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INFLUENCE OF THE VIRTUAL INERTIA OF VIBRATING BOTTOM PANEL ON THE HULL NATURAL FREQUENCIES

By Toyoji KUMAI*

The two-dimensional calculation of the rate of increase of the virtual added inertia of water induced by a bottom vibration in the vertical vibration of a midship section of Lewis' form is considered first. A three-dimensional approaches attempted there by the use of the two-dimensional theory, where the modes of vibrations of hull and bottom centre line are assumed as sinusoidal. On the other hand, considering that the relation between the increase of the amplitude of the bottom panel and the hull natural frequency is that of coupled vibration system, the author verified it from the results of vibration test of the bottom panel on board a bulk carrier. Lastly, the author presents a new methed of correction, which is applicable to the natural frequency of the hull vibration coupled with bottom vibration, in which are included the natural frequency of the bottom panel and other theoretical factors. Some numerical examples are illustrated.

1. Introduction

The phenomena of the vibration of ship hull coupled with that of bottom panel have aroused the interest of so many researchers, among whom are R. C. Leibowitz[1], Professor Y. Watanabe[2] and Yamakoshi-Onuma[3] who commented on them theoretically, T. Suhara[4] who carried out the model experimental study, Suetsugu-Fujii[5] made measurements on board actual ships and Ohtaka[6] whose study is new. Except Matsuura[7], however, as far as the author's knowledge goes, none of them have taken up the problem of the increase of the virtual added inertia induced by the flexural vibration of bottom panels.

In the present paper the two-dimensional calculation of the virtual mass, which is induced by the bottom vibration, is considered by using the midship section of Lewis' form. The three-dimensional virtual added inertia is approached from two-dimensional theory, where modes of vibrations of the hull and the bottom panel are assumed sinusoidal except the entrance and the run of ship, and the rate of increase of the virtual inertia of water and also the apparent mass of cargo in hold induced by bottom vibration is obtained as the function

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of an amplitude ratio of the bottom and hull vibrations. Next, the relation between the vibrating bottom panel and the natural frequencies of the ship hull is assumed to be of a forced coupling vibration, which is verified by the test results on board a bulk carrier. Using the above relation, the author represented the rate of increase of the virtual inertia as the function of the coupled natural frequencies of ship hull and the bottom panel, and the correction factor for the natural frequencies of hull coupled with bottom vibration is obtained as the function of the increase of the apparent mass.

2. The two-dimensional calculation of the rate of increase of virtual inertia induced by bottom vibration

The parametric equations of Lewis' form[8] is presented that,

$$\left.\begin{aligned} x&=b_0\{(e^{\alpha}+a_1e^{-\alpha})\cos\beta+a_3e^{-3\alpha}\cos 3\beta\},\\ y&=b_0\{(e^{\alpha}-a_1e^{-\alpha})\ \sin\beta-a_3e^{-3\alpha}\sin 3\beta\}.\end{aligned}\right\} \tag{1}$$

Where, x axis or $\beta=0$ and $=\pi$ presents water line and when $\alpha=0$, the figure presents the hull section. The parameters are shown as follows;

$$\left.\begin{aligned} &\frac{b_0}{b}=\frac{1}{4}\left[3\left(1+\frac{d}{b}\right)-\sqrt{\left(1+\frac{d}{b}\right)^2+8\frac{d}{b}\left(1-4\frac{\sigma}{\pi}\right)}\right],\\ &b=\text{half breadth}=b_0(1+a_1+a_3),\\ &d=\text{draught}\qquad =b_0(1-a_1+a_3),\\ &\sigma=\text{sectional area coefficient},\\ &a_1=\frac{b}{2b_0}\left(1-\frac{d}{b}\right),\quad a_3=\frac{b}{2b_0}\left(1+\frac{d}{b}\right)-1,\end{aligned}\right\} \tag{2}$$

a_1, a_3 and b_0/b are calculated for given values of σ and d/b. Typical midship section is illustrated in Figure 1, in which $\sigma=0.99$ and $b/d=1$.

The normal velocity at the boundary under consideration is presented by the sum of the velocity of vertical hull vibration and that of the flexural vibration of the bottom panel. The athwartship distribution of the normal velocity on the surface of the bottom is assumed to be of the parabola and denoting $\pm\delta$ the amplitude ratio of the bottom vibration with ship hull vibration, the normal velocity is shown by,

$$V_n=\omega\left[\frac{\partial y}{\partial\alpha}\pm\delta\left\{\frac{\partial y}{\partial\alpha}\left(1-\frac{x^2}{b^2}\right)\right\}\right]_{\alpha=0}\cos\,\omega t, \tag{3}$$

where, $+\delta$ presents inphase and $-\delta$ out-of-phaes and $\omega\,\cos\omega t$ is unit vibration velocity. Figure 1 shows the mode shapes of inphase and out-of-phase vibrations of given section.

