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https://doi.org/10.5109/7170134

出版情報：Reports of Research Institute for Applied Mechanics. 16 (55), pp.459-482, 1968. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. XVI, No. 55, 1968

ON THE GROWTH OF THE SPECTRUM OF WIND-GENERATED WAVES (I)

By

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When turbulent air passes over water surface water waves which have a spectral structure are generated, and they develop with fetch, wind duration and wind speed respectively. The growth of the spectrum of wind-generated waves at short fetch has been investigated experimentally. Based mainly on the similarity hypothesis proposed by Kitaigorodskii (1962), the form of the spectrum in a low frequency part (steep forward face) which develops with fetch or wind speed, is determined by using the data of accurately measured wave spectra and their generating conditions. Combinning the newly determined wave spectrum in a low frequency part with the equilibrium wave spectrum (Phillips 1958) in a high frequency part, the new spectral form is proposed for the wind waves at short fetch. The fetch graphs are determined both from the proposed wave spectrum and from the measured wave data, and they are compared with those given by so called SMB curves. Finally, several discussions are conducted on the wave spectra normalized by the scheme employed by Hydy & Plate (1965). In connection with the discussions on the growth of wave spectrum, peculiar characteristics of the spectrum near the saturated range have been pointed out.

1. Introduction

Since the concept of the spectrum of wind-generated wave was introduced actively by Neumann (1952) and Pierson (1952) many studies have been conducted on the spectrum of wind-generated waves. That is, many laboratory and field observations of wind waves were conducted to make clear the fundamental properties of wind waves (field observations; Burling 1959, Kinsman 1960, Cote etal 1960, Snyder & Cox 1966, laboratory observations; Hamada 1963, Mitsuyasu 1964, Colonell 1966, Hidy & Plate 1966), and various so called "theoretical" wave spectra were proposed for the practical purpose of synoptic wave forcasting, (Neumann 1952, Darbyshire 1959, Bretshneider 1963, Pierson & Moskowitz 1964)**).

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**) Many of those studies also include the field observations of waves.

In spite of those quite many studies, there still remains some uncertain points on the fundamental properties of wave spectrum and on the relations between the wave spectra and their generating conditions such as wind speed and fetch. Those circumstances are considered to be arisen mainly from the following reasons: (i) In the laboratory studies the wind waves and their generating conditions can be measured fairly accurately, but the fetch is limited very shortly and the scale of wind wave is very small. Therefore, some uncertainity is remained on the correspondence between the ocean waves and the laboratory wind waves which have very small energy and fairly high frequency range *). On the other hand, in the ocean, the generating condition of wind wave is not so simple and regular as in the controlled laboratory experiment. For examples, the wind field is not so uniform and stationary, the fetch is difficult to define so uniquely as in the laboratory, and the different wave systems generated originally in the different wind area co-exist in many cases. From those reasons, it is not so easy to obtain the accurately defined spectrum of wind waves generated under the simple definite conditions, or to determine the accurate relations between the observed wave spectra and their generating conditions. It is the intent of this paper to fill up those gaps between the laboratory studies and the field observations, and to obtain the accurate informations on the spectrum of wind waves generated by the simple definite conditions for the wide range of their generating conditions.

Concerning the high frequency part of wave spectrum the equilibrium form proposed by Phillips (1958) can provide fairly appropriate informations, although some problems are still remained as will be discussed in this paper. However, as far as we know, there are few quantitative data on the low frequency part of wave spectrum, except for several theoretical treatments of this part of wave spectrum (Hino 1963, Wiegel & Cross 1966, Inoue 1966), and apart from various "theoretical" wave spectra. Therefore, particular attentions are paid for determinning the form of wave spectrum in a low frequency side, i. e., the form of steep forward face of wave spectrum, by using accurately measured data of wind waves and their generating conditions. Wave data from two sources, i. e., laboratory measurements and field measurements at Hakata Bay, are effectively used for the determination of wave spectrum. Those data cover the range of a dimensionless fetch; gF/u_*^2 $10^2\sim10^6$, where g denote the acceleration of gravity, F the fetch and u_* the friction velocity of wind. The similarity hypothesis (Kitaigorodskii 1962) is successfully applied for the analysis of data, obtainning a simple form of spectrum for the steep forward face of wave spectrum.

2. Sources of wind and wave data

The wind and wave data from two sources have been used in the present

*) Questions are existing not only on the dynamics of wind-generated waves but also on the mechanism of their generations and developments.

study. One is the laboratory data of wind and waves generated in a wave tank of relatively large scale, and the other is the field data of wind and waves measured at Hakata Bay which has relatively simple geometrical form. As the detailed descriptions on the facilities and the methods of observations have been or will be published elsewhere (Mitsuyasu 1968), only their essential part will be described here.

2-1) Laboratory measurements

A series of laboratory experiments has been conducted and it is still now in progress to clarify the mechanism of interaction between wind and waves (Mitsuyasu 1966, 1967). Fig. 1 shows the general layout of the wind-wave facility used in the experiment. As the original wind blower generates the wind field of wall jet type which diverges horizontally and vertically toward the downstream, the wave tank was divided tentatively by constructing the dividing wall and the ceiling connecting to the wind blower, so as to form the wind-wave channel and to get the relatively uniform wind field. The width of the divided wave tank is 1.8 m, the height of the air gap is 0.64 m, and the length of the divided part of the tank is 20m. The wind and waves were measured at the locations of 10 m, 15 m, and 20 m from the outlet mouth of

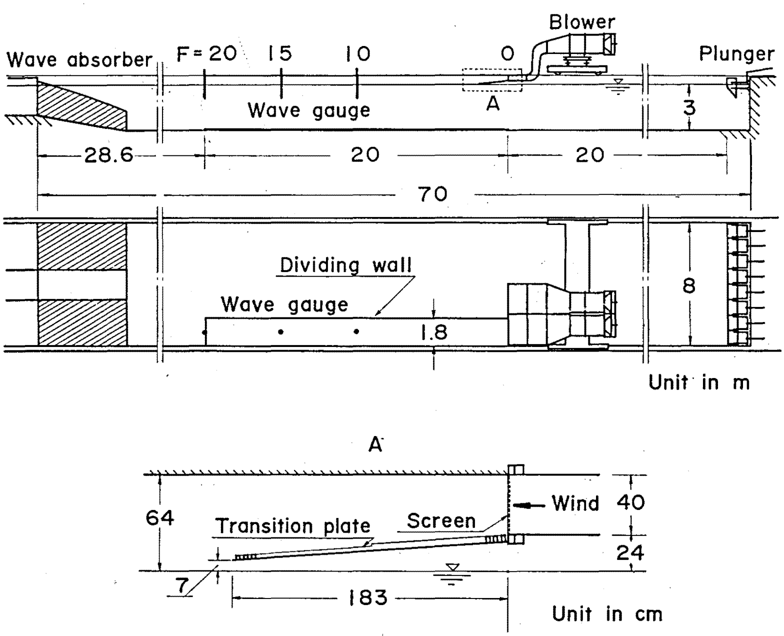

Fig. 1 Wind-wave facility. Plunger is not used and only the divided part of the tank is used for the present experiment.

the wind blower *). The air velocity profile was measured at each station by traversing the pitot-tube. The friction velocity u_* was determined by the application of logarithmic law to the measured wind profiles. The wind speed and thus the friction velocity changes slightly toward the downstream due both to the development of the boundary layer of the air flow over the water surface and to the change of the conditions of water surface. Therefore, as the friction velocity corresponding to the wind wave at the definite station, the averaged friction velocity $\bar{u}_*$ was used which is determined by averaging the value at the definite station and those at upward stations. Waves were measured by using capacitance-type wave gages which were calibrated accurately, and the spectral analysis of wave data was conducted in the ordinary method by using the data of each two minutes. The degrees of freedom of the measured spectra were approximately 90.

