ON THE SWAYING, YAWING AND ROLLING MOTIONS OF SHIPS IN OBLIQUE WAVES

TASAI, Fukuzo
Research Institute for Applied Mechanics, Kyushu University

https://doi.org/10.5109/7165106

出版情報：Reports of Research Institute for Applied Mechanics. 14 (47), pp.23-50, 1966. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. XIV, No. 47, 1966

ON THE SWAYING, YAWING AND ROLLING MOTIONS OF SHIPS IN OBLIQUE WAVES

By Fukuzō TASAI

Contents

Summary

In this paper an approximate method is introduced to calculate the swaying force, yawing and rolling moments acting upon a ship navigating in oblique waves. It next treats of linear coupled equations of swaying, yawing and rolling, where the ship has a finite velocity, in order to derive an approximate method for solving the problems of forced oscillation.

As the first step for studing such coupled motions in three degrees of freedom in lateral plane, the author investigated mutual coupling actions among swaying, yawing and rolling when the ship has zero velocity.

That is to say, the author worked out a numerical calculation for three kinds of hull forms by means of the linear coupled equations of swaying, yawing and rolling which he derived in 1965 for the case of zero ship velocity.

The results can be epitomized as follows:—

a) In case of a hull form with cut up stern, there is a remarkable coupling action between yawing and rolling.
Also in this case, if yawing is restrained, rolling will find its maximum value in quartering seas.

b) Depending on hull shapes, it may be possible that we may find rolling at its maximum in quartering seas because of the coupling actions of swaying and yawing.

c) When a ship has a cross section of large beam/draft ratio, the coefficient of effective wave slope γ sometimes becomes much larger than that of Dr. Watanabe's theoretical calculation.

I. Introduction

The author introduced in literature [1]* certain linear coupled equations of swaying, yawing and rolling at beam seas and made an investigation, neglecting yawing, on the cross coupling action of swaying and rolling. Then in [2] he showed that in the case where the beam of a ship to wave length ratio is very small the following two solutions almost coincide: one is to solve rolling with the coupled equations of swaying and rolling derived from the calculated values of hydrodynamic force and moment which act upon a restrained body; and the other, with the ordinary equations of motion based upon the Froude-Krilov's theory.

So far as it is concerned with the rolling motion at beam seas, therefore, the difference of the solutions will be negligible whether the theory of restrained body is applied or that of Froude-Krilov is resorted to.

Regarding the coupled motions of swaying and yawing in oblique waves, we have Eda's method [3] for an approximate calculation, which gives us the results showing a good coincidence with experiments.

Lately, O. Grim and Takaishi [4] introduced, for a mathematical hull form of fore-and-aft symmetry, coupled equations of sway and roll in oblique waves and when ship velocity is zero. Here they calculate total rolling moment in which the rolling moment based upon swaying inertia force is considered.

They point out that, when G_0M is small, the maximum value of rolling moment sometimes occurs at oblique waves of some 45 degrees instead of beam sea. It has been already known to a certain extent even with Froude-Krilov's theory that such phenomena can happen.

On the other hand, it is stated in literature [5] that, when ship velocity is zero, depending on hull shapes, the maximum value of rolling amplitude is to be found not at beam sea but at the state of quartering seas. The validity of such a tendency is also seen in the results of experiment [6].

In this paper, discussions are made on the coupled equations of swaying, yawing and rolling at oblique waves. Furthermore, the mutual coupling actions of these motions are investigated by means of numerical calculations made on three kinds of hull shapes.

II. Exciting Force and Moment in Oblique Waves

Assume in Fig. 1 that the ship is navigating at a constant velocity V in the direction at an angle χ to the propagating direction of a regular wave.

* Numbers in brackets designate References at the end of the paper.

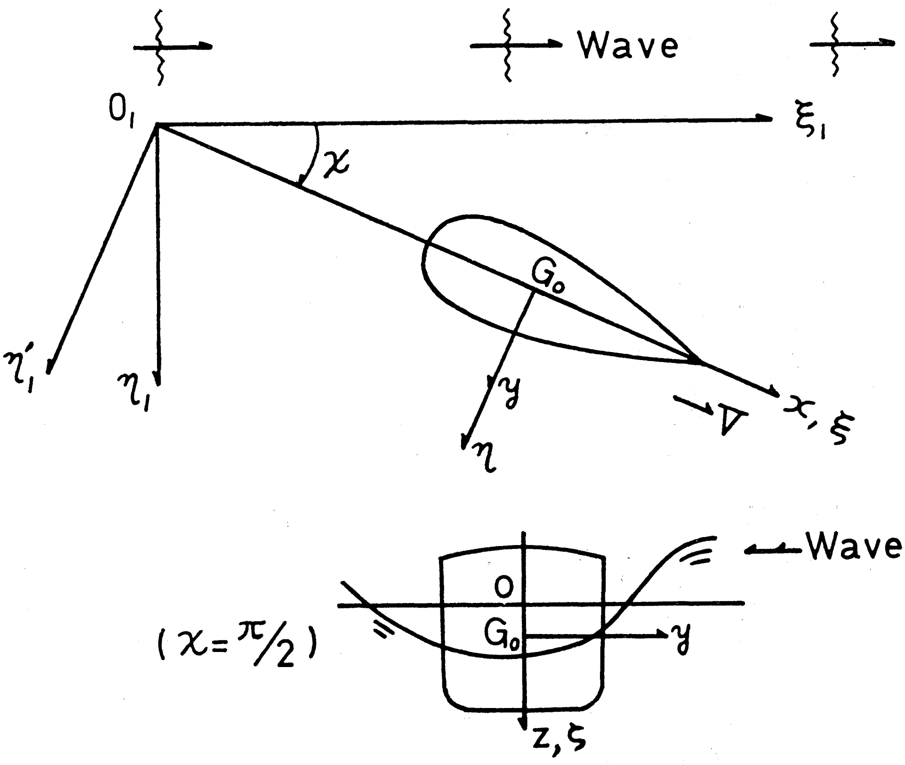


Fig. 1. Wave and coordinates systems.

Let O_1-$\xi_1\eta_1\zeta_1$ and O_1-$\xi\eta'_1\zeta_1$ be space-fixed coordinates, G_0-xyz (where G_0 is the center of gravity of the ship) hull-fixed coordinates, and O-$\xi\eta\zeta$ the coordinates with their origin at 0 (where O is the intersection of $G_0 Z$ with the painted load water line) and in parallel with O_1-$\xi\eta'_1\zeta_1$.

Suppose the ship makes small motions around O-$\xi\eta\zeta$.

The equation of regular wave can be expressed by the formula

$$\zeta w = he^{-K\zeta}\cos(K\xi_1-\omega t) \tag{2.1}$$

where h=wave amplitude, $k=\omega^2/g=2\pi/\lambda$, λ=wave length.

The orbital velocity in the η direction is

$$V_\eta = h\omega \sin\chi e^{-K\zeta}\cos(K\xi\cos\chi - K\eta\sin\chi-\omega_e t)$$
$$\omega_e=\omega(1-\tau\cos\chi),\ \tau=\omega V/g \tag{2.2}$$

Then the orbital acceleration is

$$\dot{V}_\eta = h\omega^2 \sin\chi e^{-K\zeta}\sin(K\xi\cos\chi - K\eta\sin\chi-\omega_e t) \tag{2.3}$$

In the above equation, ξ, η and ζ can be approximately replaced by x, y and z respectively since ship motion is small.

In this paper, the author will calculate the exciting force by means of the strip method. Following the method of Dr. Watanabe in his theory on pitch and heave [7], we can put the swaying force acting upon the unit length cross section of a ship approximately as

$$F_{\eta e}' \doteqdot F_{\eta 1}' + m'\overline{\dot{V}}_\eta + N_\eta' \,\overline{V}_\eta - V\frac{dm'}{dx}\overline{V}_\eta \tag{2.4}$$

The first term of (2.4) is the force based upon the Froude-Krilov's theory, the second term orbital acceleration, the third the force based upon orbital velocity, and the fourth the influence of ship velocity. Here m' and N_η' signify the swaying added mass and the coefficient of damping of the unit length cross section respectively. $\overline{\dot{V}}_\eta$ and $\overline{V}_\eta$ signify the mean values of $\dot{V}_\eta$ and V_η respectively, and they can be approximated by the values at $\zeta = T/2$ on the centre plane of the hull. That is to say, we can describe as follows:

$$\begin{aligned}\overline{\dot{V}}_\eta &= h\omega^2 \sin\chi e^{-1/2\xi d} \sin(Kx \cos\chi - \omega_e t)\\ \overline{V}_\eta &= h\omega \sin\chi e^{-1/2\xi d} \cos(Kx \cos\chi - \omega_e t)\end{aligned} \tag{2.5}$$

where $\xi_d = \frac{\omega^2}{g}T$, and T is the draft of the cross section.

Eda [3] used for $F_{\eta 1}'$, $F_{\eta 2}'$ and $F_{\eta 3}'$ in (2.4) the following approximation:

$$F_{\eta_1}' \doteqdot \rho g S w \Theta w \sin\chi e^{-1/2\xi d} \sin(Kx \cos\chi - \omega_e t) \tag{2.6}$$

where Sw is the immersed sectional area of the cross section, and Θw maximum wave slope,

$$F_{\eta_2}' \doteqdot m'\overline{\dot{V}}_\eta = \rho g S w K_\eta' \Theta w \sin \chi e^{-1/2\xi d} \sin(Kx \cos \chi - \omega_e t) \tag{2.7}$$

$$F_{\eta_3}' \doteqdot N_\eta' \overline{V}_\eta = N_\eta' h\omega \sin \chi e^{-1/2\xi d} \cos(Kx \cos \chi - \omega_e t) \tag{2.8}$$

On the other hand, the pressure variation due to waves is

$$P = -\rho g h e^{-k\zeta} \cos(K\xi \cos \chi - k\eta \sin \chi - \omega_e t) \tag{2.9}$$

Now integrating the pressure in (2.9) for a two-dimensional body with the Lewis form section, we will obtain the swaying force based upon the Froude-Krilov's theory as follows:

$$F_{\eta_1}' = \rho g \Theta_w S_w \sin\chi \cdot S \sin(kx \cos\chi - \omega_e t) \tag{2.10}$$

where

$$S = \frac{2}{S_w K \sin\chi}\int_{KD} e^{-KZs} \sin(ky_s \sin\chi) dz_s \tag{2.11}$$

and y_s and z_s are the coordinates of the contour of the Lewis form section with 0 as the origin (Fig. 2).