The two-dimensional velocity potential which satisfies Laplace equation and the boundary conditions in the domain of an infinite distance from the figure and at the free surface of water is written in refence to (1) and (3) as follows;

$$\phi=\phi_1\pm\delta\phi_2=-b_0\omega\left\{\sum_{r=1}^{\infty}A_{2r-1}e^{-(2r-1)\alpha}\sin(2r-1)\beta\right.$$

$$\left.\pm\delta\sum_{r=1}^{\infty}B_{2r-1}e^{-(2r-1)\alpha}\sin(2r-1)\beta\right\}\cos\omega t,\ (r=1,2...), \tag{4}$$

The normal velocity at $\alpha=0$ becomes

$$\left(\frac{\partial\phi}{\partial\alpha}\right)_{\alpha=0}=b_0\omega\left\{\sum_{r=1}^{\infty}(2r-1)A_{2r-1}\sin(2r-1)\beta\right.$$

$$\left.\pm\delta\sum_{r=1}^{\infty}(2r-1)B_{2r-1}\sin(2r-1)\beta\right\}\cos\omega t. \tag{5}$$

The boudary condition at $\alpha=0$ is shown by

$$\left(\frac{\partial\phi}{\partial\alpha}\right)_{\alpha=0}=V_n. \tag{6}$$

From the above relation the coefficients A_{2r-1} and B_{2r-1} are determined as follows;

$$\left.\begin{aligned}
&A_1=1+a_1,\quad A_3=a_3,\\
&B_1=(1+a_1)-\frac{b_0^2}{4b^2}(1+a_1)\left\{(1+a_1)^2+(1+a_1)a_3+2a_3^2\right\},\\
&B_3=a_3-\frac{b_0^2}{12b^2}\{(1+a_1)^3+6(1+a_1)^2a_3+3a_3^3\},\\
&B_5=-\frac{b_0^2}{4b^2}(1+a_1)(1+a_1+a_3)a_3,\\
&B_7=-\frac{b_0^2}{4b^2}(1+a_1)a_3^2,\\
&B_9=-\frac{b_0^2}{12b^2}a_3^3.
\end{aligned}\right\} \tag{7}$$

The velocity potential which satisfies the boundary conditions is thus obtained as shown in (4) and (7). The pressure distributions calculated from the above potential with different phases are shown for examples in Figure 1.

The energy of the vibrating water under consideration is expressed by

$$2T=-\rho\int_0^{\pi}\left(\phi\frac{\partial\phi}{\partial\alpha}\right)_{\alpha=0}d\beta, \tag{8}$$

and also in the case of $\delta=0$ by

$$2T_{\delta=0}=-\rho\int_0^{\pi}\left(\phi\frac{\partial\phi}{\partial\alpha}\right)_{\substack{\alpha=0\\ \delta=0}}d\beta. \tag{9}$$

The rate of increase of the virtual inertia is then written by

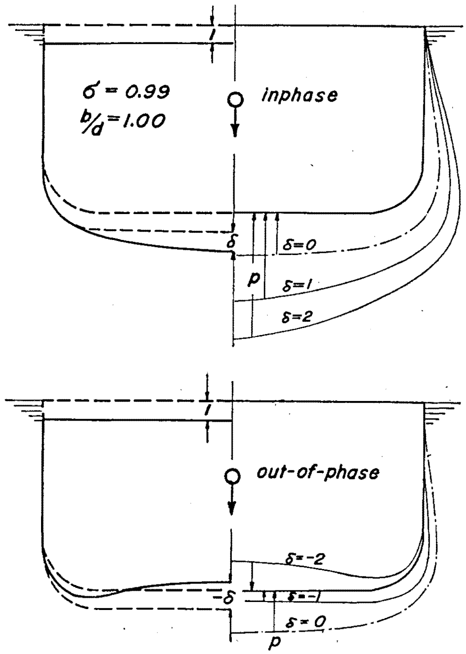

Fig. 1. Distributions of two-dimensional hydrodynomical pressure in the vibration of hull coupled with bottom vibration.

$$\varepsilon_{II}=\frac{T}{T_{\delta=0}}-1=\frac{\pm 2\delta(A_1B_1+3A_3B_3)+\delta^2\sum_{r=1}^{5}(2r-1)B^2_{2r-1}}{A_1^2+3A_3^2}. \tag{10}$$

What is to be noted is that the rate of increase of the virtual added inertia is calculated from the energy of the bottom vibration but that the first term of the numerater of (10) is indispensable. It is of interest that the rate of increase changes negative in some range of δ in out-of-phase as seen in Figure 3.

