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A STUDY ON THE MOTIONS OF A SEMI-SUBMERSIBLE CATAMARAN HULL IN REGULAR WAVES

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This paper deals with the periodic motions of a semi-submersible catamaran hull in regular waves.

An approximate calculation based on the theory of ship motion showed a fairly good coincidence with the model experiments in the water tank.

1. Introduction

Today, for the exploration and mining of oil, gas and all sorts of mineral resources in the sea bed and the substratum, various types of semi-submersible drilling platforms are being constructed.

These marine structures should be operated stably around the fixed position. Their construction and shape are therefore planned to keep them firm against wave forces in general.

Such a device for stabilization, however, makes them less movable or less ready to provide against any contingencies.

As a result, these drilling platforms are often exposed to severe environmental conditions and forced to keep operation enduring rough seas.

Promotion of basic studies on the calculation method of the wave exciting force and structural strength etc. for these marine structures is now under consideration, but there are few designs reliable enough.

For these reasons the safety of such marine structures may be said to be far from being assured[1][2].

In this point of view, the dynamics of the motion of marine structures should be studied fundamentally and systematically.

In this paper, the periodic motion of the Semi-Submersible Catamaran Hull (hereinafter referred to as S. S. C. H.) which is floating in regular progressive waves is taken up for consideration to carry out approximate calculation based on the theory of ship motions and to compare the results with the model expe-

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riments in the water tank.

The investigations carried out on the S. S. C. H. are focused on the periodic motions under the conditions of beam sea and longitudinal waves.

It has been revealed through these results that the approximate calculation method is considerably effective.

For the prediction of the motions of an actual marine structure, however, further study on the scale effect caused by the viscous force will be necessary.

2. Model and Experiments

As shown in Fig. 1, the model of Semi-Submersible Catamaran Hull (the abreviation S. S. C. H.) has two caissons, eight columns and an operation deck on the upper part of these columns. The caisson is about two meters in length.

The main particulars of the model is given in Table 1.

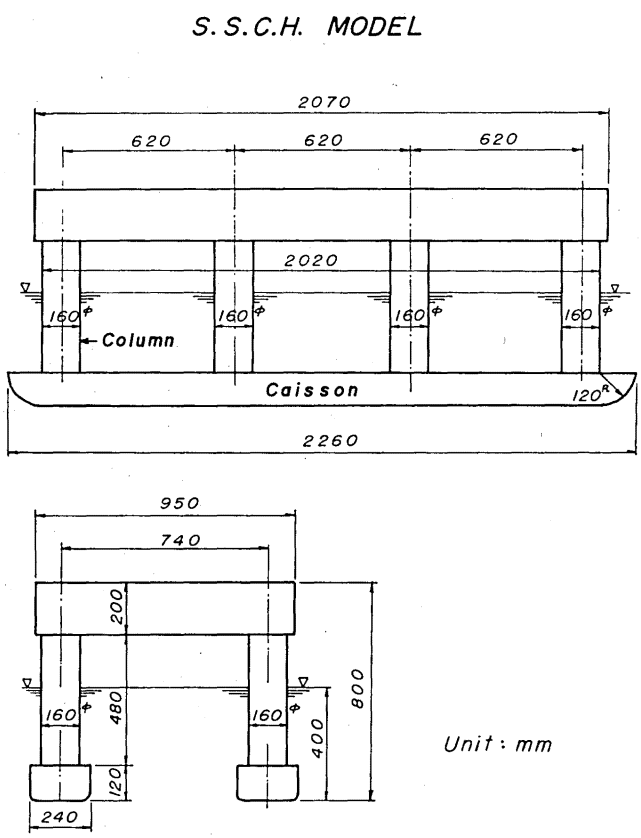

Fig. 1. The model of Semi-Submersible Catamaran Hull

It is supposed that the dimension of the actual structure is about fifty times as large as the model.

The model is under no such restraint as caused by the mooring chain.

We measured the amplitudes of periodic motions of S. S. C. H. floating freely under the conditions of beam sea and longitudinal waves.

Table 1.

Main Particulars	
$L=2.26$m	$d=0.16$m
$x_1=0.93$m	$f_m=0.4765$m
$x_2=0.31$m	$W=169.48$kg
$h_0=0.40$m	$KB=0.1114$m
$h=0.28$m	$KG=0.1835$m
$l_1=0.12$m	$BM=0.1514$m
$f_1=0.2165$m	$GM=0.0793$m
$b=0.37$m	$BM_l=0.4560$m
$b_1=0.12$m	$GM_l=0.3839$m

For measuring motions of the model, a similar instrument to that described in the reference (3) was used and this instrument was set in the center of the model deck (the point Q in Fig. 2). The measuring instrument is shown in Fig. 3.

The wave height was measured by the wave height meter of ultrasonic type.

The model experiment was performed at the experimental tank ($L\times B\times D\times d=80\text{m}\times 8\text{m}\times 3.5\text{m}\times 3\text{m}$) of the Research Institute for Applied Mechanics of Kyushu University.

3. Approximate Calculation Method of Motions

In this paper we will discuss about the motions of S. S. C. H. freely floating in the regular wave of small wave height. Moreover it is assumed that the water depth is sufficiently deep and we can apply the wave theory of deep water.

Let M be the mass of a floating body and x the displacement of motion.

Then the equation of motion in sinusoidal waves can be approximately written as follows:

$$M\ddot{x}=-C_1x-C_2\dot{x}-C_3|\dot{x}|\dot{x}-C_4\ddot{x}+C_5|\dot{\xi}_w|\dot{\xi}_w$$
$$+F_1\cos(\omega t+\varepsilon_1)+F_2\cos(\omega t+\varepsilon_2)+F_3\cos(\omega t+\varepsilon_3), \qquad (3.1)$$

in which

ω = circular frequency of regular waves

$\dot{\xi}_w$ = mean orbital velocity of wave in x direction

$-C_1x$ = restoring force

$-C_2\dot{x}$ and $-C_3|\dot{x}|\dot{x}$ = linear and non-linear damping force

$-C_4\ddot{x}$ = added inertia force

$F_1\cos(\omega t+\varepsilon_1)$ = Froude-Kriloff's force

$F_2\cos(\omega t+\varepsilon_2)$ = exciting force in phase with $\dot{\xi}_w$

$F_3\cos(\omega t+\varepsilon_3)$ = exciting force in phase with $\ddot{\xi}_w$.

Since S. S. C. H. has the smaller water plane area and larger volume of under water part than an usual ship, and since its natural period of motion is relatively long, it is now supposed that the influence of free surface on the hydrodynamic force is generally small in the neighbourhood of resonant frequency.

In the region of small wave frequency, therefore, we can assume that the added mass coefficient C_4 is constant independently of ω. F_2 is less than F_1, F_3 and is approximately regarded as the force of the negligible order.

When the wave height is small, we can infer that $C_5|\dot{\xi}_w|\dot{\xi}_w$ is a place less than F_1, F_3[4].

We discuss about the motions of S. S. C. H. in waves of small wave height, and therefore, we can neglect the force $C_5\ \dot{\xi}_w|\dot{\xi}_w|$, too.

By the consideration and approximation the equation (3. 1) can be written in the following form,

$$\begin{aligned}(M+C_4)\ddot{x}+C_2\dot{x}+C_3|\dot{x}|\dot{x}+C_1x \\ =F_1\cos(\omega t+\varepsilon_1)+F_3\cos(\omega t+\varepsilon_3).\end{aligned} \tag{3.2}$$

In the equation (3. 2) the damping forces include the wave making force and viscous one. But nowadays these forces are difficult for us to make up the theoretical estimation.

We will reserve how to theoretically calculate these damping forces in the future, and in this paper we have decided to calculate the damping forces by using C_1, C_3 obtained by the model experiment of free rolling: the way to be used in the analysis of ship rolling.

It was found that such damping forces in cases of small amplitude of heaving, pitching and rolling, as obtained in the model experiments of S. S. C. H., can be sufficiently expressed only in the linear term, and therefore, the calculation of motion was eventually made linearly in the following equation,

$$(M+C_4)\ddot{x}+C_2\dot{x}+C_1x=F_1\cos(\omega t+\varepsilon_1)+F_3\cos(\omega t+\varepsilon_3). \tag{3.3}$$

Moreover, neglecting the effect of the interference between the column and caisson, we comptuted by the strip theory[5], which has been usually used in the calculation of ship motions.

4. Heaving Motion in Beam Sea Condition

In Fig. 2, let us introduce the coordinate system 0_1-$\xi\eta\zeta$ fixed in space and the coordinate system 0-$x_by_bz_b$ fixed in the body.