2-2 Field measurements

The measurements of wind and waves have been conducted and they are still now in progress in the observation station at Hakata Bay (Mitsuyasu 1968). Fig. 2 shows the plan of Hakata Bay and the site of the observation station. The water depth in the inner part of the bay is relatively uniform and the depth at the observation station is 5 m. As shown in Fig. 3, the observation station is a simple recht triangular tower which is composed

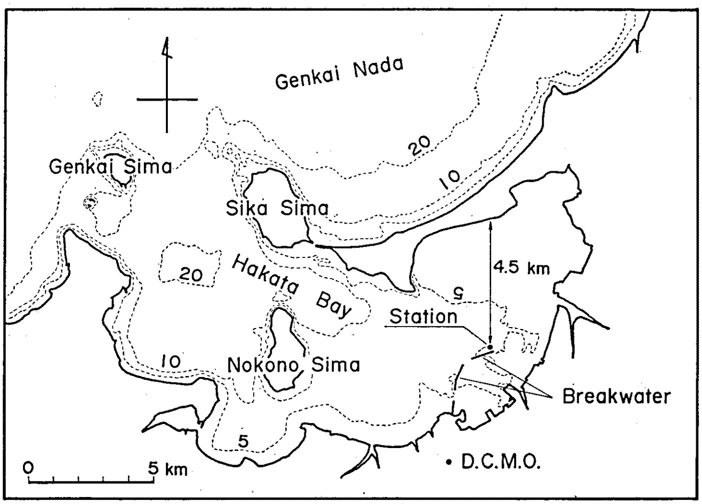

Fig. 2 Chart of area of the field measurements. D. C. M. O denotes Fukuoka District Central Meteorological Observatory.

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*) As is shown in Fig. 1 a roughend transition plate is installed in the wind-wave facility at the bottom side of the rectangular air exit in the test section. Therefore, effective fetch lengthes become approximately 8m, 13m, and 18m respectively.

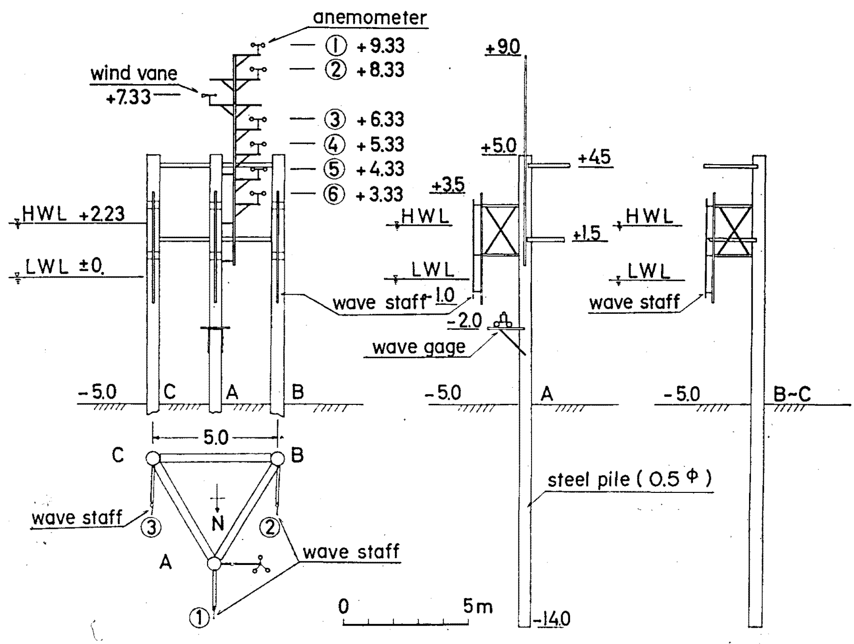


Fig. 3 Observation tower at Hakata Bay. (Unit in m)

essentially from three vertical piles (ϕ: 0.5 m) driven into the sea bottom. Capacitance-type wave gage is fixed in each pile with the arm of 1.5m length so as to reduce the effect of disturbance from the vertical main pile. Wave gages compose the recht triangular array with 5 m separation in order to determine the directional properties of wind waves. Wind speed and its vertical distribution were measured by five Robinson type 3-cup anemometers in the different heights (①: 9.33m, ②: 8.33m, ③: 6.33m, ④: 5.33m, ⑤: 4.33 m, above the datum line). The heights of anemometers relative to the water surface in each measuring time were determined by correcting the effect of tide simultaneously measured. Wind and waves were recorded at the recording station constructed at the east end on the brake-water which was apart approximately 150m from the observation tower. The friction velocity of wind was determined by the application of logarithmic law to the measured wind profile. *) The macroscopic uniformity of the wind field was checked by referring the wind data obtained at the Fukuoka District Central Meteorological Observatory (D. C. M. O). The data used in the present analysis were obtained on September 21, 1967 and October 10, 1967. In both days weathers were fine, wind directions were almost constant (NNW～N) during the observations

*) The logarithmic law was substanciated in the measured wind profiles over the sea surface up to approximately 5m height (Mitsuyasu 1968).

of wind and wave, and the wind speeds showed the tendencies to increase slowly from morning to noon and again to decrease. The fetch corresponding to the wind direction N is 4.5 km. Wave data of each ten minutes were analyzed by the same way as in the laboratory wave data.

3. Data and preliminary considerations

3-1) General aspect of the growth of wave spectrum

Fig. 4 shows the general trend of the growth of wind wave spectra observed in our laboratory tank. To examine the characteristics of the wave spectrum in a developing stage more clearly, a group of spectra measured at different fetches under the same wind speed was selected from Fig. 4 and

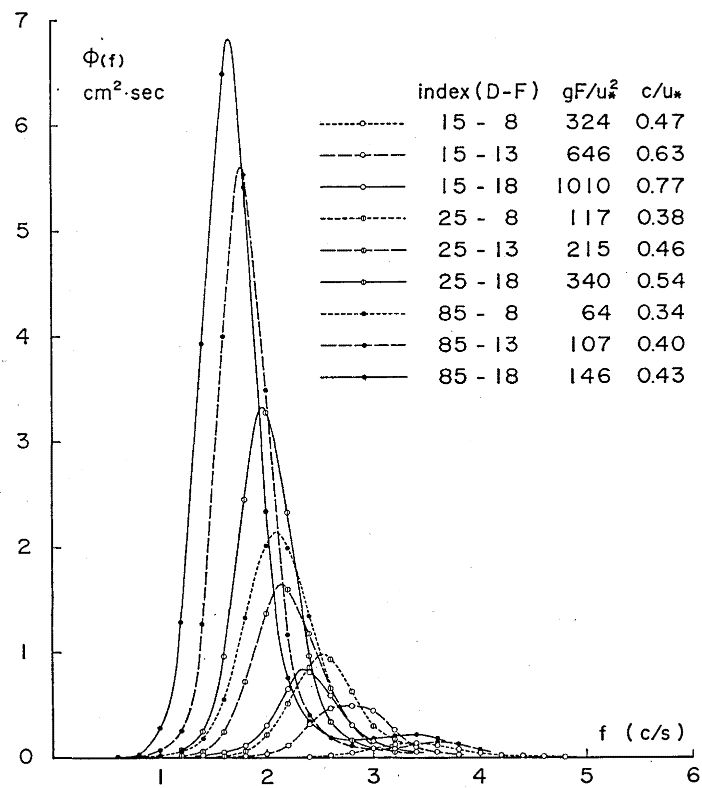


Fig. 4 Growth of spectra of wind-generated waves.*) (laboratory measurements)

