MINIMUM L_1-NORM ESTIMATION FOR ORNSTEIN-UHLENBECK TYPE PROCESS DRIVEN BY A GAUSSIAN PROCESS

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Abstract

We investigate the asymptotic properties of the minimum L_1-norm estimator of the drift parameter for Ornstein-Uhlenbeck type process driven by a general Gaussian process.

Key Words and Phrases: Minimum L_1-norm estimation, Ornstein-Uhlenbeck type process, Fractional Brownian motion, Gaussian Process.

1. Introduction

Diffusion processes and diffusion type processes satisfying stochastic differential equations driven by Wiener processes are used for stochastic modeling in a wide variety of sciences such as population genetics, economic processes, signal processing as well as for modeling sunspot activity and more recently in mathematical finance. Statistical inference for diffusion type processes satisfying stochastic differential equations driven by Wiener processes have been studied earlier and a comprehensive survey of various methods is given in Prakasa Rao (1999). There has been a recent interest to study similar problems for stochastic processes driven by a fractional Brownian motion to model processes involving long range dependence (cf. Prakasa Rao (2010)). Le Breton (1998) studied parameter estimation and filtering in a simple linear model driven by a fractional Brownian motion. Kleptsyna and Le Breton (2002) studied parameter estimation problems for fractional Ornstein-Uhlenbeck process. The fractional Ornstein-Uhlenbeck process is a fractional analogue of the Ornstein-Uhlenbeck process, that is, a continuous time first order autoregressive process $X = \{X_t, t \geq 0\}$ which is the solution of a one-dimensional homogeneous linear stochastic differential equation driven by a fractional Brownian motion (fBm) $W^H = \{W_t^H, t \geq 0\}$ with Hurst parameter H. Such a process is the unique Gaussian process satisfying the linear integral equation

$$X_t = x_0 + \theta \int_0^t X_s ds + \sigma W_t^H, t \geq 0. \tag{1}$$

They investigate the problem of estimation of the parameters θ and σ^2 based on the observation $\{X_s, 0 \leq s \leq T\}$ and study the asymptotic behaviour of these estimators as $T \to \infty$.

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In spite of the fact that maximum likelihood estimators (MLE) are consistent and asymptotically normal and also asymptotically efficient in general, they have some short comings at the same time. Their calculation is often cumbersome as the expression for MLE involve stochastic integrals at times which need good approximations for computational purposes. Further more MLE are not robust in the sense that a slight perturbation in the noise component will change the properties of MLE substantially. In order to circumvent such problems, the minimum distance approach is proposed. Properties of the minimum distance estimators (MDE) were discussed in Millar (1984) in a general frame work. Kutoyants and Pilibossian (1994) studied the problem of minimum L_1-norm estimation for the Ornstein-Uhlenbeck process. Prakasa Rao (2005) investigated the problem of minimum L_1-norm estimation for the fractional Ornstein-Uhlenbeck process driven by a fractional Brownian motion.

Our aim in this paper is to obtain the minimum L_1-norm estimator of the drift parameter of a Ornstein-Uhlenbeck type process driven by a centered Gaussian process and investigate the asymptotic properties of such estimators. El Machkouri et al. (2015), Chen and Zhou (2020), Lu (2022) study parameter estimation for an Ornstein-Uhlenbeck type process driven by a Gaussian process. Nonparametric estimation of linear multiplier in stochastic differential equations driven by Gaussian processes is studied in Prakasa Rao (2023).

2. Minimum L_1-norm Estimation

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$ be a stochastic basis satisfying the usual conditions and the processes discussed in the following are $(\mathcal{F}_t)$-adapted. Further the natural filtration of a process is understood as the P-completion of the filtration generated by this process. We consider a centered Gaussian process $G \equiv \{G_t, 0 \leq t \leq 1\}$ with the covariance function $K(t,s)$. We assume that the Gaussian process has Holder continuous paths of positive order and that integration of a non-random function with respect to the Gaussian process G is defined as a Young integral (cf. Nourdin (2012)). Note that the Young integral obeys integration by parts formula (cf. Nourdin (2012)). This class of Gaussian processes includes fractional Brownian motion, sub-fractional Brownian motion and bifractional Brownian motion (cf. Mishura and Zili (2018)).