In calm water, the 0-x_by_b plane coincides with the 0_1-$\xi\eta$ plane.

The velocity potential of regular waves progressing in the opposite direction of η in a fluid of infinite depth is given as follows:

$$\phi=\frac{\omega}{k}\zeta_a e^{-k\zeta}\sin\ (k\eta+\omega t) \tag{4.1}$$

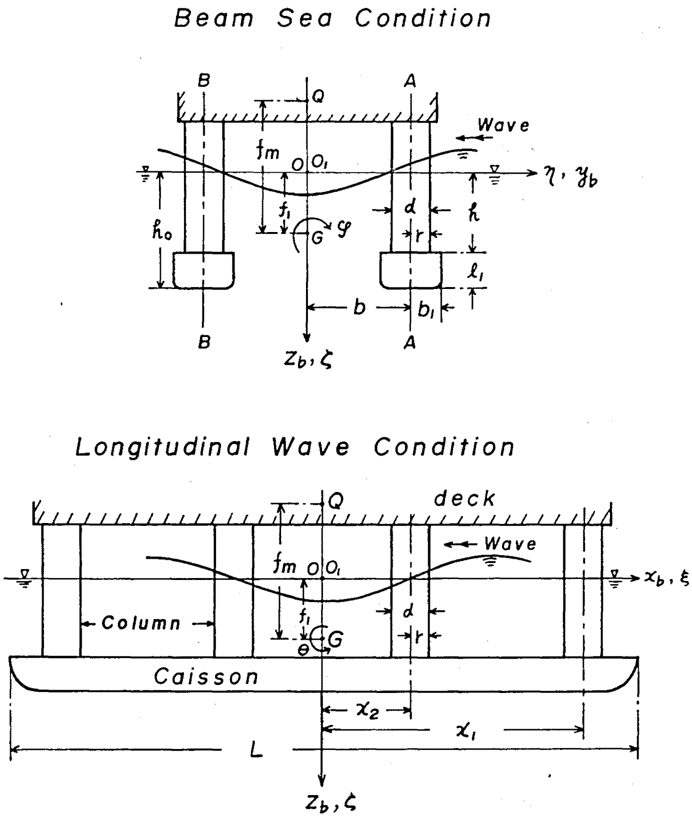

Fig. 2. System of coordinates and regular waves

The equation of subsurface of waves is

$$\zeta_w = \zeta_a e^{-k\zeta} \cos(k\eta + \omega t). \tag{4.2}$$

where ζ_a=wave amplitude, $k=2\pi/\lambda=\omega^2/g$, λ=wave length, $\omega=2\pi/T_w$ and T_w =wave period.

According to (3.2) the equation of heaving motion is given by

$$(M+m_z)\ddot{z}+N_z\dot{z}+\rho g A_w z=|F_{z1}|\cos\omega t+|F_{z2}|\sin\omega t. \tag{4.3}$$

In the equation (4.3) z is the heaving displacement, m_z the added mass and A_w the water plane area. Moreover $|F_{z1}|$, $|F_{z2}|$ are the amplitudes of wave exciting force for heaving.

As for m_z, it is enough to consider only the added mass of caissons.

As discussed in the preceding section, we can approximately neglect the effect of free surface and evaluate the added mass by assuming that the caissons heave in infinite fluid.

Then the sectional added mass m_1 in ζ direction is given by

$$m_1 = \rho\pi b_1^2 \beta. \tag{4.4}$$

Making use of the strip theory, m_z is computed by the equation

$$m_z = 2\rho\pi b_1^2 \int_L \beta(x_b)dx_b$$

In this paper, however, we assume that $\int_L \beta(x_b)dx_b \doteqdot L$ and then m_z is given by

$$m_z \doteqdot 2\rho\pi b_1^2 L. \tag{4.5}$$

The natural heaving period T_z may approximately be evaluated by putting $N_z=0$ in the equation (4.3), that is,

$$T_z = 2\pi\sqrt{(M+m_z)/\rho g A_w}. \tag{4.6}$$

$T_z=3.07$ seconds is obtained by using $A_w=8\pi d^2/4$ and m_z given by the (4.5). The corresponding experimental value is 3.20 seconds, and therefore, there is a difference of about 4 percent.

The extinction curve obtained from the experiments of free heaving may be expressed with such a linear term as $\Delta_z = a_1 z_m$ if the heaving displacement is small. We obtained $a_1=0.094$.

By the same method as used in the analysis of rolling motion of a ship[6], N_z is evaluated as follows:

$$N_z = \frac{2}{\pi}\omega_z a_1(M+m_z) = 4a_1(M+m_z)/T_z \tag{4.7}$$

In the next place, we treat of the wave exciting force.

First the Froude-Kriloff's force can be expressed as follows:

$$F_{z11} = \rho g \frac{\zeta_a}{k}\left[4Le^{-kh}(e^{-kl_1}-1)\sin kb_1 + \frac{n\pi d^2}{4}\cdot k\cdot e^{-kh}\right]\cos kb\cdot\cos\omega t \tag{4.8}$$

where n, which means the number of column, is equal to eight in case of this model.

The wave orbital velocity and acceleration in ζ direction are

$$\dot{\zeta}_w = -\zeta_a\omega e^{-k\zeta}\sin(k\eta+\omega t),\quad \ddot{\zeta}_w = -\zeta_a\omega^2 e^{-k\zeta}\cos(k\eta+\omega t). \tag{4.9}$$

As the mean value of $\ddot{\zeta}_w$ we adopted the values in the center of caisson $y_b = \pm b$ and in the depth of $h+\dfrac{l_1}{2}$.

Accordingly, the average value $\overline{\ddot{\zeta}_w}$ in the left caisson of Fig. 2 is

$$\overline{\ddot{\zeta}}_w = -\zeta_a \omega^2 e^{-k(h+l_1/2)} \cos(-kb+\omega t) \qquad (4.10)$$

, and for the right caisson

$$\overline{\ddot{\zeta}}_w = -\zeta_a \omega^2 e^{-k(h+l_1/2)} \cos(kb+\omega t).$$

The sectional exciting force in phase with $\ddot{\zeta}_w$ is given by

$$f_z = -m_1(-\overline{\ddot{\zeta}}_w) = m_1 \overline{\ddot{\zeta}}_w. \qquad (4.11)$$

From the equations (4.4) and (4.11) the force acting upon the two caissons is put as

$$F_{z12} = -2\pi \rho g k \zeta_a b_1^2 L e^{-k(h+l_1/2)} \cos kb \cdot \cos \omega t. \qquad (4.12)$$

On the other hand, F_{z2} is the force in phase with $\dot{\zeta}_w$. As we discussed in the third section, it is very small in the region of small wave frequency. And so this force can be neglected.

From the above results the exciting force for heaving motion is written as follows:

$$|F_{z1}| \cos \omega t = (|F_{z11}| + |F_{z12}|) \cos \omega t. \qquad (4.13)$$

These exciting forces are shown in Fig. 4. As seen from Fig. 4, in the $\lambda/2b \geqslant 6$, F_{z11} is in adversed phase with F_{z12}. Putting

$$z = z_a \cos(\omega t - \varepsilon_z) \qquad (4.14)$$

we calculated the non-dimensional amplitude of heaving from the equation (4.3).

In Fig. 5 the computed results are compared with experiments.

It can be said that the measured and calculated values of z_a/ζ_a show considerably good agreement.

In correspondence to the wave period of the model experiments, the wave period and wave length for the actual structure are given in table 2.

Table 2.

T_w(for model)	(sec)	1.0	2.0	3.0	4.0	5.0
T_w(for actual structure)	(sec)	7.06	14.12	21.18	28.24	35.30
λ (for actual structure)	(m)	78	312	722	1248	1950

5. Swaying and Rolling Motions in Beam Sea Condition

The rolling and swaying motions of S.S.C.H. must be dealt with as the coupled motions. This is the same as the case of rolling and swaying motions of a ship.