*) The index (D-F) in Fig. 4 means the opening of blower's damper by D (%) and fetch by F(m). Therefore, the data with index 15-8 were obtained at fetch F=8 m under the wind speed corresponding to the opening of damper D=15%.

shown in Fig. 5a. Fig. 5b shows a group of spectra which were measured at the fetch F=13 m in the different wind speed. It can be seen from those figures that the spectrum of wind waves develops with fetch almost in the same way with wind speed within a range of our experiment, and that the frequency range of the spectrum shift toward the low frequency part by the

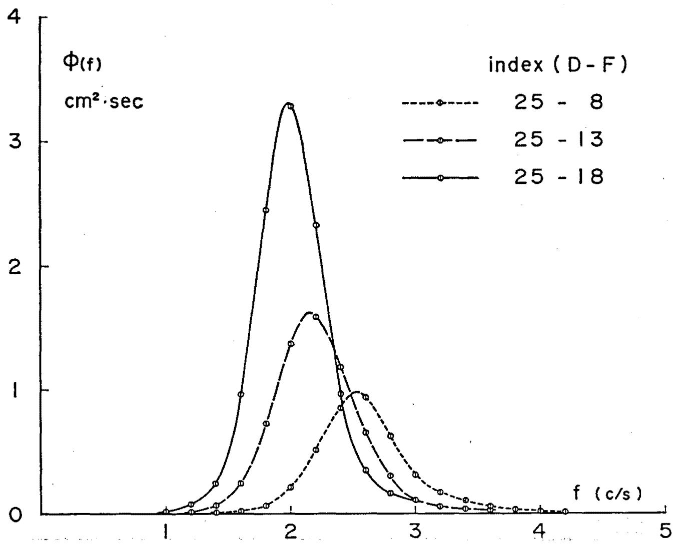


Fig. 5a Growth of wave spectra with fetch. (laboratory measurements. u_*=const)

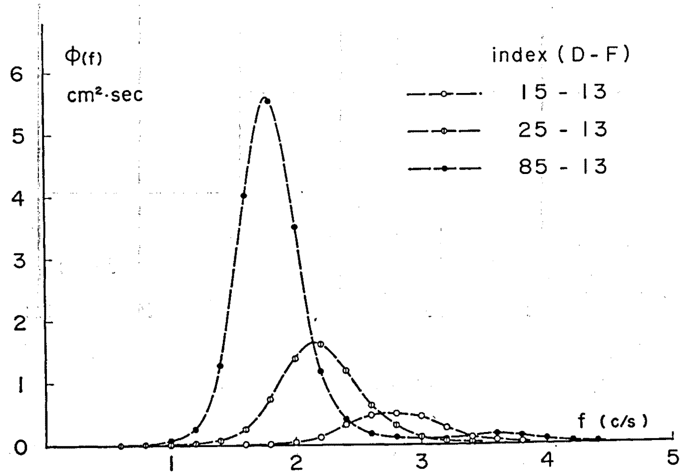


Fig. 5b Growth of wave spectra with wind speed. (laboratory measurements. F=const)

growth of spectrum. It should be pointed out here that an appriciable amount of energy loss can be seen in the frequecy part near the dominant peak of the spectrum in a growth stage. In a previous study (Mitsuyasu 1964) similar phenomena were observed in the wave spectra measured at decay area where the wind was not blowing. That is, the following two interesting characteristics were found in the measured wave spectra: (i) Near the dominant peak of the wave spectrum some excess energy was observed as compared with the equilibrium form of wave spectrum proposd by Phillips (1958)*). (ii) The dissipation of wave energy much greater than that due to viscous dissipation was observed in the frequency range where the excess energy was found. The nonlinear energy transfer seemed to be responsible for those phenomena, but definite conclusions were not obtained. **)

Returning to the present data, Figs 5a and 5b show that the excess wave energy near the dominant peak of the spectrum at some equilibium condition is dissipated very rapidly even when the condition is changed to increase the gross transfer of energy form wind to wave, i. e., the wind speed or the fetch is increased. In other words, there seems to be some optimum conditions which transfers the energy from wind to a spectral component of wind waves most effectively and keeps an equilibrium state of high level compensating for the much amount of energy loss from that frequency component. If the conditions are changed the energy loss becomes greater than the energy transferred from the wind, and new equilibrium state of lower level is attained.

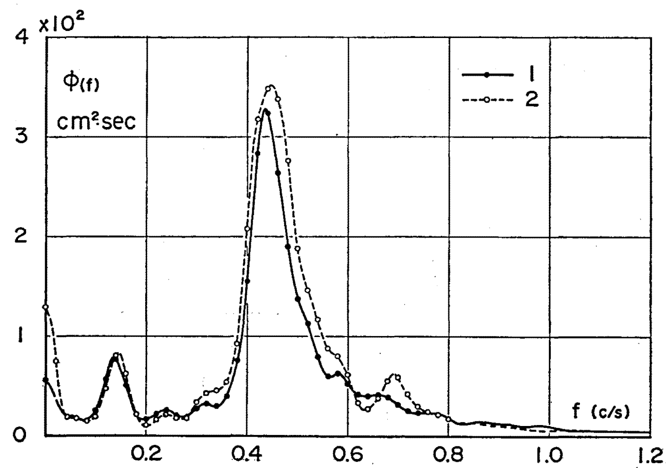


Fig 6 Spectra of wind waves at Hakata Bay. (Sept. 21, 1967, F=4.5km, $gF/u_*^2=9.11\times10^5$)

*) This phenomenon is considered to be relating to the overshoot effect observed recently by Barnett (1966) and also by Sutherland (1968).

**) Recently, Mitsuyasu (1969) made some discussions on this problem based mainly on the Hasselmann's theory (Hasselmann 1962, 1963) and Barnett's study (Barnett 1966).

When we consider the development of wave spectrum, these phenomena should be mentioned in addition to the dominant phenomena such as the increase of total wave energy and the downward shift of the frequency range of wave spectrum. However, we will not enter into this problem further at this time.

In Fig. 6. some examples of wave spectra measured at Hakata Bay is presented.*) The small peak near the frequency $f=0.14$ c/s in the spectrum corresponds to the swell which comes from the open sea into the bay, and the spectrum with dominant peak near $f=0.45$ c/s corresponds to the wind waves generated in the bay.**) As the frequency range of each spectrum is clearly separated in each others, the low frequency part of the spectrum of wind-generated waves will be not so affected by the swell spectrum except for some minor effect of nonlinear interaction.

3-2) The high frequency part of the wave spectrum

With reference to the high frequency part of the wave spectrum, Phillips (1958), from a consideration of limiting configuration of surface waves, showed that an equilibrium range may exist, and by dimensional argument he derived the following spectral form:

$$\phi(\omega) = \beta g^2 \omega^{-5} \tag{1}$$

where β is dimensionless constant, ω the angular frequency.
According to Burling's data (Burling 1959), the value of the dimensionless constant β is 1.48×10^{-2}. Phillips (1966), recently, has shown that the mean of the various observed values of β is 1.17×10^{-2}.