Let us consider a stochastic process $\{X_t, t \in [0,1]\}$ defined by the stochastic integral equation

$$X_t = x_0 + \theta \int_0^t X_s ds + \varepsilon\, G_t, 0 \leq t \leq 1, \tag{2}$$

on the probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$ where θ is an unknown drift parameters respectively. Let $P_\theta^{(\varepsilon)}$ be the probability measure generated by the process $\{X_t, t \in [0,1]\}$ when θ is the true parameter. For convenience, we write the above integral equation in the form of a stochastic differential equation

$$dX_t = \theta X_t dt + \varepsilon\, dG_t, X_0 = x_0, 0 \leq t \leq 1, \tag{3}$$

driven by the Gaussian process G. Existence and uniqueness of the solution $\{X_t, 0 \leq t \leq 1\}$, for the differential equation given above, follows whenever the process G has paths of p-variation for $p < 2$ (cf. Baudoin (2012)). Here after we assume that the solution of the equation (3) exists and it is unique.

We now consider the problem of estimation of the parameter θ based on the observation of Ornstein-Uhlenbeck type process $X = \{X_t, 0 \leq t \leq 1\}$ satisfying the stochastic differential equation

$$dX_t = \theta X_t dt + \varepsilon\, dG_t, X_0 = x_0, 0 \leq t \leq 1 \tag{4}$$

where $\theta \in \Theta \subset R$ and study its asymptotic properties as $\varepsilon \to 0$.

Let $x_t(\theta)$ be the solution of the above differential equation with $\varepsilon = 0$. It is obvious that

$$x_t(\theta) = x_0 e^{\theta t}, 0 \leq t \leq 1. \tag{5}$$

Let

$$S_1(\theta) = \int_0^1 |X_t - x_t(\theta)| dt \tag{6}$$

We define $\hat{\theta}_\varepsilon$ to be a *minimum L_1-norm estimator* if there exists a measurable selection $\hat{\theta}_\varepsilon$ such that

$$S_1(\hat{\theta}_\varepsilon) = \inf_{\theta \in \Theta} S_1(\theta). \tag{7}$$

Conditions for the existence of a measurable selection are given in Lemma 3.1.2 in Prakasa Rao (1987). We assume that there exists a measurable selection $\hat{\theta}_\varepsilon$ satisfying the above equation. An alternate way of defining the estimator $\hat{\theta}_\varepsilon$ is by the relation

$$\hat{\theta}_\varepsilon = \arg \inf_{\theta \in \Theta} \int_0^1 |X_t - x_t(\theta)| dt. \tag{8}$$

Let $G_1^* = \sup_{0 \leq t \leq 1} |G_t|$. If G is a fractional Brownian motion or a sub-fractional Brownian motion, then the maximal inequalities for such processes are known and are reviewed in Prakasa Rao (2014, 2017, 2020) following the property of self-similarity for such processes. Maximal inequalities for general Gaussian processes are presented in Li and Shao (2001), Berman (1985), Marcus and Rosen (2006) and Borovkov et al. (2017) among others. We will now present a maximal inequality for Gaussian processes due to Nordin (2012) which will be used in the sequel.

THEOREM 2.1. *Suppose G is a centered and continuous Gaussian process on the interval* $[0, 1]$*. Let* $\sigma^2 = \sup_{0 \leq t \leq 1} E[G_t^2]$*. Suppose that* $0 < \sigma^2 < \infty$*. Then* $m = E[\sup_{0 \leq t \leq 1} G_t]$ *is finite and for all* $x > m$,

$$P(\sup_{0 \leq t \leq 1} G_t \geq x) \leq \exp(-\frac{(x-m)^2}{2\sigma^2}).$$

From the fact that G is a centered Gaussian process, it follows that

$$P(\sup_{0 \leq t \leq 1} |G_t| \geq x) \leq 2\, \exp(-\frac{(x-m)^2}{2\sigma^2})$$

under the conditions stated in Theorem 2.1.

