepsilon_H^O} e^{i(\omega t - KX_R)}, \quad (39)$$

and for the swaying

$$\zeta_\infty = \frac{2(1-e^{i(2\varepsilon_H^O - 2KP)})}{2-(e^{i(2\varepsilon_H^O - 2KP)} + e^{i(2\varepsilon_S^O - 2KP)})} \overline{A}_S^0 \cdot e^{i\varepsilon_S^O} e^{i(\omega t - KX_R)}, \quad (40)$$

We can know from these expressions of the waves at infinity that the zero

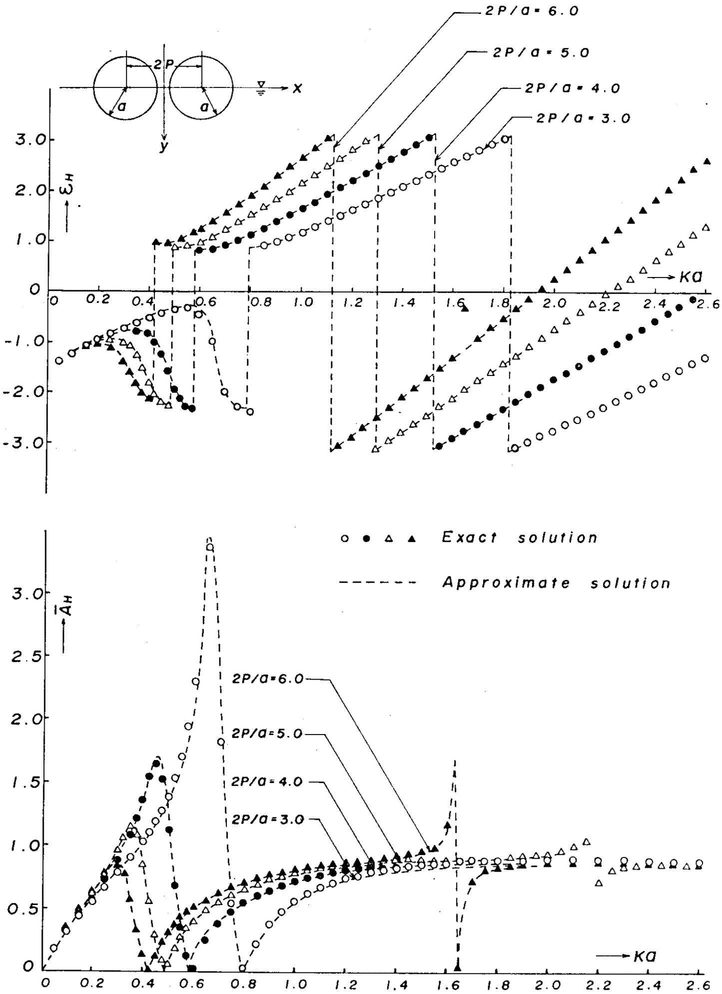

Fig. 2 $\bar{A}_H$, Ratio of wave amplitude to amplitude of heaving motion, ε_H, phase lag of the motion behind wave motion for two circular cylinders.

amplitude of the waves diverging at infinity from the oscillating two cylinders occurs at Ka where $2(\varepsilon_S^0-KP)=(2n+1)\pi$ are satisfied in the case of the heaving and where $2(\varepsilon_H^0-KP)=2n\pi$ in the case of the swaying. Further since $\varepsilon_S^0 \doteqdot \varepsilon_H^0$ when Ka is large as compared with unity, then the amplitude of the waves tends to that of one cylinder $\bar{A}_H^0$, $\bar{A}_S^0$ respectively unless the absolute values of the denominators of the equations (39), (40) are far from zero (this condition is satisfied when $2(\varepsilon_S^0-KP)$ is far from $(2n+1)\pi$ for the heaving and $2(\varepsilon_H^0-KP)$ far from $2n\pi$ for the swaying). On the other hand at Ka where $2(\varepsilon_S^0-KP)$ is nearly equal to $(2n+1)\pi$ and therefore $2(\varepsilon_H^0-KP)$ nearly equal to $(2n+1)\pi$, this