Fig. 7 shows some examples of our measured spectra in which laboratory and field data are included. As will be seen from Fig. 7 the high frequency part of the spectrum measured at Hakata Bay is very close to the equations determined from Burling's data. However, for the case of laboratory wind waves, the energy densities in a high frequency part are generally greater than that determined from Burling's data and their slopes are slightly steeper than f^{-5}, at least locally, although it is not so unatural to apply the "f^{-5} law" to this range as a first approximation. The high level of energy densities is corresponding to the excess energy accumuration discussed in the preceding section.***) At any rate, those results in Fig. 7 show that the equilibrium range of the wave spectrum shows a frequency dependence of f^{-5} as a first approximation, but dimensionless constant β is not an universal constant. Our observed data suggest that the value of β depend on the wind speed and fetch. Although it is difficult to find the physical mechanism for such phenomena,

*) In Fig. 6 the wave spectra denoted by 1 and 2 correspond respectively to the wave spectra measured simultaneously by two separate staffs 1 and 2 (cf. Fig. 3).

**) Those facts were substantiated by studying the directional properties of waves (Mitsuyasu 1968).

***) It should be noted that similar phenomena have been commonly observed in the laboratory experiments (Hamada 1963, Mitsuyasu 1964).

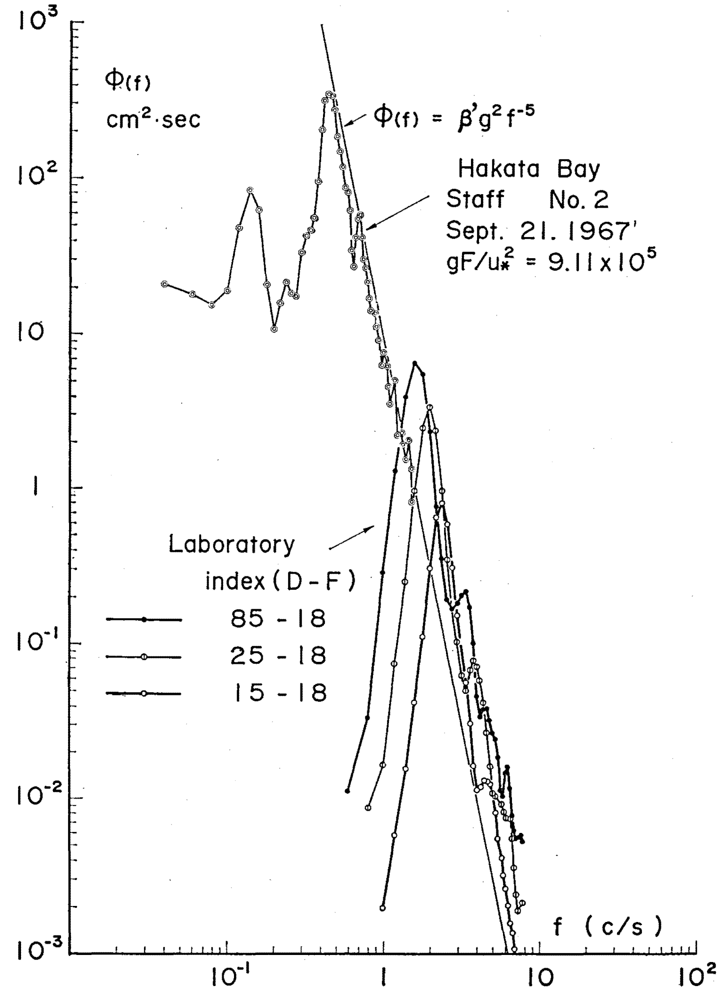


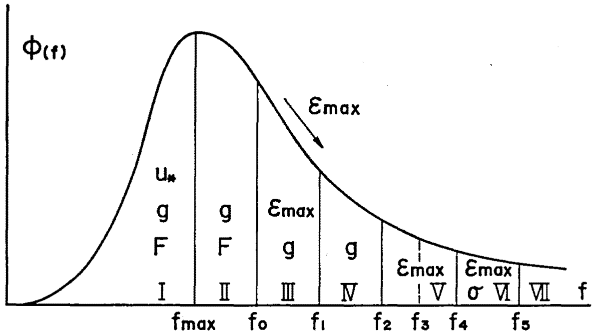
Fig. 7 High frequency parts of the wave spectra (equilibrium range) $\beta'=(2\pi)^{-4}\beta$ ($=9.51\times10^{-6}$ Burling 1959)

several discussions will be done later in the section 5 of this paper, showing that β depend approximately on a dimensionless fetch $gF/u^2{}_*$.

4. The growth of the spectrum of wind-generated waves

4-1) Similarity theory of Kitaigorodskii

Kitaigorodskii (1962) extensively applied the similarity theory to the description of wave spectra. Bay a close analogy with the spectrum of turbulence, he divided the wave spectrum into various frequency ranges which respectively



I linear interval
II intermediate interval
III interval of small scale turbulence
IV equilibrium interval (breaking)
V interval of small scale isotropic turbulence
VI capillary turbulence interval
VII dissipation interval
ε_{max}: rate of energy dissipation from the frequency range $f_{max} \sim f_5$
σ : surface tension

Fig. 8 Schematic representation of the narrow spectrum of wind waves. (Kitaigorodskii 1962)

have different main controlling factors as schematically shown in Fig. 8. He conjectured further that the wave spectrum near the frequency of maximum energy density could be expressed as

$$\phi\,(\omega) = g^2\omega^{-5}F_1\left(\frac{u_*\omega}{g},\ \frac{gF}{u^2{}_*}\right) \tag{2}$$

Further, he determined, based mainly on Burling's data, the form of the wave spectrum in the frequecy range small than dominant peak as

$$log\frac{\phi\,(\omega)\,\omega^5}{g^2} = \boldsymbol{a} + \boldsymbol{b}\ \frac{u_*\,\omega}{g} \tag{3}$$

where **a** and **b** are dimensionless constants and may depend only on the dimensionless fetch $gF/u^2{}_*$. He determined the values of **a** and **b** by using Burling's data, but their dependences on the dimensionless fetch were not clear because the observed range of the dimensionless fetch was not so wide in the Burling's data. Later, Mitsuyasu (1964) has shown from his experimental data that Eq. (3) can be applied to the spectrum of laboratory wind wave, and that the values of **a** and **b** are smaller in the laboratory data than in the field data presented by Burling (1959).

4-2) Low frequency part of the wave spectrum

Fig. 9 shows our present results expressed in the same universal form as Kitaigorodskii's one. The dimensionless frequency ranges of the spectra measured at Hakata Bay are so narrow that those spectra are reproduced in the same figure with enlarged obscissa (×10) and with the same ordinate *). As suggested by Kitaigorodskii (1962), the spectrum in the low frequency part which develops with fetch or wind speed could well be distinguished by

$$log \frac{\phi_{(f)} f^5}{g^2} = A + B \frac{u_* f}{g} \tag{4}$$

or by Eq. (3). Fig. 10 show the relations between the dimensionless constants **a**, **b** and the dimensionless fetch gF/u^2_*, which are determined from our present

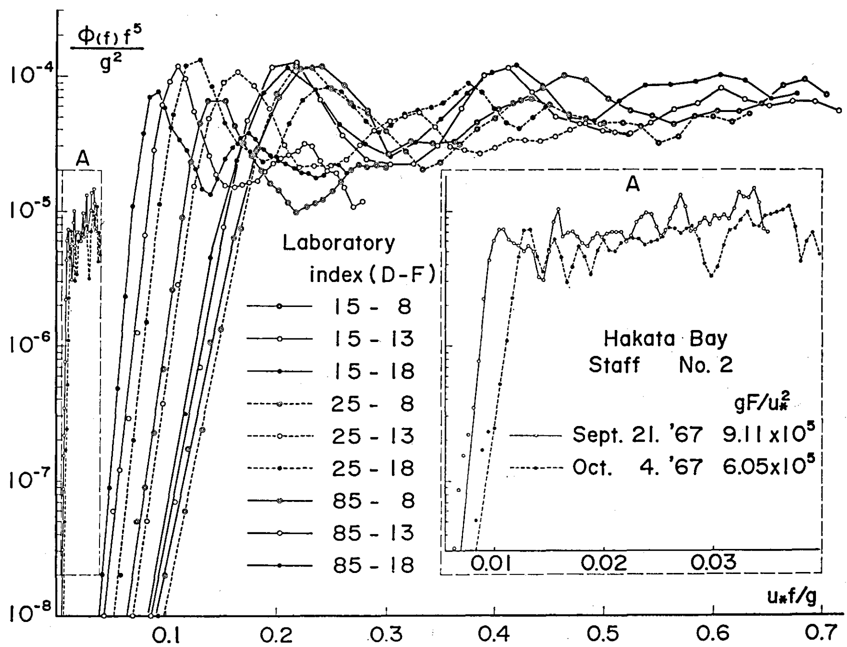


Fig. 9 Wave spectra normalized by Eq. (3). Frequency f is used in this figure instead of ω.