3. Consistency of the estimator

Let θ_0 denote the true parameter. For any $\delta > 0$, define

$$g(\delta) = \inf_{|\theta-\theta_0|>\delta} \int_0^1 |x_t(\theta) - x_t(\theta_0)|dt. \tag{9}$$

Note that $g(\delta) > 0$ for any $\delta > 0$.

THEOREM 3.1. *Suppose G is a centered and continuous Gaussian process on the interval $[0,1]$. Let $\sigma^2 = \sup_{0\le t\le 1} E[G_t^2]$. Suppose that $0 < \sigma^2 < \infty$. Let $m = E[\sup_{0\le t\le 1} G_t]$. Then there exists a positive constant C such that for every $\delta > 0$,*

$$P_{\theta_0}^{(\varepsilon)}(|\hat{\theta}_\varepsilon - \theta_0| > \delta) = O(e^{-C[g(\delta)]^2\varepsilon^{-2}}).$$

Proof: Let $||.||$ denote the L_1-norm. Then

$$\begin{aligned}
P_{\theta_0}^{(\varepsilon)}(|\hat{\theta}_\varepsilon - \theta_0| > \delta) &= P_{\theta_0}^{(\varepsilon)}(\inf_{|\theta-\theta_0|\le\delta} ||X - x(\theta)|| > \inf_{|\theta-\theta_0|>\delta} ||X - x(\theta)||) \\
&\le P_{\theta_0}^{(\varepsilon)}(\inf_{|\theta-\theta_0|\le\delta}(||X - x(\theta_0)|| + ||x(\theta) - x(\theta_0)||) \\
&\qquad > \inf_{|\theta-\theta_0|>\delta}(||x(\theta) - x(\theta_0)|| - ||X - x(\theta_0)||)) \\
&= P_{\theta_0}^{(\varepsilon)}(2||X - x(\theta_0)|| > \inf_{|\theta-\theta_0|>\delta} ||x(\theta) - x(\theta_0)||) \\
&= P_{\theta_0}^{(\varepsilon)}(||X - x(\theta_0)|| > \frac{1}{2}g(\delta)).
\end{aligned}$$

Since the process X satisfies the stochastic differential equation (2), it follows that

$$\begin{aligned}
X_t - x_t(\theta_0) &= x_0 + \theta_0 \int_0^t X_s ds + \varepsilon G_t - x_t(\theta_0) \\
&= \theta_0 \int_0^t (X_s - x_s(\theta_0))ds + \varepsilon G_t
\end{aligned}$$

a.s. $P_{\theta_0}^{\epsilon}$ since $x_t(\theta) = x_0 e^{\theta t}$. Let $U_t = X_t - x_t(\theta_0)$. Then it follows from the equation given above that

$$U_t = \theta_0 \int_0^t U_s \, ds + \varepsilon G_t. \tag{10}$$

Let $V_t = |U_t| = |X_t - x_t(\theta_0)|$. The relation given above implies that

$$V_t = |X_t - x_t(\theta_0)| \le |\theta_0| \int_0^t V_s ds + \varepsilon |G_t|. \tag{11}$$

Applying the Gronwall-Bellman Lemma, it follows that

$$\sup_{0\le t\le 1} |V_t| \le \varepsilon e^{|\theta_0|} \sup_{0\le t\le 1} |G_t|. \tag{12}$$

Hence

$$
\begin{aligned}
P_{\theta_0}^{(\varepsilon)}(||X - x(\theta_0)|| > \frac{1}{2}g(\delta)) &\leq P[\sup_{0\leq t\leq 1}|G_t| > \frac{e^{-|\theta_0|}g(\delta)}{2\varepsilon}] \\
&= P[G_1^* > \frac{e^{-|\theta_0|}g(\delta)}{2\varepsilon}].
\end{aligned}
$$