Well, wave orbital velocity and acceleration in η direction are as follows from the equation (4.1):

$$\dot{\eta}_w=\partial\phi/\partial\eta=\zeta_a\omega e^{-k\zeta}\cos(k\eta+\omega t),$$
$$\ddot{\eta}_w=\partial\dot{\eta}_w/\partial t=-\zeta_a\omega^2e^{-k\zeta}\sin(k\eta+\omega t). \tag{5.1}$$

5.1. Wave Exciting Force for Swaying

In the same way as used in the section four, the wave exciting force for swaying is approximately composed of the Froude-Kriloff's force $F_y(F.K.)$ and $F_y(\ddot{\eta}_w)$ in phase with $\ddot{\eta}_w$, that is

$$F_y=F_y(F.K.)+F_y(\ddot{\eta}_w). \tag{5.2}$$

$F_y(F.K.)$ is divided into two forces: the one is the force $F_y(F.K.)_1$ acting upon eight columns and the other is the force $F_y(F.K.)_2$ acting upon two caissons. They are given by the following equations:

$$F_y(F.K.)_1=-8\pi\rho g\frac{\zeta_a}{k}dJ_1(kr)(1-e^{-kh})\cos kb\cdot\sin\omega t$$

$$F_y(F.K.)_2=-4\rho g\frac{\zeta_a}{k}Le^{-kh}(1-e^{-kl_1})\sin kb_1\cdot\cos kb\cdot\sin\omega t \tag{5.3}$$

Secondly, let us calculate the force $F_y(\ddot{\eta}_w)_1$ which is in phase with $\ddot{\eta}_w$ and acts upon the columns.

If we consider a case where wave length is very long compared with the column diameter d and neglect the free surface effect, the sectional added mass m_2 in the horizontal direction will be given by

$$m_2=\rho\pi\left(\frac{d}{2}\right)^2. \tag{5.4}$$

Then the force acting upon the columns in the right side of Fig. 2 can be calculated by the equation

$$4\int_0^h m_2\overline{\ddot{\eta}}_w dz_b\doteqdot-4\int_0^h\rho\pi\left(\frac{d}{2}\right)^2\zeta_a\omega^2e^{-kZb}\sin(kb+\omega t)dz_b, \tag{5.5}$$

where $\overline{\ddot{\eta}}_w$ is the mean value of the horizontal wave orbital velocity which was obtained by putting $\eta=\pm b$ in the equation (5.1).

In the same way, the force acting upon the columns of the left side is as follows:

$$-4\int_0^h\rho\pi\left(\frac{d}{2}\right)^2\zeta_a\omega^2e^{-kZb}\sin(-kb+\omega t)dz_b \tag{5.6}$$

From the equations (5.5) and (5.6), we get

$$F_y(\ddot{\eta}_w)_1=-2\pi\rho g\zeta_a d^2(1-e^{-kh})\cos kb\cdot\sin\omega t. \tag{5.7}$$

Now, by the same approximate method as the cases of m_1, m_2, the sectional added mass m_3 of the caisson in the horizontal direction is given by

$$m_3 = \rho\pi\left(\frac{l_1}{2}\right)^2 \beta. \tag{5.8}$$

As the mean value of β in this case $\beta \doteqdot 1.5$ was used.

Then, the force $F_y(\ddot{\eta}_w)_2$ acting upon the two caissons can be written as follows:

$$F_y(\ddot{\eta}_w)_2 = -1.5\frac{\pi}{2}\rho gk\zeta_a l_1^2 Le^{-k(h+l_1/2)}\cos kb\cdot\sin\omega t \tag{5.9}$$

As mentioned above the wave exciting force for swaying doesn't include the term $\cos\omega t$. This is due to the fact that the force in phase with $\dot{\eta}_w$ is omitted.

The exciting forces for swaying motion are shown in Fig. 6.

5.2. Wave Exciting Moment for Rolling

According to the basic ideas in the section 3, the wave exciting moment for rolling about the center of gravity G can be approximately written as follows:

$$M_\varphi = M_\varphi(F.K.) + M_\varphi(\ddot{\eta}_w) + M_\varphi(\ddot{\zeta}_w) \tag{5.10}$$

Furthermore, $M_\varphi(F.K.)$ can be divided as follows:

$$M_\varphi(F.K.) = M_\varphi(F.K.)_1 + M_\varphi(F.K.)_2 + M_\varphi(F.K.)_3 \tag{5.11}$$

where $M_\varphi(F.K.)_1$ is the moment due to the horizontal force acting on the columns, $M_\varphi(F.K.)_2$ and $M_\varphi(F.K.)_3$ are the ones due to the horizontal and vertical force acting on the caissons respectively.

With the short calculations, each term mentioned above is given by

$$M_\varphi(F.K.)_1 = 8\pi\rho gd\frac{\zeta_a}{k}J_1(kr)\left\{-e^{-kh}\left(h-f_1+\frac{1}{k}\right)+\frac{1}{k}-f_1\right\}\cos kb\cdot\sin\omega t$$

$$M_\varphi(F.K.)_2 = 4\rho g\frac{\zeta_a}{k}Le^{-kh}\left\{\left(h-f_1+\frac{1}{k}\right)-e^{-kh}\left(h-f_1+l_1+\frac{1}{k}\right)\right\}$$
$$\times\cos kb\cdot\sin kb_1\cdot\sin\omega t$$

$$M_\varphi(F.K.)_3 = 4\rho g\frac{\zeta_a}{k}Le^{-kh}(1-e^{-kl_1})\left(\frac{1}{k}\cos kb\cdot\sin kb_1 + b\ \sin kb\cdot\sin kb_1\right.$$
$$\left.-b_1\ \cos kb\cdot\cos kb_1\right)\sin\omega t - 2\pi\rho g\zeta_a bd^2 e^{-kh}\sin kb\cdot\sin\omega t. \tag{5.12}$$

Secondarily, $M_\varphi(\ddot{\eta}_w)$ has two parts: the moment $M_\varphi(\ddot{\eta}_w)_1$ due to the force $F_y(\ddot{\eta}_w)_1$ and the moment $M_\varphi(\ddot{\eta}_w)_2$ due to the force $F_y(\ddot{\eta}_w)_2$.

These moments are given by

$$M_\varphi(\ddot{\eta}_w)_1 = 8\rho\pi\left(\frac{d}{2}\right)^2\zeta_a\omega^2\cos kb\cdot\sin\omega t\int_0^h e^{-kz_b}(z_b-f_1)\,dz_b$$

$$=2\pi\rho g\zeta_a d^2\left\{e^{-kh}\left(f_1-h-\frac{1}{k}\right)+\frac{1}{k}-f_1\right\}\cos kb\cdot\sin\omega t, \quad (5.13)$$

and

$$M_\varphi(\ddot{\eta}_w)_2=\frac{1.5\pi}{2}\rho gk\zeta_a l_1^2 Le^{-k(h+l_1/2)}\left(h+\frac{l_1}{2}-f_1\right)\cos kb\cdot\sin\omega t. \quad (5.14)$$

Lastly $M_\varphi(\ddot{\zeta}_w)$, which is the moment in phase with $\ddot{\zeta}_w$ (the equation (4.11)), is given by

$$M_\varphi(\ddot{\zeta}_w)\doteqdot 2\pi\rho gb_1^2 bLk\zeta_a e^{-k(h+l_1/2)}\sin kb\cdot\sin\omega t. \quad (5.15)$$

As given above, the rolling moment M_φ doesn't include the term $\cos\omega t$. This fact is due to the same reason as in the case of swaying. The results of calculation made on these exciting moments are given in Fig. 7.

5.3. Coupled Force and Moment

Now by designating the swaying displacement y and the rolling angle φ, we will put as follows:

$$y=y_a\cos(\omega t-\varepsilon_y)$$
$$\varphi=\varphi_a\cos(\omega t-\varepsilon_\varphi) \quad (5.16)$$

where y_a is the amplitude of swaying and φ_a that of rolling, and both ε_y and ε_φ are phase lags.

When S.S.C.H. sways, there occurs added inertia force $-m_2\ddot{y}$ on the unit length section of the column (Fig. 8).

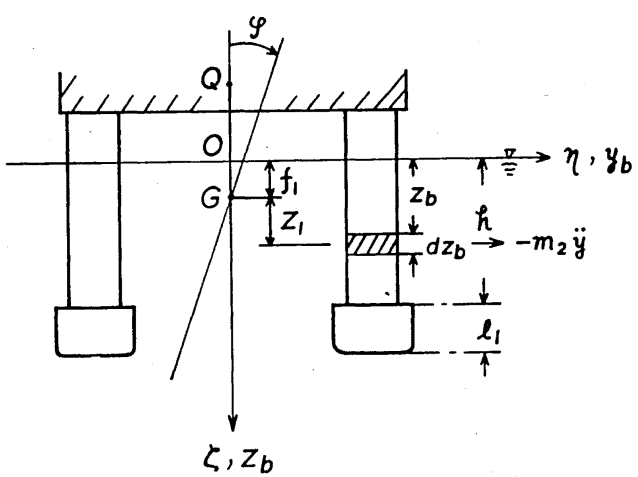

Fig. 8.

