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VORTEX EXCITATION OF A CIRCULAR CYLINDER TREATED AS A BINARY FLUTTER

by Yasuharu NAKAMURA*

Vortex excitation of a two-dimensional circular cylinder immersed in a uniform flow is treated as a self-excited oscillation of a linear binary system consisting of the cylinder and the flowing fluid. Birkhoff's oscillator model for the dead air region behind the cylinder is adopted as representing the fluid subsystem. Flutter analysis shows that a binary flutter is possible in a limited range of the flow velocity including the value at which the uncoupled frequency of the vortex shedding coincides with the cylinder natural frequency. The lift coefficient during flutter with respect to the cylinder displacement is obtained. The variation of the lift coefficient during flutter with the flow velocity is characteristic of a binary flutter, and is different from that for a forced-oscillating cylinder.

Notations

C_L = Lift coefficient $= \dfrac{L}{\frac{1}{2}\rho V^2 d}$

d = Diameter of circular cylinder

f_y = Uncoupled characteristic frequency of circular cylinder

f_α = Uncoupled characteristic frequency of dead air region

f_m = Coefficient of lift force due to Magnus effect ($L=-f_m\alpha$)

F = External force per unit length

h = Thickness of dead air region

I = Moment of inertia of dead air region about the centre of circular cylinder per unit length

k = Coefficient of fluid dynamical restoring moment

K = Spring constant

$2l$ = Length of dead air region

L = Lift force per unit length

M = Mass of circular cylinder per unit length

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R = Reynolds number
S = Strouhal number
t = Time
U = Peripheral velocity of rotating circular cylinder
V = Flow velocity
$\bar{V}$ = Non-dimensional flow velocity $= \frac{V}{f_y d}$
y = Transverse displacement of circular cylinder
y_0 = Amplitude of transverse displacement of circular cylinder
α = Angular displacement of dead air region relative to the direction of uniform flow
α_0 = Amplitude of angular displacement of dead air region relative to the direction of uniform flow
$\bar{\lambda}$ = Imaginary part of σ
μ = Non-dimensional mass ratio $= \frac{\rho d^2}{M}$
ρ = Fluid density
σ = Non-dimensional frequency $= \frac{\omega}{\omega_y}$
$\bar{\sigma}$ = Real part of σ
ϕ = Phase angle between lift force and cylinder displacement
ω = Circular frequency
ω_y = Uncoupled characteristic circular frequency of circular cylinder
ω_α = Uncoupled characteristic circular frequency of dead air region

1. Introduction

Vortex excitation of a bluff body such as a circular cylinder immersed in a uniform flow has long attracted attention of many investigators from both academic and practical points of view. A concise history of investigation is found, for example, in a review paper by Marris.[1]

Observations show that the transverse displacement of the body is greatly amplified when the frequency of the vortex shedding is close to that of the body displacement. Nevertheless, the mechanism of excitation of the body does not permit to be explained as a forced-oscillation due to the vortex shedding. The reason for this is that according to the observations, the shedding of the vortex itself is then greatly amplified also; that is, the lift force due to the vortex shedding may well be considered to be dependent on the transverse displacement of the body. In other words, the oscillation seems to be of a self-excited type. For more detailed experimental evidence, see, for example, Bishop and Hassan.[2]

Recently, Funakawa put forward a mathematical model for the vortex excitation of a circular cylinder.[3] This is based on Birkhoff's oscillator model for the dead air region behind the cylinder. Present paper treats the same problem and proposes another new model. We start with a model similar to Funakawa's in the sense that both assume the fluid system to be represented by Birkhoff's model.

However, there is an essential difference between the present model and the model proposed by Funakawa. In our model the system is assumed to be a binary one, consisting of the displacements of the cylinder and the dead air region, whereas Funakawa considers that the motion of the dead air region is a forced-oscillation which is driven by the cylinder oscillation. Thus, each model finds a different avenue of approach, and necessarily arrives at flutter of a different type. The self-excited oscillation in our model is a flutter typical to a binary system, that is, a binary flutter, whereas Funakawa treated it as a flutter in one degree of freedom.

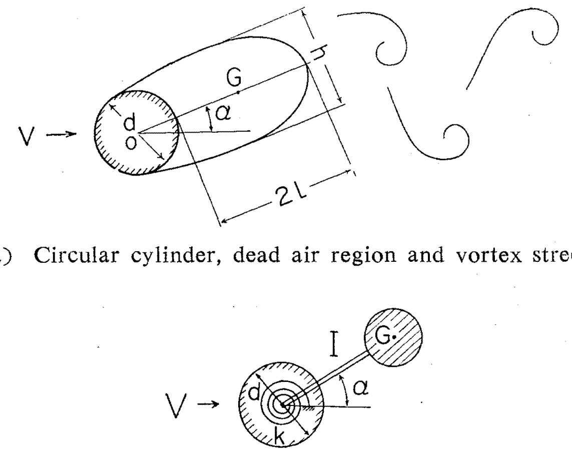
In § 2, Birkhoff's model is first introduced. In § 3, after specifying the coupling terms between the motions of the cylinder and the dead air region, the equations of motion for the system under consideration are formulated. Flutter analysis is made in § 4, and numerical examples are presented. In order to investigate the mechanism of flutter in details, the lift force during flutter is obtained in § 5. In § 6, the lift force for a forced-oscillating cylinder is obtained and compared with the result in § 5. Discussions follow in § 7, where the method of checking the model experimentally, the limitations of the model and the difference between the present model and the model proposed by Funakawa are outlined. Finally, § 8 presents the conclusions.

