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https://doi.org/10.5109/7170821

出版情報：Reports of Research Institute for Applied Mechanics. 16 (54), pp.239-250, 1968. 九州大学応用力学研究所

Reports of Research Institute for Applied Mechanics
Vol. XVI, No. 54, 1968

ON THE ESTIMATION OF NATURAL FREQUENCIES OF VERTICAL VIBRATION OF SHIPS*

By

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The paper deals with the estimation of natural frequencies of the vertical vibration of ship hull, taking into account the recent investigations on the effects of fineness of the hull, shear deflection and rotatory inertia and the virtual inertia coefficient of the water. Some empirical factors obtained from the observations on board actual ships are shown. Here are also put forward new empirical formulae for estimating inertia coefficient of the water surrounding a vibrating ship based on the data obtained from several ship-shaped models.

1 Introduction

Quite a number of empirical formulae have been so far put forward by many authorities for a quick estimation of natural frequencies of a ship hull in the design stage, for instance, Schlick formula, Todd formula, Prohaska's investigations on the corrections to existing knowledge on ship vibration and recently Dieudonné's formula and other formulae by numerous researchers[1),2),3)]. Great efforts have been exerted on the subject of estimating natural frequencies of ship vibration in the past. Such an upshot of the problem may be due to the reason that the natural frequencies of ship hull are determined on the basis of the free-free bar corresponding to a prismatic ship hull in which the following basic parameters are accounted for;

(1) Effect of variable cross section and distribution of load.
(2) Virtual added mass of water surrounding ship.
(3) Shear deflection and rotatory inertia of the section of ship hull.
(4) Effective breadth of the section or the reduction of rigidity of the hull in the higher mode of hull vibration.
(5) Effects of coupled vibration of bottom panels and superstructure upon the main hull vibration in the higher frequency.

In the present paper the author considers the first three effects for estimating natural frequencies, including the higher frequency of the hull vibra-

* The paper was read before the Spring Meeting of the Society of Naval Architects of Japan in Tokyo on May 1967.

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tion and taking into account more rational consideration than the current empirical formulae. Any considerations are, however, left out here either on the reduction of rigidity or the effect of coupled vibration of the bottom panel of the hull and deck house on ship hull vibration, both in the higher frequency of the vibration. For that further study about (4) and (5) would be necessary.

While the effect of variable cross section and that of the virtual inertia of water on the natural frequencies are derived from energy principle, the effects of shear deflection and rotatory inertia are separately considered from an approximate solution of Timoshenko equation to the empirical formula for estimating the natural frequency of a ship hull vibration in the present investigation. With regard to the virtual inertia coefficient of the water, new empirical formulae from the author's investigations and experiments made on a model ship are put forward in the present paper.

2 Calculation of natural frequency of the vertical vibration of ship hull

Depending on energy principle we can calculate natural circular frequency of the flexural vibration of such an elastic beam of variable cross section as belonging to a ship that is kept afloat on water, that

$$\int_L EIy''^2dx=\omega^2 \cdot \int_L (m+Jm_e)y^2dx \tag{1}$$

where, ω natural circular frequency of flexural vibration of ship
EI flexural rigidity of ship
m mass per unit length
m_e two-dimensional virtual mass of water per unit length
x length co-ordinate of ship
y normal function of the vibration of ship
$y''=d^2y/dx^2$
J three-dimensional local reduction factor of m_e
L length of ship

from the above equation, ω is written by

$$\omega=\sqrt{\frac{EI_0\int_L \frac{I}{I_0}y''^2dx}{\bar{m}\int_L \frac{m}{\bar{m}}y^2dx\left(1+\frac{\int_L Jm_e y^2dx}{\int_L my^2dx}\right)}} \tag{2}$$

where, EI_0 flexural rigidity at midship section
I/I_0 distribution function of I along length of the ship
$\bar{m}$ mean mass of the ship per unit length
$m/\bar{m}$ mass distribution function along the ship length