Then the added inertia force acting on the column is totally $-y \int m_2 dz_b$, and $\int m_2 dz_b$ is the added mass of the column for swaying.

Next, the rolling moment about G induced by $-m_2\ddot{y}$ is as follows:

$$-(-m_2\ddot{y}z_1)=m_2z_1\ddot{y} \tag{5.17}$$

On the other hand, rolling causes horizontal acceleration $-z_1\ddot{\varphi}$ to the cross section of the column.

The sectional swaying force caused by this acceleration is

$$-m_2(-z_1\ddot{\varphi})=m_2z_1\ddot{\varphi}. \tag{5.18}$$

As above mentioned, the coupled rolling moment from swaying is $m_2z_1\ddot{y}$ and the coupled sectional swaying force from rolling is $m_2z_1\ddot{\varphi}$.

In the same way, for the unit length cross section of caisson, the coupled rolling moment from swaying is $m_3z_1\ddot{y}$ and the coupled swaying force from rolling $m_3z_1\ddot{\varphi}$.

Then the total coupled rolling moment $M_{y\varphi}$ induced by swaying is as follows:

$$M_{y\varphi}=\ddot{y}\left[8\int_0^h m_2(z_b-f_1)dz_b+2Lm_3\left(\frac{h+l_1}{2}-f_1\right)\right]$$

Putting now

$$M_{y\varphi}=C_{y\varphi}\cdot\ddot{y}, \tag{5.19}$$

we obtain

$$C_{y\varphi}=\rho\pi\left[d^2h(h-2f_1)+\frac{L}{2}l_1^2\left(\frac{h+l_1}{2}-f_1\right)\times 1.5\right]. \tag{5.20}$$

Hence the total coupled swaying force $F_{\varphi y}$ induced by rolling is also

$$F_{\varphi y}=C_{y\varphi}\cdot\ddot{\varphi}. \tag{5.21}$$

5.4. Comparison of Calculation and Experiments

The coupled equations of swaying and rolling under the condition of beam sea are approximately given as follows:

$$\left.\begin{aligned}&(M+m_y)\ddot{y}-C_{y\varphi}\cdot\ddot{\varphi}=F_y\\&(J_\varphi+I_\varphi)\ddot{\varphi}+N_\varphi\cdot\dot{\varphi}+W\cdot GM\cdot\varphi-C_{y\varphi}\cdot\ddot{y}=M_\varphi\end{aligned}\right\} \tag{5.22}$$

where m_y is the added mass of swaying and given by

$$m_y=8\pi\rho h\left(\frac{d}{2}\right)^2+2\rho L\pi\left(\frac{l_1}{2}\right)^2\times 1.5, \tag{5.23}$$

and

J_φ=mass moment of inertia of rolling about G
I_φ=added mass moment of inertia of rolling about G
N_φ=damping coefficient of rolling
W=displacement
GM=metacentric height.

The natural rolling period T_φ of the model was 4.5 seconds.

From the value of $W\cdot GM$ and measured T_φ, we calculated $J_\varphi+I_\varphi$ of the model by using the relation $W\cdot GM/(J_\varphi+I_\varphi)=(2\pi/T_\varphi)^2$.

The extinction curve of the free rolling has shown a non-linear form. But when the motions were small we obtained the linear damping $\Delta\varphi=a_1\varphi_m$ and $a_1=0.33$.

Using $N_\varphi=(J_\varphi+I_\varphi)4a_1/T_\varphi$ we can solve the equation (5.22).

Calculation results of φ_a, y_a and ε_φ are shown in Figs. 9, 10 and 11.

Now, let y_m be the swaying displacement of the point Q.

We can compute the swaying motion of the measuring position by the following equation:

$$y_m=y+f_m\cdot\varphi=y_{ma}\cos(\omega t-\varepsilon_{ym}) \tag{5.24}$$

The calculated value of y_{ma} is shown by the chain line in Fig. 10.

Moreover, the experimental results of φ_a and y_{ma} are also given in Figs. 9 and 10.

In Fig. 9, the measured and calculated values of φ_a show good agreement for $T_w<4.0$ seconds.

In the experiment $T_w>4.0$ seconds, regrettable to say, we failed to obtain any analyzable data because of the bad experimental condition of the model.

As for y_{ma}, calculation and experiments show good agreement.

6. Pitching in Longitudinal Wave Condition

Now we consider the case when S. S. C. H. makes pitching motion about the center of gravity G in longitudinal wave condition (Fig. 2).

The subsurface equation of regular waves progressing in the direction of $-\xi$ is given by

$$\zeta_w=\zeta_a e^{-k\zeta}\cos(k\xi+\omega t) \tag{6.1}$$

, and the pressure variation in waves is

$$p=-\rho g\zeta_a e^{-k\zeta}\cos(k\xi+\omega t). \tag{6.2}$$

From the equation (6.1) the vertical components of orbital velocity and acceleration of waves are given as follows:

$$\left.\begin{aligned}\dot{\zeta}_w&=-\zeta_a\omega e^{-k\zeta}\sin(k\xi+\omega t)\\ \ddot{\zeta}_w&=-\zeta_a\omega^2 e^{-k\zeta}\cos(k\xi+\omega t)\end{aligned}\right\} \tag{6.3}$$

In the same way, as for the horizontal components we obtain

$$\left.\begin{aligned}\dot{\xi}_w&=\zeta_a\omega e^{-k\zeta}\cos(k\xi+\omega t)\\ \ddot{\xi}_w&=-\zeta_a\omega^2 e^{-k\zeta}(k\xi+\omega t).\end{aligned}\right\} \tag{6.4}$$

The Froude-Kriloff's pitching moment due to the forces acting upon the caissons can be written as follows:

$$M_\theta(F.K.)_1=M_\theta(F.K.)_{11}-M_\theta(F.K.)_{12}$$

$M_\theta(F.K.)_{11}$ is a moment in case of no column and $M_\theta(F.K.)_{12}$ is the correction term owing to the column.

These moments are expressed as the following equation,

$$M_\theta(F.K.)_{11}=4\rho g\zeta_a b_1 l_1 L e^{-kh}\left(\cos æ-\frac{\sin æ}{æ}\right)\sin\omega t, \tag{6.5}$$

$$M_\theta(F.K.)_{12}=-\frac{\pi}{2}\rho g\zeta_a d^2 L e^{-kh}\{\xi_1\sin(\xi_1 æ)+\xi_2\sin(\xi_2 æ)\}\sin\omega t, \tag{6.6}$$

where

$$æ=kL/2=\pi L/\lambda=\omega^2 L/2g, \tag{6.7}$$

$$\xi_1=2x_1/L,\ \xi_2=2x_2/L. \tag{6.8}$$

The pitching moment caused by the pressure P which acts upon the column, is given by

$$M_\theta(F.K.)_2=8\pi\rho g\zeta_a r\cdot J_1(kr)\{\cos(\xi_1 æ)+\cos(\xi_2 æ)\}P_0\sin\omega t, \tag{6.9}$$

where

$$P_0=\frac{1}{k}\left[e^{-kh}\left(h-f_1+\frac{1}{k}\right)-\frac{1}{k}+f_1\right]. \tag{6.10}$$

Next, the force which is in phase with $\ddot{\zeta}_w$ and acts upon the caisson's unit length section, is $m_1\ddot{\zeta}_w$.

Then the pitching moment acting on S.S.C.H. is

$$\begin{aligned}M_\theta(\ddot{\zeta}_w)&=-2\int_L m_1\ddot{\zeta}_w x_b dx_b\\ &\doteqdot 2\rho g\zeta_a\pi b_1^2 L e^{-kh}\left(1-\frac{kl_1}{2}\right)\left(\cos æ-\frac{\sin æ}{æ}\right)\sin\omega t.\end{aligned} \tag{6.11}$$

In the above calculation we used $\ddot{\zeta}_w$ at the depth of $h+\frac{l_1}{2}$.

In the same way, the force in ξ direction, which is in phase with $\ddot{\xi}_w$ and acts upon the column's unit length section, is $m_2\ddot{\xi}_w$.

Then the pitching moment $M_\theta(\ddot{\xi}_w)$ by 8 columns is

$$M_\theta(\ddot{\xi}_w)=8\pi\rho g\zeta_a r\cdot\frac{kr}{2}\{\cos(\xi_1 æ)+\cos(\xi_2 æ)\}P_0\sin\omega t. \tag{6.12}$$

When kr is small, $J_1(kr) \doteqdot \frac{kr}{2}$, so that $M_\theta(F.K.)_2$ comes to be nearly equal to $M_\theta(\ddot{\xi}_w)$.