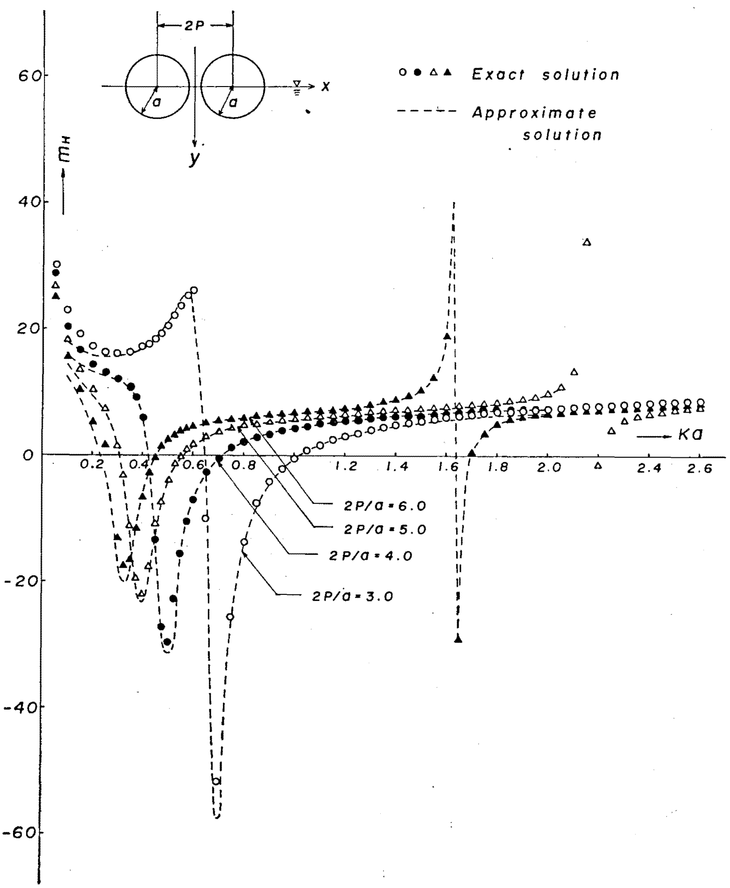

Fig. 3 m_H, Added mass coefficient of heaving motion for two circular cylinders.

amplitude due to the heaving two cylinders becomes very large. As mentioned above near this *Ka* the zero amplitude also occurs and accordingly the variation of the amplitude with respect to *Ka* is ought to be abrupt (though continuous). Such an occurence is found in Fig. 2. As for the swaying it is similar. Since $\varepsilon_S^0 > \varepsilon_H^0$, for the heaving two cylinders the maximum amplitude occurs at *Ka* a little smaller than that where the zero amplitude are realized and for the swaying two cylinders a little larger *Ka*.

As for added mass coefficient such an abrupt change happens but this change may be not necessarily discontinuous known from the equations (16), (25) etc.

The reason why such a phenomenon occurs is that at comparatively large *Ka* reflection coefficient (the amplitude of reflected wave/that of incident wave) is nearly equal to unity (not accurately) and the condition that $2(\varepsilon_H^0 - KP) =$

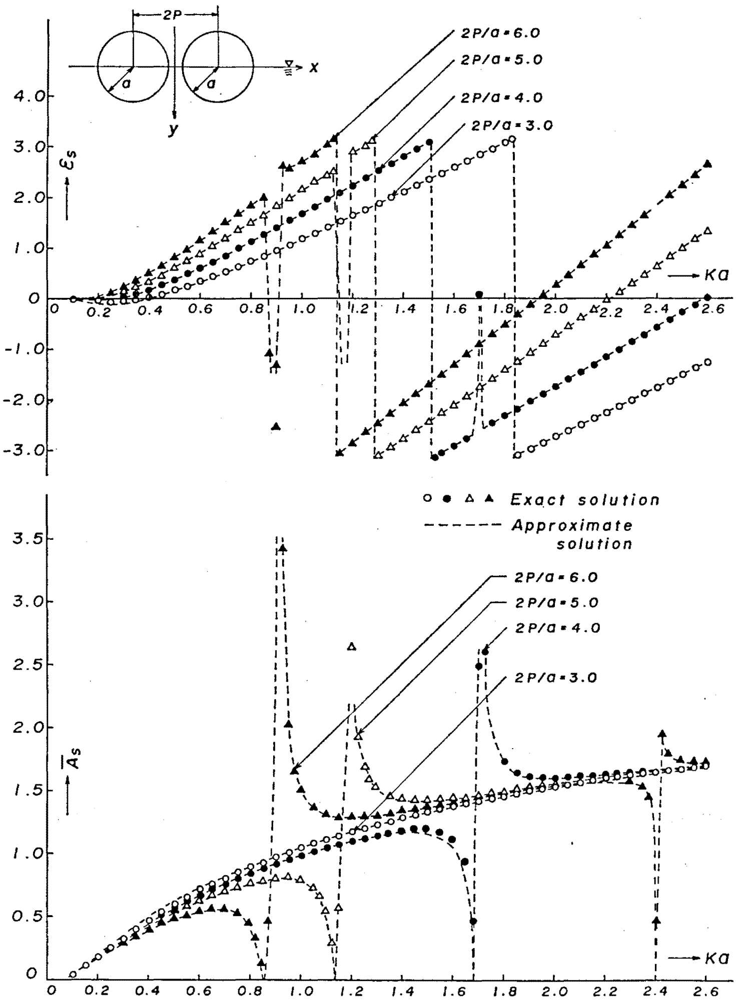

Fig. 4 $\overline{A}_S$, Ratio of wave amplitude to amplitude of swaying motion, ε_S, phase lag of the motion behind wave motion for two circular cylinders.

$(2n+1)\pi$ for heaving and $2(\varepsilon_S^0 - KP) = 2n\pi$ for swaying are satisfied so that the effect of the reflected wave may be accumulated and in the resuts the very high wave comes upon each of the cylinders. Sometimes this high wave are cancel out at infinity by the interference with transmitted wave by the cylinder of this high wave. Even in this case, however, the very high wave exists between the cylinders (this is confirmed through the experiments in the previous paper,).