3. Three-dimensional approach

If the ship hull is considered under the state of the translational oscillation with unit amplitude, then the mode shape of the vibration of the bottom centre line will become,

$$\eta=(1\pm\delta\eta_2),$$

where, η_2 is the mode shape of the vibration of bottom centre line, which should always take a positive sign.

By putting η_1 for the mode of hull vibration, the mode of vibration of the bottom centre line is presented by

$$\eta=\eta_1(1\pm\delta\eta_2). \tag{11}$$

As seen the above expression, η becomes zero at the nodal point of η_1 and η equals to η_1 at the position of bulkhead, and when η_1 takes positive (or negative), η becomes positive (or negative). One of the examples of the mode shapes are shown in Figure 2.

Now, the three-dimensional velocity potential for vibration of the hull coupled with bottom vibration will be assumed taking into account the two-dimensional potential and the modes of vibration as

$$\Phi=\phi_1\eta_1\pm\delta\phi_2\eta_1\eta_2, \tag{12}$$

where, ϕ_1 and ϕ_2 are the two-dimensional potentials already obtained in (4) and (7), and η_1 and η_2 are the mode shapes of vibrations of hull and bottom centre line respectively.

The rate of increase of the virtual inertia of the bottom vibration to that of ship hull vibration is calculated from the above three-dimentional potential Φ by integrating the energy of each section of the ship. It will be assumed that the hull section of the cargo holds is to be constant. The rate of increase of inertia of water under consideration is then represented by

$$\varepsilon=\frac{\pm 2\delta b_0^2(A_1B_1+3A_3B_3)\int_{\zeta 1}^{\zeta 2}\eta_1^2\eta_2 d\zeta+\delta^2 b_0^2\sum_{r=1}^{5}(2r-1)B^2_{2r-1}\int_{\zeta 1}^{\zeta 2}\eta_1^2\eta_2^2 d\zeta,}{\int_0^{\zeta 1} b_0^2(A^2_{1\zeta}+3A^2_{3\zeta})\eta_\zeta^2 d\zeta+\int_{\zeta 2}^{\zeta 1} b_0^2(A^2_{1\zeta}+3A^2_{3\zeta})\eta_\zeta^2 d\zeta+b_0^2(A_1^2+3A_3^2)\int_{\zeta_1}^{\zeta_2}\eta_1^2 d\zeta}, \tag{13}$$

where, $\zeta=z/L$,

z = length co-ordinate with the origine at A. P.

L = length of ship

ζ_1 = after end of cargo hold

ζ_2 = fore end of cargo hold

It is to be noted that the coefficient A_{2r-1} and b_0 of the first two terms of the denominater of the above equation are varied in length co-ordinate.

For the evaluation of the integrals in the above equation, the mode of hull vibration is assumed to be the solution of a shearing vibration of the beam with trapezoidal distributions of the rigidity and weight, and these are also assumed to be symmetrical about midship section, as seen in Figure 2. So far as the simple mathematical expression of the modes of ship hull vibrations including higher frequencies are concerned, the solutions mentioned above will be represen-

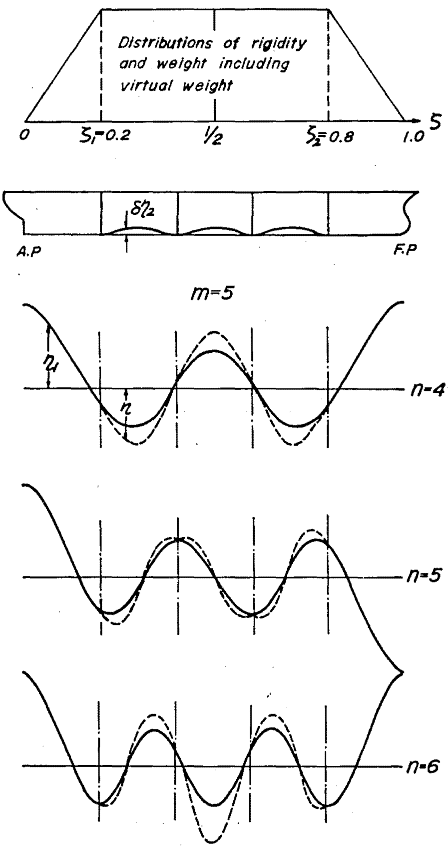

Fig. 2. Assumed distributions of rigidity and weight, and modes of vibrations of hull and bottom centre line.

ted almost similar to that measured on board actual ships, where as the natural frequencies of shearing vibration should be corrected for bending moment[9].

