A NOTE ON THE NONLINEAR ENERGY TRANSFER IN THE SPECTRUM OF WIND-GENERATED WAVES

MITSUYASU, Hisashi
Research Institute for Applied Mechanics, Kyushu University : Associate Professor

https://doi.org/10.5109/7170823

出版情報：Reports of Research Institute for Applied Mechanics. 16 (54), pp.251-264, 1968. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. XVI, No. 54, 1968

A NOTE ON THE NONLINEAR ENERGY TRANSFER IN THE SPECTRUM OF WIND-GENERATED WAVES

By

Hisashi MITSUYASU*

In a previous study, the rate of energy transfer in the continuous wave spectrum was determined from the observation of the spectra of wind-generated waves in a decay area of a large scale wind-wave facility [Mitsuyasu 1964]. The experimental results have been compared, in this paper, with the prediction of the theory of nonlinear wave-wave interaction, particularly with that of parameterized equation which has been obtained recently by Barnett (1966, 1968) by parameterizing the theoretical results of Hasselmann (1962, 1963a, 1963b).

Near the dominant peak of the wave spectrum, the measured rate of energy transfer was very close to that predicted theoretically, and the appreciable amount of energy loss in a frequency range near the dominant peak of the spectrum was considered to be attributed to the nonlinear energy transfer from that frequency range to the other frequency ranges of the spectrum. However, the dissipation mechanism of wave energy remained unsolved, because the conservation of wave energy which should be a consequence of wave-wave interaction theory was not satisfied in the measured rate of energy transfer.

1. Introduction

Since the existence of the mechanism of resonant interaction of water wave was demonstrated theoretically by Phillips (1960)** many studies on the nonlinear interaction among discrete surface waves have been made mainly from the theoretical approaches (Longuet-Higgins 1962, Longuet-Higgins & Phillips 1962, Benny 1962 and Bretherton 1964). Meanwhile, studies were extended to such cases as the resonant interaction between surface and internal wave modes (Ball 1964, Thorpe 1966) and second-order energy interchanges among gravity-capillary waves (McGoldrick 1965). In other directions, Hasselmann (1962, 1963a 1963b) studied, in his series of paper, the resonant transfer of energy for a continuous wave spectrum and found theoretically that for ocean waves the modification of the wave spectrum by this mechanism should be appreciable. More recently, the theory of nonlinear interaction has been generalized physically and mathematically. Hasselmann (1967) proposed general theory of weak interaction, which yields the energy transfer for all

* Associate Professor, Member of Research Institute for Applied Mechanics, Kyushu University.

** At about the same time Litvak (1960) has studied the interaction between random trains of magneto-hydrodynamic waves in an ionized gas free of collisions.

expansible interactions between the wave field and the atmosphere and ocean. Benny (1967) studied the mathematical features common to a variety of physical situations of weakly interacting nonlinear waves.

In contrast with the development of such growing body of theory there have been quite a few experimental studies, expect for those by Longuet-Higgins & Smith (1966) and by McGoldrick, Phillips, Huang and Hodgson (1966). They studied the resonant interaction among two trains of waves in mutually perpendicular directions, and identified the growing resonant third-order interaction product. However, corresponding controlled experiments have been, to our knowledge, never done for the case of random waves with continuous spectrum. The present paper is an account of an attempt to compare the author's previous results of experiment with the prediction of a theory of nonlinear wave-wave interaction. The experiments were carried out in 1962 by using the wind-wave facility schematically shown in Fig-1 to clarify the attenuation mechanism of wind wave in the area of no wind.

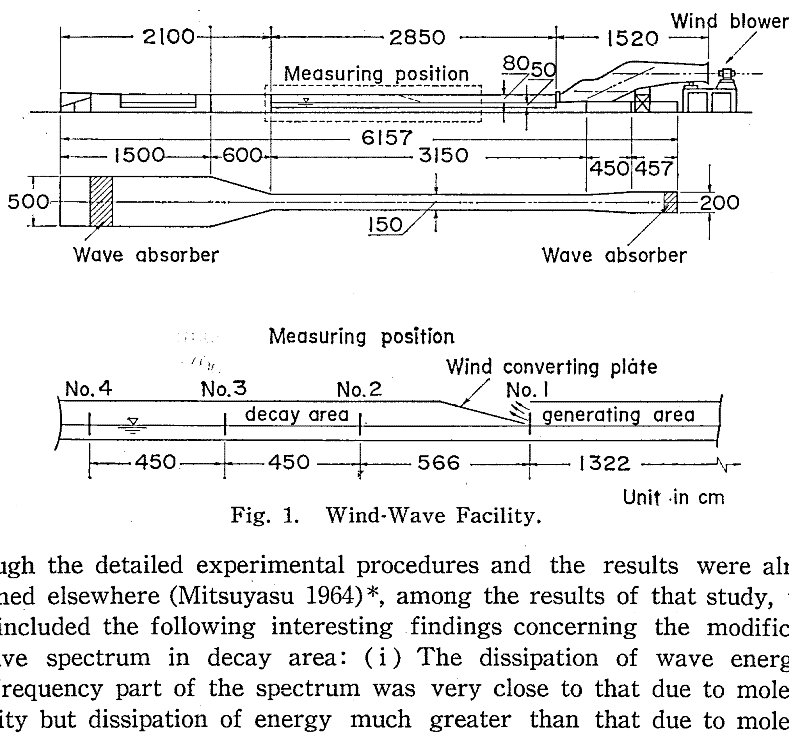

Fig. 1. Wind-Wave Facility.