*) On each day, waves are measured simultaneously by two or three wave staffs separated each other, but only one record of each case was presented in Fig. 9 to avoid the congestion, although for the determinations of the values of **a** and **b** all data were used.

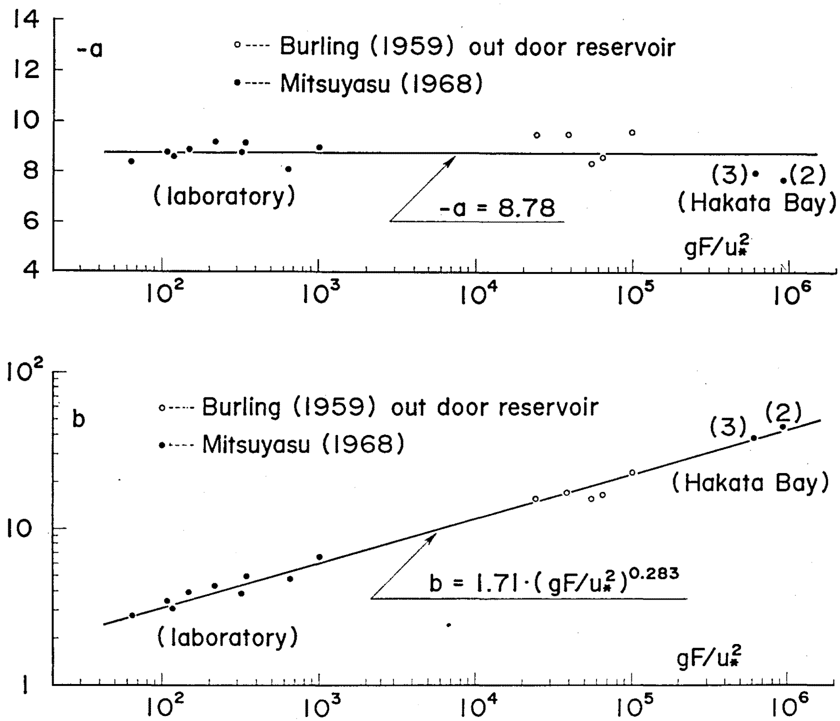


Fig. 10 The dimmensionless constants **a** and **b** in Eq. (3) versas gF/u^2_*.

data and Burling's data analyzed by Kitaigorodskii (1962). It is noteworthy that, fairly simple relations are existing between **a**, **b** and gF/u^2_* for a wide range of dimensionless fetch $gF/u^2_* \fallingdotseq 10^2 \sim 10^6$.

From Eq. (3) and the results shown in Fig. 10, the form of the wave spectrum in the frequency range lower than dominant peak is determined as

$$\frac{\phi_{(\omega)}\omega^5}{g^2} = 1.66 \times 10^{-9} exp\left[39.4\left(\frac{gF}{u^2_*}\right)^{0.283} \cdot \frac{u_*\omega}{g}\right] \tag{5}$$

4-3) Proposed spectrum of wind waves at short fetch and fetch graph

In the preceding sections, based on the accurately measured wave spectra and their generating conditions, we have experimentally determined the forms of the spectrum of wind waves at short fetch respectively for the high frequency part and for the low frequency part. Applying Eq. (1) for the spectrum in the frequency range higher than the frequency f_m where the spectral density has a maximum, and Eq. (5) for the spectrum in the frequency range lower than f_m, and definning conventionally the peak of the wave spectrum as the intersection of both spectra, one can get the spectral form of wind waves schematically shown in Fig. 11. If the frequencies are measured in cyle

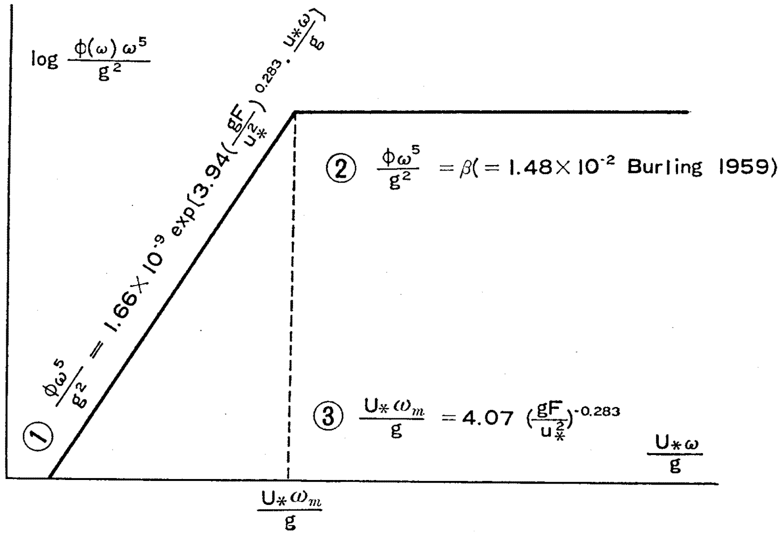


Fig. 11 Schematic representation of the proposed spectrum.

(not radian) per second, then the form of the wave spectrum is represented as

$$\frac{\phi_{(f)} f^5}{g^2} = 1.07 \times 10^{-12}\, exp\left[24.7\left(\frac{gF}{u^2_*}\right)^{0.283} \cdot \frac{u_* f}{g}\right] \tag{6}$$

for $f \leq f_m$

$$\frac{\phi_{(f)} f^5}{g^2} = (4\pi)^{-4}\beta = \beta' \; (= 9.51 \times 10^{-6}, \; \textit{Burling } 1959)$$

for $f \geq f_m$ (7)

At the intersection of the equations Eq. (1) and Eq. (5) [or Eq. (6) and Eq. (7)], which corresponds to the peak of the wave spectrum, the following relations are satisfied;

$$\frac{u_* \omega_m}{g} = 4.07\left(\frac{gF}{u^2_*}\right)^{-0.283} \tag{8}$$

or

$$\frac{u_* f_m}{g} = 6.47 \times 10^{-1}\left(\frac{gF}{u^2_*}\right)^{-0.283} \tag{9}$$

In the derivations of Eq. (8) and Eq. (9) the value of β was assumed tentatively to be 1.48×10^{-2}. Therefore, some corrections will be needed when the accurate properties of β are determined by further studies. Eq. (8) or Eq. (9) represents the fetch graph by which the dimensionless wave frequency $u_*\omega_m/g$ or u_*f_m/g, of peak spectral density can be determined as a function of the