Errors from the approximate formula (2.6) will become large at a cross section of broader beams. In this paper the formula (2.10) will be used throughout the calculation in which the hull section is to be approximated by the Lewis form section.

On the other hand, there is a value exactly calculated by Tamura [8] concerning the exciting force of waves acting upon a two-dimensional body under restrained condition at beam sea. It must be noted here that an attempt to apply the results of [8] to the case, $\chi \fallingdotseq \pi/2$ and $\omega_e \fallingdotseq \omega$, is not appropriate considering its boundary-conditions. It can be however used approximately for the cases where χ is nearly equal to $\pi/2$, ship velocity is low, and ξ_d is small. But the fourth term of (2.4) cannot be obtained through this method.

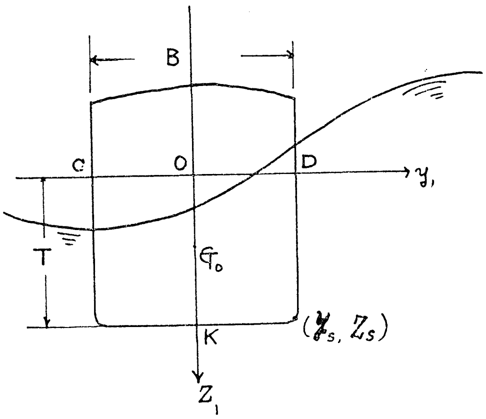


Fig. 2.

In this paper, therefore, the author will also obtain a consistent formula of approximate calculation based upon the idea of (2.4) concerning rolling moment, independently of V, χ and the period of encounter. That is to say, the rolling moment on a restrained body can be written analogously to (2.4) as follows:

$$M_{\theta e}' = M_{\theta 1}' + M_{\theta 2}' + M_{\theta 3}' + M_{\theta 4}' \tag{2.12}$$

Now, $M_{\theta 1}'$ the rolling moment based upon the Froude-Krilov's theory can be obtained for the Lewis form section as:

$$M_{\theta 1}' = [\overline{OG}_0 \cdot S + (P-R)T]\, \rho g S_w \Theta_w \sin\chi \, \sin(Kx \cos\chi - \omega_e t) \tag{2.13}$$

where

$$P-R = \frac{2}{T \cdot S_w K \sin\chi}\left[\int_0^{B/2} e^{-KZ_s} \sin(Ky_s \sin\chi) y_s dy_s\right] - \int_0^T e^{-KZ_s} \sin(Ky_s \sin\chi) Z_s \, dz_s \tag{2.14}$$

Substituting further $(R-P)T/S = l_1$, (2.15)

we obtain

$$M_{\theta 1}' = \rho g S_w \Theta_w \sin\chi \cdot S(\overline{OG}_0 - l_1) \sin(Kx \cos\chi - \omega_e t) = F'_{\eta 1}(\overline{OG}_0 - l_1) \tag{2.16}$$

From the above equation it can be seen that $S(\overline{OG}_0 - l_1)/\overline{G_0 M}$ is the coefficient of effective wave slope γ. Also, (2.13) is one and the same with the principal term of rolling moment (formula (20), p. 82, of [9]) which Dr. Watanabe used for calculating γ.

Both (2.11) and (2.14) can be calculated exactly. For example, Fig. 3 and Fig. 4 show the values of S and P−R for a broad beam section of $H_0 = B/2T = 4.0$ and $\sigma = S_w/BT = 0.8$, while the values of S and $P-R$ in the case of $\chi = \pi/2$ are indicated in Tables I and II.

As can be seen from these tables, $P-R$ is negative for a section of small H_0; however, with H_0 becoming larger it becomes positive, and it grows very

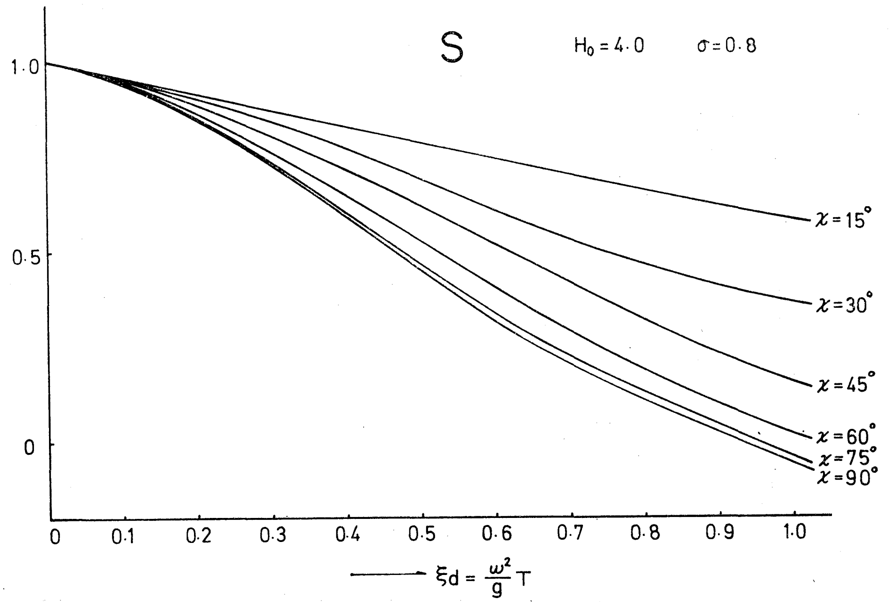


Fig. 3. Calculated values of S for the section of $H_0=B/2T=4.0$ and $\sigma=S_w/BT=0.8$.

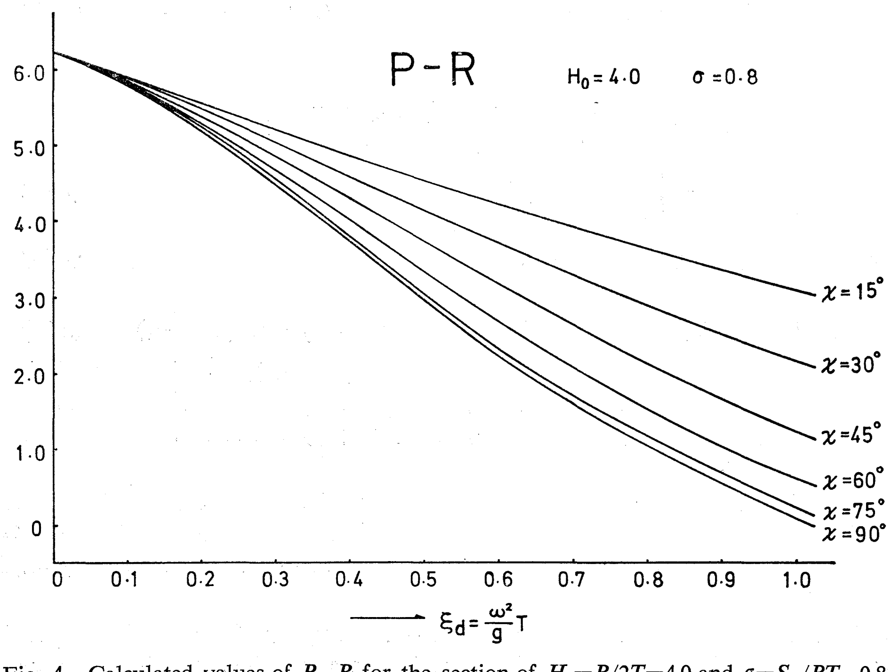


Fig. 4. Calculated values of $P-R$ for the section of $H_0=B/2T=4.0$ and $\sigma=S_w/BT=0.8$.

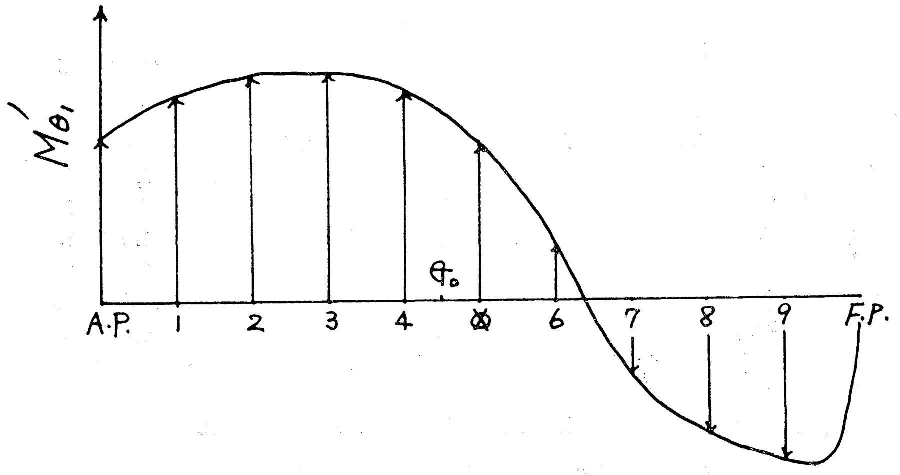


Fig. 5. Longitudinal distribution of the rolling moment based upon Froude-Krilov's theory.

large for $H_0=3$ and 4. Hence in the case of a ship with diversified cross section in longitudinal direction, $M_{\theta 1}'=\rho g S_w \sin\chi \cdot S(\overline{OG_0}-l_1)$ may change its sign in going from fore to aft of G_0(Fig. 5).

Next, the moment of the second and the third terms of (2.12) can be, as discussed in [2], represented by the following approximate formulae:

$$M_{\theta 2}' \doteqdot F_{\eta 2}'(\overline{OG_0}-l_\eta) \tag{2.17}$$

and

$$M_{\theta 3}' \doteqdot F_{\eta 3}'(\overline{OG_0}-l_w) \tag{2.18}$$

Also, similarly to the case of swaying, the fourth term can be written;

$$M_{\theta 4}' \doteqdot -V\frac{d}{dx}\{m'(\overline{OG_0}-l_\eta)\}\dot{V}_\eta \tag{2.19}$$

where l_η and l_w are the moment lever due to swaying force.