Although the detailed experimental procedures and the results were already published elsewhere (Mitsuyasu 1964)*, among the results of that study, there were included the following interesting findings concerning the modification of wave spectrum in decay area: (i) The dissipation of wave energy at high frequency part of the spectrum was very close to that due to molecular viscosity but dissipation of energy much greater than that due to molecular viscosity was observed near the dominant peak of the spectrum. (ii) A part

* As shown in Fig. 1, air flow was diverted to the outside of the flume at the middle part of the tank so as to realize the decay area in the rear part of the flume. Wind waves were measured at the observation station, No. 1, No. 2, No. 3 and No. 4, and their data were analyzed in various ways.

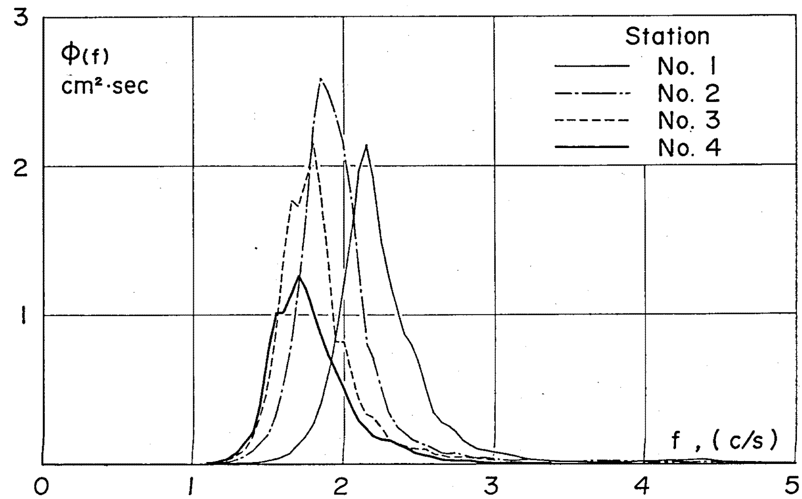

Fig. 2. Spectra of Wind Waves. (r. p. m. 400)

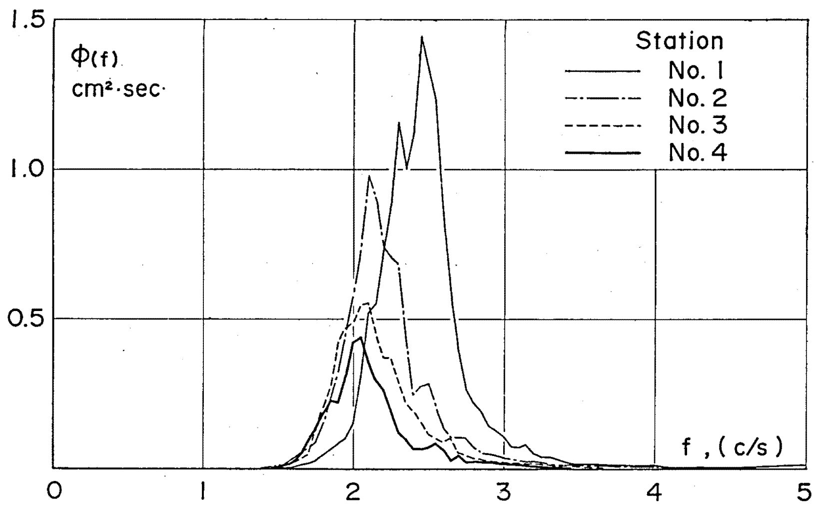

Fig. 3. Spectra of Wind Waves. (r. p. m. 300)

of the energy was seemed to be transfered from the frequency range near the dominant peak of the spectrum to the lower frequency range (cf. Figs. 2 and 3)*, and the transfers were conspicuous especially when the waves were breaking at the initial part of the decay area. Although such phenomena

* In Figs. 2 and 3 r. p. m. means the rotation of blower's propeller per minute, and the correspondences between the values of r. p. m. and the mean wind speeds at the generating area of the flume are as follows; r. p. m. $400 \sim \bar{U} = 10.80$m/sec and r. p. m. $300 \sim \bar{U} = 8.12$m/sec.

were considered to be relating to the nonlinear interaction among wave components, an attempt to compare that results with the theory was not done at that time mainly due to the following reason : Time scale of the actual phenomena was seemed to be too short as compared the characteristic time Tc of the energy flux estimated by the Hasselmann's theory, i. e., $Tc \sim TS^{-4}$ (T: suitably defined mean wave period, S: root mean square wave slope)* Therefore, the theory of Hasselmann (1962) was thought at that time to be not so hopefull for the explanation of our observed results, as compared the hard labours for the long sophisticated computations.