2. Birkhoff's oscillator model for the dead air region behind a circular cylinder

Direct visual observation indicates that the dead air region, namely, the early wake existing between a circular cylinder and the vortex street behind it, swings from side to side, somewhat like the tail of a swimming fish. For example, see Taneda's experiment.[4] In his paper on vortex street, Birkhoff mentioned this experimental feature and put forward a simple oscillator model for the motion of the dead air region behind the cylinder.[5] The model somewhat modified by Funakawa and used in the present paper is described as follows.

Consider a stationary two-dimensional circular cylinder of unit length, shown in Fig. 1a, with a diameter d, placed normal to the direction of a uniform flow which has a velocity V. Suppose that the dead air region occupies the area $h\times 2l$, and G denotes its center of gravity. Let us assume that with the shedding of the vortex, this dead air region undergoes a rigid-body oscillatory rotation about the center of the cylinder O, its angular displacement with respect to the direction of the uniform flow being denoted by α. Then we obtain an equation governing the motion of the dead air region,

$$I\ddot{\alpha}+k\alpha=0, \tag{1}$$



a) Circular cylinder, dead air region and vortex street

b) Birkhoff's oscillator model for the dead air region

Fig. 1. Stationary circular cylinder immersed in a uniform flow

where I denotes the moment of inertia of the dead air region with respect to O, and k, the coefficient of the fluid dynamical restoring moment, respectively. The corresponding oscillator model is illustrated in Fig 1b. The values of I and k are specified as follows.

$$I=\rho\cdot 2lh\left(\frac{d}{2}+l\right)^2, \tag{2}$$

$$k=\frac{1}{2}\rho V^2\cdot 2\pi\left(\frac{d}{2}+l\right). \tag{3}$$

Equations (1), (2) and (3) enable us to calculate the characteristic frequency of the oscillating dead air region as,

$$f_\alpha=\frac{1}{2\pi}\sqrt{\frac{k}{I}}=\frac{V}{\sqrt{4\pi h\left(\frac{d}{2}+l\right)}}. \tag{4}$$

Putting rather arbitrarily,

$$h=1.25\,d, \tag{5}$$

$$l=1.1\,d, \tag{6}$$

we get the following non-dimensional frequency which is numerically equal to the Strouhal number for a stationary circular cylinder at subcritical Reynolds numbers.

$$S=\frac{f_\alpha d}{V}=0.2. \tag{7}$$

The relations (5) and (6) are found to be roughly consistent with the flow photographs. For example, see Taneda's experiment.[4] It appears that the Birkhoff model, though quite simple, retains the essentials of the complicated physical process which is occurring in the periodic wake behind the cylinder.

3. Cylinder-fluid as a binary system

Consider the case of a two-dimensional circular cylinder of unit length oscillating transversely in a direction perpendicular to a uniform flow which is shown schematically in Fig. 2. We first remark that the system under consideration consists of the oscillating cylinder and the flowing fluid, and that the flowing fluid is in itself a subsystem in which the periodic wake is present even when the cylinder is placed stationary.

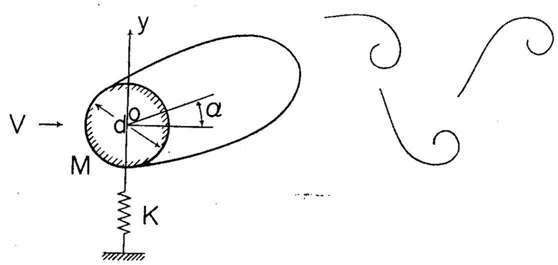


Fig. 2. Circular cylinder oscillating transversely in a direction perpendicular to a uniform flow

Thus, there must be two equations, one governing the cylinder motion and the other, the fluid motion. Exactly speaking, the equation governing the fluid motion may be a time-dependent Navier-Stokes equation. At present, however, it seems to be difficult to treat the problem in such an exact way. It is necessary for us to look for a suitable simplified model which, nevertheless, can provide a correct magnitude for the lift force acting on the cylinder. We thus arrive at the Birkhoff model as the one which may describe the fluid motion most simply and correctly.

In other words, our present system is assumed to be a binary system, the state of which is specified by the transverse displacement of the cylinder, y, and the angular displacement of the dead air region with respect to the uniform flow, α, respectively. The problem is then reduced to find the couplings between the cylinder motion and the motion of the dead air region; namely to find the reaction force on the cylinder due to the angular displacement of the dead air region (the lift force), and vice versa.