Thus entire exciting moment for pitching is as follows:

$$M_\theta = M_\theta(F.K.)_{11} - M_\theta(F.K.)_{12} + M_\theta(F.K.)_2 + M_\theta(\ddot{\zeta}_w) + M_\theta(\ddot{\xi}_w) \tag{6.13}$$

The calculated M_θ is shown in Fig. 12.

Assuming that the coupling effect of surging is small, we calculated the pitching motion by the following uncoupled equation,

$$(J_\theta + I_\theta)\ddot{\theta} + N_\theta \cdot \dot{\theta} + W \cdot GM_l \cdot \theta = M_\theta, \tag{6.14}$$

where

J_θ = mass moment of inertia of pitching
I_θ = added mass moment of inertia of pitching
N_θ = linear damping coefficient
GM_l = longitudinal metacentric height.

Since we don't have any instruments at present to measure J_θ of such a model as S. S. C. H., we can not give the model a difinite value of J_θ.

The natural period of pitching obtained by model experiments was 2.88 seconds.

Using the known $W \cdot GM_l$ and $T_\theta = 2.88$ seconds, we evaluated $J_\theta + I_\theta$ by the equation $T_\theta = 2\pi\sqrt{(J_\theta + T_\theta)/W \cdot GM_l}$.

On the other hand, I_θ is approximately computed by the following equation:

$$I_\theta \doteqdot \frac{1}{6}\rho\pi b_1^2 L^3 \tag{6.15}$$

Thus we obtain $0.24L$ as the radius of gyration of model under the experimental condition.

Putting

$$\theta = \theta_a \cos(\omega t - \varepsilon_\theta) \tag{6.16}$$

and using $a_1 = 0.227$ obtained from the extinction curve of free pitching, we calculated the equation (6. 14).

Fig. 13 is the comparison of experimental results and calculated ones.

It will be evident from this figure that the calculation and experiment show fairly good agreement.

7. Discussion

The calculation method in this investigation will be summarised as follows:

1) We considered the forces based on Froude-Krilloff's theory and also those in phase with wave orbital acceleration. Therefore, the force in phase

with wave orbital velocity and that which is proportional to the square of wave orbital velocity are neglected.

2) As for the added mass, the influence of free surface was neglected.

3) Damping coefficients of motions were not only obtained from model experiments, but they were also approximately expressed only in the linear term.

4) Neglecting the three dimensional effect and the effect of interference between columns and caissons, the strip method was used.

The motion amplitudes which were calculated by the above method resulted in good coincidence with the experiments.

The theoretical method of estimation of the damping forces including the viscous drag is not yet established at present.

Though the calculation introduced in this paper does not cover everything, we think the method is practically effective.

In relation to the damping force for such a marine structure as S. S. C. H., the most important problem may be the scale effect caused by the viscous force.

Now, we define the Reynolds number $R_e = \frac{d\dot{\eta}_{aw}}{\nu}$, in which d is the diameter of the column and $\dot{\eta}_{aw}$ the horizontal component of wave orbital velocity at $\zeta = 0$ in Fig. 2. In the model experiments maximum R_e will be the order of 3×10^4.

On the other hand, it results in $R_e = 10^7$ in case of the actual structure of fifty times as large as the model.

As seen from the experiments made on the circular cylinder (Roshko 1960[7]), it is smaller than critical Reynolds number in case of model experiment, while it is considered to be in supercritical or transcritical region in case of the actual structure.

Therefore, in case of the structure framed mainly by the round body, the viscous resistance obtained by the model experiments will be larger than that of an actual structure.

Consequently, there is a tendency to underestimate the motion of an actual structure in predicting it from the model experiment.

When a marine structure is made to float freely in a fluid, there is no resonance period in the horizontal oscillation.

Therefore, the amplitude of swaying can be determined chiefly by exciting force of waves and inertia force, and the effect of damping force remain only in the second order.

On the other hand, when the structure is moored by the chain, the resonance condition comes to exist owing to the restoring force by the chain. In consequence, the problem on the scale effect for damping force is very important in estimating the swaying in a moored structure.

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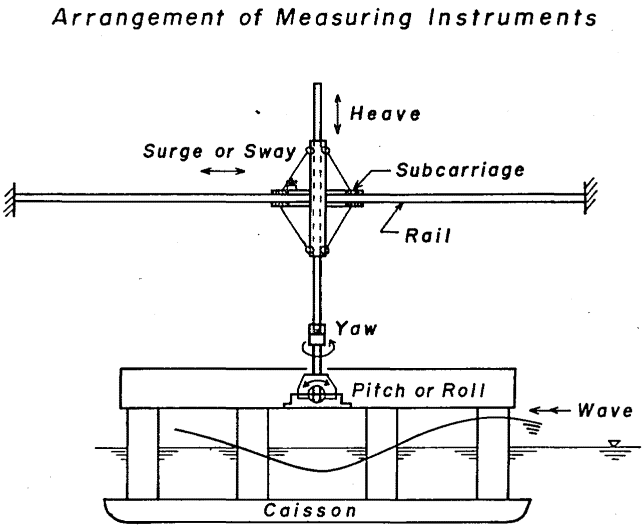