Recently, however, Barnett (1966, 1968) derived the parameterized equation of the Hasselmann's results, which enables the easy computation of the nonlinear energy transfer in a wave spectrum. ** Computations of nonlinear energy transfer were carried out by the parameterized equation and they were compared with the observed results in the author's previous experiment. Very close agreements were found between the theory and experiment, and they were beyond the expectation. Before we enter into the detailed comparison of the experimental results with the theoretical prediction, the parameterized equation for the nonlinear energy transfer will be outlined briefly.

2. A parameterized equation for the nonlinear energy transfer.

According to the studies of Hasselmann (1962, 1963a, 1963b) energy transfer equation can be written in the form [equations (1. 1), (1. 2) Hasselmann 1963b Part 3]

$$\frac{\partial F(\boldsymbol{K}_4)}{\partial t} = \iiiint_{-\infty}^{\infty} a'\delta\,(\omega_1+\omega_2-\omega_3-\omega_4)\,\omega_4\,\{\omega_4 F(\boldsymbol{K}_1)\,F(\boldsymbol{K}_2)\,F(\boldsymbol{K}_3)$$

$$+\,\omega_3 F(\boldsymbol{K}_1)\,F(\boldsymbol{K}_2)\,F(\boldsymbol{K}_4)-\omega_1 F(\boldsymbol{K}_2)\,F(\boldsymbol{K}_3)\,F(\boldsymbol{K}_4)$$

$$-\,\omega_2 F(\boldsymbol{K}_1)\,F(\boldsymbol{K}_3)\,F(\boldsymbol{K}_4)\,\}\,d^2k_1\,d^2k_2 \tag{1. 1}$$

where $$\boldsymbol{K}_3 = \boldsymbol{K}_1+\boldsymbol{K}_2-\boldsymbol{K}_4,\quad a' = \pi\left(\frac{3gD}{2\rho\,\omega_1\omega_2\omega_3\omega_4}\right)^2 \tag{1. 2}$$

and $$D = D^{+}_{\boldsymbol{K}_1},\ {}^{+}_{\boldsymbol{K}_2},\ {}^{-}_{-\boldsymbol{K}_3}$$

The evalution of this equation involves cubic integration and it requires long sophisticated numerical computations.

To be compatible with a practical wave prediction scheme, Barnett (1966,

* In the previous paper (Mitsuyasu 1964) there had been some misunderstanding on the definition of characteristic time, and the time scale determained experimentally was not adequate to compare the result of theoretical prediction.

** Informations about the parameterized equation have been communicated by T. P. Barnett in an early time of his publication. The author is indebted to his suggestions.

1968) has obtained the parameterized form of the Hasselmann's transfer equation, in a manner that will readily used for rapid and accurate calculation. The essential part of his equation and its derivation are as follows.

The wave-wave interaction effect on any particular spectral component was thought of as the sum of two processes as has been originally demonstrated by Hasselmann (1962). In the first, the component is "active" and is transferring its energy to other components which satisfy resonant interaction criteria. The energy loss is proportional to the magnitude of the component and a parameter called the decay time, the inverse of which will be designated by τ and will have unit of t^{-1}. In the second process, the spectral component is "passive" and receives energy from the other components. This rate of energy transfer is linear in time and will be denoted by the parameter T having unit of length squared. Combining these two effects one has

$$\frac{\partial \phi(f,\theta)}{\partial t} = T - \tau\phi(f,\theta) \tag{2}$$

where $\phi(f,\theta)$ is the directional spectrum and the functional forms of T and τ have yet to be determined.

To arrive at quantitative expressions for T and τ, using dimensional analysis and the qualitative features of Hasselmann's theory, forms of T and τ were determined for the case of fully developed sea, so that eq. (2) could give energy transfer that corresponded closely with those derived from numerical calculation of Hasselmann's transfer equation, (Fig. 2, Hasselmann 1963b, Part 3). The final forms of T and τ were given below with all units being C. G. S.;

$$T = \frac{4.4\times 10^8 E^3 f_0^{\,8}}{g^4} \cos^4(\theta-\theta_0)\,(1-0.42\frac{f_0}{f})^3 \exp\left[-4\,(1-\frac{f_0}{f})^2 + 0.1\,(\frac{f_0}{f})^5\right]$$