When the ship oscillates with the circular frequency of encounter ω_e, the values of K_η', N_η', l_η and l_w will be obtained as the function of ω_e from the TAMURA's table [8].

Though we cannot check in detail the degree of precision of the swaying force and rolling moment thus obtained, in the case of $\chi=\pi/2$, we can investigate it by comparing these with the results of Tamura [8]. One example is shown in Table III.

Through the comparison with Tamura [8] we can derive the following points: the approximate values in this paper show a good coincidence with the cases of full sections, except for the cases of fine sections. The reason for this descrepancy can be ascribed to the failure in the approximation of $K_\eta' e^{-1/2\xi d}$, $N_\eta' e^{-1/2\xi d}$ of swaying force. It is natural, too, that with increasing ξ_B their approximation should grow worse. It is, however, possible to approximate total swaying force and total rolling moment, when $\xi_B \leqq 0.4$, with an error of about 10

Table III.

	H_0=1.2, σ=1.0					H_0=1.0, σ=0.5				
$\xi_B=\omega^2 B/2g$	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
S	1.0	0.917	0.830	0.740	0.656	1.0	0.934	0.869	0.801	0.734
K_η'	0.9699	1.314	1.175	0.694	0.369	1.2324	1.440	1.404	1.190	0.988
$S+K_\eta' e^{-1/2\xi d}$	1.9699	2.126	1.823	1.379	0.995	2.2324	2.237	2.019	1.550	1.393
$\overline{K_r'}$ (Tamura)	1.9699	2.061	1.704	1.170	0.838	2.2324	2.091	1.735	1.331	1.023
$-F_{\eta 3}'$		−0.265	−0.760	−0.835	−0.696		−0.159	−0.409	−0.498	−0.467
$\overline{K_i'}$ (Tamura)		−0.248	−0.645	−0.637	−0.450		−0.141	−0.305	−0.296	−0.174
$(P-R)-K_\eta' e^{-1/2\xi d} l_\eta/T$		−0.253	−0.238	−0.195	−0.144		0.514	0.496	0.435	0.376
$-\overline{K_r'}(l_w/T)$(Tamura)		−0.262	−0.248	−0.187	−0.143		0.490	0.447	0.365	0.299
$\overline{M_{\theta 3}'}$		0.0366	0.109	0.133	0.119		0.037	0.104	0.136	0.130
$-\overline{K_i'}(l_w/T)$(Tamura)		0.0315	0.094	0.102	0.076		0.031	0.078	0.079	0.051

($H_0=B/2T$, σ=Sectional area coefficient)

percent. On the other hand, there is Motora's study [10] on the approximate calculation of swaying force. The author however, will leave here the problems of such corrections in order to carry out calculations by the approximate method introduced in this paper.

Now that we have obtained here the swaying force and rolling moment which act on any cross section of hull, we can obtain by means of the strip method the swaying force, yawing moment and rolling moment, for the whole ship, as follows.

Swaying force:

$$F_{\eta e}=W\Theta_w \sin\chi(F_c \cos\omega_e t-F_s \sin\omega_e t)$$

$$\left.\begin{aligned} F_c&=\overline{S_1}+\overline{S_3}+\overline{S_6}-V\overline{S_8}\\ F_s&=\overline{S_2}+\overline{S_4}-\overline{S_5}+V\overline{S_7}\end{aligned}\right\} \qquad (2.20)$$

Yawing moment:

$$M_{\phi e}=W\Theta_w L \sin\chi(Y_c \cos\omega_e t-Y_s \sin\omega_e t)$$

$$\left.\begin{aligned} Y_c&=\overline{Y_1}+\overline{Y_3}+\overline{Y_6}-V\overline{Y_8}\\ Y_s&=\overline{Y_2}+\overline{Y_4}-\overline{Y_5}+V\overline{Y_7}\end{aligned}\right\} \qquad (2.21)$$

Rolling moment:

$$M_{\theta e}=W\Theta_w \sin\chi(M_c \cos\omega_e t-M_s \sin\omega_e t)$$

$$\left.\begin{aligned} M_c&=F_c\cdot\overline{OG_0}-(\overline{M_1}+\overline{M_3}+\overline{M_6}-V\overline{M_8})\\ M_s&=F_s\cdot\overline{OG_0}-(\overline{M_2}+\overline{M_4}-\overline{M_5}+V\overline{M_7})\end{aligned}\right\} \qquad (2.22)$$

The terms $\overline{S_1}$, $\overline{S_3}$,......$\overline{M_7}$ in the equations (2.20), (2.21) and (2.22) are to be given by the following equations with the substitution of $K\cos\chi=K_1$: (2.23)

$$\overline{S_1}=\int_{-l_1}^{l_2} S_w\cdot S\cdot\sin(K_1 x)dx/V_0$$

$$\left.\begin{aligned}
\overline{S_3}&=\int S_w K_\eta' e^{-1/2\xi d}\sin(K_1x)dx/V_0\\
\overline{S_5}&=\int N_\eta' e^{-1/2\xi d}\sin(K_1x)dx/m_0\omega\\
\overline{S_7}&=\int \left(\frac{dm'}{dx}\right)e^{-1/2\xi d}\sin(K_1x)dx/m_0\omega\\
\overline{Y_1}&=\int S_w\cdot S\cdot x\sin(K_1x)dx/V_0L\\
\overline{Y_3}&=\int S_w\cdot K_\eta' e^{-1/2\xi d}x\sin(K_1x)dx/V_0L\\
\overline{Y_5}&=\int N_\eta' e^{-1/2\xi d}x\sin(K_1x)dx/m_0\omega L\\
\overline{Y_7}&=\int \left(\frac{dm'}{dx}\right)e^{-1/2\xi d}x\sin(K_1x)dx/m_0\omega L\\
\overline{M_1}&=\int S_w(R-P)T\sin(K_1x)dx/V_0\\
\overline{M_3}&=\int S_w K_\eta' e^{-1/2\xi d}\, l_\eta\sin(K_1x)dx/V_0\\
\overline{M_5}&=\int N_\eta' e^{-1/2\xi d}\, l_w\sin(K_1x)dx/m_0\omega\\
\overline{M_7}&=\int \frac{d}{dx}(m'l_\eta)e^{-1/2\xi d}\sin(K_1x)dx/m_0\omega
\end{aligned}\right\}\qquad(2.24)$$

The terms of even suffixes, $\overline{S_2}$, $\overline{S_4}$......$\overline{M_8}$ are to be obtained by substituting $\sin(k_1x)$ in the integrand of $\overline{S_1}$, $\overline{S_3}$......$\overline{M_7}$ respectively with $\cos(k_1x)$. In the equations (2.24), V_0 and m_0 signify the volume of the displacement and mass of the ship respectively.

With a hull shape which is symmetrical in fore and aft about the center of gravity G_0,

$$\overline{S_1}=\overline{S_3}=\overline{S_5}=\overline{S_7}=\overline{Y_2}=\overline{Y_4}=\overline{Y_6}=\overline{Y_8}=\overline{M_1}=\overline{M_3}=\overline{M_5}=\overline{M_7}=0 \qquad (2.25)$$

With the hull shaped quite asymmetrical in fore and aft, the terms of (2.25) may sometimes become considerably large.

Also, referring to Fig. 1, the signs of $\overline{S_1}$, $\overline{S_3}$......$\overline{M_7}$ will be reversed accordind as $\chi<90°$, (i.e. the state of following waves, $k_1>0$), or $\chi>90°$ (i.e. the state of head sea, $k_1<0$).

Now let us consider about rolling moment. In the first place, following the

Froude-Krilov's theory, we get the terms M_c and M_s in equations (2.22) as the following equations:

$$M_{co}=\overline{S}_1\cdot\overline{OG}_0-\overline{M}_1=\int S_w\{S\cdot\overline{OG}_0+(P-R)T\}\sin(k_1x)dx/V_0$$

$$M_{so}=\overline{S}_2\cdot\overline{OG}_0-\overline{M}_2=\int S_w\{S\cdot\overline{OG}_0+(P-R)T\}\cos(k_1x)dx/V_0$$

Between the two states of "following wave" and "head sea", while the sign of M_{co} changes, that of M_{so} doesn't. In this case, however, M_{co} will retain the same absolute value. On the other hand, at beam sea where $\chi=90°$, $M_{co}=0$ will hold. As a result, according to hull shapes, $M_0=\sqrt{M_{co}^2+M_{so}^2}$ may often become larger than in the case of $\chi=90°$ when $\chi=75°$ or $\chi=105°$.

The values of M_0 are equal regardless of whether $\chi=75°$ or $\chi=105°$. That is, M_0 has a symmetrical distribution about $\chi=90°$. On the other hand, as M_{co} changes its sign in going from $\chi=75°$ to $\chi=105°$ or inversely, the phase difference between rolling moment and wave will change its sign too.

As can be seen from the equations, (2.20), (2.21) and (2.22), however, the terms $\overline{S}_5$, $\overline{S}_6$, $\overline{Y}_5$, $\overline{Y}_6$, $\overline{M}_5$ and $\overline{M}_6$ do not vanish even at $V=0$, and therefore swaying force, yawing moment and rolling moment in the case of $\chi=75°$ must differ from the values in the case of $\chi=105°$. This is due to the existence of the terms $N_\eta' e^{-1/2\xi d}$ and $N_\eta' e^{-1/2\xi d}\ l_w$. These values are large in sections of smaller H_0, and become relatively larger in comparison with $K_\eta' e^{-1/2\xi d}$ and $K_\eta' e^{-1/2\xi d}\ l_\eta$ with the increase of ξ_d.

Hence in the case of a hull shape of fore-and-aft asymmetry, rolling moment has different magnitude in two cases of following wave and head sea, even when $V=0$. However, since ξ_d is generally small in the vicinity of rolling's resonance point, the asymmetry of rolling moment about $\chi=90°$ is naturally small.