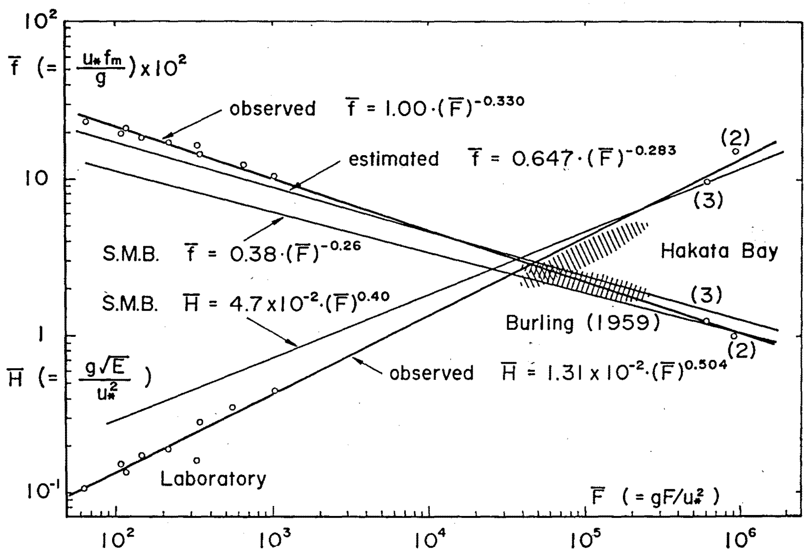


Fig. 12 Fetch graph (1): Dimensionless parameters $u_* f_m/g$ and $g\sqrt{E}/u^2_*$ versas gF/u^2_*. In this figure f̄, H̄, and F̄ represent only dimensionless parameters and not the averaged quantites. The data with perenthesized number mean the mean values of data of the indicated number.

dimensionless fetch gF/u^2_*. In Fig. 12 Eq. (9) is shown as "estimated" one, in association with the following relation which is determined directly from the measured data of u_*, F and f_m

$$\frac{u_* f_m}{g} = 1.00 \left(\frac{gF}{u^2_*}\right)^{-0.330} \tag{10}$$

Fig. 11 also included the relation derived from the following SMB curve

$$\frac{gT_{\frac{1}{3}}}{2\pi U} = 8.5 \times 10^{-2} \left(\frac{gF}{U^2}\right)^{0.26} \tag{11}$$

under the assumptions

$$U = U_{10} \doteqdot 25\, u_* \tag{12}$$

$$T_{\frac{1}{3}} = \frac{1}{1.05 f_m} \tag{13}$$

where U_{10} is wind velocity measured at 10 m above the sea surface and $T_{\frac{1}{3}}$ is the significant wave period. Eq. (12) was determined by the assumption that drag coefficient γ^2_{10} has a constant value 0.0016 at relatively low wind speed, and Eq. (13) was determined from the present experimental data.

It is difficult to derive the fetch graph for the wave energy or wave height from our proposed spectrum, because Eq. (6) can not be integrated analytically. Therefore, in Fig. 12 there are presented the fetch graphs for wave height which derived respectively from our measured data and from the following SMB curve

$$\frac{gH_{\frac{1}{3}}}{U^2} = 4.0 \times 10^{-3}\left(\frac{gF}{U^2}\right)^{0.40} \tag{14}$$

where $H_{\frac{1}{3}}$ is the significant wave height. In the derivation of fetch graph for wave height in our notation from Eq. (14) Eq. (12) and the following relation obtained by Longuet-Higgins (1952) have been used:

$$H_{\frac{1}{3}} = 4.01\sqrt{E} \tag{15}$$

In Fig. 12, for comparison, the scattered ranges of Burling's data are shown with hatches, although his data have not been used for the determination of our present relations. It can be seen from Fig. 12 that the SMB curve give larger values than the measured one in a short fetch not only for the wave period but also for the wave height, although the differences are relatively small at large fetch, and that the values given by Eq. (8), which is derived from the proposed spectrum, is very close to those given by Eq. (10) which is determined directly by using the measured data.

5. Normalized spectra of wind waves

Various schemes have been suggested for normalizing the wave spectra to find the similarity between the ocean waves and the laboratory wind waves. Here, we try to normalize the wave spectra by the scheme employed by Hidy & Plate (1965) and Colonell (1966). The dimensionless spectral density is computed by multiplying the peak energy frequency f_m to the original spectral density $\phi_{(f)}$ and dividing by the total energy of the spectrum E. A dimensionless frequency is found by dividing the actual frequency f by the peak energy frequency f_m. That is, the normalized wave spectrum is represented as

$$\frac{\phi_{(f)} f_m}{E} = \Phi\left(\frac{f}{f_m}\right) \tag{16}$$

Fig. 3 shows some examples of our measured spectra normalized by the above scheme, in which the following normalized form of Pierson & Moskowitz spectrum is included;

$$\frac{\phi_{(f)} f_m}{E} = 5\left(\frac{f}{f_m}\right)^{-5} exp\left[-\frac{5}{4}\left(\frac{f}{f_m}\right)^{-4}\right] \tag{17}$$

Fig. 13 shows that normalized three spectra roughly coinside each others, but there are existing several disagreements among them. For examples, at high frequency part the energy density of the normalized spectrum of laboratory

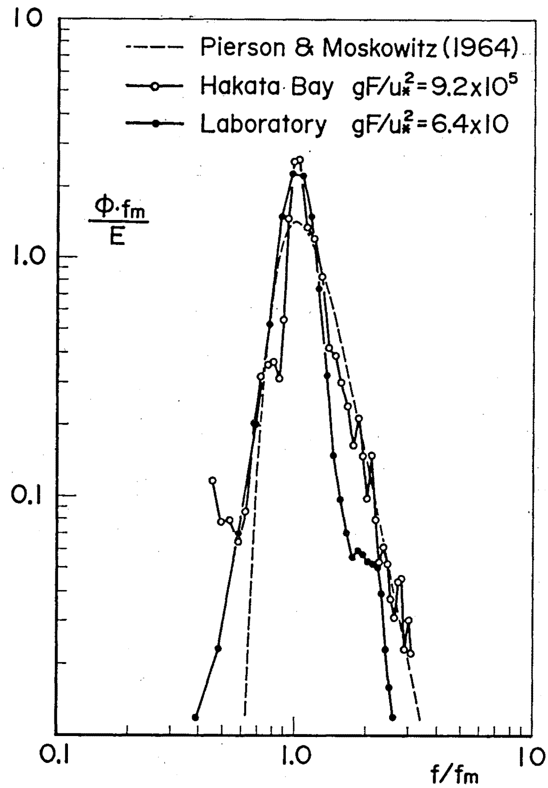


Fig. 13 Normalized wave spectra (1)
The data shown in this figure have been picked up from Figs. 14a and 14b respectively.

wind wave is smaller than those of ocean waves, and the coincidence between the normalized spectrum measured at Hakata Bay and that of Pierson & Moskowitz spectrum is not so good near the peak energy frequency f_m and in a low frequency part, although their coincidence is good in a high frequency part. Figs. 14a and 14b shows the normalized spectra respectively for the laboratory wind waves and for the waves measured at Hakata Bay. As will be seen from Fig. 14a, the normalized spectra of laboratory wind waves measured at various different conditions are so similar that it is difficult to find their differences. The same thing can be said about the spectra measured at Hakata Bay (Fig. 14b). Therefore, the slight differences in the normalized spectra mentioned above seems to depend on the scale of phenomena.