3.1 Lift force acting on a circular cylinder

First, we mention that the angular displacement of the dead air region with respect to the direction of the uniform flow corresponds to forward and rearward movements of the separation points at the surface of the cylinder, and hence, it is associated with the change in the flow circulation. Therefore, the assumption is made that the lift force due to Magnus effect which is proportional to the angular displacement of the dead air region acts on the cylinder. Thus,

$$L=-f_m\alpha, \tag{8}$$

where f_m is a constant.

The coefficient f_m was given by Funakawa on the basis of the experimental results on circular cylinders rotating with constant angular velocities. Fig. 3 shows the experimental lift coefficient versus the ratio of peripheral to free-stream velocity of a rotating circular cylinder at subcritical Reynolds numbers.[6] On the other hand, the flow photograph of a rotating circular cylinder[7] indicates that for the velocity ratio, $U/V=0.5$, the angular displacement of the dead air region α was 20 degrees approximately. Therefore, as a rough estimate of f_m, we get

$$f_m=\frac{1}{2}\rho V^2 d\frac{dC_L}{d\alpha}\fallingdotseq 0.58\rho V^2 d. \tag{9}$$

It must be admitted that the present estimate of f_m is of a tentative nature, and there is a need for a further refinement.

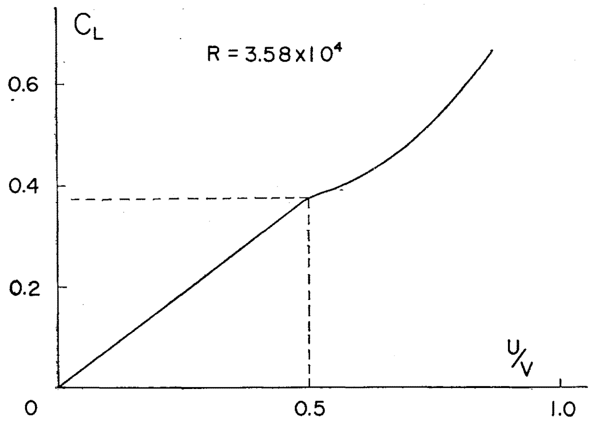


Fig. 3. Experimental lift coefficient versus ratio of peripherical to free-stream velocity of a rotating circular cylinder ...... ref. 6

3.2 Effects of cylinder motion on the dead air region

In deriving an equation governing the angular displacement of the dead air region for the case of an oscillating cylinder, let us first consider the motion of a compound pendulum with its axis of rotation undergoing horizontal movement. A simple consideration shows that the pendulum is exerted by a moment which is proportional to the accelleration of the horizontal movement of the axis.

The assumption is then made that the motion of the dead air region with its axis of rotation undergoing transverse movement is similar to that of the equivalent compound pendulum with its axis of rotation undergoing horizontal movement.

3.3 Equations of motion governing the cylinder-fluid system

Discussions developed in the preceding subsections lead us to the following equations governing the cylinder-fluid system shown in Fig. 2.

$$M\ddot{y}+Ky=-f_m\alpha, \tag{10}$$

$$I\ddot{\alpha}+k\alpha=-\frac{I\ddot{y}}{\left(\frac{d}{2}+l\right)}. \tag{11}$$

For simplicity, the structural and the fluid dynamical dampings which are inherent in the system and assumed to be positive, are neglected in equations (10) and (11).

The assumption of a positive fluid dynamical damping may require some explanation. In the actual flow, the dead air region for a stationary circular cylinder swings from side to side at a small amplitude; that is, the oscillation of the dead air region has a stable limit cycle. If it would be triggered over the limit-cycle amplitude of oscillation, a damped oscillation of the dead air region would occur and the limit cycle would again be reached. In other words, the fluid dynamical damping for the oscillation of the dead air region would be positive for amplitudes over the limit-cycle value. The limit-cycle amplitude of oscillation of the dead air region is considered to be so small that it is neglected in the present model.

Inspection of equations (10) and (11) reveals that the coupling terms on the right-hand sides of the both equations are not symmetric with respect to y and α. Knowledge on aeroelasticity tells us that the onset of a self-excited oscillation, namely, a binary flutter, will be possible in such a system, where due to the phase difference between the two degrees of freedom, the rate of work done by the fluid dynamical lift force may be positive. It is worth while noting here that the effects of the dampings, structural and fluid dynamical, on a binary flutter are, in most cases, considered to be small and the elimination of these dampings from equations (10) and (11) may not invalidate the subsequent discussions seriously. The most essential feature of flutter of the present type is the presence of the oscillatory fluid dynamical force acting on a stationary body; in conventional flutter, it is simply due to the displacement of the body.

4. Flutter analysis

We shall now study the stability of the system which is governed by equations (10) and (11). First, equations (10) and (11) are transformed into

$$\ddot{y}+\omega_y^2 y=-\frac{f_m}{M}\alpha, \tag{12}$$

$$\ddot{\alpha}+\omega_\alpha^2\alpha=-\frac{\ddot{y}}{\left(\frac{d}{2}+l\right)}, \tag{13}$$

where $\frac{\omega_y}{2\pi}$ and $\frac{\omega_\alpha}{2\pi}$ are the uncoupled characteristic frequencies of the cylinder and the fluid oscillations, respectively.