$$f > 0.42 f_0 \ \ and \ \ |\theta-\theta_0| < \frac{\pi}{2} \tag{3}$$

$$= 0 \qquad \text{otherwise}$$

$$\tau = \frac{7.5\times 10^7 E^2 f_0^{\,7}}{g^4 f}\{1+16\,|\cos(\theta-\theta_0)|\}\,(f-0.53 f_0)^3$$

$$f > 0.53 f_0 \tag{4}$$

$$= 0 \qquad \text{otherwise}$$

where E, f_0 and θ_0 are parameters which characterize the spectrum and are defined respectively by the following equations:

$$E = \iint \phi(f,\theta)\, df d\theta \qquad \text{total energy} \tag{5}$$

$$f_0 = \left\{\frac{1}{E}\iint f^2\phi(f,\theta)\,dfd\theta\right\}^{\frac{1}{2}} \qquad \text{mean frequency*} \qquad (6)$$

$$\theta_0 = \frac{1}{E}\iint \theta\phi(f,\theta)\,dfd\theta \qquad \text{mean direction} \qquad (7)$$

At this time, however, Eq. (2) should be converted into the equation for one dimensional wave spectrum, because the measured rates of energy transfer were concerned with one dimensional wave spectra. This can be easily done by integrating Eq. (2) with respected to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, i.e., by

$$\frac{\partial\phi(f)}{\partial t} = \left(\int_{-\pi/2}^{\pi/2} \frac{\partial\phi(f,\theta)}{\partial t}d\theta\right) = \int_{-\pi/2}^{\pi/2} T d\theta - \int_{-\pi/2}^{\pi/2} \tau\phi(f,\theta)\,d\theta \qquad (8)$$

Assuming the $\frac{8}{3\pi}\cos^4\theta$ spreading factor** of wave spectrum and inserting Eqs. (3) and (4) into the right-hand side of Eq. (8), we finally obtain

$$\frac{\partial\phi(f)}{\partial t} = T' - \tau'\phi(f) \qquad (9)$$

where $T' = \frac{4.4\times 10^8 E^3 f_0^8}{g^4}\,\frac{3\pi}{8}\,(1-0.42\frac{f_0}{f})^3 \,exp\left[-4(1-\frac{f_0}{f})^2 + 0.1(\frac{f_0}{f})^5\right]$

$$f > 0.42 f_0 \qquad (2)$$

$= 0$ otherwise

$$\tau' = \frac{7.5\times 10^7 E^2 f_0^7}{g^4 f}\,(1-\frac{8\times 16^2}{45\pi})\cdot(f-0.53 f_0)^3$$

$$f > 0.53 f_0 \qquad (11)$$

$= 0$ otherwise

3. Comparison of theory and observation

In the previous paper (Mitsuyasu 1964) the rate of energy transfer T_* was defined by the equation.

$$\frac{\partial\phi(f,x)}{\partial x} = T_* - D\phi(f,x) \qquad (12)$$

where
$$D = \left\{\frac{256\pi^5\nu}{g^3} f^5 + \frac{8\pi}{bg}\sqrt{\pi\nu}\,f^{\frac{3}{2}}\right\} \qquad (13)$$

---

* According to Barnett (1968), the definition of f_0 is

$$f_0 = \frac{1}{E}\iint f\phi(f,\theta)\,dfd\theta$$

However, this seems to be misprint according to the results of the present author's re-comparison of his equation with the Hasselmann's results.

** In the laboratory experiment, only one dimentional wave spectra were measured. Therefore, directional spreading factor was assumed tentatively as $\frac{8}{3\pi}\cos^4\theta$ as a first approximation.

g: acceleration of gravity, ν: kinematic viscosity of water,
b: width of wave tank

In Eq. (12) $\phi(f. x)$ is one dimentional wave spectrum, and the second term in the right-hand side of the equation represents a viscous dissipation of the wave energy. The rate of energy transfer T_* in Eq. (12) was determined approximately by measuring directly $\frac{\triangle\phi}{\triangle x}$ and adding the viscous dissipation $D\{\phi(f, x)+\phi(f, x+\triangle x)\}/2$.

From Eq. (12) the energy balance at a definite station can be written as

$$T'_*(=CgT_*)=Cg\left\{\frac{\partial\phi}{\partial x}-D\phi\right\} \tag{14}$$

The rate of energy transfer T'_* such obtained corresponds approximately to $\frac{\partial\phi}{\partial t}$ in Eq. (9) and T'_* will be denoted hereafter by $\left(\frac{\partial\phi}{\partial t}\right)_{\text{measured}}$ Figs. 4, 5, 6 and 7 show the measured rate of energy transfer $\left(\frac{\partial\phi}{\partial t}\right)_{\text{measured}}$ and