When such a high wave comes upon the cylinder and in spite of it the force of some phase is totally small, it is expected that the accuracy of the

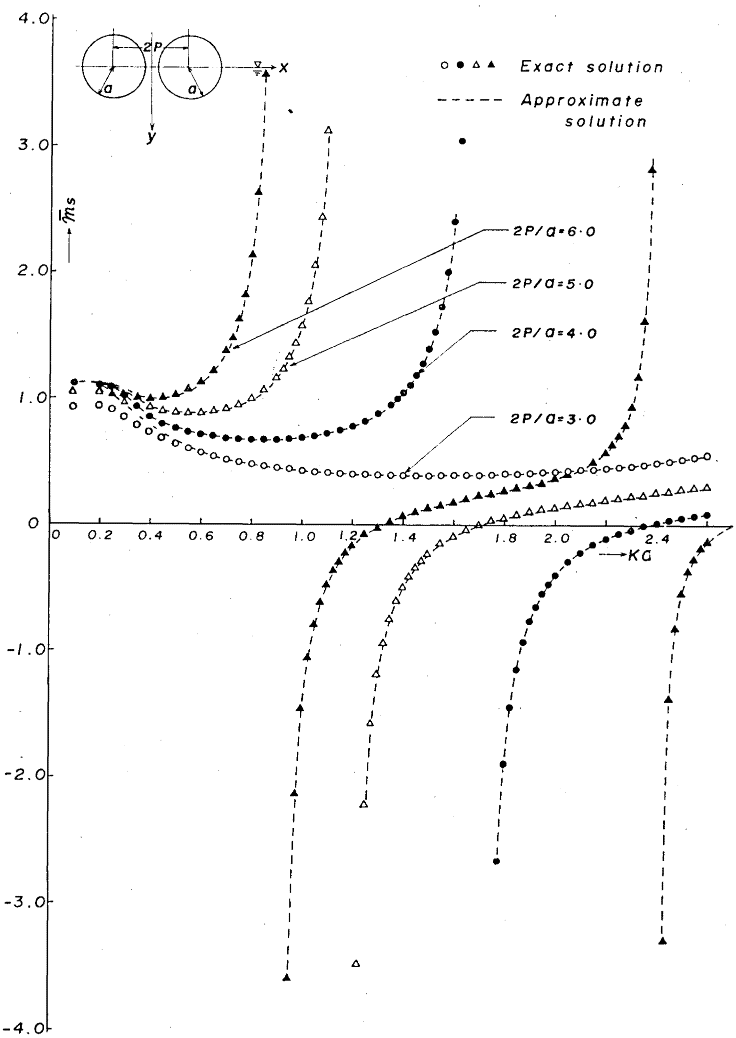

Fig. 5 $\bar{m}_S$, Added mass coefficient of swaying motion for two circular cylinders.

numerical integration of the hydrodynamic pressure along the surface of the cylinder is very much lower. With this fact taking into account our approximate solution is better than the exact solution at such Ka.

ii). Four circular cylinders

Laying 2-cylinders of radius a, the hydrodynamic coefficients of which are known from the calculation of i), at left and at right we can obtain the coefficients for four cylinders.

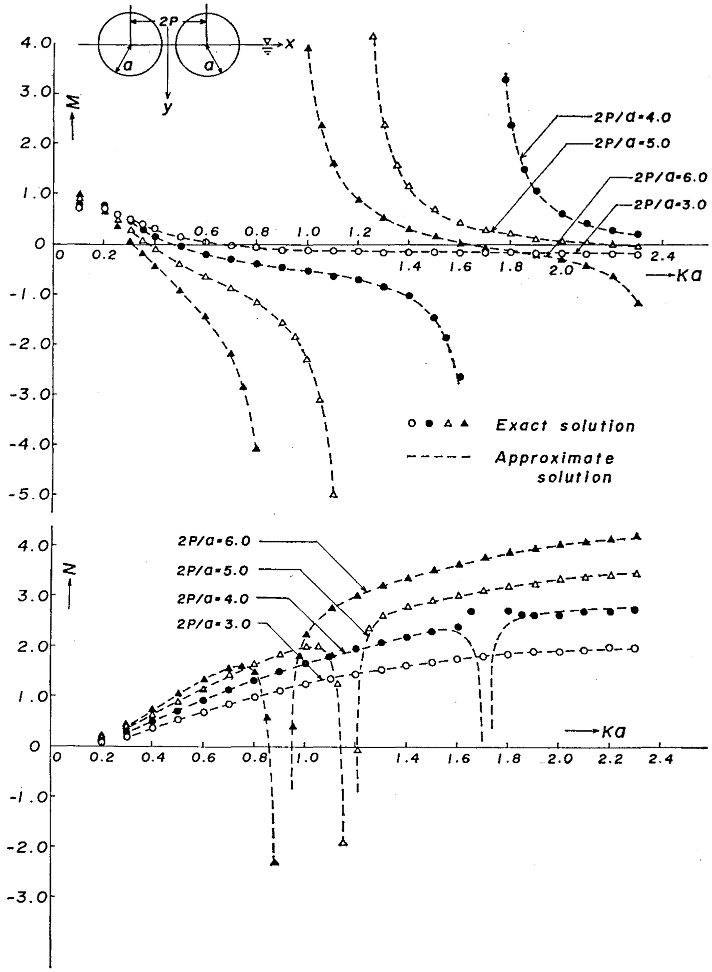

Fig. 6 *M, N,* Coupling term of swaying for two circular cylinders.