Let us express the displacements of the system as

$$y=y_0 e^{i\omega t}, \tag{14}$$

$$\alpha=\alpha_0 e^{i\omega t}. \tag{15}$$

Note that ω, y_0 and α_0 may be generally complex numbers. Substitution of equations (14) and (15) into equations (12) and (13) yields the following homogeneous equations for y_0 and α_0.

$$(-\omega^2+\omega_y^2)y_0+\frac{f_m}{M}\alpha_0=0, \tag{16}$$

$$-\frac{\omega^2}{\left(\frac{d}{2}+l\right)}y_0+(-\omega^2+\omega_\alpha^2)\alpha_0=0. \tag{17}$$

From equations (16) and (17), we get the following characteristic equation.

$$\Delta=\begin{vmatrix} -\omega^2+\omega_y^2 & \frac{f_m}{M} \\ -\frac{\omega^2}{\left(\frac{d}{2}+l\right)} & -\omega^2+\omega_\alpha^2 \end{vmatrix}=0. \tag{18}$$

By expanding the left-hand side of equation (18), we get

$$(-\omega^2+\omega_y^2)(-\omega^2+\omega_\alpha^2)+\frac{f_m}{M}\frac{\omega^2}{\left(\frac{d}{2}+l\right)}=0. \tag{19}$$

In order to arrive at a covenient non-dimensional formulation, the following relations are utilized;

$$\sigma=\frac{\omega}{\omega_y}, \tag{20}$$

$$\frac{\omega_\alpha^2}{\omega_y^2}=\frac{k}{I}\frac{1}{\omega_y^2}=1.58\frac{V^2}{\omega_y^2 d^2}=0.04\frac{V^2}{f_y^2 d^2}=0.04\ \overline{V}^2, \tag{21}$$

$$\overline{V}=\frac{V}{f_y d}, \tag{22}$$

where σ and $\overline{V}$ denote a non-dimensional frequency and a non-dimensional flow velocity, respectively. Also,

$$\frac{f_m}{M}\frac{\omega^2}{\left(\frac{d}{2}+l\right)}=0.00915\left(\frac{\rho d^2}{M}\right)\overline{V}^2\omega_y^2\omega^2=0.00915\,\mu\,\overline{V}^2\omega_y^2\omega^2, \tag{23}$$

$$\mu=\frac{\rho d^2}{M}, \tag{24}$$

where μ denotes a non-dimensional mass ratio.
Equation (19) now leads to

$$(-\sigma^2+1)(-\sigma^2+0.04\overline{V}^2)+0.00915\mu\overline{V}^2=0. \tag{25}$$

Hence, we get

$$\sigma^4-[(1+0.04\,\overline{V}^2)-0.00915\,\mu\overline{V}^2]\sigma^2+0.04\,\overline{V}^2=0. \tag{26}$$

Equation (26) is quadratic in σ^2. Therefore, the roots are easily obtained.

$$\sigma^2=\frac{1}{2}\Big\{[(1+0.04\,\overline{V}^2)-0.00915\,\mu\overline{V}^2]$$
$$\pm\sqrt{[(1+0.04\,\overline{V}^2)-0.00915\mu\overline{V}^2]^2-4\times0.04\,\overline{V}^2}\Big\}$$
$$=\frac{1}{2}[(1+0.04\,\overline{V}^2-0.00915\,\mu\overline{V}^2)$$
$$\pm\sqrt{(1-0.04\overline{V}^2)^2-2\times(1+0.04\overline{V}^2)\times0.00915\mu\overline{V}^2+(0.00915\mu\overline{V}^2)^2}\,]. \tag{27}$$

Let us consider the variation of σ^2 with the increase in $\overline{V}$. First, in equation (27), the uncoupled characteristic frequencies are obtained by neglecting the terms associated with μ. That is,

$$\sigma^2=1 \quad \text{or} \quad 0.04\overline{V}^2, \tag{28}$$

which respectively correspond to the cylinder characteristic frequency and the vortex shedding frequency for a stationary circular cylinder. Equation (28) indicates that the coalescence of frequency occurs at $\overline{V}=5$.

Next, if we know that the numerical value of μ is generally quite small in comparison with unity, the effects of coupling on σ^2 are found to be small in the range of small values of $\overline{V}$. In other words, at low speeds, the system consists of two sinusoidal, that is, stable oscillations with frequencies not very much different from those of the uncoupled modes.

However, with the increase in speed, these two frequencies become close to and eventually coincide with each other, because the radical on the right-hand side of equation (27) decreases and becomes eventually zero. The critical speeds are obtained by equating the radical in equation (27) to zero,

$$\bar{V}=\frac{0.2\pm0.0957\sqrt{\mu}}{0.04-0.00915\,\mu}. \tag{29}$$

Note that there are two numerical values for the critical speed, namely, the upper and the lower ones. The corresponding characteristic frequencies of oscillation are given by

$$\sigma^2=\frac{1}{2}\left[1+\frac{(1\pm0.478\sqrt{\mu})^2}{1-0.228\,\mu}\right]. \tag{30}$$

Between the two critical speeds shown by equation (29) the radical in equation (27) is negative, so that the σ^2 are then conjugate complex numbers.