Now, putting

$$\frac{\int_L \frac{I}{I_o} y''^2 dx}{\int_L \frac{m}{\overline{m}} y^2 dx} \equiv \frac{c_1^2}{L^4} \tag{a}$$

$$\frac{\int_L J m_e y^2 dx}{\int_L m y^2 dx} \equiv \tau \tag{b}$$

$$\overline{m} g L \equiv \Delta \tag{c}$$

where, c_1 variable section coefficient including eigen value
τ virtual inertia coefficient of water surrounding vibrating ship
Δ displacement of ship
g gravitational acceleration

Substituting (a), (b) and (c) into (2), following current formula is obtained,

$$\omega = c_1 \sqrt{\frac{g E I_o}{\Delta L^3 (1+\tau)}} \tag{3}$$

Since distributions of I, Jm_e are known for a given type of ship, ω could be calculated provided that y were assumed in the above expression, by means of numerical integration of equation (a). Estimation of c_1-value in two node vertical vibration of ships, the investigation was carried out in detail by Prof. Prohaska [1] for various loading distributions of cargo.

On the other hand, regarding the effect of shear deflection and rotatory inertia, the natural frequency for the vibration of uniform bar is estimated through some calculations on Timoshenko equation, provided that the mode of the vibration is assumed to be a normal function $\cos n\pi x/L$ ($n=2, 3, 4, \ldots$) as an approach. Hence the natural frequency in equation (3) will be written by correcting shear deflection and rotatory inertia as follows [4];

$$N_{\mathrm{cpm}} = \frac{60}{2\pi} \cdot c_n \cdot n^2\pi^2 \sqrt{\frac{g E I_o}{\Delta L^3 (1+\tau)\{1+(\alpha+\beta) n^2\pi^2\}}} \tag{4}$$

where, $n^2\pi^2$ eigen value
n number of nodes of vertical vibration of the hull

$$\alpha = \frac{E I_o}{k' G A_o L^2}\ ; \quad k'GA_o : \text{shear rigidity of midship section} \tag{d}$$

$$\beta = \frac{r_o^2}{L^2}\ ; \quad r_o : \text{radius of gyration of the midship section} \tag{e}$$

$$c_n = \frac{L^2}{n^2\pi^2} \sqrt{\frac{\int_L \frac{I}{I_o} y''^2 dx}{\int_L \frac{m}{\overline{m}} y^2 dx}}\ ; \quad \text{variable section coefficient} \tag{f}$$

Strictly speaking, α and β are variables along ship length, but in the present paper, these values are assumed to be constant to make an approach easy. If we prepare the ship plan, we could calculate all terms presented by (b), (c), (d), (e) and (f) under a certain assumption of the normal mode of the vibration y and by means of numerical integration of equations (b) and (f), and obtain the natural frequencies numerically from (4). However, if the above formula is used for a quick estimation in the stage of initial design, c_n would be considered as an empirical factor, and the terms other than τ-value would be estimated from the ship plan.

3 Estimation of virtual inertia coefficient of ship hull

Dr. Todd defined the virtual inertia factor as

$$\text{V. I. F} = \frac{\text{displacement} + \text{added virtual weight}}{\text{displacement}} \qquad (3.1.7)$$

in his text book[2] (p. 51), however, this factor is treated differently from the author's coefficient $(1+\tau)$ here as is seen in the expression (b). The virtual inertia factor defined as V. I. F is available for the translational oscillation of ship hull. If the normal mode y were constant for length coordinate x in the expression (b), $(1+\tau)$ would coinside with Todd's V. I. F. As seen in the expressions (2) and (b), virtual inertia coefficient τ in the flexural vibration of the ship will be defined as the ratio of energies of the virtual added mass and that of ship mass in the flexural vibration of the hull, so that both integrands may involve the square of normal mode of the vibration. Expression (b) distinctly shows how discrepancy is raised between virtual inertia coefficient given by the author[5] and that defined by Todd and other researcher[6]. The well known Todd's formula, $0.2+B/3d$ therefore has a value considerably higher than the present τ in the flexural vibration of ship hull as shown in Figure 1.