Some examples of this calculation are shown in Fig. 7 and 8. It should be noted that the waveless condition are realized at two kinds of *Ka.* One is the *Ka* where both left and right 2-cylinders do not make a progressing wave at infinity shown in i) and so this four cylinders composed of two of these 2-cylinders. Another is the *Ka* at which the interference between the left 2-cylinders and the right 2-cylinders cause the zero amplitude of the wave.

iii). Two and three cylinders with a Lewis form cross section

Some of numerical results for two cylinders and three cylinders with a Lewis form cross section ($a_1=0.0$, $a_3=-0.1$), where three cylinders are con-

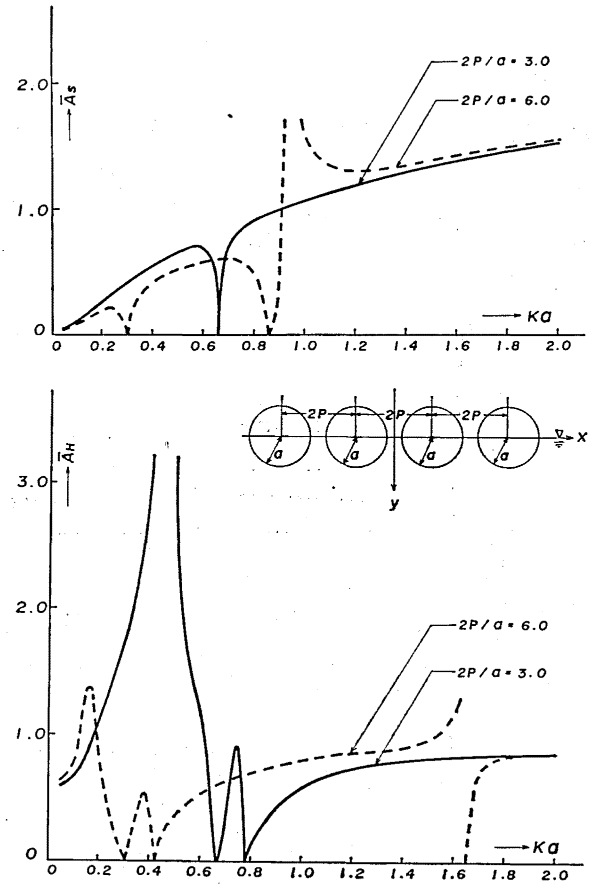

Fig. 7 $\overline{A_H}$, $\overline{A_S}$, Ratio of wave amplitude to amplitude of heaving and swaying motion for four circular cylinders.

sidered to be the combination of a left cylinder (two cylinders) and a right cylinder (one cylinder), and the hydrodynamic coefficients are calculated from those of two cylinders and one cylinder which are previously obtained, are shown in Fig. 9, 10, 11 and Fig. 12, where k_L, k_R is considered to be $\pi/2$.

Thus we can comparatively easily compute the hydrodynamic coefficients for multiple cylinders with a Lewis form cross section and accordingly applying the strip method we can treat theoretically as well the motions of catamaran as ordinary ships.

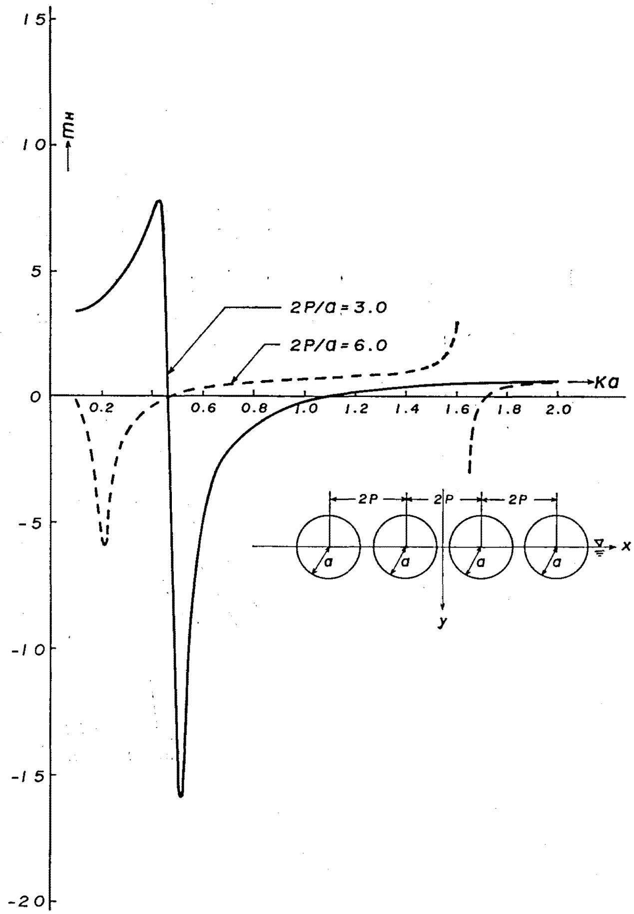

Fig. 8 m_H, Added mass coefficient of heaving motion for four circular cylinders.