$$\sigma^2=\frac{1}{2}\Big[(1+0.04\,\bar{V}^2-0.00915\,\mu\bar{V}^2)$$

$$\mp i\sqrt{4\times0.04\,\bar{V}^2-(1+0.04\,\bar{V}^2-0.00915\mu\bar{V}^2)^2}\Big]. \tag{31}$$

Hence,

$$\sigma=\bar{\sigma}\mp i\,\bar{\lambda}, \tag{32}$$

where

$$\bar{\sigma}=\frac{1}{2}\sqrt{(1+0.2\,\bar{V})^2-0.00915\,\mu\bar{V}^2}\,, \tag{33}$$

$$\bar{\lambda}=\frac{\sqrt{4\times0.04\,\bar{V}^2-(1+0.04\,\bar{V}^2-0.00915\,\mu\bar{V}^2)^2}}{2\sqrt{(1+0.2\,\bar{V})^2-0.00915\,\mu\bar{V}^2}}\,. \tag{34}$$

Equations (32) to (34) show that the system is now characterized by the superposition of two oscillatory modes, one stable and the other unstable, both being of the type, $e^{\lambda t}\sin\omega t$. In other words, equations (32) to (34) indicate the onset of a binary flutter. It is clear from equation (29) that flutter is possible within a limited speed range covering the value at which the uncoupled characteristic frequency of the vortex shedding is equal to that of the cylinder. With further increase in $\bar{V}$ beyond the upper critical value, two stable oscillations are again reached, of which one becomes close to the uncoupled vortex shedding, and the frequency of the other becomes eventually zero.

Equation (29) shows that an increase in the value of mass ratio μ results in a wider flutter range. On the other hand, equation (33) indicates that the frequency of oscillation during flutter is approximately equal to the mean value of the uncoupled characteristic frequencies of the cylinder and the vortex shedding. Fig. 4 is a stability diagram of the present cylinder-fluid system for two values of mass ratio. The case of the larger value of μ (μ=0.16) roughly corresponds to a brass circular cylinder oscillating in water.

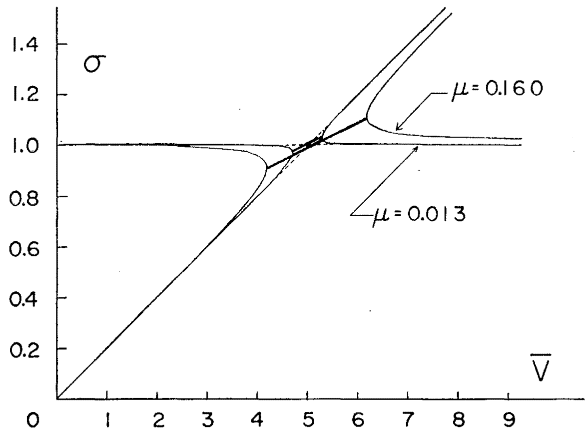


Fig. 4. Stability curves showing the onset of vortex excitation of a circular cylinder in a limited velocity range

5. Lift force acting on a circular cylinder during flutter

In order to investigate the mechanism of flutter in more detail let us observe the variation of the lift force with the transverse displacement of the cylinder. The lift force is obtained from equation (10)

$$\begin{aligned} L &= -f_m \alpha = M\ddot{y} + Ky \\ &= M(-\omega^2 + \omega_y^2)y \\ &= M\omega_y^2(-\sigma^2+1)y. \end{aligned} \tag{35}$$

First, it must be mentioned that outside of the flutter range, the phase angle between the lift force and the transverse displacement of the cylinder is zero or 180 deg, because σ^2 is then a real number. During flutter, equation (35) becomes

$$\begin{aligned} L &= M\omega_y^2\Big\{-\frac{1}{2}[(1+0.04\,\bar{V}^2-0.00915\,\mu\bar{V}^2 \\ &\quad - i\sqrt{4\times 0.04\,\bar{V}^2-(1+0.04\,\bar{V}^2-0.00915\,\mu\bar{V}^2)^2}]+1\Big\}y \\ &= \frac{M}{2}\omega_y^2[(1-0.04\bar{V}^2+0.00915\,\mu\bar{V}^2 \\ &\quad + i\sqrt{4\times 0.04\,\bar{V}^2-(1+0.04\,\bar{V}^2-0.00915\mu\bar{V}^2)^2}]y. \end{aligned} \tag{36}$$

From equation (36), we can calculate the lift coefficient during flutter. The phase angle of the lift coefficient relative to the cylinder displacement is defined as,

$$\tan\phi=\frac{\sqrt{4\times 0.04\,\overline{V}^2-(1+0.04\,\overline{V}^2-0.00915\,\mu\overline{V}^2)^2}}{1-0.04\,\overline{V}^2+0.00915\,\mu\overline{V}^2}\,. \tag{37}$$

Clearly:

1) At low speeds below the lower critical, $\tan\phi=0$. That is, $\phi=0$ deg.

2) During flutter, ϕ changes from 0 to 180 deg. Note that the denominator of the right-hand side of equation (37) changes sign from positive to negative at a speed near $\overline{V}=5$.