The mode mentioned above is written by

$$\begin{aligned} \eta_\zeta &= CJ_0(\lambda\zeta), \qquad 0 \leqq \zeta \leqq \zeta_1, \\ \eta_1 &= {\cos \atop \sin}\left(\frac{1}{2}-\zeta\right)\lambda, \quad \zeta_1 \leqq \zeta \leqq \frac{1}{2}, \end{aligned} \tag{14}$$

where, J_0 is Bessel function of zeroth order, C is the amplitude ratio of the end to parallel part, cosine or sine presents symmetric or antisymmetric modes respectively and λ the eigen value, which are determined by

$$\frac{J_1(\lambda\zeta_1)}{J_0(\lambda\zeta_1)}=\begin{matrix}-\tan\\ \cot\end{matrix}\left(\frac{1}{2}-\zeta_1\right)\lambda,$$

and
$$C=\frac{\begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{1}{2}-\zeta_1\right)\lambda}{J_0(\lambda\zeta_1)}.$$

The four-, five- and six-noded modes in which $\zeta_1=0.2$ are shown as examples in Figure 2.

With regard to the mode of the vibration of the centre line of the bottom panel, η_2, this mode should always takes a positive sign. A simple expression of such a mode will be considered as

$$\eta_2=\sin^2 m\pi\zeta,\quad \zeta_1\leqq\zeta\leqq\frac{1}{2},\tag{15}$$

where, m is the number of equidivisions of cargo holds between ship length, for instance, $m=5$ as given in Figure 2.

By using (14) and (15) the integrals in equation (13) are evaluated as follows;

$$\left.\begin{aligned}
k_1&=2\int_{\zeta_1}^{1/2}\eta_1{}^2\eta_2 d\zeta=\frac{1}{2}\left(\frac{1}{2}-\zeta_1\right)\pm\left\{\frac{1}{4\lambda}+\frac{1}{4}\frac{\lambda}{(m\pi)^2-\lambda^2}\right\}\sin(1-2\zeta_1)\lambda,\\
k_2&=2\int_{\zeta_1}^{1/2}\eta_1{}^2\eta_2{}^2 d\zeta=\frac{3}{8}\left(\frac{1}{2}-\zeta_1\right)\pm\left\{\frac{3}{16\lambda}+\frac{1}{4}\frac{\lambda}{(m\pi)^2-\lambda^2}\right.\\
&\qquad\left.-\frac{1}{16}\frac{\lambda}{(2m\pi)^2-\lambda^2}\right\}\sin(1-2\zeta_1)\lambda,\\
k_3&=2\int_0^{\zeta_1}\frac{\zeta}{\zeta_1}\eta_\zeta^2 d\zeta+2\int_{\zeta_1}^{1/2}\eta_1{}^2 d\zeta\\
&=C^2\cdot\zeta_1\{J_0{}^2(\lambda\zeta_1)+J_1{}^2(\lambda\zeta_1)\}+\left(\frac{1}{2}-\zeta_1\right)\pm\frac{1}{2\lambda}\sin(1-2\zeta_1)\lambda,
\end{aligned}\right\}\tag{16}$$

where, + or − should be used for symmetric or antisymmetric mode respectively.

Now we put,

$$\left.\begin{aligned}
K_1&=2(A_1B_1+3A_3B_3)k_1,\\
K_2&=\sum_{r=1}^{5}(2r-1)\,B_{2r-1}^2\cdot k_2,\\
K_3&=(A_1{}^2+3A_3{}^2)\cdot k_3,
\end{aligned}\right\}\tag{17}$$

the rate of increase of the virtual inertia induced by bottom vibration presented by (13) becomes,

$$\varepsilon=\frac{\pm K_1\delta+K_2\delta^2}{K_3}.\tag{13)'$$

The above equation is also represented by the following simple form,

$$\varepsilon=\varepsilon_1\frac{\delta}{\delta_1}\left(\frac{\delta}{\delta_1}+2\right), \tag{18}$$

where,

$$\delta_1=\frac{K_1}{2K_2}, \qquad \text{(a)}$$

$$\varepsilon_1=\frac{K_1^{\,2}}{4K_2K_3}. \qquad \text{(b)}$$

The calculated curve of ε versus δ, in which mean values of δ_1 and ε_1 for the number of nodes respectively are used, is shown as an example in Figure 3 under the assumption made in $\zeta_1=0.2$ and $m=10$ with $\sigma=0.99$.

It is evident that the rate of increase of the virtual inertia of water induced by bottom vibration, ε, in the three-dimensional one is less than that in the two-dimentional, as shown in Figure 3. The values ε_1 and δ_1 in respective number of nodes become slightly different from each other as seen in Table II. It is to be noted that ε becomes negative, that is to say, the virtual inertia decreases between $\delta=0$ and $\delta=-2\delta_1$ in out-of-phase.