An approximate τ-value of ship hull will be obtained from the observation of natural frequencies of the vibration of a ship-shaped model as follows;

$$\tau = \left(\frac{f_a}{f_w}\right)^2 - 1 \qquad (5)$$

where, f_a and f_w are natural frequencies measured on ship model in air and in water respectively. It is proved by a theoretical approach that the τ-value of the vibration of a half-immersed elliptical cylinder is proportional to B/d, where B and d denote the breadth of cylinder at the water line and the draught respectively, which is shown by[7]

$$\tau = k\frac{B}{d} \qquad (6)$$

k-value takes about 0.35 in the two node vertical vibration of an elliptical cylinder, provided that the three-dimensional reduction factor is assumed to be of 0.70 at $L/B=7.5$. The result shows good agreement with the theoretical one[8]. In the experiments on the ship-shaped model, however, τ-value is not proportional to B/d, namely k-value is not apparently constant but depends on the draught d in the same model ship, because in consequence of the draught changes to shallow distribution patterns of m_e becomes sharp or m_e concentrate near the midship section with little variation of the distribution of m in the relation (b), whereas in the half-immersed elliptical cylinder, the above distribution does not change for B/d. From the result of recent study in author's laboratory, the τ-value of a cargo ship model is to be expressed empirically as follows,

$$\tau = 0.3\frac{B}{d} - 0.033\left(\frac{B}{d}\right)^2 \tag{7}$$

This is an empirical formula for quick estimation of τ-value of cargo ship. Since $\tau=0.24\ B/d$ which was presented in the author's previous paper[7] shows a mean value, unlike the previous formula, the above expression (7) shows correct value of a typical cargo ship.

Different empirical coefficients of B/d and $(B/d)^2$ for the τ-value were determined from experiments on tanker models that

$$\tau = 0.4\frac{B}{d} - 0.035\left(\frac{B}{d}\right)^2 \tag{8}$$

The results of observations to τ-values, which are represented in (6), (7) and (8) are shown in Figure 1.

On the other hand, it is to be noted that the τ-value of ship hull in the vertical vibration in water is to be calculated under some assumptions of the J-value and y by means of numerical integral of equation (b). In this equation, the two-dimensional virtual mass of the ship section m_e is easily obtained from the well known Lewis' figures[9] or Prohaska's chart[10] for given value of B/d and area coefficient of the ship section. Regarding the local three-dimensional correction factor J, the author[11] has recently presented a method for estimating local J-value of ship-section by the use of a chart obtained from computed results of the theoretical approach on the three-dimensional virtual mass of Lewis-formed cylinder.

Next, though it seems to be difficult to assume normal mode it will be possible to assume, for instance, from the mean from ship data of the normal mode of main hull vibration, as an approach, which obtained by A. J. Johnson and P. W. Ayling[12]. The τ-value of actual ship will be thus estimated by the use of above numerical results. If we represent the values in equation (b) by series of trigonimetric function of length co-ordinate, we can quickly estimate τ-value by means of analytical integral of (b).

With regard to the inertia coefficient τ-values in the higher modes of hull vibrations, τ-value becomes different from that of two-node with the increase of number of nodes in the vertical vibration of the structural model of a tanker. This mainly depends on the three-dimensional virtual mass of water and its decreasing or increasing rate changes for B/d and area coefficient of the section of the hull[8), 11)]. It is to be noted that the inertia coefficient τ in the higher mode becames somewhat larger than that in the two node vibration in some experimental results of measurements on wooden models of cargo ship and also in the result of calculation of two-dimensional inertia coefficient under some assumption of load distribution[7)]. Figure 2 shows measured τ-values versus number of nodes of vertical vibration of the structural model of a tanker made by thin brass plate in ballast condition and of the wooden model of a cargo ship in full load condition as two examples.