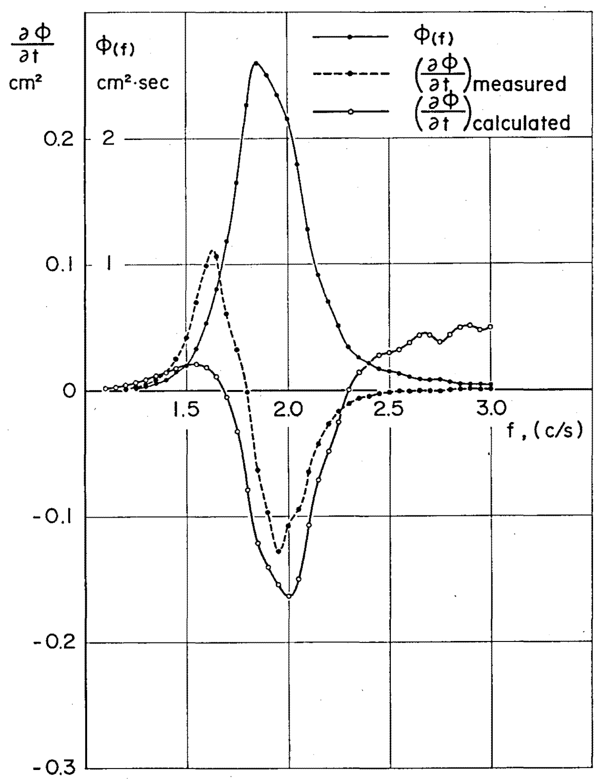

Fig. 4. Wave Spectrum and Rate of Energy Transfer.
(r. p. m. 400, Station: No. 2→No. 3)

calculated one $\left(\frac{\partial\phi}{\partial t}\right)_{\text{calculated}}$ $(=T'-\tau'\phi)$ in association with the wave spectrum at each upward station. It can be seen from Figs. 4 through 7 that the agreements between the theory and the experiment are qualitatively even quantitatively good on the whole. Those results, therefore, can be considered as supporting the view that the appreciable amount of energy loss near the dominant peak of the wave spectrum is largely attributed to the nonlinear wave-wave interactions.

4. Discussion and conclusions.

In spite of the extended application of the parameterized equation which was originally derived for the case of the fully arisen Neumann spectrum, fairly good agreements between the obserevation and the theory were obtained for the modifications of spectra of laboratory wind waves which were slightly different from the fully arisen Neumann spectrum. When we consider the accuracies of the measured spectra which have the degree of freedom of 90,

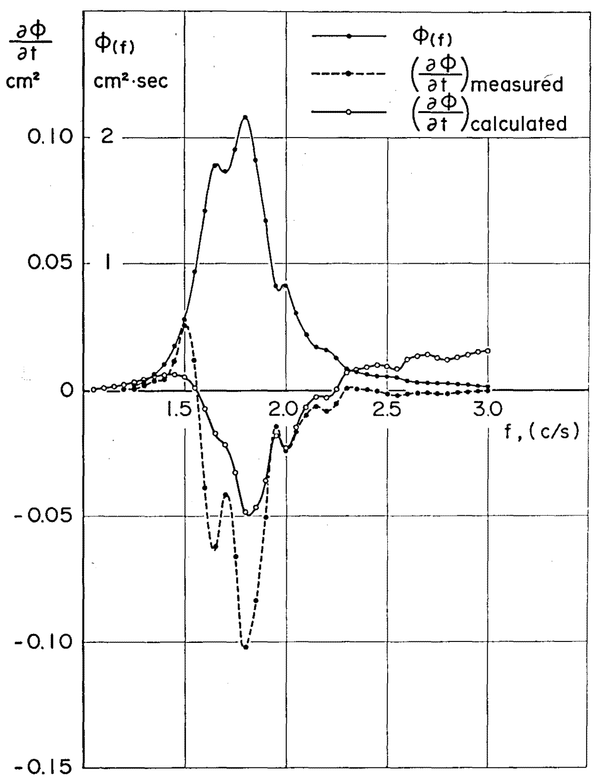

Fig. 5. Wave Spectrum and Rate of Energy Transfer. (r.p.m. 400, Station: No. 3→No. 4)

those agreements are more than we expected.

However, it should be mentioned here that the following several discrepancies are found when the results are carefully examined: (i) In Fig. 4, the calculated positive energy transfer in the low frequency range of the spectrum is much smaller than the measured one. (ii) General disagreements between the theory and the experiment are found in the high frequency range of each spectrum.