III. The Coupled Equations of Sway, Yaw and Roll in Oblique Waves

Let η, ϕ and θ be displacements of swaying, yawing and rolling motions respectively.

In the case of $V=0$, the coupled equations of swaying, yawing and rolling in oblique waves can be written as follows as given in [1]:

$$\begin{aligned}
&m_0(1+\overline{K}_\eta)\ddot{\eta}+\overline{N}_\eta\dot{\eta}+m_0\overline{K}_\eta\,\overline{x}_1\ddot{\varphi}+\overline{N}_\eta\,\overline{x}_2\dot{\varphi}+m_0\overline{K}_\eta\overline{x}_4\ddot{\theta}+\overline{N}_\eta\,\overline{x}_5\dot{\theta}=F_{\eta e}\\
&(J_z+I_z)\ddot{\varphi}+\overline{N}_\varphi\dot{\varphi}+m_0\overline{K}_\eta\,\overline{x}_6^2\ddot{\theta}+\overline{N}_\eta\overline{x}_7^2\dot{\theta}+m_0\overline{K}_\eta\,\overline{x}_1\ddot{\eta}+\overline{N}_\eta\,\overline{x}_2\dot{\eta}=M_{\varphi e}\\
&(J_x+I_x)\ddot{\theta}+\overline{N}_{\theta e}\dot{\theta}+W\overline{G_0M}\,\theta+m_0\overline{K}_\eta\,\overline{x}_4\ddot{\eta}+\overline{N}_\eta\,\overline{x}_5\dot{\eta}+m_0\overline{K}_\eta\overline{x}_6^2\ddot{\varphi}+\overline{N}_\eta\overline{x}_7^2\dot{\varphi}=M_{\theta e}
\end{aligned} \tag{3.1}$$

where,

$$m_0\overline{K}_\eta=\int_{-l_1}^{l_2}\rho S_w K_\eta' dx=\int m' dx,\quad \overline{N}_\eta=\int N_\eta' dx$$

$$m_0\overline{K}_\eta\bar{x}_1=\int m'\cdot x\cdot dx,\ \overline{N}_\eta\bar{x}_2=\int N_\eta' x\cdot dx$$

$$m_0\overline{K}_\eta\bar{x}_4=\int m'(\overline{OG}_0-l_\eta)dx,\ \overline{N}_\eta\bar{x}_5=\int N_\eta'(\overline{OG}_0-l_w)dx$$

$$I_z=\int m'x^2dx,\ \overline{N}_\varphi=\int N'_\eta x^2\cdot dx$$

$$m_0\overline{k}_\eta\bar{x}^2{}_6=\int m'(\overline{OG}_0-l_\eta)x\ dx,\ \overline{N}_\eta\bar{x}^2{}_7=\int N_\eta'(\overline{OG}_0-l_w)x\cdot dx \qquad (3.2)$$

J_z=mass moment of inertia of yawing,

J_x+I_x=virtual mass moment of inertia of rolling,

and $\overline{N}_{\theta e}$=equivalent linear damping coefficient of rolling.

Now, using $F_{\eta e}$, $M_{\varphi e}$ and $M_{\theta e}$ (putting $V=0$) as were given in the preceding chapter, we can calculate from (3.1) the motions of swaying, yawing and rolling in oblique waves for the case of $V=0$.

In the case of $V\neq 0$, each term on the left side of (3.1) should be changed in general. As a few examples from the literatures on the calculation of coupled equations for swaying and yawing in the case $V\neq 0$, we can cite the works of Rydill [11], Eda [3], Motora [12], etc. All these authors make their calculation on the assumption that the linear forces and moments which are generally used for the theory of maneuverability in still water can simply be applied to the case of periodic motion, too. Moreover, Eda [3] shows a good coincidence between his calculation and experimental results. On the other hand, A. I. Raff [13] calculated swaying and yawing motions at the time of $V\neq 0$ applying the force derivatives he had obtained regarding the periodic motion at the time of $V=0$: here this author assumed that the values of these derivatives were almost constant regardless of whether $V\neq 0$ or $V=0$. He made further calculations on the lateral bending moment, the results of which show a general coincidence with Eda's calculation [3].

If steering is kept out of consideration, there will be no natural periods for swaying and yawing motions in waves; and in general there occur no such resonance oscillations as are seen in the cases of pitching, heaving and rolling. Therefore, the solutions of forced oscillation can be determined in this case chiefly by exciting force of waves and such inertia forces as $m_0(1+K_\eta)\ddot{\eta}$ and $(J_z+I_z)\ddot{\phi}$; and the effects of damping force and other coupled terms remain only in the second order. That the results of Eda [3] and Raff [13] almost coincide mutually can be explained by the general coincidence of their estimations for both external force and inertia force.

Little is known, however, about whether or not the coefficients on the left sides of (3.1) can be represented by the sum of what is based upon the wave-making phenomenon as discussed in this paper and what is due to the circulation around the hull [14].

Concerning the case when the ship has a finite velocity ($V \neq 0$) and is swaying and yawing periodically, experimental studies on the force and moment derivatives are now being proceeded by both G. Van Leeuwen [15] and Motora [16] with the method of forced oscillation. Here Leeuwen [16] employed a model ship (Todd 60 Series $C_B=0.70$), for which the author of this paper carried out a numerical calculation. Now let us compare the results from both cases.

When the ship has a finite velocity, there arises the swaying inertia force $m_0 V\dot{\phi}$. Taking this force into consideration, therefore, the fourth term of the first equation of (3.1) must be modified to the following form,

$$(m_0 V+\overline{N}_\eta \bar{x}_2)\dot{\varphi} \tag{3.3}$$

The results of Leeuwen's experiment made under the conditions, $F_n=0.20$ and provided with neither rudder nor screw, are compared with the author's calculation, as shown in Table IV.

As can be seen from the table, the tendency of variation of $-N_v$ and

Table IV.

		$\omega=5$ $\xi_d=0.325$	$\omega=8$ $\xi_d=0.832$
Leeuwen	$m_0-Y_{\dot{V}}$ (kg sec²/m)	13.4	8.7
Author	$\overline{K}_\eta$	1.3	0.38
	$m_0(1+\overline{K}_\eta)$ (kg. sec²/m)	15.4	9.2
Leeuwen	$J_z-N_{\ddot{\psi}}$ (kg. m. sec²)	3.8	3.2
Author	I_z/J_z	1.44	0.53
	J_z+I_z (kg. m. sec²)	4.9	3.1
Leeuwen	$-Y_V$ (kg. sec/m)	27	47
Author	$N_\eta\sqrt{gL}/W$	1.8	3.2
	$\overline{N}_\eta$ (kg sec/m)	25.2	44.8
Leeuwen	$-N_{\dot{\psi}}$ (kg. m. sec)	8	13.5
Author	$\overline{N}_\varphi\sqrt{gL}/WL^2$	0.082	0.210
	$\overline{N}_\varphi$ (kg. m. sec)	5.9	15.0
Leeuwen	$-N_V$ (kg. sec)	2	−2
Author	$\overline{N}_\eta \bar{x}_2\sqrt{gL}/WL$	0.041	0.106
	$\overline{N}_\eta \bar{x}_2$ (kg. sec)	1.3	3.35
Leeuwen	$m_0V-Y_{\dot{\psi}}$ (kg. sec)	8.2	10.60
Author	$m_0V+\overline{N}_\eta \bar{x}_2$ (kg. sec)	7.6	9.65

$\overline{N_\eta x_2}$ with respect to ξ_d is different from each other. Although J_z+I_z and $\overline{N_\varphi}$ show an error of about 20% for $\omega=5$, all other terms are roughly coincidental in the order of magnitude. That is to say, a fairly good approximation will be attained merely by introducing $m_0 V\dot{\phi}$ as the effect of ship velocity into the equations (3.1), in which only wave-making resistance is considered.

The results of Leeuwen [15], however, make us assume that within the range, $\xi_d<0.10$, it is better, as shown in [3], [11] and [12], to employ the derivatives for the case, $\omega\to 0$, neglecting wave-making phenomena.

On the other hand, with the case when the ship velocity is not zero, we are almost ignorant about the hydrodynamic rolling moment to be induced from swaying and yawing motions, and everything should be left for future studies.

If a ship has a finite velocity, therefore, one possible approximation at the present stage would be: to modify the forth term of the first equation of (3.1) as shown in (3.3), to calculate the solutions of forced oscillations, and to investigate the mutual coupling influences of swaying, yawing and rolling.

IV. Numerical Calculation and Consideration

In the case of $V=0$, numerical calculations are made, following the methods described in chapters II and III, on three kinds of hull shapes.

A. The Roundhaul Netter of 79 ton type

The main particulars of this boat are $L_{pp}=24.54$ m, $B=5.6$ m and $D=2.5$ m and the body plan is shown in Fig. 6. The calculation was made for the full-loaded conditions of the boat when it starts the fishing-ground, that is:

$W=184.5^{K.T.}$, $C_B=0.584$, ⊠$G_0=-1.86$ m, $G_0M=0.62$ m,
$dm=2.06$ m, $df=1.23$ m, $da=2.89$ m, $J_z=m_0(0.242\ L_{pp})^2$,
natural rolling period $T_\theta \doteqdot 6.0$ sec., equivalent linear damping coefficient $a_e=0.32$ (deduced from the data of [9]).

In the first place, the values of $F_{\eta e}$, $M_{\varphi e}$ and $M_{\theta e}$ were calculated for the wave periods $T_w=5.0$, 6.0 and 6.5 sec. Here $F_{\eta e}$ and $M_{\theta e}$ were found to have maximum value at $\chi=90°$, while $|M_{\varphi e}|$ showed, as in Fig. 7, two maximums in the vicinities of $\chi=45°$ and 135°. Moreover, each of the curves $F_{\eta e}$, $|M_{\varphi e}|$ and $M_{\theta e}$ took almost symmetrical form in relation to $\chi=90°$.

Next, θ_0/Θ_w shown in Fig. 8 is the solution of rolling obtained by Equations (3.1). There is seen hardly any difference in its magnitude between two conditions of head sea and following waves.