4. Apparent mass of dry cargo in holds

The apparent mass of cargo in each hold will also increase in the vertical vibration of hull coupled with bottom vibration as well as that of the virtual added inertia of water. The suggestion concerning with this problem has already given in the author's previous paper[12].

The rate of increase of the apparent weight of a dry cargo in the hull vibration under consideration is similarly computed as the virtual added inertia of water which described in the previous chapter in reference to the author's paper[12] that,

$$\varepsilon_c=\varepsilon_1'\frac{\delta}{\delta_1'}\left(\frac{\delta}{\delta_1'}+2\right), \tag{18$'$}$$

where,

$$\delta_1'=\frac{K_1'}{2K_2'}, \qquad \text{(a)}'$$

$$\varepsilon_1'=\frac{K_1^{2\prime}}{4K_2'K_3'}, \qquad \text{(b)}'$$

$$\left.\begin{aligned} K_1'&=\frac{4}{3}k_1,\\ K_2'&=\frac{8}{15}k_2,\\ K_3'&=\left(\frac{1}{2}-\zeta_1\right)\pm\frac{1}{2\lambda}\sin(1-2\zeta_1)\lambda. \end{aligned}\right\} \tag{17$'$}$$

The results of numerical calculation of ε_c is compared with that of the virtual added inertia of water as seen in Figure 3 and Table II. If the ship is vibrating under the loaded condition, the rate of increase of an apparent dead weight should be taken into account as well as that of the virtual added inertia of water induced by the bottom vibration. It is to be noted that the rate of increase of a liquid cargo in the holds is expected slightly less than that of dry cargo in the inphase, however, it conspicuously decreases that of dry cargo in the case of out-of-phase as will be seen in the author's previous investigation[12].

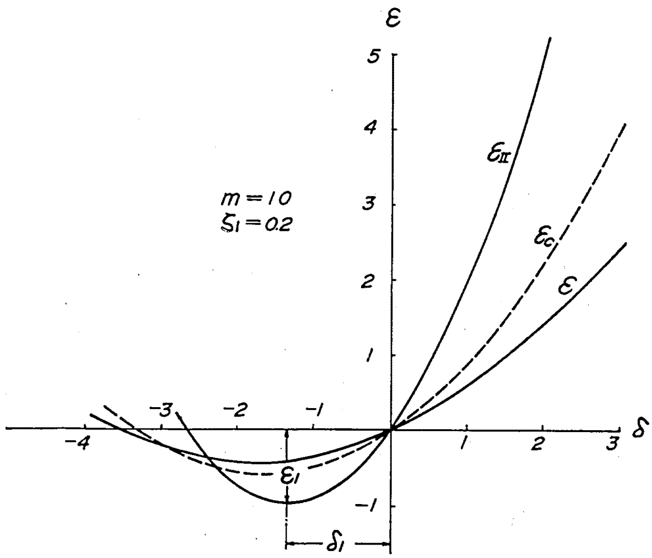

Fig. 3. Rate of increase of virtual inertia of water and cargo in holds versus amplitude of bottom vibration.

5. Relation between δ and the natural frequencies of ship hull

The vibration of the bottom panel now examined appears to be a forced coupling vibration system, in which the exciting force is considered to be given from the hull vibration. If N denotes the coupled natural frequency of the hull vibration and P the natural frequency of the bottom panel, the amplitude of the forced vibration will be written in reference to that of the coupled vibration of a mass-spring system of two degree of freedom[10] as follows;

$$\delta=\frac{1}{\frac{P^2}{N^2}-1}. \tag{19}$$

If the measurements of the bottom vibration are attempted on board an actual ship with respect to different natural frequencies of hull vibration, the unknown quantity P in the above equation will be easily obtained. The values P obtained from measurements on board a bulk carrier[13] are shown as an example in Table I. As seen in Table, values P are appeare almost constant for measured natural frequencies of hull vibration N.

Table I Measured values of N and δ, and calculated P-values.

N cpm	δ	P cpm
147	1.25	197
177	3.60	200
202	8.80	213
286	-2.50	222
Mean		208

Measured on board a bulk carrier[13]

The relation between δ and P/N assumed by equation (19) will be thus confirmed by the experimental analysis as mentioned above. So we may replace δ in equation (18) and (18)′ by the function P/N as follows;

$$\varepsilon=\varepsilon_1\frac{1}{\delta_1\left(\frac{P^2}{N^2}-1\right)}\left\{\frac{1}{\delta_1\left(\frac{P^2}{N^2}-1\right)}+2\right\}, \tag{20}$$

$$\varepsilon_c=\varepsilon_1'\frac{1}{\delta_1'\left(\frac{P^2}{N^2}-1\right)}\left\{\frac{1}{\delta_1'\left(\frac{P^2}{N^2}-1\right)}+2\right\}. \tag{20$'$}$$

The rates of increase of the virtual added inertia of water and that of the cargo in holds induced by the bottom vibration are thus presented as the function of P/N. It is to be noted that the values N in the above equations are the coupled hull frequencies measured on board actual ships.