4 Estimations of shear rigidity and rotatory inertia

The shear rigidity $k'GA$ is roughly estimated by the Hovgaard's formula [13)] as that of web area of ship girder. The exact calculation of $k'GA$ for the ship section was presented first by Y. Watanabe [14)]. The result of exact calculation of $k'GA$ is slightly less than that of web area method. Some examples of the numerical results of $k'GA$ of cargo ship, bulk carrier and tankers are shown by Ohtaka[15)]. (see also Table I)

As the rotatory inertia factor $\beta = r^2/L^2$ of ship hull shows a small value compared with the shear deflection factor α of the same ship, the effect of

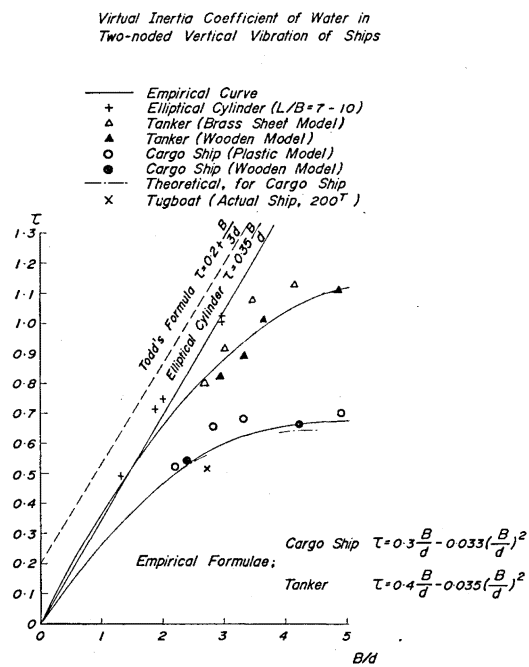

Fig. 1 Results of τ-values obtained from observations of two-node vertical vibration of several ship-shaped models and empirical curves.

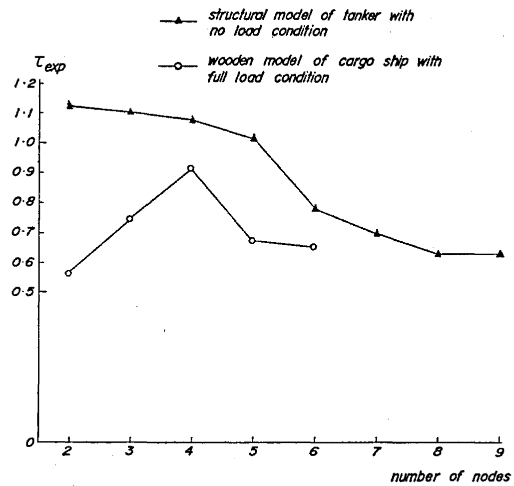

Fig. 2 Observation of τ-values for higher modes of vertical vibrations of the structural model of a tanker and wooden model of a cargo ship.

β on the natural vibration may be ignored in the estimate of natural frequency of the vertical vibration of ship hull in practice.

Recently the effect of the bending rigidity due to the effective breadth of the planting of a ship hull upon the natural frequencies in the higher mode have been discussed by Pleß [16] and Ohtaka [17]. This effect is of importance for estimating higher modes of the natural frequency of ship hull, though the problem is not considered in the present investigation.

5 Numerical examples of empirical factor c_n on actual ships

As one of the recent observations, Ohtaka et al. [15),17] measured natural frequencies of several types of ship on a trial trip and the calculations of natural frequencies were carried out by an electronic computer. By the use of results of their measurements of natural frequencies and the shear rigidities and other data of the section of respective ship, the result of the c_n-values computed from the fomula estimated by the author were obtained by following the above method of assumptions of each factor. The results of the c_n-value obtained from actual ships are shown in Table I. As seen in the table, c-value (c denote mean value of c_n) of cargo ships clearly different from that of tanker or ore carrier, and these values in ballast and full load conditions are also distinctly different. Moreover, the c_n-values in the higher mode up to five-noded vibration are almost constant with each ship.