Also amplitudes of swaying and yawing showed, almost symmetrical distribution about $\chi=90°$, similarly as in Fig. 8.

Though this hull shape presents a large fore-and-aft asymmetry at a first glance, the rolling moment is distributed almost symmetrically in the x-direction about G_0; and, further in this case, the value of $|M_{\theta e}|$ is very large.

On the other hand, the coupled terms $m_0\overline{K}_\eta\bar{x}_1$, $m_0\overline{K}_\eta\bar{x}_6^2$, etc. are so small that yawing motion can hardly have any influences upon rolling motion, and therefore, it is quite natural that the rolling amplitude has maximum value at $\chi=90°$.

Let us next try a comparison between the numerical calculation based on Tamura's [8] data and the calculation of this paper about the external force in the case, $\chi=90°$ and $Tw=6.0$ sec., the results of which are shown in the following table:

	Tamura's method	this treatise	Froude-Krilov
(1) $\lvert F_{\eta e}\rvert/\Theta_W$	298^t	328^t	
(2) $\lvert M_{\theta e}\rvert/\Theta_W$	$224^{t\cdot m}$	$243^{t\cdot m}$	
(3) $\lvert\overline{M}\rvert/\Theta_W$	$141^{t\cdot m}$ ($\gamma=1.23$)	$154^{t\cdot m}$ ($\gamma=1.34$)	
(4) $\lvert\overline{M}_0\rvert/\Theta_W$			$153^{t\cdot m}$ $\gamma=(1.335)$

Here, the term (3) represents the rolling moment in which swaying inertia force has been taken into consideration, namely

$$\overline{M}=M_{\theta e}-\frac{\overline{K}_\eta(\overline{OG_0}-\bar{l}_\eta)}{1+\overline{K}_\eta}F$$

On the other hand, the term (4) signifies the rolling moment of the Froude-Krilov's theory obtained by employing the exact calculation values for the Lewis form section (Table I, II), namely,

$$\overline{M}_0=\rho g\int S_w[S\cdot\overline{OG_0}+(P-R)T]\,dx$$

As seen from this table, there is made about 10%'s over-estimation on each of $|F_{\eta e}|$ and $|M_{\theta e}|$ in the approximation of this paper.
Furthermore, the coefficient of effective wave slope γ is larger than unity. From a calculation after Dr. Watanabe's approximate method in which mean draft at trimmed conditions is used, we obtained $\gamma=0.727$. This value is about 56% of the values from the term (4). In Dr. Watanabe's method equivalent mean draft is adopted for calculation and there every cross section is assumed to be rectangular for simplification. While, the greater part of the stern of this boat has, with $H_0=B/2T=3\sim4$, generally a wider section. This seems to be the reason why there has been caused such a heavy discrepancy between the results of this treatise and the results by Dr. Watanabe's method.

B. The Submarine Chaser

The profile of this ship is shown in Fig. 9. The main particulars are as follows: $L_{pp}=L_{WL}=59.0$ *m*, $B=7.1$ *m*, $d=2.33$ m, Trim$=0$, $W=480^{K.T.}$, $KG_0=2.707$ *m*, $OG_0=-0.377$ m, $G_0M=0.736$ *m*. It is assumed that the radius of gyration for yawing is $=0.25\ L_{pp}$, and $a_e=0.32$.

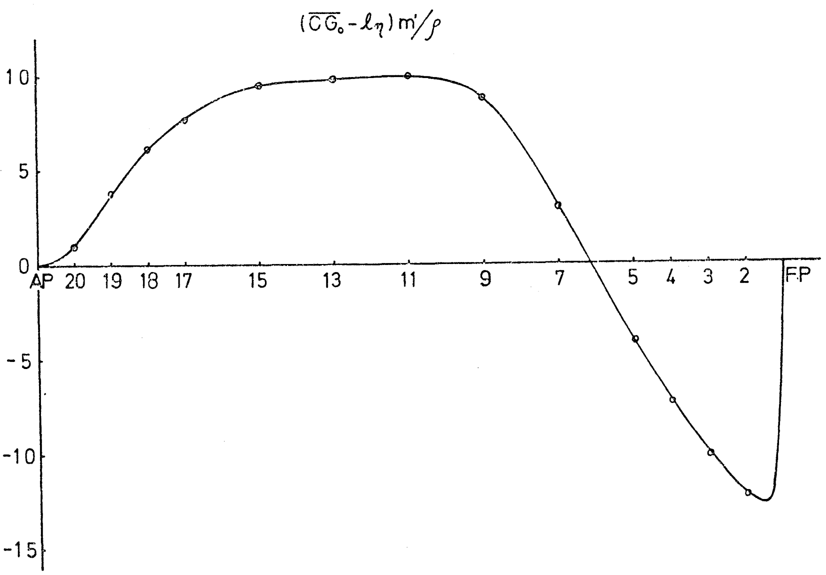


Fig. 10 Longitudinal distribution of the rolling moment derived from swaying inertia force

Assuming $T_\theta \doteqdot 7.0$ sec., calculations were carried out for the case where the ship oscillates in the waves of which $T_W=7$ sec. i. e., $\lambda=76$ m. In Fig. 10 is shown the distribution in the x-direction of the rolling moment due to swaying inertia force about G_0(Fig. 10).

Next, the coefficients on the left side of the equations of motion (3.1) become as follow:

$$
\begin{aligned}
&m_0(1+\overline{K}_\eta)=99^{t\ \mathrm{sec^2/m}}, && \overline{N}_\eta=8^{t.\mathrm{sec/m}},\\
&m_0\overline{K}_\eta\overline{x}_1=254^{t\ \mathrm{sec^2}}, && \overline{N}_\eta\overline{x}_2=19^{t\ \mathrm{sec}},\\
&m_0\overline{K}_\eta\overline{x}_4=21^{t\ \mathrm{sec^2}}, && \overline{N}_\eta\overline{x}_5=4.7^{t\ \mathrm{sec}},\\
&J_z+I_z=21593^{t\ m\ \mathrm{sec^2}}, && \overline{N}_\varphi=1209^{t.m.\mathrm{sec}},\\
&m_0\overline{K}_\eta\overline{x}^2{}_6=-433^{t\ m\ \mathrm{sec^2}}, && \overline{N}_\eta\overline{x}^2{}_7=-43^{t\ m\ \mathrm{sec}},\\
&J_x+I_x=437^{t\ m\ \mathrm{sec^2}}, && \overline{N}_{\theta e}=80^{t.m.\mathrm{sec}}, \qquad (4.1)
\end{aligned}
$$

In contrast with the case of the fishing-boat in the preceding section, $m_0\overline{K}_\eta\overline{x}_1$, $m_0\overline{K}_\eta\overline{x}_6^2$ and $\overline{N}_\eta\overline{x}_7^2$ are very large.

The swaying force, yawing moment and rolling moment under restrained conditions are shown in Fig.s 11, 12 and 13. The distributions of absolute values of both swaying force and yawing moment have almost symmetric form in relation to $\chi=90°$. The same tendency is observed in the distribution of $|M_{\theta e}|$ except that it is slightly double-peaked. However, the curve of the rolling moment based upon the Froude-Krilov's theory has the maximum at $\chi=90°$, and besides, it is of the single-peaked type. In Fig. 13 is shown by a dotted line the rolling moment $\overline{M}$ in which swaying inertia force is taken into consideration. Here the double-peaked type is emphasized.

Shown in Fig.s 14, 15 and 16 are the curves of the amplitudes and phase-differences of swaying, yawing and rolling calculated from the equations (3.1).

First of all, the swaying amplitude curve of Fig. 14 has the maximum at $\chi=90°$. In this ship, following waves cause greater influence upon swaying than head sea. This is mainly due to the coupling effects of yawing.

Next, in the yawing amplitude curve of Fig. 15, the peak at head sea 135° is much greater than that at following waves 45°. Now, from the equations (3.1), we can obtain the yawing moment which has taken into consideration the coupling moments of swaying and rolling, as follows:

$$M_\varphi = M_{\varphi e} - m_0\overline{K}_\eta \overline{x}^2{}_6\, \ddot{\theta} - \overline{N}_\eta \overline{x}^2{}_7 \cdot \dot{\theta} - m_0 \overline{K}_\eta \overline{x}_1 \cdot \ddot{\eta} - \overline{N}_\eta \overline{x}_2 \cdot \dot{\eta} \tag{4.2}$$

Putting further $M_\varphi = \overline{Y}_c \cos \omega t - \overline{Y}_s \sin \omega t$ and inserting the solutions of θ and η into the equation (4.2) we can obtain the $\overline{Y}_c$ and $\overline{Y}_s$. We can find that each of $m_0\overline{K}_\eta\overline{x}^2{}_6\ddot{\theta}$ and $m_0\overline{K}_\eta\overline{x}_1\ddot{\eta}$ has a great influence upon $\overline{Y}_c$ and $\overline{Y}_s$ respectively. Moreover $|\overline{Y}_c|$ is much greater than $|\overline{Y}_s|$. Therefore, due to the fact that $m_0 \overline{K}_\eta\overline{x}^2{}_6\ddot{\theta}$ is not only large but also $m_0\overline{K}_\eta\overline{x}^2{}_6<0$, the value of M_φ in (4.2), and therefore, the peak value of yawing amplitude ϕ_0 at head sea has become much greater than that at following waves.

Now let us consider about rolling motion. Putting first $\phi=0$, and solving the coupled equation of swaying and rolling, we can obtain the amplitude curve, as indicated by black circles in Fig. 16, in an emphasized double-peaked type. The maximum is obtained here at quartering seas $\chi=60°$.

On the other hand the values of θ_0 obtained from equations (3.1) are shown by white circles in Fig. 16, which yield the maximum at $\chi=75°$. However, in this case the amplitude curve becomes flattened, and the degree of asymmetry moderated.

This is due to the fact that, in this ship, although $m_0\overline{K}_\eta\overline{x}^2{}_6$ and $\overline{N}_\eta\overline{x}^2{}_7$ are large and therefore the copuling action of yawing on rolling is large too, the terms $m_0\overline{K}_\eta\overline{x}^2{}_6$, etc. have negative signs and therefore tend to lessen the coupling effects of swaying on rolling.