3) At high speeds beyond the upper critical, $\tan\phi=0$. Hence, $\phi=180$ deg.

The magnitude of the lift coefficient is also obtained by using equation (36).

$$\begin{aligned}C_L&=\frac{L}{\frac{1}{2}\rho V^2 d}\\ &=\frac{M\omega_y{}^2}{\frac{1}{2}\rho V^2 d\times 2}[(1-0.04\,\overline{V}^2+0.00915\,\mu\overline{V}^2)^2+4\times 0.04\overline{V}^2\\ &\qquad -(1+0.04\,\overline{V}^2-0.00915\,\mu\overline{V}^2)^2]^{1/2}y\\ &=\frac{7.54}{\sqrt{\mu}\,\overline{V}}\frac{y}{d}.\end{aligned} \tag{38}$$

Thus, the magnitude of the lift coefficient is inversely proportional to $\overline{V}$ and $\sqrt{\mu}$. It is interesting to point out that the lift coefficient is dependent on the mass ratio. This feature may not be observed in one-degree-of-freedom flutter, but is characteristic of a binary flutter. Equations (37) and (38) provide a means of obtaining the lift coefficient during flutter from the experimental time history of the transverse displacement of the cylinder.

6. Lift force acting on a circular cylinder oscillating sinusoidally

In this section we shall treat the case where the cylinder is forced to oscillate in a prescribed sinusoidal mode. It will be interesting to see the variation of the lift coefficient with respect to the cylinder displacement when the frequency of oscillation is raised through the speed range in which flutter would otherwise occur.

Fig. 5 shows the system under consideration. Equations of motion for the present case are

$$M\ddot{y}+Ky=-f_m\alpha+F, \tag{39}$$

$$I\ddot{\alpha}+k\alpha=-\frac{I\ddot{y}}{\left(\frac{d}{2}+l\right)}, \tag{40}$$

where F is an external force acting on the cylinder. The displacement of the

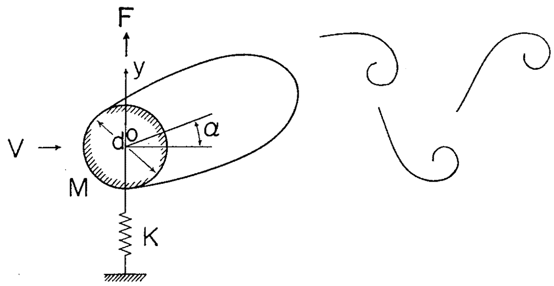


Fig. 5. Circular cylinder oscillating inexorably in a sinusoidal mode

cylinder is specified as

$$y=y_0e^{i\omega t}. \tag{14}$$

Then, the solution to equation (40) is easily obtained

$$\alpha=\frac{\omega^2}{-\omega^2+\omega_\alpha{}^2}\frac{y}{\left(\frac{d}{2}+l\right)}=\frac{\sigma^2}{-\sigma^2+0.04\overline{V}^2}\frac{y}{\left(\frac{d}{2}+l\right)}. \tag{41}$$

This solution does not include the characteristic oscillation, because a positive fluid dynamical damping in the actual flow is assumed. (See the discussion in § 3.3.)

The lift coefficient with respect to the cylinder displacement is

$$\begin{aligned} C_L&=-\frac{f_m\alpha}{\frac{1}{2}\rho V^2 d} \\ &=-0.725\frac{\omega^2}{-\omega^2+\omega_\alpha{}^2}\frac{y}{d} \\ &=-0.725\frac{\sigma^2}{-\sigma^2+0.04\,\overline{V}^2}\frac{y}{d}. \end{aligned} \tag{42}$$

Equations (41) and (42) indicate the possibility of resonance. Namely, at the driving frequency equal to the uncoupled characteristic frequency of the vortex shedding, i.e., at $\sigma=0.2\ \overline{V}$:

1) The magnitude of the lift curve slope, that is, the lift coefficient divided by $\left(\frac{y}{d}\right)$ tends to infinity.

2) The phase angle of the lift coefficient changes from 180 deg to zero deg abruptly.

The magnitude and the phase angle of the lift curve slope for $\overline{V}=5$ are shown in Figs. 6a and 6b.

Measurements of the lift coefficient for a circular cylinder oscillating sinusoidally have been made by Bishop and Hassan,[2] Tanaka and Takahara.[8] The