Table I Examples of empirical factors of actual ships[15),17)] on their trial trips.

ship	type	L(m)	Δ(ton)	C_b	α	τ_2	N_2(cpm) c_2	N_3(cpm) c_3	N_4(cpm) c_4	N_5(cpm) c_5	$\bar{c}$
A	Cargo	145	8,100			0.678	107 0.700	191 0.699	— —	— —	0.699
B	〃	145	11,400	0.667	0.022	0.615	100.7 0.717	182 0.729	260 0.719	— —	0.722
C	〃	145	15,900			0.521	86 0.757	152 0.600	— —	— —	0.739
D_1	〃	150	8,667	0.559	0.029	0.682	96 0.566	178 0.600	— —	— —	0.583
D_2	〃	150	11,356			0.660	93 0.623	— —	— —	— —	0.623
F_1	Tank-er	218	29,030	0.811	0.012	1.131	54 0.703	118 0.794	174 0.778	— —	0.758
F_2	〃	218	66,289			0.855	48.8 0.943	96 0.853	148 0.875	195 0.880	0.887
I	Ore Car.	214	32,167	0.816	0.011	1.127	56 0.734	115 0.765	— —	242 0.801	0.766

6 Experimental study on the structural model of a tanker

A structural model of one hundredth of 40,000 ton d. w. taker was tested for estimating natural frequencies and corresponding virtual inertia coefficients up to nine node mode of vertical vibration of the model itself in air and in water. The structural model is made by brass sheet of 0.3 mm in thickness. The particulars of the model ship and its midship section are shown in Table II and in Figure 3 respectively.

Table II Particulars of the structural model of a tanker.

Scale 1/100; 40,000 t.d.w. Tanker	
$L_{BP}=214.9$cm	$I=270\times10^4$mm^4
$L_{OA}=224.0$cm	$k'A^*=188$mm^2
$B=30.2$cm	$E=10{,}000$kg/mm^2
$D=16.0$cm	$G=3{,}900$kg/mm^2
$\Delta_L=35.1$kg, $d_m=7.5$cm	$\alpha=0.00733$
$\Delta_F=58.1$kg, $d_m=11.3$cm	$\beta\doteqdot0$

* Sum of sectional area of side shell and of longitudinal bulkhead

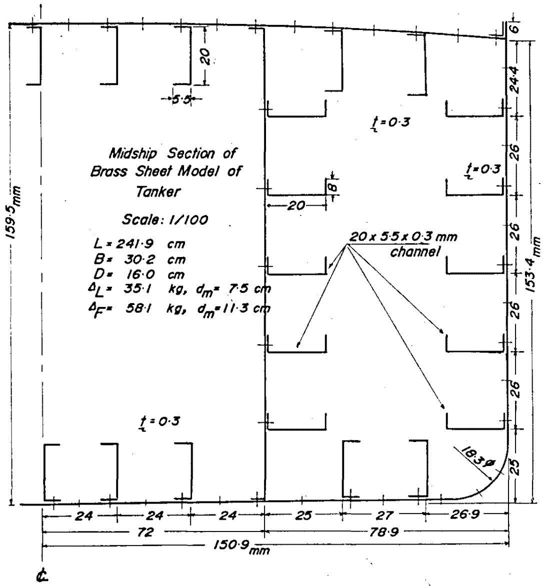

Fig. 3 Midship section of the structural model of a tanker.