Next, the phase angle ε_θ' at head sea becomes greater than the one at following waves, for either case of white and black circles. This shows us that, in the case of head sea, the deck edge is nearer to the surface of waves.

On the other hand, the curve of θ_0 based upon the Froude-Krilov's theory becomes that of the symmetrical single-peaked type. Moreover, at $\chi=90°$, we have $\gamma=0.945$. However, if Dr. Watanabe's method is followed, $\gamma=0.823$ will be obtained. The difference between both results amounts to about 15%.

C. The Destroyer

The profile of this ship is shown in Fig. 9. The main particulars are as follows: $L_{pp}=115$ m, $B=12$ m, $d=4$ m, $W=2{,}890^t$, Trim$=0$, $G_0M=1.01$ m. The calculation was made, by assuming $T_\theta \doteqdot 10$ sec., about the case of wave of which $T_w=10$ sec. and $\lambda=156$ m, with the assumption $a_e=0.30$, and $J_z=m_0(0.25L)^2$. Shown in Fig. 17 are the calculation results of rolling amplitude and phase angle.

In the case of this hull shape, the same trend as in the case of the subchaser was observed on any motions of swaying, or rolling.

V Conclusion

The principal purpose of this paper is to investigate whether, in the case of zero ship ship velocity, following waves participate in yielding the maximum value of rolling amplitude or not. In this paper, further discussion was made on the linear equations of motion of swaying, yawing and rolling in oblique waves. We are, however, still in the first stage in this field. The principal conclusions to be obtained from the calculations of this paper may be summarized as follows:

1) Regarding the swaying force, yawing and rolling moments acting on the ship hull in oblique waves, the method discussed in Chapter II gives considerably good approximation.

2) It is ascribable to $N_\eta' e^{-1/2\xi d} l_w$ that the rolling moment acting on the ship hull in following waves at $V=0$ is different from the case in head sea.

3) The coupled equations of motion for swaying, yawing and rolling in the case $V=0$ are given by (3.1). In the case $V \neq 0$, however, it will be assumed that for the coefficients of the swaying and yawing motions, equations (3.2) can be employed approximately, provided that $m_0 V\dot{\phi}$ is taken into consideration.

4) In the case of an Roundhaul Netter, which apparently has a heavy fore-and-aft asymmetry, hardly any difference of rolling amplitude can exist between the two conditions of head sea and following waves. This is because $M_{\theta e}$ is very large, and the cross coupling terms between swaying and yawing or yawing and rolling are all very small.

5) In the cases of the subchaser and the destroyer, the amplitude curves for swaying, yawing and rolling showed asymmetric distributions about $\chi=90°$, due to the fact that $m_0\bar{K}_\eta\bar{x}_1$, $m_0\bar{K}_\eta\bar{x}_6^2$ and $\bar{N}_\eta\bar{x}_7^2$ are large. Yawing showed the maximum peak in head sea due to the inertia couple of rolling. The degree of asymmetric property of rolling about $\chi=90°$, however, decreased on account of yawing. It is to be noted however, that a large peak of rolling would be produced in quartering seas should yawing be restrained by some methods.

6) In the experiments [6] in which yawing was restrained, a rolling peak was produced in quartering seas. It seems to be possible that, according to hull shapes, the coupling actions of swaying and yawing on rolling might be united to produce a peak in quartering seas.

7) We should be careful, as can be seen from the example of the fishing-boat and Table III, that the rolling moment of the Froude-Krilov's theory might become very large for a ship with sections of large H_0 value.

8) The transverse bending moment, twisting moment, etc. of ships in oblique waves can be calculated from equations (3.1).

We heard Dr. Watanabe often referred in his lecture to a hull shape yielding the maximum rolling in quartering seas. It is concluded that a reason thereof might consist in the coupling action of swaying and yawing on rolling, as discussed in this paper. It seems to be necessary hereafter to investigate, on various types of hull shapes, the coupling action by means of calculation and to examine it experimentally too.

In concluding this paper, the author heartily appreciates kind cooperations of Mr. Tsuchiya of the Fishing-boat Laboratory, Mr. Kuwabara and Mr. Yoshimoto, both belonging to the Usuki Iron Woorks, Mr. Kaneko, engineer of the Kure Dockyard, who presented lines and other data to the author. Also thanks are due to Miss Okazaki and Miss Mori who assisted the author in numerical calculation.

VI. References

[1] F. Tasai: "Ship Motions in Beam Seas" Report of the Research Institute for Applied Mechanics, Kyushu University Vol. XIII, No. 45, 1965

[2] F. Tasai: "On the Equation of Rolling Motion of a Ship", Bulletin of the Research Institute for Applied Mechanics, Kyushu University, Vol. 25, 1965

[3] H. Eda and C. Lincoln Crame: "Steering Characteristics of Ships in Calm Water and Waves", S. N. A. M. E., 1965

[4] O. Grim and Y. Takaishi: "Das Rollmoment in Schtäglaufender Welle", Schiff und Hafen, Heft 10, 1965

[5] "Shipbuilding handbook, Vol. I" (published by Society of Naval Architects of Japan), page 570

[6] K. Taniguchi: "Experimental Results in Waves on a model of Fishing Boat", Journal of SEIBU ZŌSENKAI, No. 13, 1957

[7] Y. Watanabe: "On the Theory of Heaving and Pitching Motions of a Ship", Technology Report of the Kyushu University, Vol. 31, No. 1, June 1958

[8] K. Tamura: "The Calculation of Hydrodynamic Forces and Moments acting on the Two-Dimensional Body—According to the Grim's Theory—", Journnl of SEIBU ZŌSENKAI, No. 26, Sept. 1963

[9] Y. Watanabe: "On the Motion of the Centre of Gravity of Ships and Effective Wave Slope", Journal of Society of Naval Architects of Japan, Vol. 49, 1932

[10] S. Motora: "Stripwise Calculation of Hydrodynamic Forces Due to Beam Seas", Journal of Ship Research, June 1964

[11] L. J. Rydill: "A Linear Theory for the Steered Motion of Ships in Waves", T. I. N. A., 1959

[12[S. Motora and T. Koyama: "On the Improvement of the Maneuverability of Ships by means of Automatic Steering", J. S. N. A. of Japan, No. 116, 1964

[13] A. I. Raff: "The Dynamic Calculation of Lateral Bending Moments on Ships in Oblique Waves", T R G Report 147, 1964

[14] F. Tasai: "A Note on the Swaying, Yawing and Rolling Motions in Oblique Wa-

ves", Report in Technical Committee of SEIBU ZŌSENKAI, 1965
[15] G. Van Leeuwen: "The Lateral Damping and Added Mass of a Horizontally Oscillating Ship Model", T N O Report No. 65 S, 1964
[16] S. Motora and M. Fujino: "On the Measurement of the Stability Derivatives by means of Forced Yawing Technique", J. S. N. A. of Japan, No. 118, 1965

(Received September 13, 1966)