6. A method of correction of hull natural frequencies coupled with bottom vibration

Putting τ for the virtual inertia coefficient of the n-th noded mode of hull vibration, the hull natural frequency is inversely proportional to $\sqrt{1+\tau}$. The τ-values will slightly decrease in the higher mode of the hull vibration[11]. If the hull vibration is coupled with bottom vibration, the virtual added inertia is conspicuously increase, as shown in the present investigation, the virtual mass coefficient for such a case will be written by

$$\sqrt{1+\tau(1+\varepsilon)}.$$

The correction factor of the natural hull freqencies coupled with bottom vibration is then presented by

$$\sqrt{\frac{1+\tau}{1+\tau(1+\varepsilon)}}\ . \tag{21}$$

On the other hand, the effect of the rate of increase of inertia of the cargo in holds, ε_c, on the hull frequencies induced by the bottom vibration will be considered as

$$\sqrt{\frac{\text{Displacement}}{\text{Hull weight}+\text{Dead weight}(1+\varepsilon_c)}}\ . \tag{21}'$$

Now, putting $\qquad \dfrac{\tau}{1+\tau}=\mu, \qquad$ (c)

and $\qquad \dfrac{\text{Dead weight}}{\text{Displacement}}=\nu, \qquad$ (d)

the correction of the natural hull frequencies for the effects of the virtual added inertia of water and that of cargo in holds are written by

$$N=N_n\sqrt{\frac{1}{(1+\mu\varepsilon)(1+\nu\varepsilon_c)}}=N_n\sqrt{\frac{1}{1+\mu\varepsilon+\nu\varepsilon_c+\mu\nu\varepsilon\varepsilon_c}}.$$

Since the fourth term of the denominater of the above equation is small compared with the sum of other three terms, this term will be neglected in practical use. The above relation thus apploximately represented as follows;

$$\mu\varepsilon+\nu\varepsilon_c=\frac{N_n^2}{N^2}-1. \tag{22}$$

For obtaining the coupled natural freqencies N, eliminate $\mu\varepsilon$ and $\nu\varepsilon_c$ from the equations (20), (20)′ and (22), if so, the following cubic equation with respect to N^2/P^2 is obtained.

$$Z^3+aZ^2+bZ+c=0, \tag{23}$$

where, $\qquad Z=\dfrac{N^2}{P^2}$

$$a=-s\left[\theta+2\left\{1-\left(\frac{\mu\varepsilon_1}{\delta_1}+\frac{\nu\varepsilon_1'}{\delta_1'}\right)\right\}\right]$$

$$b=s(1+2\theta)$$

$$c=-s\theta$$

$$\theta=\frac{N_n^2}{P^2}$$

$$s=\frac{1}{1-\left\{\frac{\mu\varepsilon_1}{\delta_1}\left(2-\frac{1}{\delta_1}\right)+\frac{\nu\varepsilon_1'}{\delta_1'}\left(2-\frac{1}{\delta_1'}\right)\right\}}$$

To solve the above equation, putting

$$p=-\left(\frac{a}{3}\right)^2+\frac{b}{3},$$

$$q=\left(\frac{a}{3}\right)^3-\frac{a\cdot b}{6}+\frac{c}{2}.$$

In the present problem, $p^3+q^2>0$ in the range of $N/P<1$, so the solution of the above equation is given by

$$Z=\sqrt[3]{-q+\sqrt{p^3+q^2}}+\sqrt[3]{-q-\sqrt{p^3+q^2}}-\frac{a}{3}. \qquad \frac{N}{P}<1, \tag{24}$$

When the ratios N/P exceed unity, p^3+q^2 become negative and the solution that saticefies present problem is shown by

$$Z=2\sqrt{-p}\cos(u/3)-\frac{a}{3}, \qquad \frac{N}{P}>1, \tag{25}$$

where, $$\cos u=\frac{q}{p\sqrt{-p}}.$$

The coupled hull frequencies, N, are thus obtained and the rate of increase of the inertia of water, ε, and that of the dry cargo, ε_c and the amplitude of bottom vibration, δ, are calculated from (20), (20)′, and (19) respectively, and the correction factor is obtained from (24) and (25) or (22) as the functions of the rate of increase of the inertia force of the system of a coupled vibration.

7. Numerical examples

The numerical values of δ_1 and ε_1, and also δ_1' and ε_1' are tabulated in Table II. The values in respective number of nodes become slightly different from each other, however, the mean of these values will be used in the present examples.