The first one should be attributed partly to the limitation of the theory which assumed the weak interaction. That is, the results shown in Fig. 4 was obtained in the initial part of the decay area for the case of wind waves generated by relatively high wind speed ($\bar{U}=10.80$ m/sec). The wind wave in this area was almost saturated condition and the breaking of waves was visually observed. Such evidence is considered to be reflecting the strong nonlinearities in the wave motion. Therefore, the approximation of weak interaction would be not so adequate in such situation. An additional explanation will be given by considering the effects of the drift current. That is, as shown in Fig. 8 the speed of the drift current attenuate in the decay area where the wind is not blowing, and this will cause the downward shift of the frequency range of the wave spectrum. Very rough calculations based on the

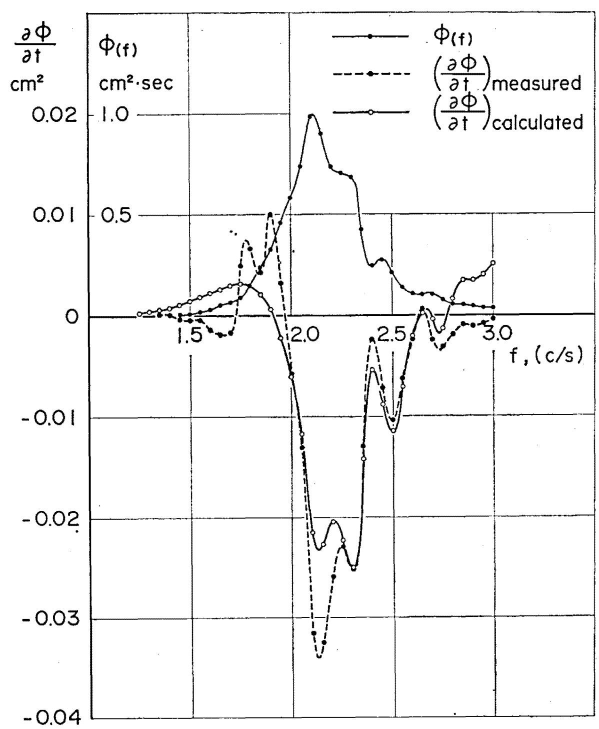

Fig. 6. Wave Spectrum and Rate of Energy Transfer.
(r. p. m. 300, Station: No. 2→No. 3)

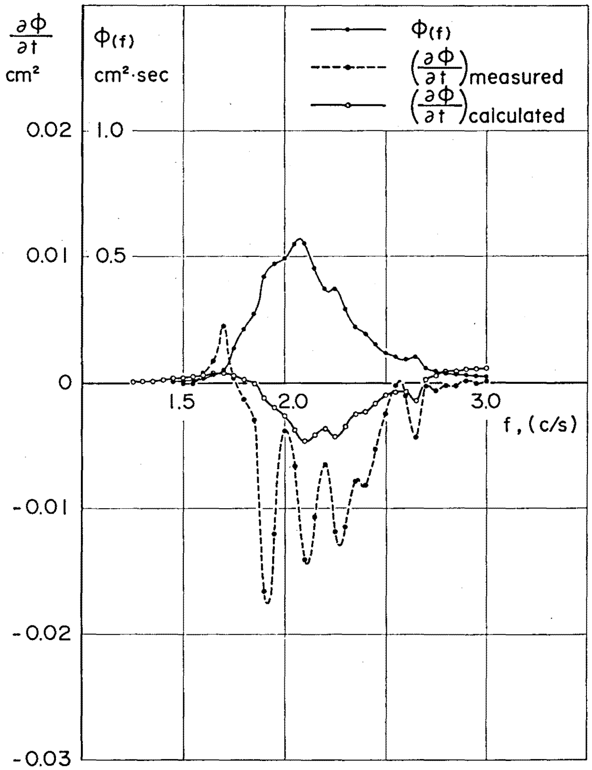

Fig. 7. Wave Spectrum and Rate of Energy Transfer.
(r. p. m. 300, Station: No. 3→No. 4)

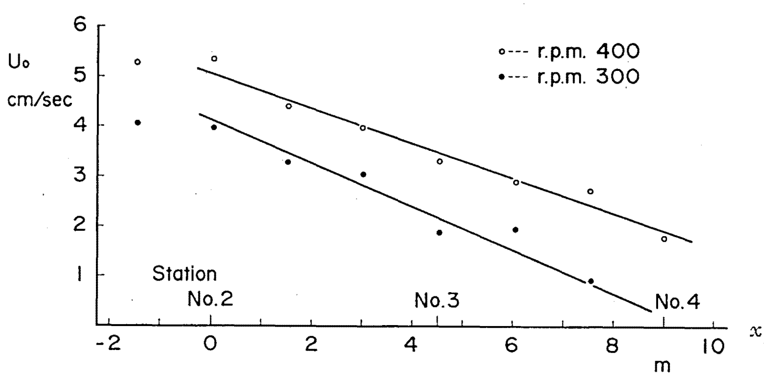

Fig. 8. Surface Speed of Drift Current.

velocity change shown in Fig. 8, showed that the change of the frequency in the spectra shown in Figs. 2 and 4 is of the order of 10^{-2} c/s.