Table III Measured and calculated natural frequency, empirical factors and inertia coefficients of water in vertical vibrations of the structural tanker model.

number of nodes, n	c_n	f_a %		f_w %		τ_n
		emp	exp	emp	exp	
2	0.928	125.3	125.3	86	86	1.12
3	0.940	264.5	268	182.5	185	1.10
4	0.928	412	412	286	286	1.07
5	0.917	562	556	396	392	1.01
6	0.847	716	653	537	489	0.782
7	0.803	868	751	667	577	0.695
8	0.784	1018	859	782	673	0.628
9	0.760	1156	947	905	740	0.628

$\bar{c} = 0.928$

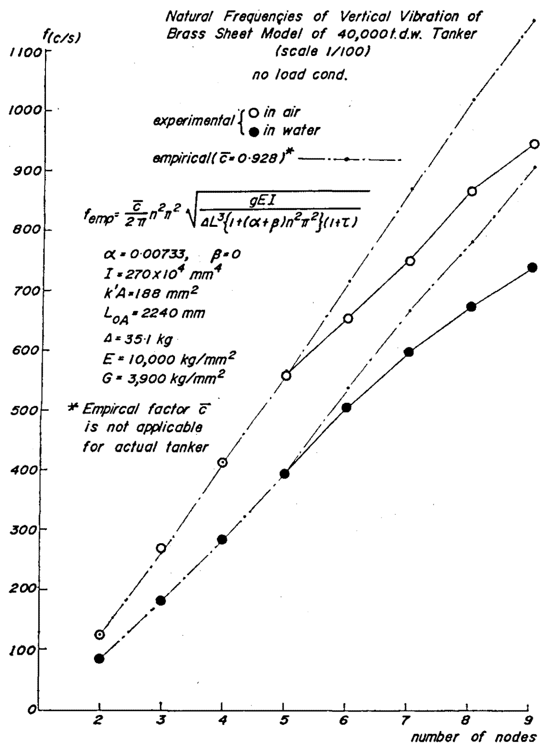

Fig. 4 Comparison of natural frequencies obtained from observation and empirical formula in the vertical vibrations of structural model of a tanker in air and in water.

Table III shows the comparison of the results of experiments and calculation according to the present empirical fomula. Figure 4 presents natural frequencies of the vertical vibration of the above model ship in air and in water. The results obtained from experiments agree with empirical formula up to 5-node mode though they are somewhat lower than that form present empirical formula in the higher frequencies form 6-node mode to 9-node one.

We may assume that the discrepancy between the calculation and experiment as seen in the Figure 4 depends either upon the panel vibration of the structural model coupled with the hull natural frequency, as already investigated by Dr. Y. Watanabe [18] and by Reibowitz [19], or upon the rigidity of shell plate which in the higher mode of the hull vibration [16),17]. Further study of the correction factor for empirical formula will be needed for that.

Regarding the discrepancy of the empirical coefficient c between actual tanker and the present model ship, we can easily assume that the cause of difference lies in the distributions of bending rigidity and in ship's mass along her length both of a model and an actual ship, as was seen in the equation (a) of previous section of this paper.

7 Conclusions

With an aim to evaluate natural frequencies of a ship hull, an estimate formula is put forward, where are used the results of recent theoretical and experimental investigations made into three parameters of ship vibration. If the normal modes are assumed by the series of cosine curves, approximate value of natural frequencies of n-noded mode will be obtained by means of numerical integral by the use of ship data.

New empirical formulae for estimating virtual inertia coefficient in the vertical vibration of typical ships are put forward here from the test results of model ship data. The empirical factors in some types of hull are obtained as examples from actual ships. The factors of each type covering higher modes up to five-noded one showed steady constancy in the observations made both on board ship and on a structural ship model. Further studies on the reduction of rigidity and the effect of local vibration on the higher natural frequency of ship vibration will be necessary.

The author wishes to express his indebtedness to the research fund granted him in this investigation by the Science Division of the Education ministry of Japan and his deep gratitude to Mr. Y. Sakurada, Mr. H. Komatsu and Miss Y. Aikawa of his laboratory for their help in carrying out the present work.

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(Received October 18, 1968)