The hull natural frequencies with no account of the effect of the bottom vibration is assumed as an approach[9] to be

Table II Numerical values of δ_1 and ε_1 for given values of ζ_1, m and n

n	$\zeta_1=0.20$						$\zeta_1=0.125$	
	$m=10$				$m=5$		$m=8$	
	δ_1	ε_1	δ_1'	ε_1'	δ_1	ε_1	δ_1	ε_1
2	1.7500	0.4975	1.6224	0.7721	1.6675	0.4548	1.7458	0.6180
3	1.8460	0.3460	1.7069	0.5661	1.7408	0.3088	1.8134	0.4571
4	1.7800	0.4140	1.6465	0.5727	2.3916	0.7456	1.6452	0.4994
5	2.0000	0.3900	1.8397	0.5332	0.9517	0.0872	1.9305	0.4776
6	1.7150	0.4000	1.5862	0.5660	1.7944	0.4419	1.6093	0.4735
Mean	1.8180	0.4095	1.6803	0.5900	1.7090	0.4077	1.7488	0.5051

$$N_n = N_2(n-1).$$

where, n is the number of nodes of the vertical vibration of a ship hull.

The virtual inertia coefficient is estimated from the author's empirical formula[11],

$$\tau_2 = 0.4\frac{B}{d} - 0.035\left(\frac{B}{d}\right)^2.$$

The rate of change of τ_n for the number of nodes is ignored in the present examples.

Figure 4 shows some numerical examples of δ, ε and N/N_n for number of nodes n. Figure 5 shows corrected natural frequencies of hull vibration in two cases of empty $\nu=0$ and loaded $\nu=0.6$ with $\mu=0.398$ versus n.

It is interesting that the hull frequencies decrease up to the resonance, however, that increase after the resonance of the bottom vibration, for decrease of apparent masses of water and cargo in hold in out-of-phase vibration of bottom panels, as seen in the Figures 4 and 5.

8. Conclusions

It may be concluded from the present investigation that the reduction of the natural frequencies of the hull vibration coupled with bottom vibration may be calculated taking into account the rate of increase of the virtual added inertia of water and that of the apparent inertia of cargo in holds induced by the bottom vibration.

The amplitude ratio of the bottom panel to that of ship hull vibration measured on board a bulk carrier was confirmed to be of a forced coupling vibration system. By putting all accounts together the theory and the experi mental results, a method of correction of the natural hull frequencies of the vertical vibration of ship coupled with bottom vibration has been proposed here.

Though reduced natural frequencies corrected by the present method appear to be qualitatively good as numerical examples in a bulk carrier, yet the auther believes it necessary to make comparison of the results with that measured on board actual ships.

The author wishes to express his gratitude to Mr. Y. Sakurada and Mr. H. Komatsu of his laboratory for their help in carrying out the numerical calculations.

References

1) Leibowitz, R. C. Natural Mode and Frequencies of Vertical Vibration of a Beam with an Attached Sprung Mass. D. T. M. B. Rep. 1958.
2) Watanabe, Y. On the Causes of Stern Vibration of a Ship. J. S. N. A. Seibu, Vol. 20, 1960.