Fig. 3. Schematic arrangement of measuring instruments

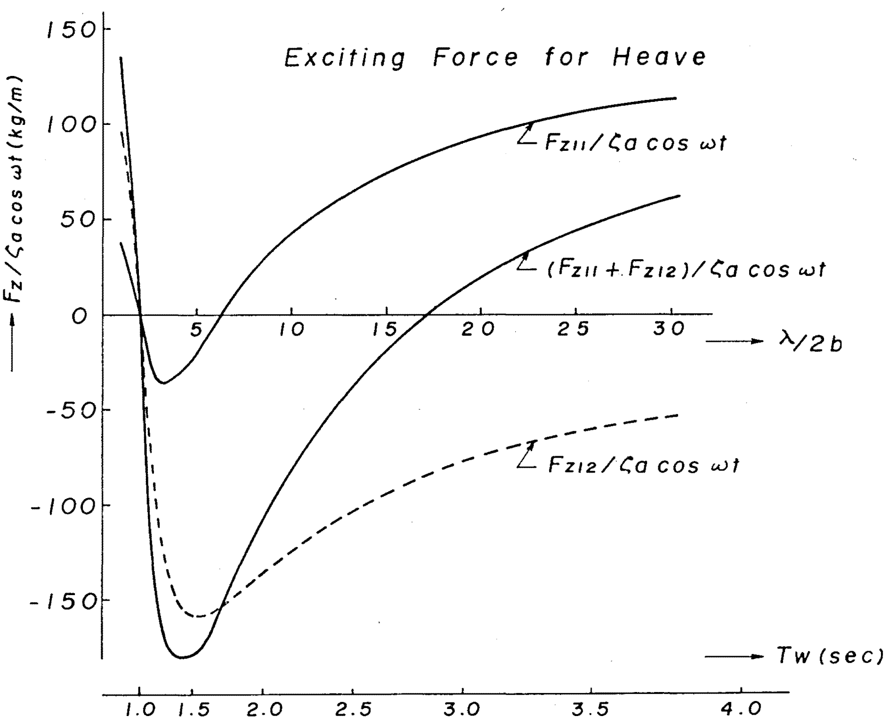

Fig. 4. Exciting force for heave

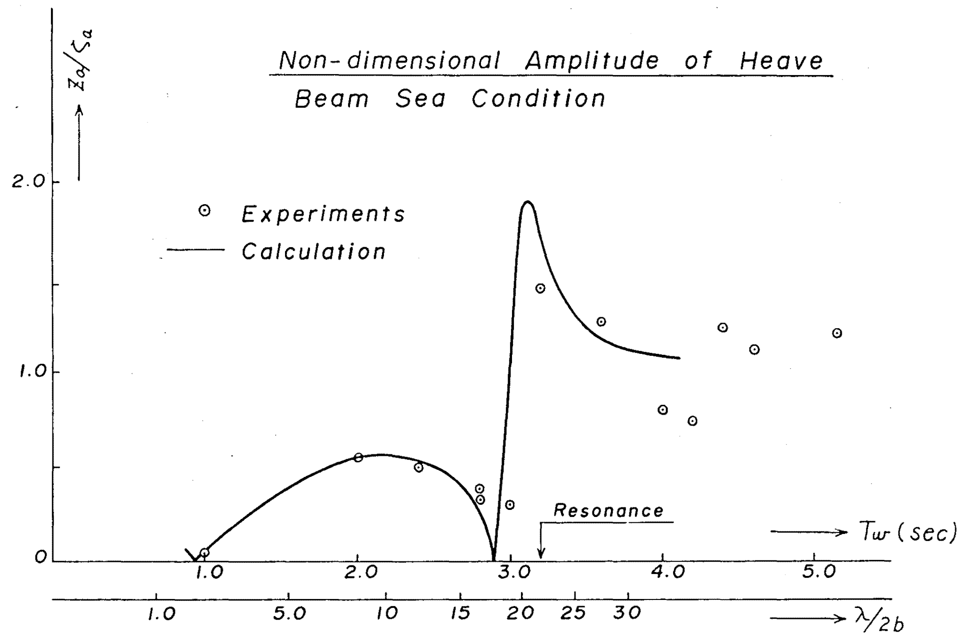

Fig. 5. Comparison of calculated and measured amplitude of heave

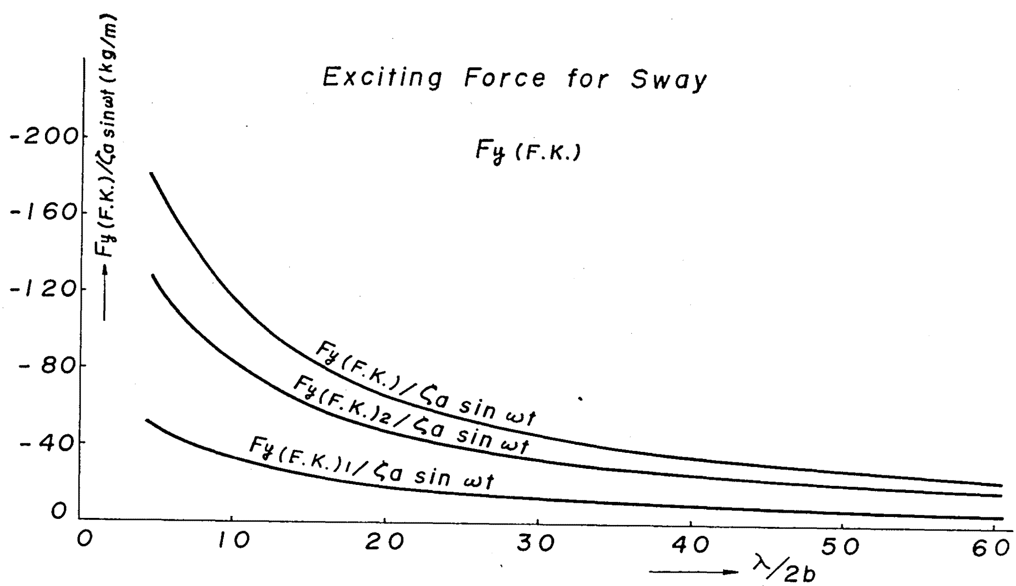

Fig. 6-1. Exciting force for sway

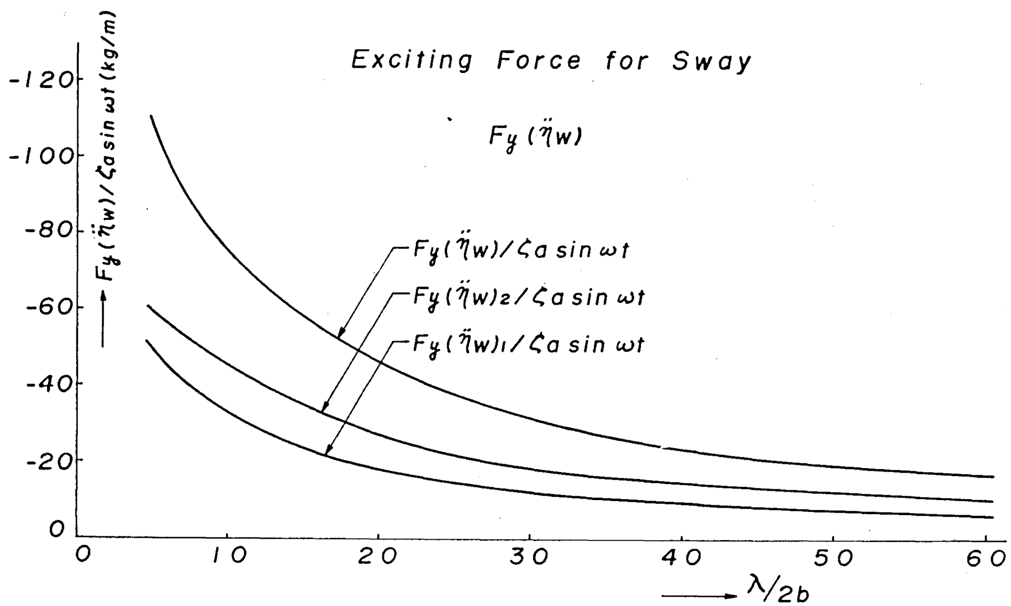

Fig. 6-2. Exciting force for sway

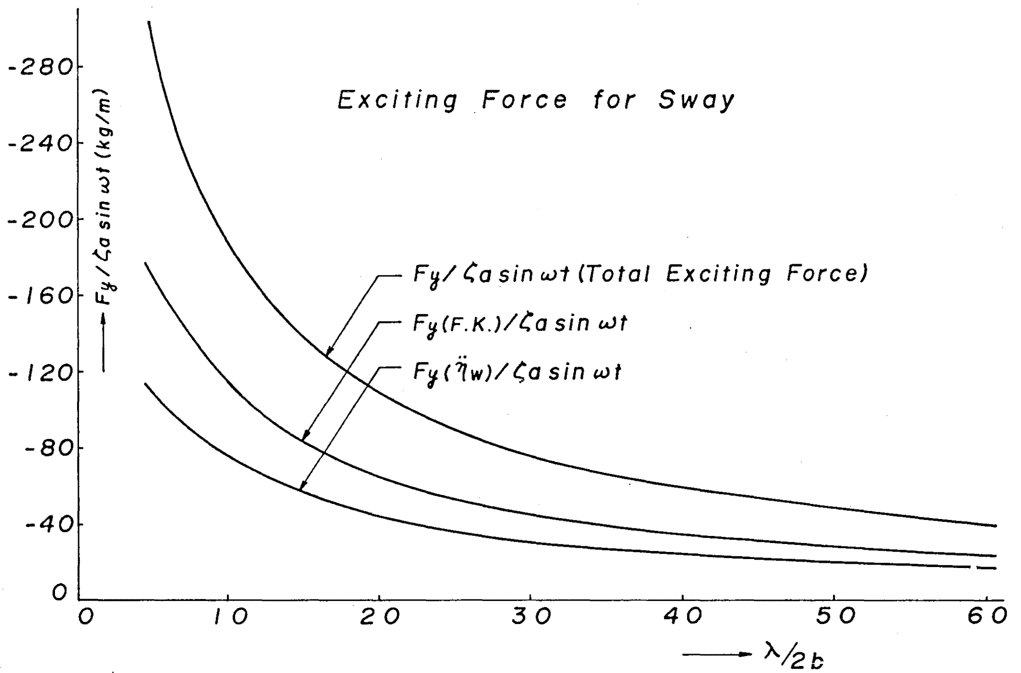