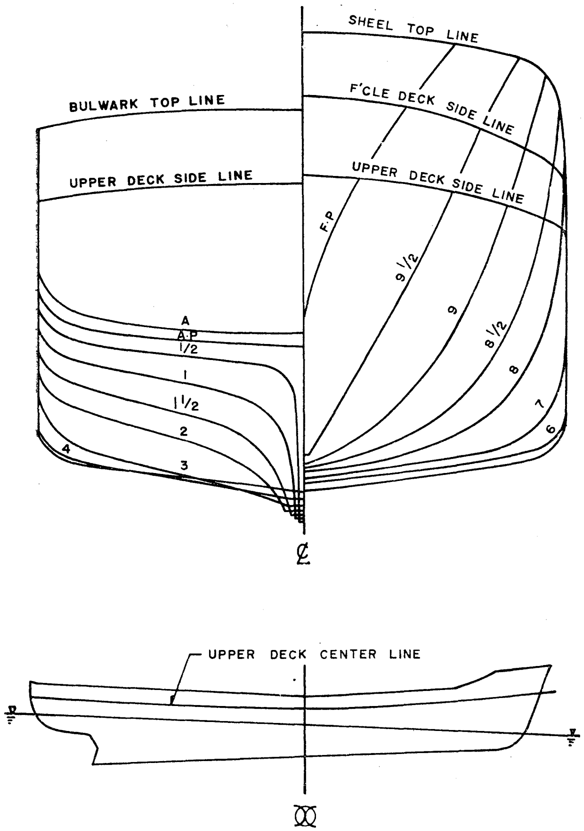


Fig. 6 The body plan and profile of the Roundhaul netter

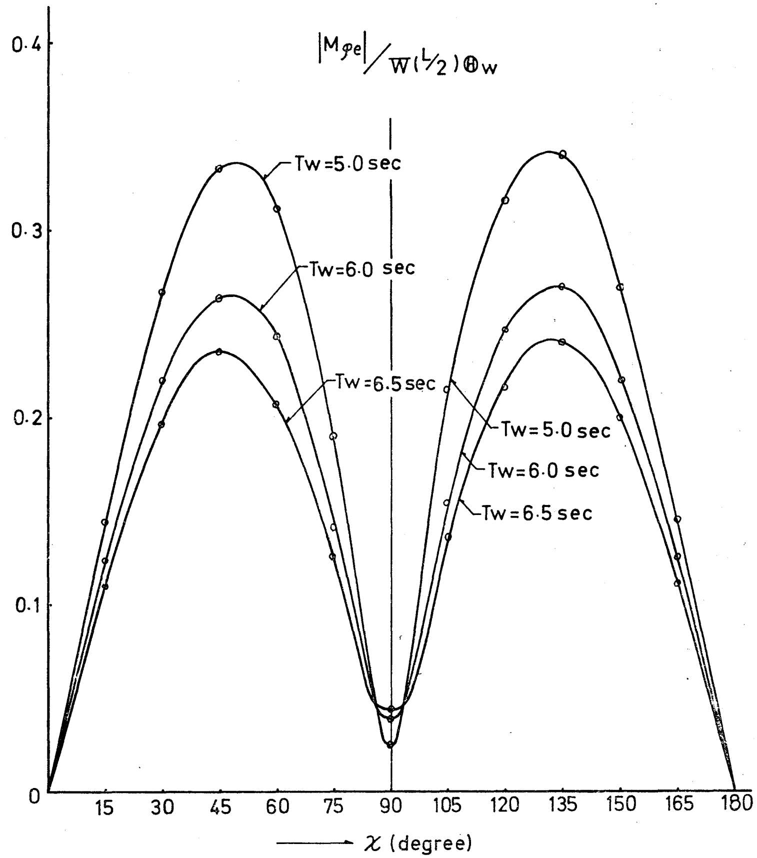


Fig. 7 Yawing moment

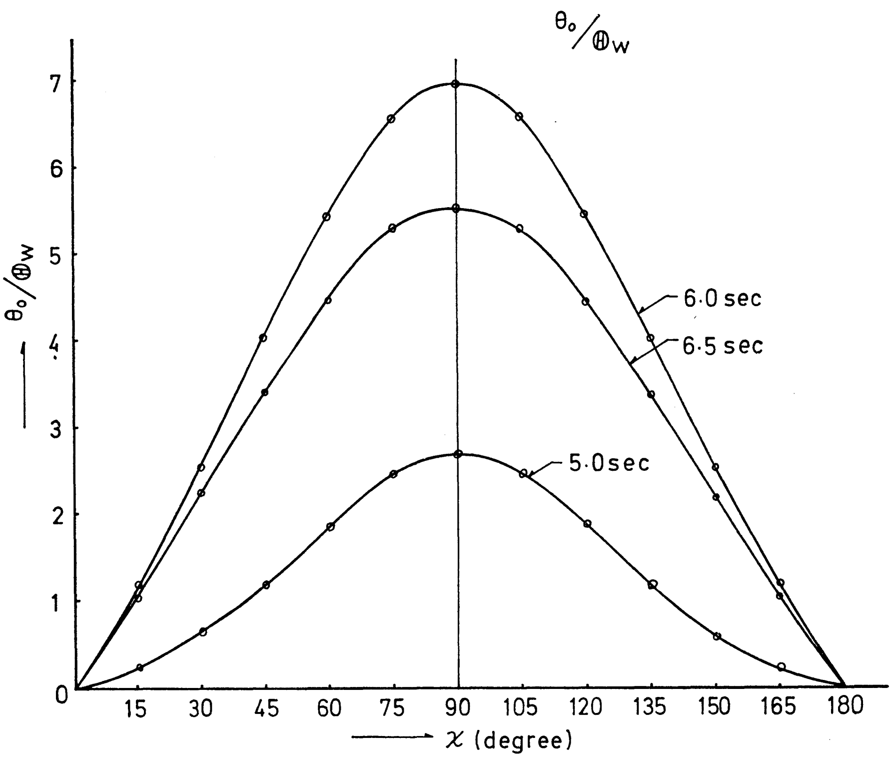


Fig. 8 Rolling amplitude

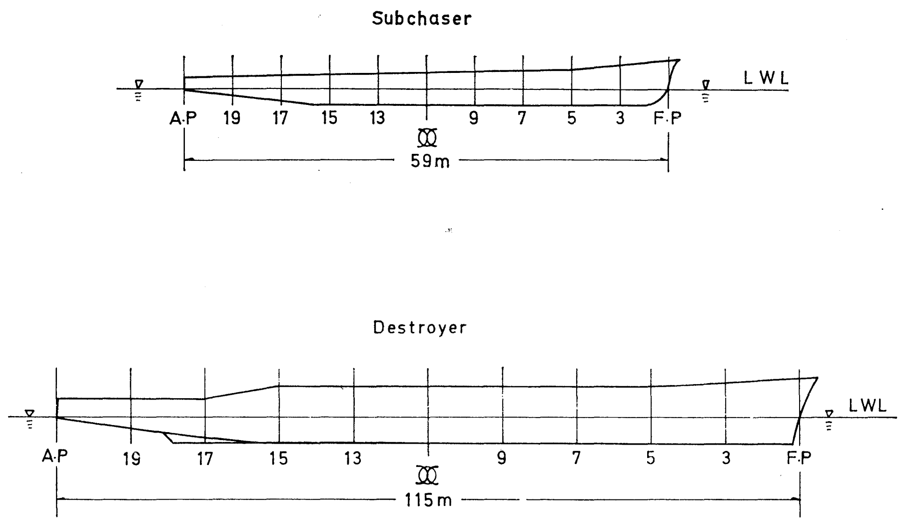


Fig. 9 Profiles of the Submarine-chaser and the Destroyer

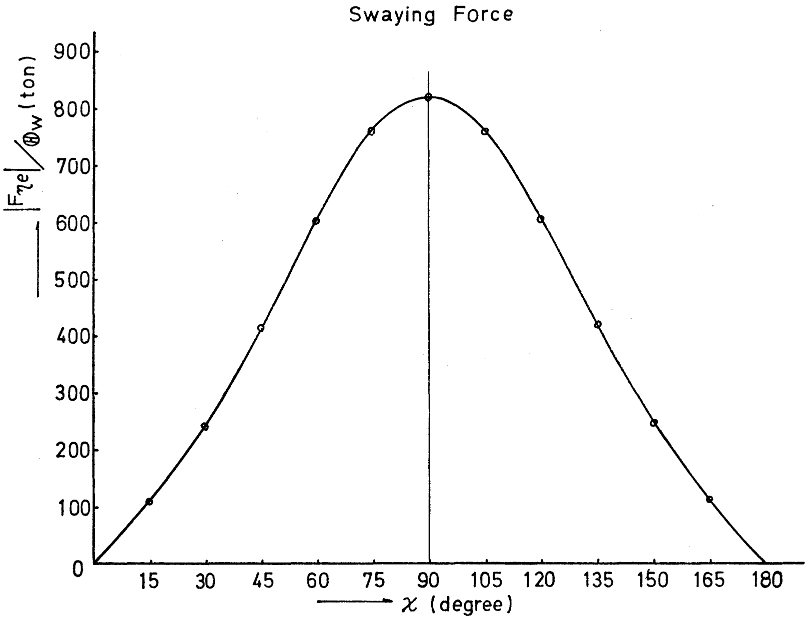

Swaying Force
$|F_{\eta e}|/\Theta_w$ (ton)
900
800
700
600
500
400
300
200
100
0
15 30 45 60 75 90 105 120 135 150 165 180
χ (degree)

Fig. 11

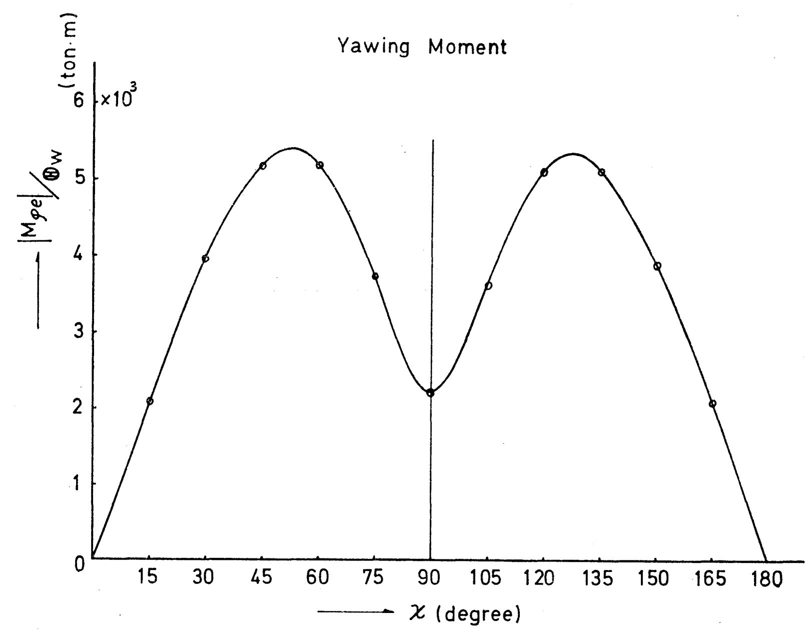
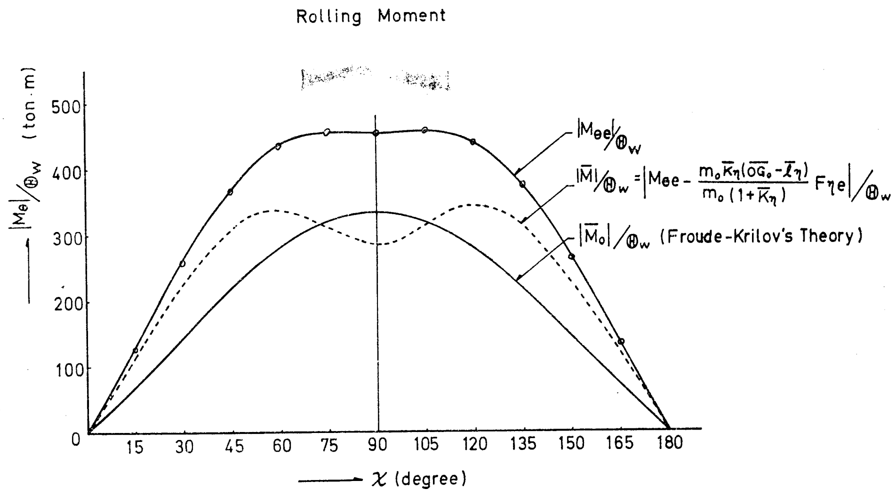
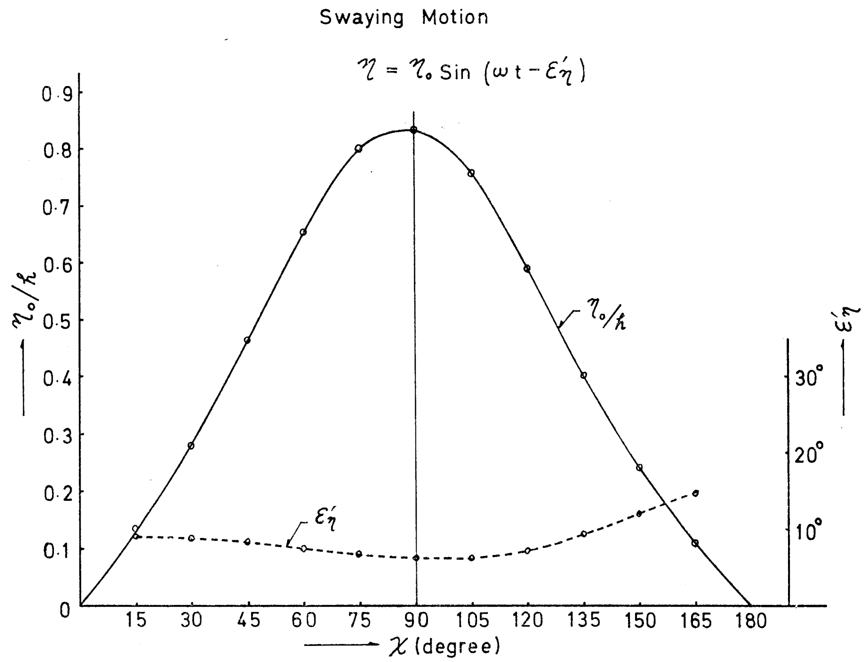