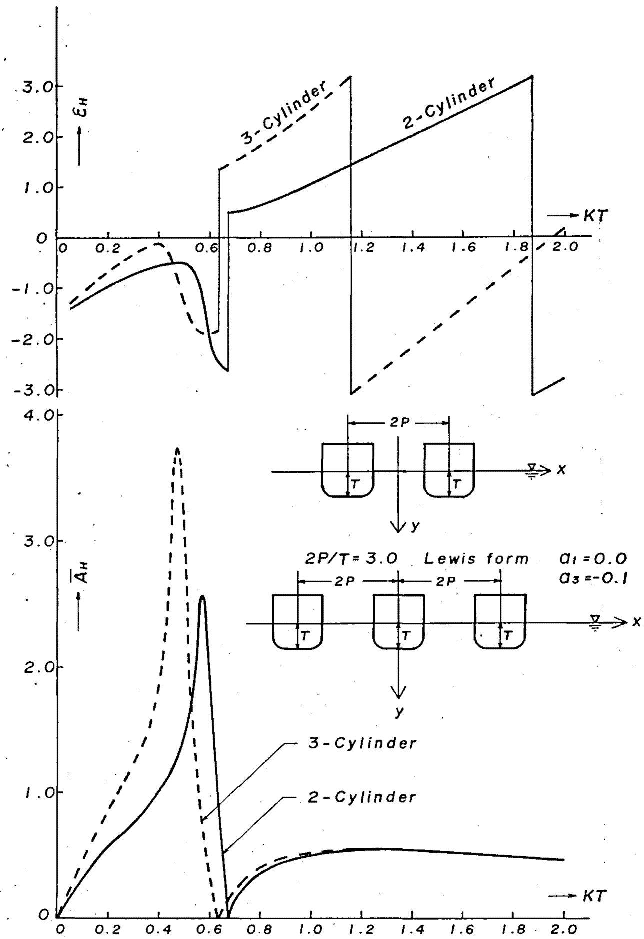

Fig. 9 $\overline{A}_H$, Ratio of wave amplitude to amplitude of heaving motion for two and three Lewis form cylinders.

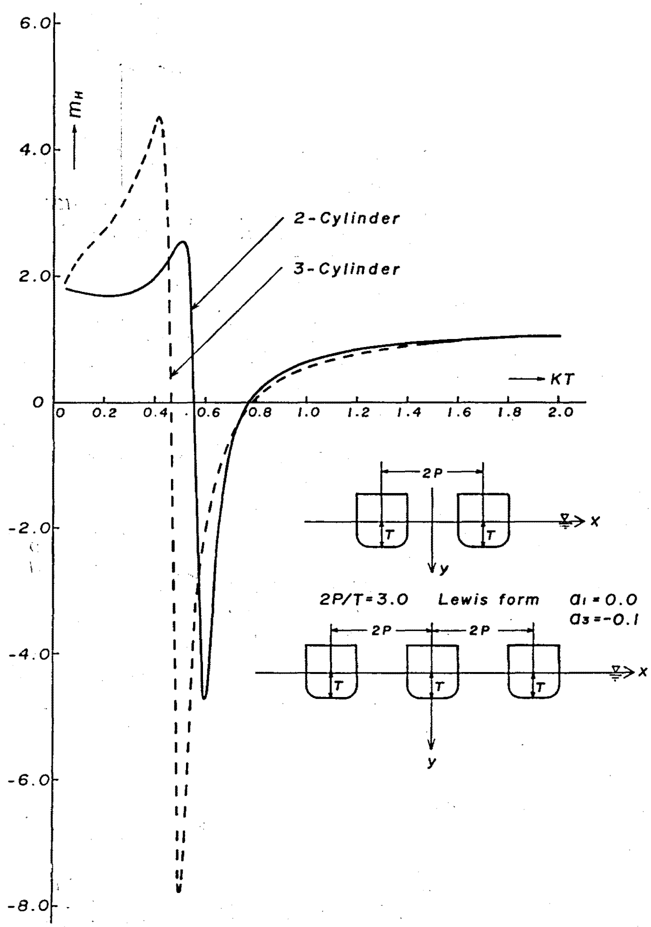

Fig. 10 m_H, Added mass coefficient of heaving motion for two and three Lewis form cylinders.

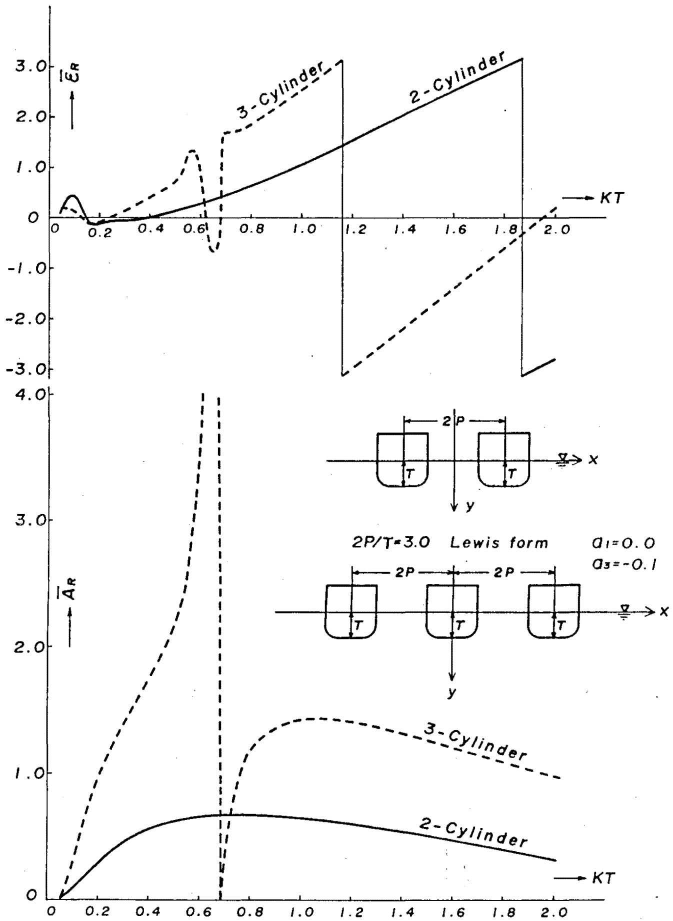

Fig. 11 $\overline{A}_R$, Ratio of wave amplitude to amplitude of rolling motion$\times T$ for two and three Lewis form cylinders.