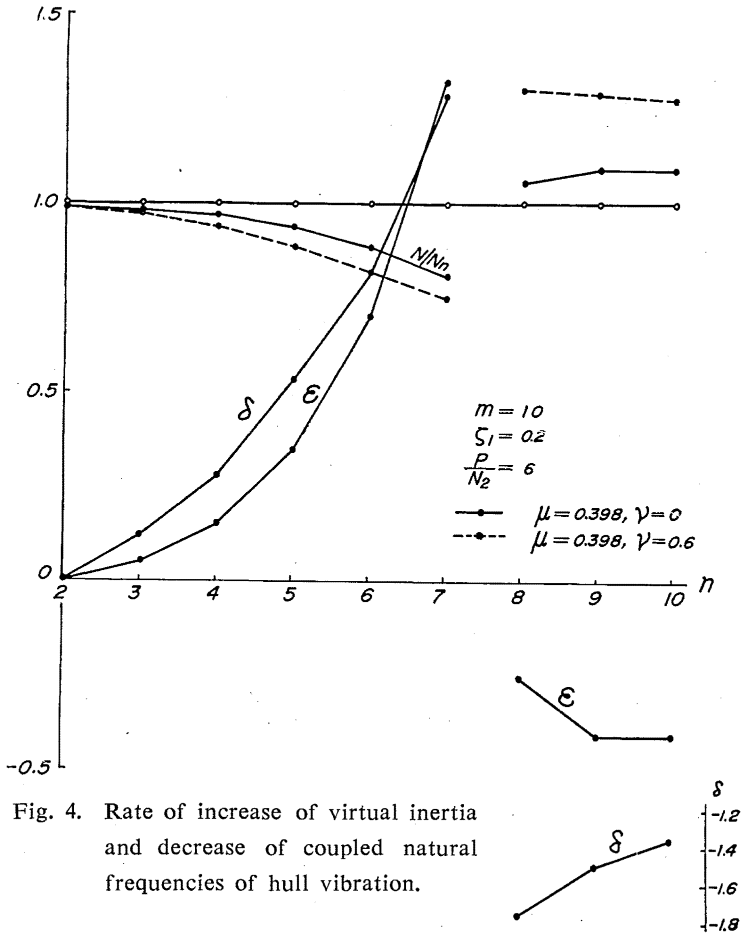

Fig. 4. Rate of increase of virtual inertia and decrease of coupled natural frequencies of hull vibration.

3) Yamakoshi, M. and S. Onuma, On the Coupling of Hull Vibration and Bottom Vibration of Ships. J. S. N. A. Japan, Vol. 118. 1965.
4) Suhara, T. Vibration Experiments on Box Beam. Rep. of S. S. Committee, Seibu, Nos. 32, 33 and 34, 1960.
5) Suetsugu, I. and K. Fujii, On the Vibration of Bottom of Ships. J. S. N. A. Kansai, Vol. 111, 1963.
6) Ohtaka, K. and T. Ohyama, Vertical Vibration of Ships Coupled with Bottom Vibration. Rep. No. 1. J. S. N. A. Japan, Vol. 126, 1969.
7) Matsuura, Y. An Analysis of Vertical Vibration of Cargo Ships. J. S. N. A. Japan, Vol. 108, 1960.
8) Lewis, F. M. The Inertia of Water Surrounding a Vibrating Ship. S. N. A. M. E. 1929.
9) Kumai, T. Shearing Vibration of Ships. Rep. of Res. Inst. Appl. Mech. Kyushu Univ. Vol. IV, No. 15, 1956.

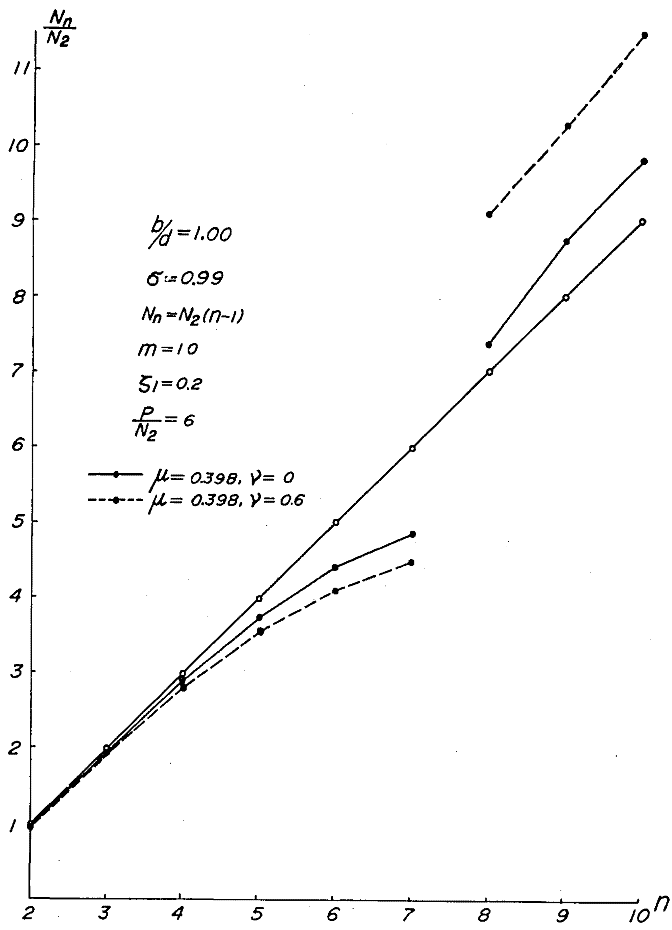

Fig. 5. Corrected hull natural frequencies.

10) Den Hartog, J. P. Mechanical Vibration, 4th Ed., p. 89.
11) Kumai, T. On the Estimation of Natural Freqencies of Vertical Vibration of Ships. J. S. N. A. Japan, Vol. 121, 1767.
12) Kumai, T. On the Apparent Mass of Cargo Oil in Vibration of a Tanker. Rep. of Res. Inst. Appl. Mech., Kyushu Univ. Vol. XIII, No. 46, 1965.
13) SR-94, J. S. R. A. Rep. Vol. I, 1967.

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