Fig. 6-3. Exciting force for sway

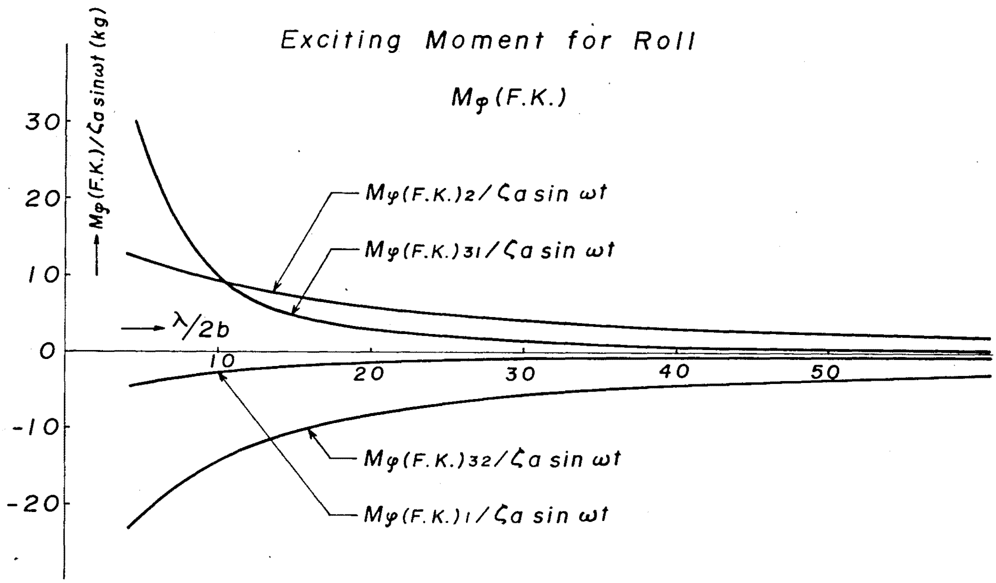

Fig. 7-1. Exciting moment for roll

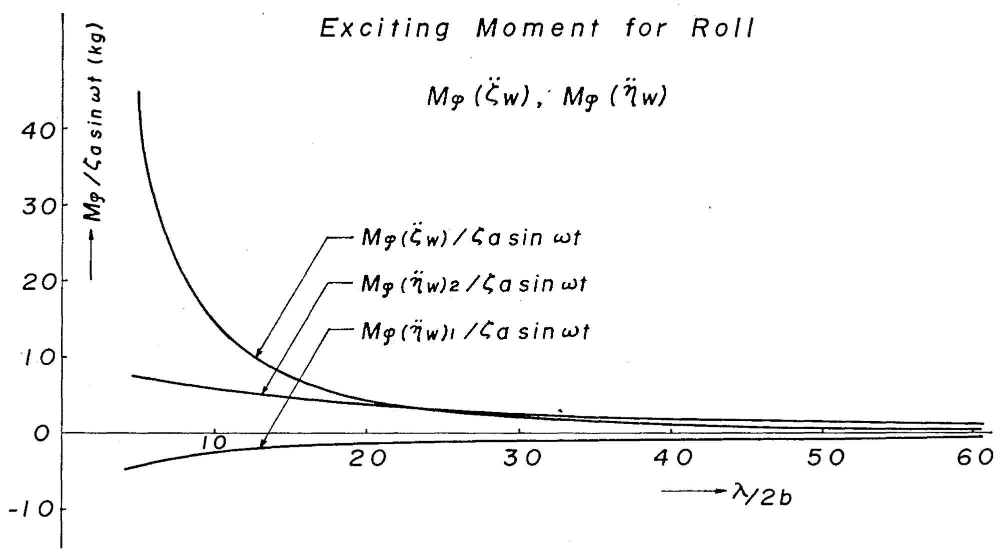

Fig. 7-2. Exciting moment for roll

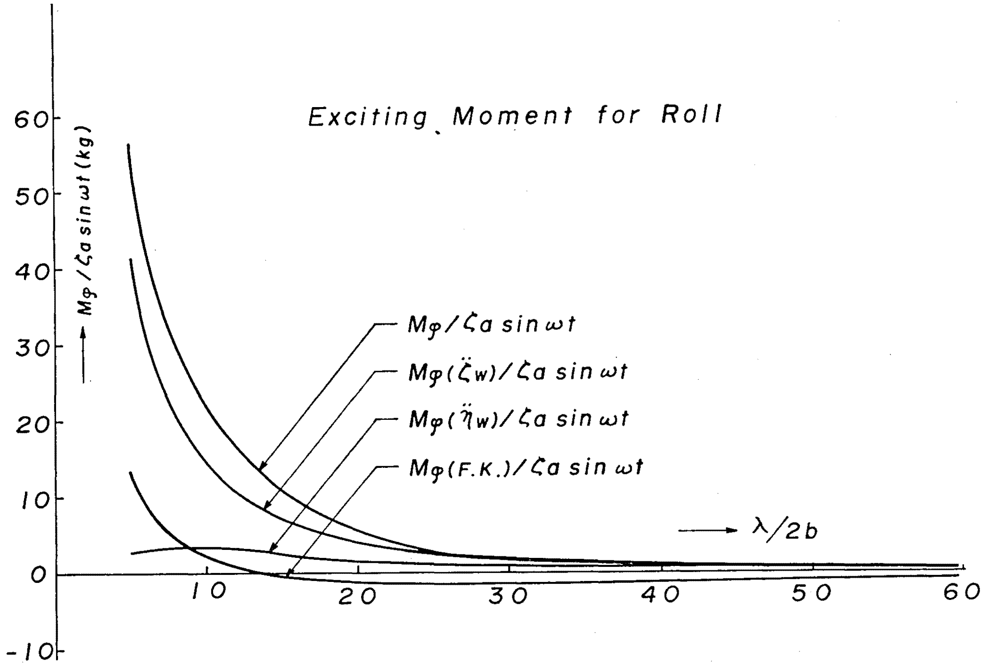

Fig. 7-3. Exciting moment for roll

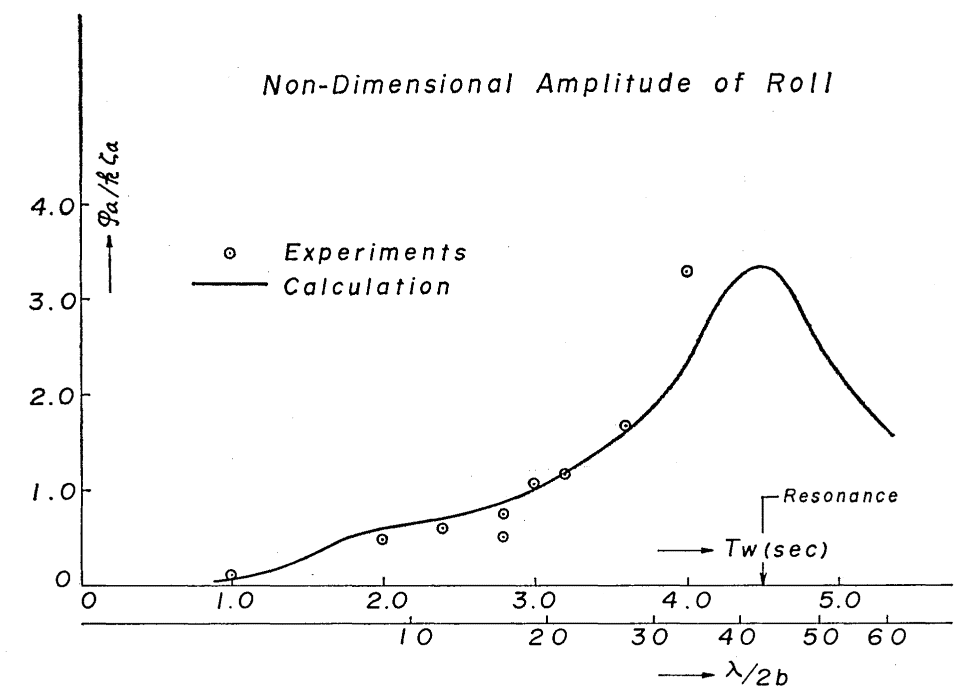

Fig. 9. Comparison of calculated and measured amplitude of roll

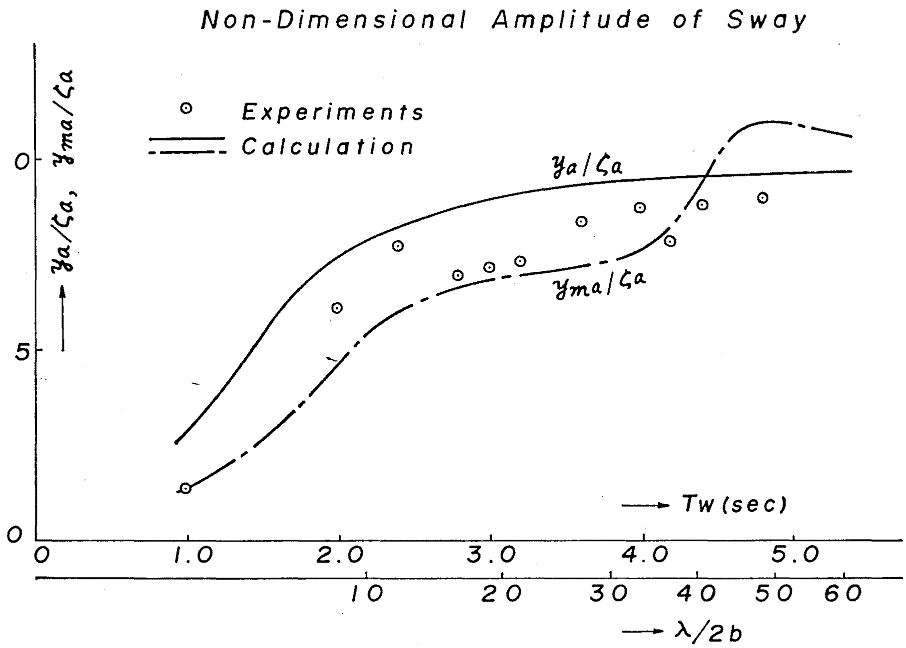