The second disagreement will be more essential because they are found almost in every cases. The following considerations will be given for this evidence. Since the equation has been derived under the assumption of no energy dissipation, energy must be conserved within the mode of wave motion.* In other words, the energy loss near the dominant peak of the wave spectrum should be compensated by the energy gains in some other frequency ranges of the wave spectrum. From such considerations, the calculated energy accumulation in high frequency range of each wave spectrum should be reasonable at least qualitatively, and the measured energy transfer should rather be contradictory. In actual system of wind wave, however, the energy must be lost from the mode of wave motion to the other modes of motions through various complicated mechanism. That is, effect of viscosity for high frequency part of wave spectrum, the breaking of wind wave, the generation of ripples overlapping on the dominant waves (Longuet-Higgins 1963) and the interactions between the wave motion and the other modes of water motions such as surface current and turbulent motion of water, almost all of such various mechanism will contribute to dissipate the wave energy**. On account of this, slight disagreement between the observation and the theory which assumed no energy dissipation from the wave motion would be inevitable at this time.

Finally, it should be necessary to make some comment on the difference between the measured spectra and the Neumann spectrum, since the explicit form of the transfer equation of Barnett (1966, 1968) was determined mainly for the case of the Neumann spectrum.*** Fig. 9 show the measured spectra at station No. 2 and the Neumann spectrum, the both of which are normalized in the same way, i. e., as

$$\frac{\phi f_m}{E} = F\left(\frac{f}{f_m}\right) \tag{15}$$

where f_m is the frequency at the dominant peak of the spectrum and E is the total energy of the spectrum. As shown in Fig. 9 the energy concentration near the dominant peak of the measured spectrum is greater than that of the

* In the parameterized equation obtained by Barnett (1966, 1968) the conservation of the wave energy is not satisfied in a strict sense, because its form was determined so as only to duplicate the physical characteristics of the interaction process as given by Hasselmann (1963b). However, in the original transfer equation given by Hasselmann, the conservation of the wave energy is satisfied (Hasselmann 1962).

** Although some correction for the energy dissipation was made in the measured rate of energy transfer, its correction was concerned only for the viscous dissipation.

*** The approximate consistency of the parameterized equation was checked by Barnett (1966) for the case of the Pierson & Moskowitz spectrum too.

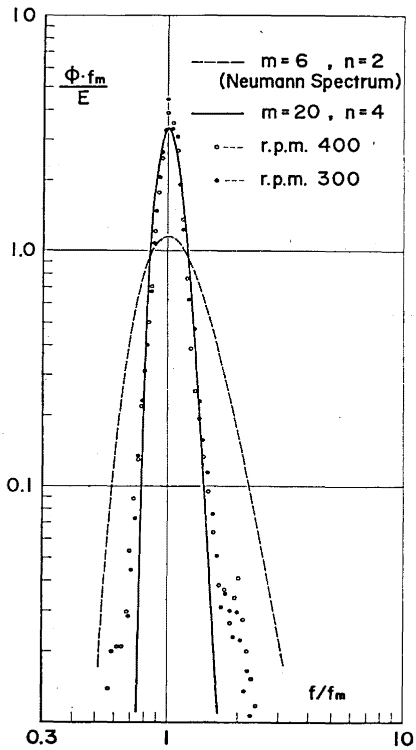

Fig. 9. Spectra of Wind Waves.
The broken and solid lines show the theoretical spectra $\phi_{(f)}=k_1 f^{-m} \cdot exp[-k_2 f^{-n}]$. *)The open and solid circles show the spectra measured at station No. 2.

Neumann spectrum, and the form of the measured spectrum is rather close to $\phi=k_1 f^{-20} \cdot exp[-k_2 f^{-4}]$ than the Neumann spectrum, $\phi=k_1 f^{-6} \cdot exp[-k_2 f^{-2}]$. Fig. 10 shows the rates of energy transfers given by the parameterized equation when the equation is applied to the various forms of the spectrum. From this figure it can be seen (i) that the parameterized equation can give the fairly consistent results, at least qualitatively, even when the equation is applied to the spectra which are different from the Neumann spectrum, and (ii) that the rate of energy transfer increases with the energy concentration near the dominant peak of the spectrum. At this time, however, those

* The spectrum of the form $\phi(f)=k_1 f^{-m} exp[-k_2 f^{-n}]$ can be normalized as

$$\frac{\phi f_m}{E}=\frac{n\left(\frac{m}{n}\right)^{\frac{m-1}{n}}}{\Gamma\left(\frac{m-1}{n}\right)}\left(\frac{f}{f_m}\right)^{-m} exp\left[-\frac{m}{n}\left(\frac{f}{f_m}\right)^{-n}\right]$$

where $\Gamma\left(\frac{m-1}{n}\right)$ is the gamma function.