Let $m = E[\sup_{0\leq t\leq 1} G(t)]$. Applying the maximal inequalities for Gaussian processes given in Theorem 2.1, we get that, for fixed $\delta > 0$, we can choose ϵ sufficiently small so that $\frac{e^{-|\theta_0|}g(\delta)}{2\epsilon} > m$. For such ε,

$$
\begin{aligned}
P_{\theta_0}^{(\varepsilon)}(|\hat{\theta}_\varepsilon - \theta_0| > \delta) &\leq 2\exp(-\frac{((e^{-|\theta_0|}g(\delta)/2\epsilon) - m)^2}{2\sigma^2}) \\
&= O(e^{-C[g(\delta)]^2\varepsilon^{-2}})
\end{aligned}
$$

for some positive constant C independent of ε.

Remarks: As a consequence of the result obtained above, it follows that

$$P_{\theta_0}^{(\varepsilon)}(|\hat{\theta}_\varepsilon - \theta_0| > \delta) \to 0 \text{ as } \epsilon \to 0$$

for every $\delta > 0$. Hence the minimum norm L_1-estimator θ_ε^* is weakly consistent for estimating the parameter θ_0.

4. Asymptotic distribution of the estimator

We will now study the asymptotic distribution if any of the estimator $\hat{\theta}_\varepsilon$ after suitable scaling. It can be checked that

$$X_t = e^{\theta_0 t}\{x_0 + \int_0^t e^{-\theta_0 s}\varepsilon dG_s\} \tag{13}$$

or equivalently

$$X_t - x_t(\theta_0) = \varepsilon e^{\theta_0 t}\int_0^t e^{-\theta_0 s}dG_s. \tag{14}$$

Let

$$Y_t = e^{\theta_0 t}\int_0^t e^{-\theta_0 s}dG_s. \tag{15}$$

Note that $\{Y_t, 0 \leq t \leq 1\}$ is a Gaussian process and can be interpreted as the "derivative" of the process $\{X_t, 0 \leq t \leq 1\}$ with respect to ε. We obtain that, P-a.s.,

$$Y_t e^{-\theta_0 t} = \int_0^t e^{-\theta_0 s}dG_s \tag{16}$$

The integral with respect to the process G is interpreted as Young integral (cf. El Machkouri et al. (2015)). In particular, it follows that the random variable $Y_t e^{-\theta_0 t}$

and hence Y_t has the normal distribution with mean zero and furthermore, for any $0 \leq t, s \leq 1$,

$$\begin{aligned} Cov(Y_t, Y_s) &= e^{\theta_0 t + \theta_0 s} E[\int_0^t e^{-\theta_0 u} dG_u \int_0^s e^{-\theta_0 v} dG_v] \\ &= e^{\theta_0 t + \theta_0 s} \int_0^t \int_0^s e^{-\theta_0 (u+v)} K(u,v) du dv \\ &= R(t,s) \ \text{(say)}. \end{aligned}$$

In particular,

$$Var(Y_t) = R(t,t). \tag{17}$$

Observe that $\{Y_t, 0 \leq t \leq 1\}$ is a zero mean Gaussian process with $Cov(Y_t, Y_s) = R(t,s)$. Let

$$\zeta = \arg \inf_{-\infty < u < \infty} \int_0^1 |Y_t - u t x_0 e^{\theta_0 t}| dt. \tag{18}$$

THEOREM 4.1. *As $\varepsilon \to 0$, the random variable $\varepsilon^{-1}(\hat{\theta}_\varepsilon - \theta_0)$ converges in probability to a random variable whose probability distribution is the same as that of the random variable ζ defined by the equation (18).*

Proof: Let $x_t'(\theta) = x_0 t e^{\theta t}$ and let

$$Z_\varepsilon(u) = ||Y - \varepsilon^{-1}(x(\theta_0 + \varepsilon u) - x(\theta_0))|| \tag{19}$$

and

$$Z_0(u) = ||Y - u x'(\theta_0)||. \tag{20}$$

Furthermore, let

$$A_\varepsilon = \{\omega : |\hat{\theta}_\varepsilon - \theta_0| < \delta_\varepsilon\}, \delta_\varepsilon = \varepsilon^\tau, \tau \in (\frac{1}{2}, 1), L_\varepsilon = \varepsilon^{\tau - 1}. \tag{21}$$