On the other hand, according to our proposed spectrum, the normalized form of the spectrum can be represented as

$$\frac{\phi_{(f)} f_m}{E} = 1.07 \times 10^{-12} \left(\frac{E f_m^4}{g^2}\right)^{-1} \left(\frac{f}{f_m}\right)^{-5} exp\left[24.7\left(\frac{gF}{u^2_*}\right)^{0.283} \cdot \left(\frac{u_* f_m}{g}\right) \cdot \left(\frac{f}{f_m}\right)\right]$$

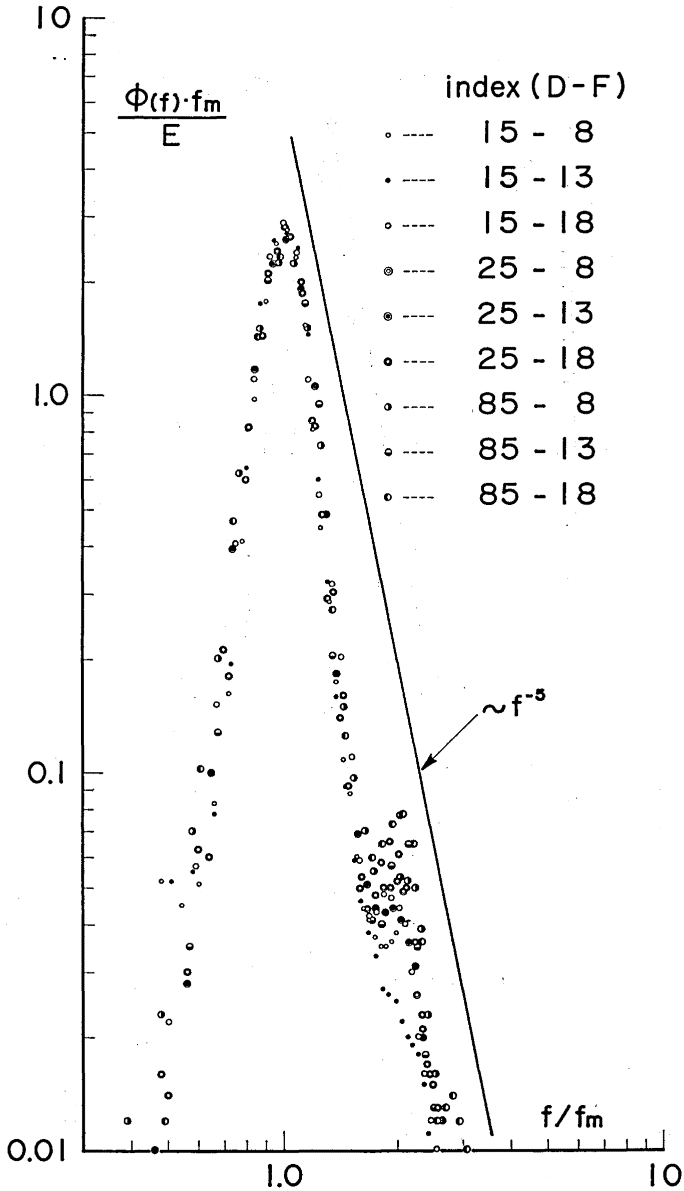


Fig. 14a Normalized wave spectra (2)
(laboratory measurements)

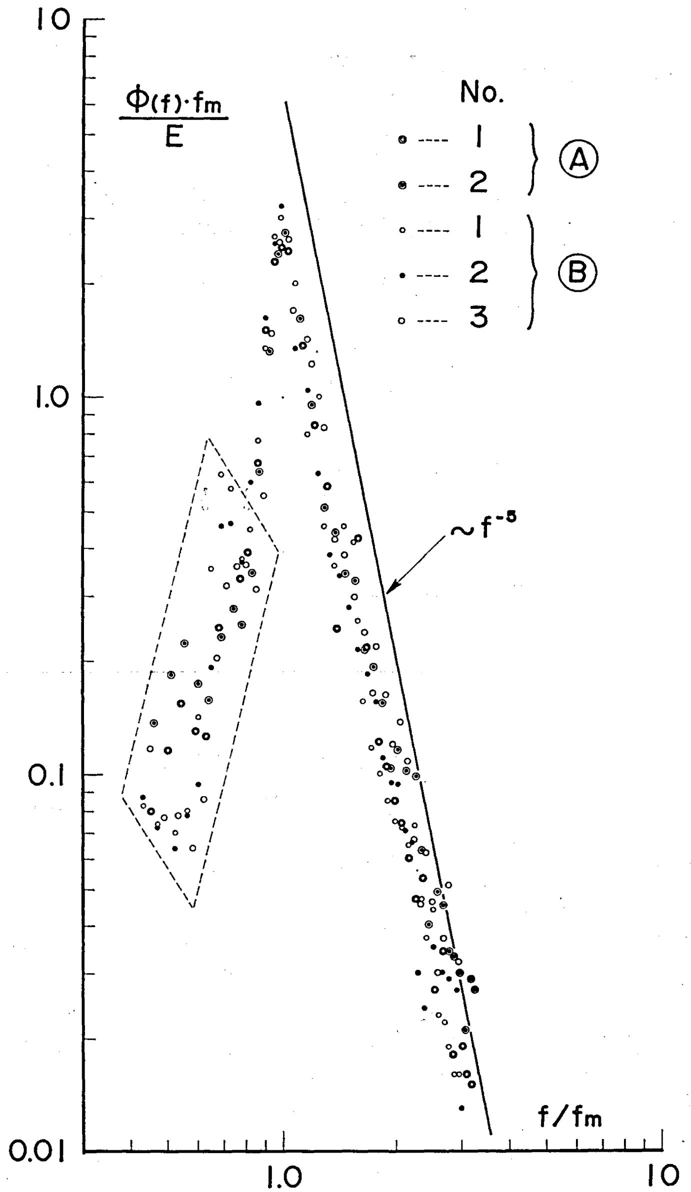


Fig. 14b Normalized wave spectra (3)
(field measurements, Hakata Bay)

"No". denote the wave staff No. (c. f. Fig. 3) and the data of A and B correspond to the data obtained respectively on Sept. 21 and Oct. 4. The data enclosed with a broken line are affected by swell.

$$\text{for} \quad f/f_m \leq 1 \tag{18}$$

$$\frac{\phi_{(f)} f_m}{E} = 9.51 \times 10^{-6} \left(\frac{E f_m^{\ 4}}{g^2}\right)^{-1} \left(\frac{f}{f_m}\right)^{-5} \tag{19}$$

$$\text{for} \quad f/f_m \geq 1$$

Therefore, if the normalized spectral density $\phi(f)\ f_m/E$ is uniquely determined by the normalized frequency f/f_m the following relations must be satisfied;

$$\frac{E f_m^{\ 4}}{g^2} = const. \tag{20}$$

$$\frac{u_* f_m}{g} \sim \left(\frac{gF}{u^2_*}\right)^{-0.283} \tag{21}$$

Eq. (21) is a kind of fetch graph similar to those discussed in the preceding section. Remembering now Eq. (13) and Eq. (15) in the preceding section, Eq. (20) means that the wave steepness $H_{\frac{1}{3}}/L_{\frac{1}{3}}$ is constant for all wind waves at short fetch. However, this statement seems not to be accepted without some verifications, because a definite relation between the wave steepness $\delta(=H_{\frac{1}{3}}/L_{\frac{1}{3}})$ and wave age $\beta_*(=C/U)$*) has been one of the most

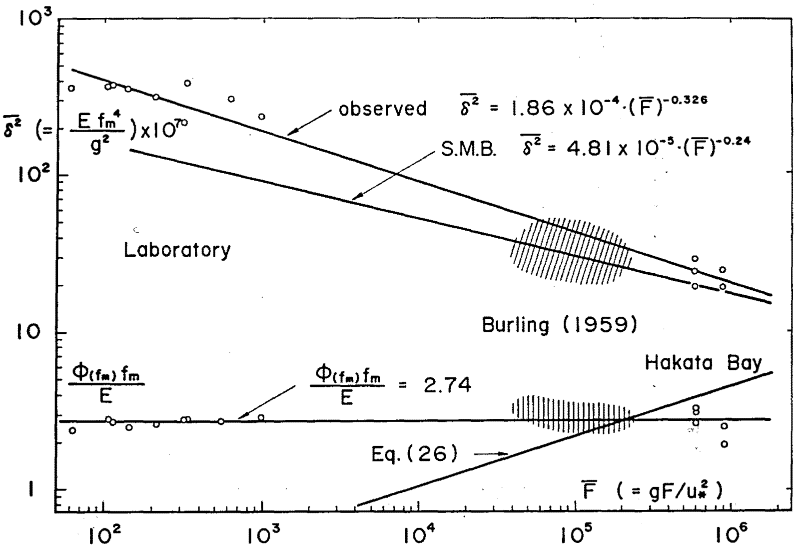


Fig. 15 Fetch graph (2): Dimensionless parameters Ef_m^4/g^2 and $\phi\ (f_m)\ f_m/E$ versas gF/u_*^2.