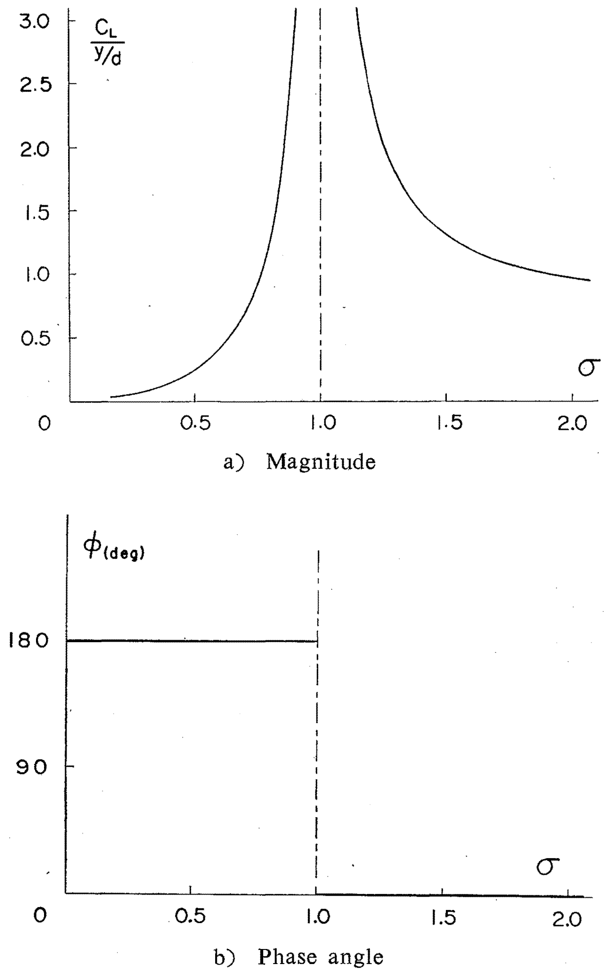


Fig. 6. Lift curve slope for a circular cylinder oscillating inexorably in a sinusoidal mode

results of reference 8 are reproduced in Figs. 7a and 7b. Comparison between Figs. 6 and Figs. 7 seems to indicate the validity of the present model.

However, it is worth while pointing out that the lift curve slope for a circular cylinder oscillating sinusoidally does not give insight into the mechanism of a binary flutter of the cylinder-fluid system. In order to see this situation, let us consider the forced-oscillation of a circular cylinder in the velocity range where flutter would otherwise occur; in the flutter range, the driving frequency is varied according to the velocity dependence of frequency for a freely-oscillating cylinder, shown as bold lines in Fig. 4. Figs. 8a and 8b show the comparison of the lift curve slope for a forced-oscillating cylinder with that for a freely-

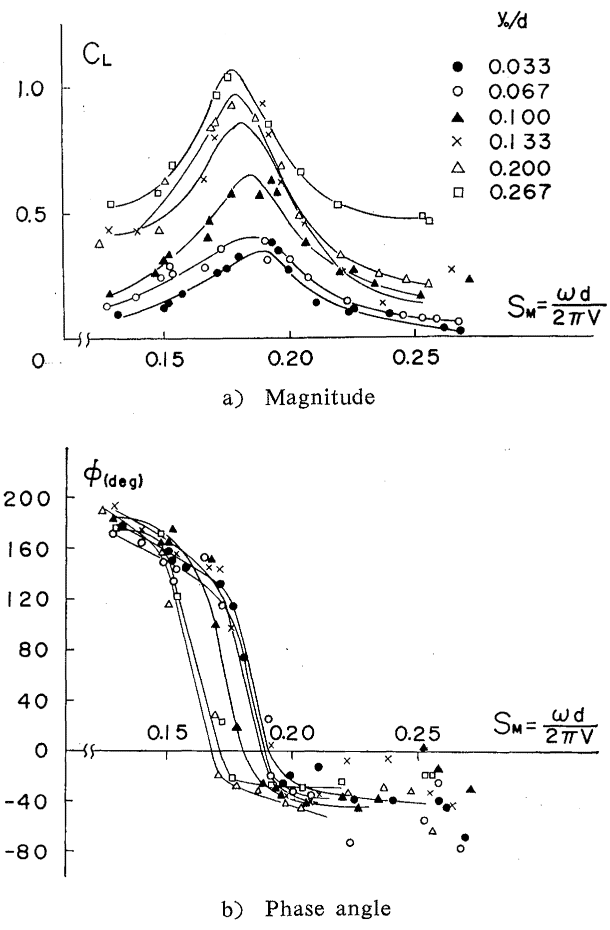


Fig. 7. Experimental lift coefficient for a circular cylinder oscillating inexorably in a sinusoidal mode ...... ref. 8

oscillating cylinder during flutter. The difference in lift curve slope between the two cases is self-explanatory.

7. Discussions

1) Checking the model by experiments —— It may be possible to check equations (29), (37), (38) and (42) by experiments.

2) Limitations of the present model —— In the following, we shall outline the limitations of the present model. First, in the present model, the lift force is assumed to be due to Magnus effect only. However, for large values of mass ratio, the lift force due to virtual mass should be added. Secondly, it may be possible that the angular displacement of the dead air region will not follow