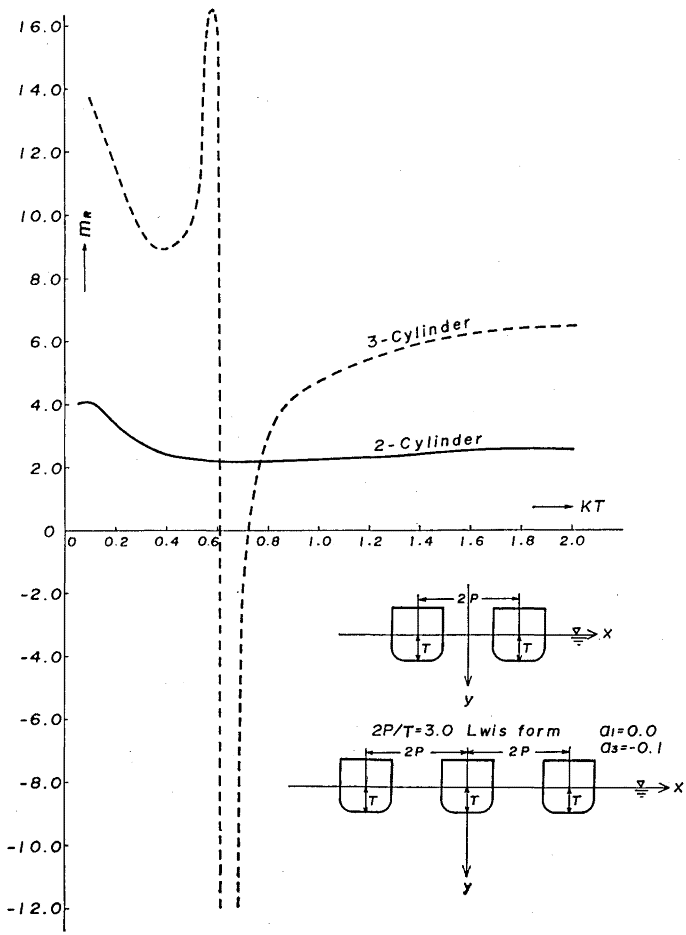

Fig. 12 m_R, Added moment of inertia coefficient of rolling motion for two and three Lewis form cylinders.

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I dedicate this paper to my daughter who passed away during I was writing this paper.

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Appendix 1

Let us suppose that two infinitely long circular cylinders with their axes on the surface of a fluid as shown Fig. A. 1 are forced to make a swaying motion $X=Re[le^{i\omega t}]$ or a rolling motion $\theta=Re\ [\Theta e^{i\omega t}]$ about the origin of the coordinate system and denote the velocity potential describing a fluid motion by $\phi\ e^{i\omega t}$ (here is assumed that a velocity potential exists and the boundary conditions on both the free surface of a fluid and the cylinder surface are linearized). Of course the rolling motion may be regarded to be a heaving

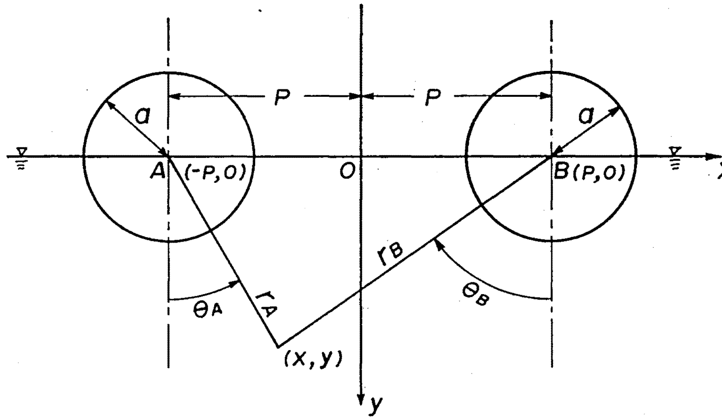

Fig. A. 1. Two circular cylinders

motion $y=-Re\,[P\Theta e^{i\omega t}]$ of the left cylinder and $y=Re\,[P\Theta e^{i\omega t}]$ of the right cylinder.

Applying the same procedure as in the previous paper[10] and defining the velocity potential ϕ_A^0, ϕ_B^0 respectively when the left cylinder is oscillating alone, that is without the existence of the right cylinder, and the right cylinder is oscillating alone, we can obtain the following expression for ϕ

$$\begin{aligned}\frac{\pi\omega}{gl}\phi=\frac{\pi\omega}{gl}(\phi_A^0+\phi_B^0)&+\zeta_1/l\{\phi_S(Kr_A,\ \theta_A)-\phi_S(Kr_B,\ \theta_B)\\&+\sum_{m=1}^{\infty}P_m[f_m(Ka,\ r_A/a,\ \theta_A)-f_m(Ka,\ r_B/a,\ \theta_B)]\}\\&+\zeta_2/l\{\phi_D(Kr_A,\ \theta_A)+\phi_D(Kr_B,\ \theta_B)\\&+\sum_{m=1}^{\infty}Q_m[g_m(Ka,\ r_A/a,\ \theta_A)+g_m(Ka,\ r_B/a,\ \theta_B)]\}\end{aligned}\tag{A. 1}$$