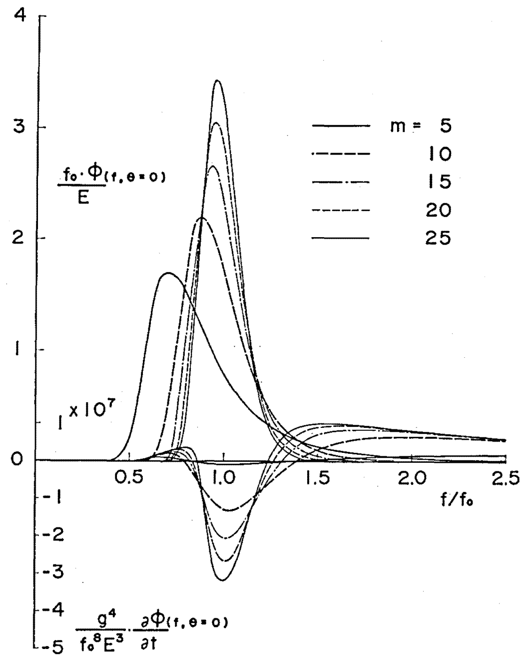

Fig. 10 Energy transfer for the spectrum of the form of
$\phi(f,\theta)=\frac{8}{3\pi}cos^4\theta \cdot k_1 f^{-m} \cdot exp[-k_2 f^{-n}](n=4,\ \theta=0)$
($m=5$, $n=4$: Pierson-Moskowitz Spectrum)

properties will not necessarily be connected with the actual phenomena, since adequacy of the extended application of the parameterized equation (beyond the Neumann spectrum) is not examined yet. It must be one of the further studies needed to evaluate the transfer equation of Hasselmann (1963b) for the case of spectra which are different from the Neumann spectrum.

The author is greately indebted to Prof. M. Kurihara of Nagasaki University for his valuable advice. He is also indebted to Prof. K. Kajiura of Tokyo University for fruitful discussions of the problem. Much of this work was assisted by Mr. T. Komori, to whom the author owe thanks, particularly for the co-operation in the numerical computations and for the drawing of the figures.

This work was partly supported by Scientific Research Funds from the Ministry of Education.

References

Ball, K. (1964): Energy transfer between external and internal gravity waves, J. Fluid Mech. 19, 465–78.

Barnett, T. P. (1966): On the generation, dissipation and prediction of ocean wind

waves, Scripps Institution of Oceanography, Doctral thesis.
Barnett, T. P. (1968): On the generation, dissipation, and prediction of ocean wind waves, J. Geophys. Res., Vol. 73, No. 2, 513–29.
Benny, D. J. (1962): Non-linear gravity waves interactions, J. Fluid Mech. 14, 577–84.
Benny, D. J. (1967): The asymptotic behaviour of non-linear dispersive waves, J. Math. and Phys. 46, 115–32.
Bretherton, F. P. (1964): Resonant interaction between waves, the case of discrete oscillations, J. Fluid Mech. 20, 457–79.
Hasselmann, K. (1962): On the non-linear enegy transfer in a gravity-wave spectrum. Part 1. General theory, J. Fluid Mech. 12, 481–500.
Hasselmann, K. (1963a): On the non-linear energy transfer in a gravity-wave spectrum. Part 2. Conservation theorem; wave-particle analogy; irrevesibility. J. Fluid Mech. 15, 273-81.
Hasselmann, K. (1963b): On the non-linear energy transfer in a gravity-wave spectrum. Part 3. Evaluation of the energy flux and sea-swell interaction for a Neumann spectrum, J. Fluid Mech. 15, 385–98.
Hasselmann, K. (1967): Weak-interaction theory of ocean waves, Institut fur Schiffbau der Universität Hamburg, Bertich Nr. 181.
Litvak, M. M. (1960): A transport equation for magneto-hydrodynamic waves, AVCO-Ererett Res., Rep. No. 92.
Longuet-Higgins, M. S. (1962): Resonant interactions between two trains of gravity waves, J. Fluid Mech. 12, 321–32.
Longuet-Higgins, M. S. & Phillips, O. M. (1962): Phase velocity effects in tertiary wave interactions, J. Fluid Mech. 12, 333–36.
Longuet-Higgins, M. S. (1963): The generation of capillary waves by steep gravity waves, J. Fluid Mech. 16, 138–59.
Longuet-Higgins, M. S. and Smith, N. D. (1966): An experiment on third-order resonant wave interactions, J. Fluid Mech. 25, 417–35.
McGoldrick, L. F. (1965): Resonant interactions among capillary-gravity waves, J. Fluid Mech. 21, 305–31.
McGoldrick, L. F. Phillips, O. M., Huang, N. E. & Hodgson, T. H. (1966): Measurement of third-order resonant wave interactions, J. Fluid Mech. 25, 437–56.
Mitsuyasu, H. (1964): Wind wave in decay area, Report of Port & Harbour Technical Research Institute, No. 5.
Phillips, O. M. (1960): On the dynamics of unsteady gravity waves of finite amplitude, Part 1. The elementary interactions, J. Fluid Mech. 9, 193–217.
Thorpe, S. A. (1966): On wave interactions in a stratified fluid, J. Fluid Mech. 24, 737–52.

(Received October 18, 1968)