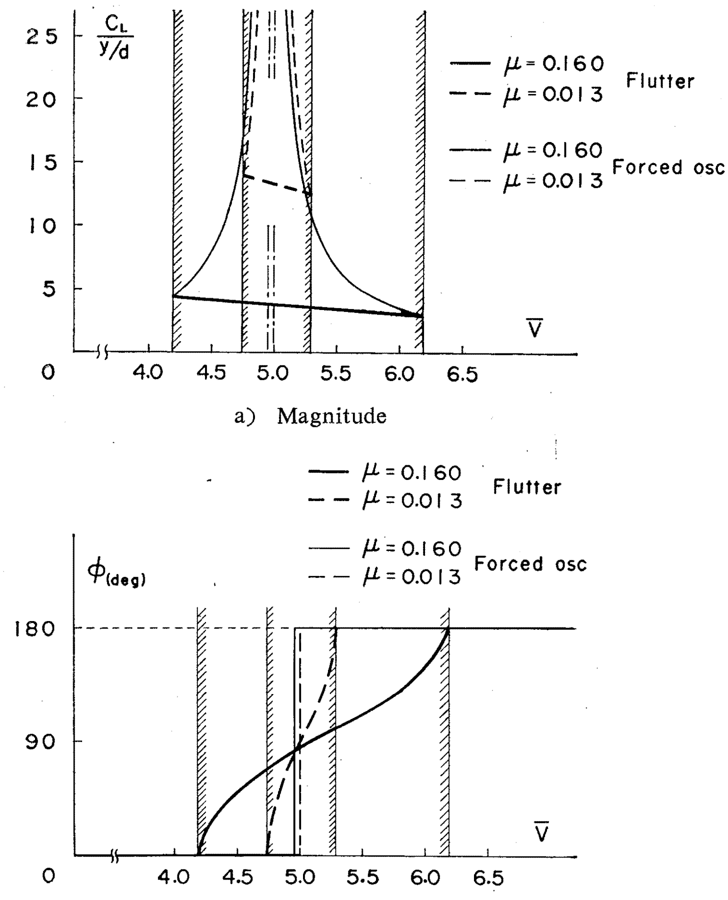
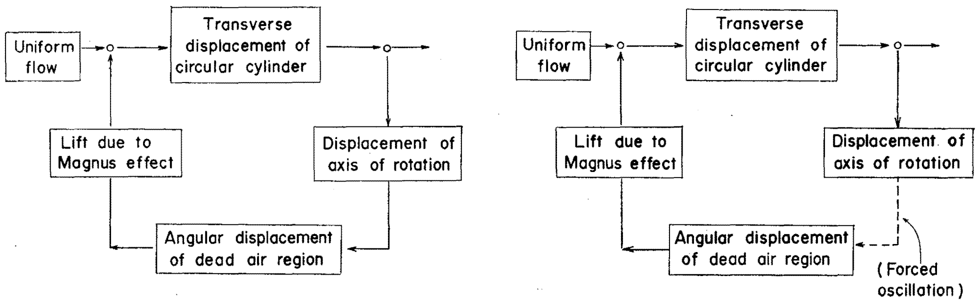


Fig. 8. Comparison of lift curve slope between two circular cylinders in different modes of oscillation

the cylinder displacement instantaneously, but with some time delay. This should be taken into account. Finally, because the present model assumes a linear system, it can not explain the experimental results mainly attributable to the non-linearity of the flow. Most observations show that in the flutter range, the frequency of the vortex shedding is locked in the uncoupled characteristic frequency of the cylinder.[2] In contrast to this, the frequency of oscillation during flutter in the present model is approximately equal to the mean value of the uncoupled frequencies of the cylinder and the vortex shedding.

3) Difference between the present model and the model proposed by Funakawa —— As stated in the Introduction, Funakawa put forward a theory for the vortex excitation of a circular cylinder which is based on a model somewhat

similar to ours. However, there is an essential difference in the mechanism of flutter between the two models. The schematic representation for the two models is given in Fig. 9. In Funakawa's model, the oscillation of the dead air region was assumed to be forced by the cylinder oscillation, so that the self-excited oscillation was treated as one-degree-of-freedom flutter. On the contrary, the system in our present model has two degrees of freedom, and hence flutter is a binary one. It appears that the lift force during flutter predicted by Funakawa's model is similar in nature to the one given in equation (42) in the present paper.



a) Present model b) Funakawa's model

Fig. 9. Illustration of the mechanism of vortex excitation of a circular cylinder

8. Conclusions

Vortex excitation of a two-dimensional circular cylinder immersed in a uniform flow is treated as a self-excited oscillation of a linear binary system consisting of the cylinder and the flowing fluid. Birkhoff's oscillator model for the dead air region behind the cylinder is adopted as representing the fluid subsystem. The whole system is thus specified by two quantities, one, the transverse displacement of the cylinder, and the other, the angular displacement of the dead air region relative to the direction of the uniform flow.

Flutter analysis shows that a binary flutter is possible in a limited range of the flow velocity covering the value at which the uncoupled frequency of the vortex shedding coincides with the cylinder natural frequency. An increase in the value of mass ratio results in a wider flutter range. In the flutter range, the magnitude of the lift curve slope with respect to the cylinder displacement is inversely proportional to both the non-dimensional flow velocity and the square root of the non-dimensional mass ratio; the phase angle of the lift curve slope with respect to the cylinder displacement is increased from zero to 180 deg with the increase in the non-dimensional flow velocity. The variation of the lift curve slope for a forced-oscillating cylinder with the non-dimensional flow velocity in the flutter range is essentially different from that for a freely-oscillating cylinder.

Flutter of the present type is most characterized by the presence of the oscillatory fluid dynamical force acting on a stationary body, whereas, in conventional flutter, it is simply due to the displacement of the body.

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