where for rolling motion l in this expression means $P\cdot\Theta$ and $\phi_S(Kr_A,\ \theta_A)$, $\phi_S(Kr_B,\ \theta_B)$ are the velocity potentials of a source at A and B and $\phi_D(Kr_A,\ \theta_A)$, $\phi_D(Kr_B,\ \theta_B)$ the velocity potentials of a dipole at A and B. r_A is a distance from A to a point $(X,\ Y)$ and r_B a distance from B to the point. θ_A, θ_B are the angles that the r_A, r_B make with a vertical line through A and a vertical line through B respectively (counter-clockwise is positive). $\phi_S(Kr,\ \theta)$, $\phi_D(Kr,\ \theta)$ are given by

$$\begin{aligned}\phi_S(Kr,\ \theta)=&i\pi e^{-Kr\cos\theta}\,e^{-iKr\,|\sin\theta|}\\&+\int_0^{\infty}\frac{\sin(Krt\cos\theta)-t\cos(Krt\cos\theta)}{1+t^2}e^{-Krt\,|\sin\theta|}\,dt\end{aligned}\tag{A. 2}$$

$$\phi_D(Kr,\ \theta)=\mp\pi e^{-Kr\cos\theta}e^{-iKr|\sin\theta|}-\frac{Kr\cdot\sin\theta}{(Kr)^2}$$

$$\pm\int_0^\infty\frac{t\sin(Krt\cos\theta)+\cos(Krt\cos\theta)}{1+t^2}e^{-Krt|\sin\theta|}dt,\ for\ \theta\gtrless 0, \qquad (A.3)$$

Further $f_m(Ka,\ r_A/a,\ \theta_A)$, $f_m(Ka,\ r_B/a,\ \theta_B)$ are wave free potentials symmetrical about A and B respectively and $g_m(Ka,\ r_A/a,\ \theta_A)$, $g_m(Ka,\ r_B/a,\ \theta_B)$ are anti-symmetrical one about A and B respectively and they are

$$f_m(Ka,\ r/a,\ \theta)=\frac{Ka\cdot\cos(2m-1)\theta}{(2m-1)(r/a)^{2m-1}}+\frac{\cos2m\theta}{(r/a)^{2m}} \qquad (A.4)$$

$$g_m(Ka,\ r/a,\ \theta)=\frac{Ka\cdot\sin(2m-1)\theta}{2m(r/a)^{2m}}+\frac{\sin(2m+1)\theta}{(r/a)^{2m+1}} \qquad (A.5)$$

ζ_1/l, ζ_2/l, P_m, Q_m are complex numbers and the wave elevation ζ due to ϕ is at $X=+\infty$

$$\zeta=Re[(\zeta_0-\zeta_1-i\zeta_2)e^{i(\omega t-K\overline{X+P})}-(-\zeta_0-\zeta_1+i\zeta_2)e^{i(\omega t-K\overline{X-P})}], \qquad (A.6)$$

where ζ_0 is the wave amplitude due to the potentials ϕ_A^0 or ϕ_B^0. The conjugate stream function ψ derived from the velocity potential ϕ is as follows

$$\frac{\pi\omega}{gl}\psi=\frac{\pi\omega}{gl}(\psi_A^0+\psi_B^0)+\frac{\zeta_1}{l}\{\psi_S(Kr_A\ \theta_A)-\psi_S(Kr_B,\ \theta_B)$$

$$+\sum_{m=1}^{\infty}P_m[\xi_m(Ka,\ r_A/a,\ \theta_A)-\xi_m(Ka,\ r_B/a,\ \theta_B)]\}$$

$$+\frac{\zeta_2}{l}\{\psi_D(Kr_A,\ \theta_A)+\psi_D(Kr_B,\ \theta_B)$$

$$+\sum_{m=1}^{\infty}Q_m[\eta_m(Ka,\ r_A/a,\ \theta_A)+\eta_m(Ka,\ r_B/a,\ \theta_B)]\} \qquad (A.7)$$

$\psi_S(Kr,\ \theta)$, $\psi_D(Kr,\ \theta)$, $\xi_m(Ka,\ r/a,\ \theta)$ and $\eta_m(Ka,\ r/a,\ \theta)$ are the conjugate harmonic functions of $\phi_S(Kr,\ \theta)$, $\phi_D(Kr,\ \theta)$, $f_m(Ka,\ r/a,\ \theta)$ and $g_m(Ka,\ r/a,\ \theta)$ respectively.

If ψ satisfies the following condition on the surface of the cylinder A, then it satisfies the condition which should be satisfied on the cylinder B because ψ and this boundary condition for ψ is symmetrical about Y-axis.

$$\frac{\pi\omega}{gl}\psi=\begin{cases}i\pi\cdot Ka\cdot\cos\theta_A+C & \text{for swaying}\\ i\pi Ka(P/a)\sin\theta_A+C & \text{for rolling}\end{cases} \qquad (A.8)$$

where C is a constant independent of θ.

ψ_A^0, of course, satisfies the condition (A.8) on the cylinder A and therefore the condition for ψ on the cylinder A becomes

$$C_0 - \frac{\pi\omega}{gl}\psi_B^0 = \frac{\zeta_1}{l}\{\psi_S(Ka,\ \theta) - \psi_S(Kr_B,\ \theta_B)$$

$$+ \sum_{m=1}^{\infty} P_m[\xi_m(Ka,\ 1,\ \theta) - \xi_m(Ka,\ r_B/a,\ \theta_B)]\}$$

$$+ \frac{\zeta_2}{l}\{\psi_D(Ka,\ \theta) + \psi_D(Kr_B,\ \theta_B)$$

$$+ \sum_{m=1}^{\infty} Q_m[\eta_m(Ka,\ 1,\ \theta) + \eta_m(Ka,\ r_B/a,\ \theta_B)]\} \tag{A. 9}$$

where

$$C_0 = \text{const.}$$

$$\theta_B = \tan^{-1}\left(\frac{\sin\theta - 2P/a}{\cos\theta}\right) \tag{A. 10}$$

$$(r_B/a)^2 = 1 - 2(2P/a)\sin\theta + (2P/a)^2 \tag{A. 11}$$

The equations about the unknown coefficients P_m, Q_m, ζ_1/l, ζ_2/l and C_0 are solved by replacing this system of infinite numbers of equations with a finite number of equations and, for example, by the least square method.