Fig. 10. Comparison of calculated and measured amplitude of sway

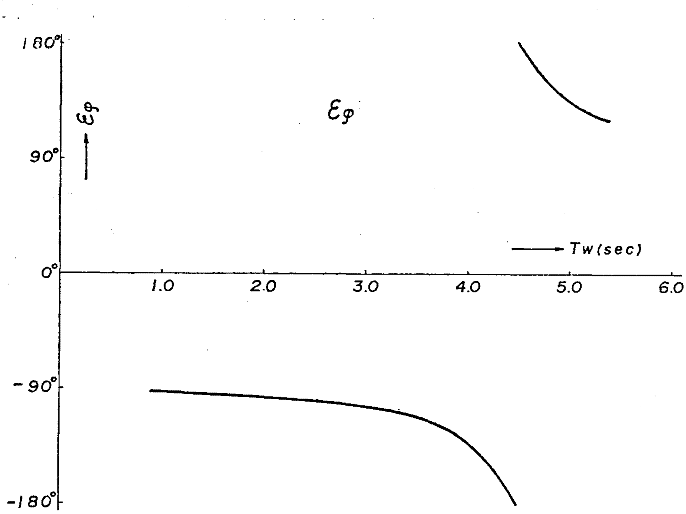

Fig. 11. Calculated frequency characteristics of phase angle between roll and waves

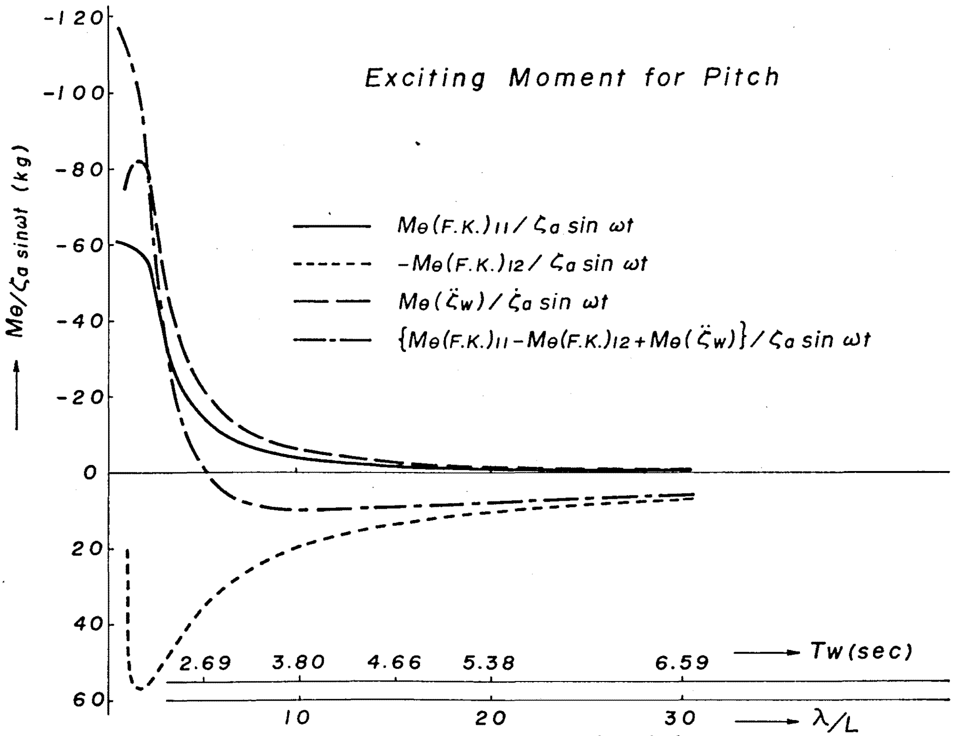

Fig. 12-1. Exciting moment for pitch

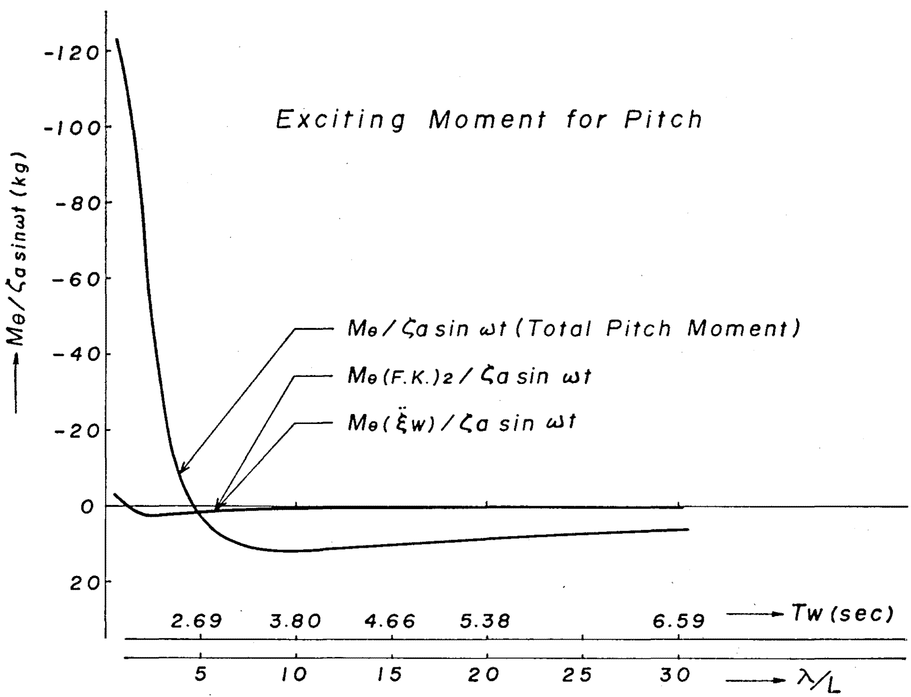

Fig. 12-2. Exciting moment for pitch

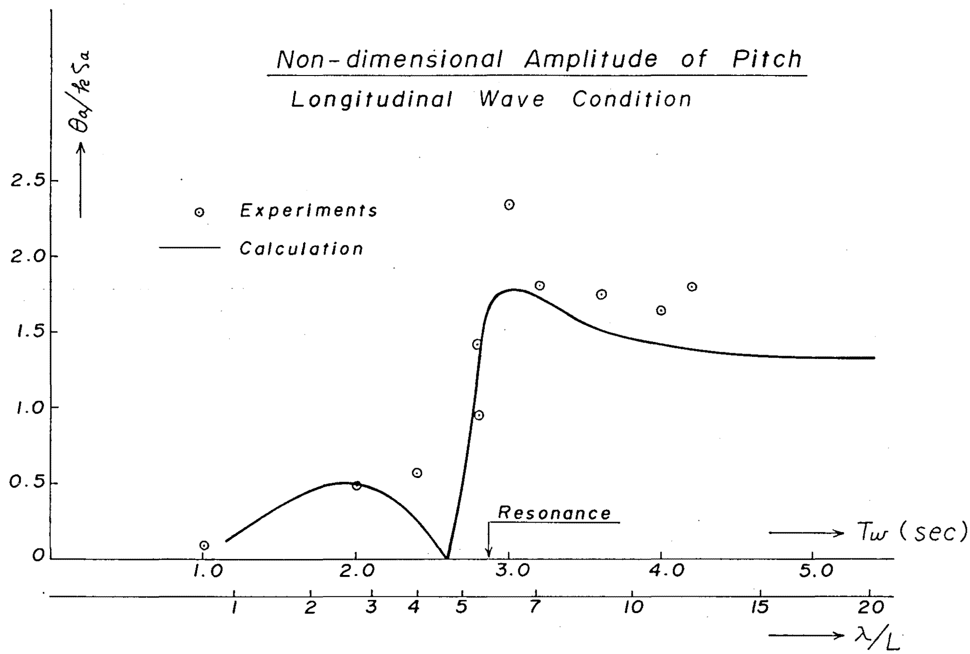

Fig. 13. Comparison of calculated and measured amplitude of pitch