Observe that the random variable $u_\varepsilon^* = \varepsilon^{-1}(\theta_\varepsilon^* - \theta_0)$ satisfies the equation

$$Z_\varepsilon(u_\varepsilon^*) = \inf_{|u| < L_\varepsilon} Z_\varepsilon(u), \omega \in A_\varepsilon. \tag{22}$$

Define

$$\zeta_\varepsilon = \arg \inf_{|u| < L_\varepsilon} Z_0(u). \tag{23}$$

Observe that, with probability one,

$$\begin{aligned} \sup_{|u| < L_\varepsilon} |Z_\varepsilon(u) - Z_0(u)| &= \sup_{|u| < L_\varepsilon} |||Y - u x'(\theta_0) - \frac{1}{2} \varepsilon u^2 x''(\tilde{\theta})|| - ||Y - u x'(\theta_0)||| \\ &\leq \frac{\varepsilon}{2} L_\varepsilon^2 \sup_{|\theta - \theta_0| < \delta_\varepsilon} \int_0^1 |x''(\theta)| dt \\ &\leq C \varepsilon^{2\tau - 1}. \end{aligned}$$

Here $\tilde{\theta} = \theta_0 + \alpha(\theta - \theta_0)$ for some $\alpha \in (0,1)$. Note that the last term in the above inequality tends to zero as $\varepsilon \to 0$. Further more the process $\{Z_0(u), -\infty < u < \infty\}$ has a unique minimum u^* with probability one. This follows from the arguments given in Theorem 2 of Kutoyants and Pilibossian (1994). In addition, we can choose the interval $[-L, L]$ such that

$$P_{\theta_0}^{(\varepsilon)}\{u_\varepsilon^* \in (-L, L)\} \geq 1 - \beta(g(L))^{-1} \tag{24}$$

and

$$P\{u^* \in (-L, L)\} \geq 1 - \beta(g(L))^{-1} \tag{25}$$

where $\beta > 0$. Note that $g(L)$ increases as L increases. The processes $Z_\varepsilon(u), u \in [-L, L]$ and $Z_0(u), u \in [-L, L]$ satisfy the Lipschitz conditions and $Z_\varepsilon(u)$ converges uniformly to $Z_0(u)$ over $u \in [-L, L]$. Hence the minimizer of $Z_\varepsilon(.)$ converges to the minimizer of $Z_0(u)$. This completes the proof.

Remarks : We have seen earlier that the process $\{Y_t, 0 \leq t \leq 1\}$ is a zero mean Gaussian process with the covariance function $Cov(Y_t, Y_s) = R(t, s)$ for $0 \leq t, s \leq 1$. Recall that

$$\zeta = \arg \inf_{-\infty < u < \infty} \int_0^1 |Y_t - utx_0 e^{\theta_0 t}| dt. \tag{26}$$

It is not clear what the distribution of the random variable ζ is. It depends on the Gaussian process G. Observe that for every u, the integrand in the above integral is the absolute value of a Gaussian process $\{J_t, 0 \leq t \leq 1\}$ with the mean function $E(J_t) = -utx_0 e^{\theta_0 t}$ and the covariance function $Cov(J_t, J_s) = R(t, s)$ for $0 \leq s, t \leq 1$. It is easy to extend the results to any Gaussian process defined on any interval $[0, T]$ for any fixed $T > 0$. The distribution of

$$u_T = \text{arginf}_{u \in R} \int_0^T |Y_t - utx_0 e^{\theta_0 t}| dt,$$

as $T \to \infty$, has been investigated in Aubry (1999) for a diffusion process, Diop and Yode (2010) for the Ornstein-Uhlenbeck process driven by a Levy process, Kutoyants and Pilibossian (1994) for the Ornstein-Uhlenbeck process and by Shen et al. (2018) for Ornstein-Uhlenbeck process driven by a fractional Levy process.

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