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*) Note that the following relations are existing;

$C/U \sim g/u_* f \sim (gF/u_*^2)^{\mathrm{R}}$

where R is a numerical constant (R=0.283 in our experimental data).

important relation in the Sverdrup-Munk theory (Sverdrup & Munk 1947).

In order to check the relation given by Eq. (20) all our measured values of Ef_m^4/g^2 were plotted against the dimensionless fetch gF/u_*^2 in Fig. 15. It is apparent from Fig. 15 that Ef_m^4/g^2 depends on gF/u_*^2. The relation between them can be approximately given by

$$\frac{Ef_m^4}{g^2} = 1.86 \times 10^{-4}\left(\frac{gF}{u_*^2}\right)^{-0.326} \tag{22}$$

For comparison, there is shown in Fig. 15 the corresponding relation

$$\frac{Ef_m^4}{g^2} = 4.85 \times 10^{-5}\left(\frac{gF}{u_*^2}\right)^{-0.24} \tag{23}$$

which is derived from SMB curves, i. e., Eq. (11) and Eq. (14) by using Eq. (13) and Eq. (15).

According to the results shown in Fig. 15 the normalized spectrum can not be determined uniquely by f/f_m, because the necessary condition Eq. (20) is not satisfied. On the constantry, combinning Eq. (9), Eq. (10), Eq. (19) and Eq. (22) the normalized form of our measured spectra can be written as

$$\frac{\phi_{(f)}f_m}{E} = 5.75 \times 10^{-9}\left(\frac{gF}{u_*^2}\right)^{0.326} \cdot \left(\frac{f}{f_m}\right)^{-5} exp\ 16.1\left(\frac{f}{f_m}\right) \tag{24}$$

$$\text{for } f/f_m \leq 1$$

$$\frac{\phi_{(f)}f_m}{E} = 5.11 \times 10^{-2}\left(\frac{gF}{u_*^2}\right)^{0.326}\left(\frac{f}{f_m}\right)^{-5} \tag{25}$$

$$\text{for } f/f_m \geq 1$$

According to Eq. (24) or Eq. (25) the normalized density at the spectral peak is given by

$$\frac{\phi_{(fm)}f_m}{E} = 5.11 \times 10^{-2}\left(\frac{gF}{u_*^2}\right)^{0.326} \tag{26}$$

In order to check the relation of Eq. (26) the measured value of the normalized spectral density at the peak of each spectrum is plotted against gF/u_*^2 in Fig. 15. As may readily be seen from Fig. 15, the measured results are quite different from Eq. (26). However, it is very interesting to find that within a range of our measurement $\phi_{(fm)}f_m/E$ is approximately constant for every spectra and takes the following value

$$\frac{\phi_{(fm)}f_m}{E} = 2.74 \tag{27}$$

in spite of the evidence that the spectra normalized in a form Eq. (16), has been slightly different in each others. Eq. (27) can be used for such practical purpose as to estimate the total energy E of wave spectrum from the peak density of the spectrum $\phi_{(f_m)}$ and the corresponding frequency f_m.

The disagreements between Eq. (26) and the measured results Eq. (27) must partly be due to the assumption of the constant value of β ($\beta = 1.48 \times 10^{-2}$). Therefore, if the measured relations of Eq. (22) and Eq. (27) are applied to the relation

$$\frac{\phi_{(f)} f_m}{E} = \beta\ (2\pi)^{-4} \left(\frac{E f_m^4}{g^2}\right)^{-1} \left(\frac{f}{f_m}\right)^{-5} \tag{28}$$

which is the original form of Eq. (25) and include the undermined constant β, one can determined β as

$$\beta = 7.92 \times 10^{-1} \left(\frac{gF}{u_*^2}\right)^{-0.326} \tag{29}$$

The observed results shown in Fig. 7 and Fig. 9 seem to be supporting the relation of Eq. (29), although more data will be needed to vertify the relation of Eq. (29).

Finally, it should be remarked that the value of $\phi_{(f_m)} f_m / E$ given by Eq. (27) is different either from the value given by Pierson & Moskowitz spectrum or from that given by Neumann spectrum (Neumann 1952). That is, the following relations can be derived respectively from those spectra;

$$\frac{\phi_{(f_m)} f_m}{E} = 1.43 \tag{30}$$

(Pierson & Moskowitz spectrum)

$$\frac{\phi_{(f_m)} f_m}{E} = 1.17 \tag{31}$$

(Neumann spectrum)

Simple calculations shows that the value of $\phi_{(f_m)} f_m/\mathrm{E}$ increases with the value of m or n in the normalized spectrum of the following form

$$\frac{\phi_{(f)} f_m}{E} = \frac{n}{\Gamma\left(\frac{m-1}{n}\right)} \left(\frac{m}{n}\right)^{\frac{m-1}{2}} \left(\frac{f}{f_m}\right)^{-m} exp \left[-\frac{m}{n}\left(\frac{f}{f_m}\right)^{-n}\right] \tag{32}$$

where Γ is a gramma function. Eq. (32) is a normalized form of the spectrum

$$\phi_{(f)} = k_1 f^{-m}\, exp\left[-k_2 f^{-n}\right] \tag{33}$$

in which the Pierson & Moskowitz spectrum is included as a case $m = 5$, $n = 2$.

6. Concluding remarks

As a result of the present experimental investigations on the spectrum of fetch-limited wind waves the form of wave spectrum in a low frequency part was determined. The spectral form in a low frequency part, i. e., the steep forward face of the wave spectrum is given, with sufficient accuracy, by Eq. (5) or Eq. (6). Combinning the newly determined wave spectrum with the equilibrium form of wave spectrum in a high frequency part those can be used extensively for the practical purposes. However, as Eq. (5) or Eq. (6) is slightly different from those expected from the recent theory of wave generation the data presented in this paper are being subjected to more detailed analysis by considering the mechanism of energy transfer from wind to waves. The results will be reported in near future.

Acknowledgement

The author wishes to express his gratitude to Prof. M. Kurihara of Nagasaki University for his invaluable advice and encouragement in the course of work. He is also indebted to Prof. K. Kajiura of Tokyo University for fruitful discussions of the problem. Appreciations are extended to a number of people, in particular, to Mr. R. Nakayama, Mr. T. Komori and Mr. K. Eto for their constant assistance in carrying out the study, and to Miss S. Ushijima for the typing the manuscript. Finally, he wishes to express his gratitude to 4th Regional Bureau of Port and Habour Constraction for giving the opportunities of wind and wave observations at Hakata Bay.

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(Received December 16, 1968)