Using the coefficients thus obtained we can calculate the hydrodynamic pressure on the cylinder surface from the velocity potential given by the equation (A. 1).

The wave amplitude ratio $\overline{A}_S$ for swaying and $\overline{A}_R$ for rolling (wave amplitude/l or $a\Theta$) and the phase of the wave ε_S, ε_R are derived from the equation (A. 6).

In addition added mass coefficient of swaying m_S (added mass/$\rho\pi a^2$), added mass moment coefficient m_R of rolling (added moment/$\rho\pi a^4$) and cross coupling moment expressed as

$$\rho\pi a^3 \cdot M\left(-\frac{d^2x}{dt^2}\right) + \frac{\rho g^2}{\omega^3}a \cdot N\left(-\frac{dx}{dt}\right) \quad , \tag{A. 12}$$

can be derived by integrating the hydrodynamic pressure on the cylinder surface.

Some results for the cases of $2p/a = 3.0$, 4.0, 5.0 and 6.0 are shown in Fig. 2, 3, 4, 5 and 6 as exact solutions.

Appendix 2

Let us restrict here the problem to two dimension for simplicity.

The so-called Haskind Newman relation[9] is a formula by which we can compute the wave exciting force or moment in a direction upon a body when it is held fixed on incoming plane waves from the wave amplitude diverging at infinity in a corresponding radiation problem. This relation, however, does not mention of the phase of the exciting force or moment, but gives only its amplitude which is insufficient to be the term on the righthandside of the equations of the motions.

Bessho[8] showed extending this relation that if we know the amplitudes and the phase (the phase lag between the motion of the waves and the body) of the waves diverging at infinity in a radiation problem, we can immediately calculate the amplitude and the phase of the corresponding component of the wave exciting force or moment.

In addition he showed a simple formula that gives the amplitude and the phase of reflected and transmitted waves by an incident wave coming upon a symmetrical body when the solutions for the swaying and heaving radiation problem of the body are previously known.

These relations are summarized in a little modified expression convenient for our use in this paper as follows.

When a symmetrical body shown in Fig. A.2 makes a swaying motion $x=Re[e^{i\omega t}]$, a heaving motion $y=Re[e^{i\omega t}]$ and a rolling motion $\theta=Re[e^{i\omega t}]$ about the origin O, the progressing waves at $X=+\infty$ are given respectively by the following expression,

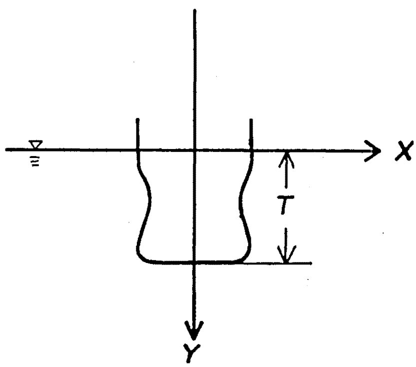

Fig. A. 2 Cylinder

$$\overline{A}_S e^{i\varepsilon_S} \cdot e^{i(\omega t-KX)}, \text{ for swaying,} \tag{A. 13}$$

$$\overline{A}_H e^{i\varepsilon_H} \cdot e^{i(\omega t-KX)}, \text{ for heaving,} \tag{A. 14}$$

$$\overline{A}_R \cdot a \cdot e^{i\varepsilon_H} \cdot e^{i(\omega t-KX)}, \text{ for rolling.} \tag{A. 15}$$

Suppose that an incident wave $l \cdot e^{i(\omega t+KX)}$ comes upon the body, then the waves at $X=+\infty$ and $X=-\infty$ are respectively

$$\zeta_{X=+\infty}=l \cdot e^{i(\omega t+KX)}+iH^{+}(K) \cdot l \cdot e^{i(\omega t-KX)}, \tag{A. 16}$$

$$\zeta_{X=-\infty}=l \cdot e^{i(\omega t+KX)}+iH^{-}(K) \cdot l \cdot e^{i(t\omega+KX)}, \tag{A. 17}$$

where $H^{\pm}(K)$ are the complex numbers dependent upon the body form and K and are given by

$$H^{\pm}(K) = ie^{i\varepsilon_H}\cos\varepsilon_H \mp e^{i\varepsilon_S}\cdot\sin\varepsilon_S, \tag{A-18}$$

The incident wave and the diffraction wave of this incident wave by the existence of the body exert upon the body the hydrodynamic force and moment which are as follows,

x direction $\quad \dfrac{i\rho g}{K} l\cdot\overline{A}_S e^{i\varepsilon_S}$

y direction $\quad \dfrac{i\rho g}{K} l\cdot\overline{A}_H e^{i\varepsilon_H}$

moment around O $\quad \dfrac{i\rho g}{K} a\cdot l\cdot\overline{A}_R e^{i\varepsilon_R}$

where ρ is the density of a fluid and g the gravity acceleration.