Yawing Moment
$|M_{\varphi e}|/\Theta_w$ (ton·m)
$\times 10^3$
6
5
4
3
2
1
0
15 30 45 60 75 90 105 120 135 150 165 180
χ (degree)

Fig. 12



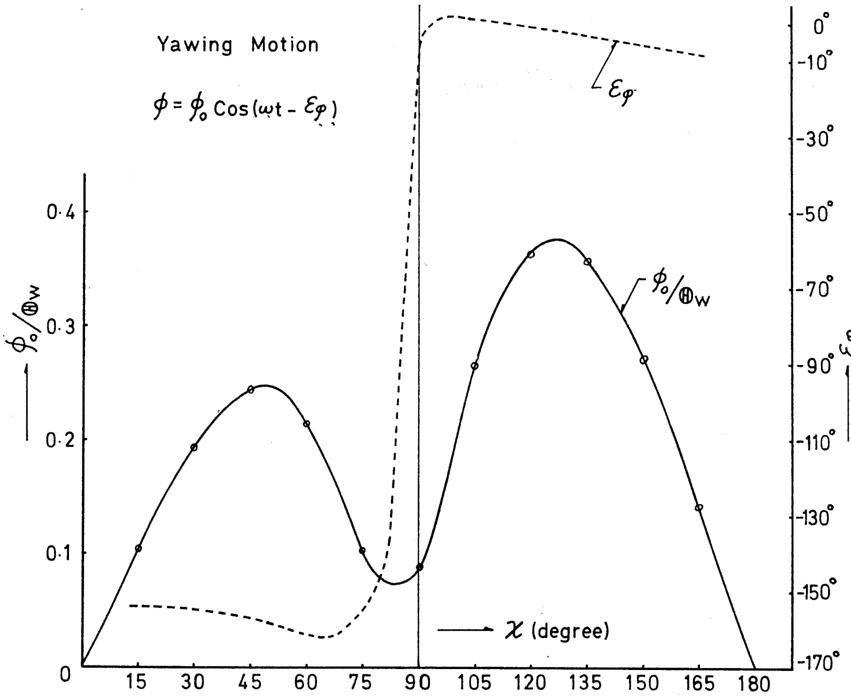
Rolling Moment
500
400
300
200
100
0
15 30 45 60 75 90 105 120 135 150 165 180
χ (degree)
|Mθ|/Θw (ton·m)
|Mθe|/Θw
|M̄|/Θw = |Mθe − m₀K̄η(OG₀−l̄η) / m₀(1+K̄η) Fηe|/Θw
|M̄₀|/Θw (Froude-Krilov's Theory)

Fig. 13



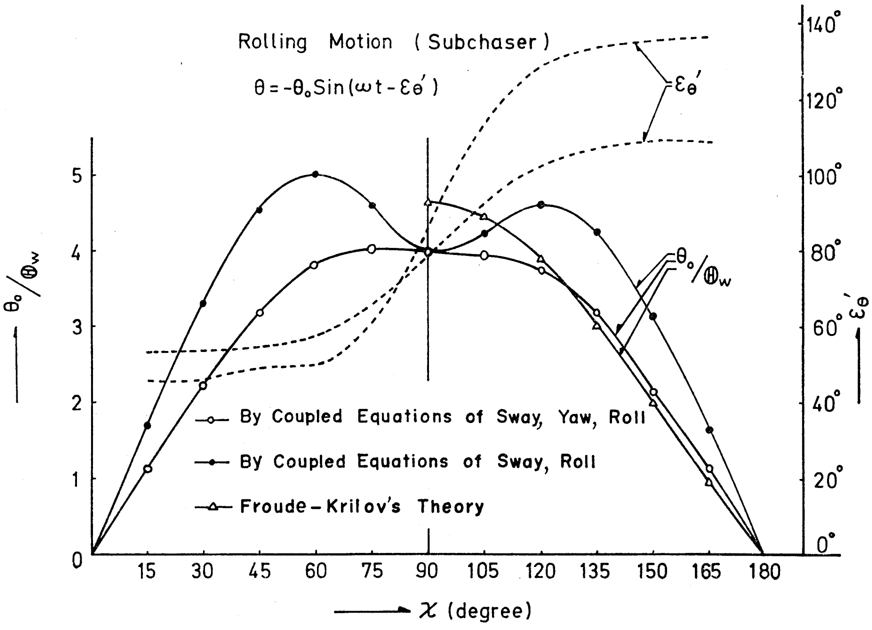
Swaying Motion
η = η₀ Sin (ωt − ε′η)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
15 30 45 60 75 90 105 120 135 150 165 180
χ (degree)
η₀/h
ε′η
30°
20°
10°

Fig. 14



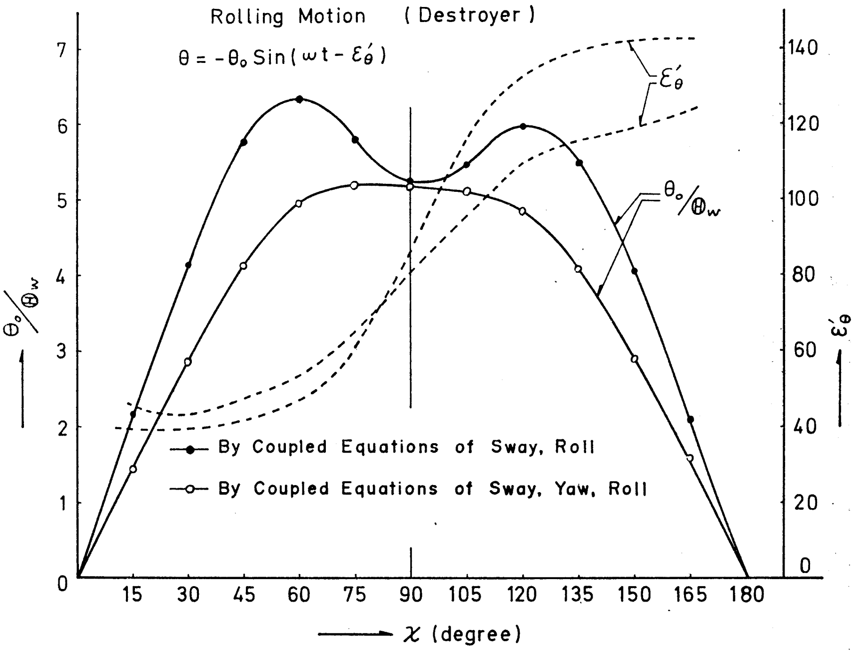
Yawing Motion
φ = φ₀ Cos(ωt - εφ)
εφ
φ₀/Θw
→ χ (degree)
0.4
0.3
0.2
0.1
0
15 30 45 60 75 90 105 120 135 150 165 180
0° -10° -30° -50° -70° -90° -110° -130° -150° -170°

Fig. 15



Rolling Motion (Subchaser)
θ = -θ₀Sin(ωt - εθ')
εθ'
θ₀/Θw
By Coupled Equations of Sway, Yaw, Roll
By Coupled Equations of Sway, Roll
Froude-Krilov's Theory
5 4 3 2 1 0
15 30 45 60 75 90 105 120 135 150 165 180
→ χ (degree)
140° 120° 100° 80° 60° 40° 20° 0°

Fig. 16



Rolling Motion (Destroyer)
θ = -θ₀ Sin(ωt - ε′θ)
ε′θ
θ₀/Θw
By Coupled Equations of Sway, Roll
By Coupled Equations of Sway, Yaw, Roll
0
1
2
3
4
5
6
7
0
20
40
60
80
100
120
140
15
30
45
60
75
90
105
120
135
150
165
180
χ (degree)

Fig. 17

Table I S (χ=90°)

H_0=0.2

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.9540	0.910	0.828	0.756	0.635
0.9	1.0	0.956	0.914	0.837	0.768	0.652
0.8	1.0	0.959	0.920	0.847	0.782	0.671
0.7	1.0	0.962	0.927	0.859	0.798	0.693
0.6	1.0	0.965	0.934	0.873	0.816	0.719
0.5	1.0	0.972	0.944	0.891	0.841	0.755

H_0=0.4

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.954	0.908	0.824	0.749	0.620
0.9	1.0	0.956	0.913	0.834	0.763	0.639
0.8	1.0	0.960	0.919	0.845	0.777	0.660
0.7	1.0	0.962	0.925	0.856	0.793	0.682
0.6	1.0	0.966	0.933	0.870	0.811	0.707
0.5	1.0	0.970	0.940	0.886	0.833	0.738

H_0=0.6

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.953	0.906	0.819	0.739	0.598
0.9	1.0	0.956	0.912	0.830	0.754	0.619
0.8	1.0	0.959	0.918	0.841	0.770	0.642
0.7	1.0	0.962	0.924	0.853	0.786	0.665
0.6	1.0	0.968	0.933	0.867	0.805	0.691
0.5	1.0	0.972	0.939	0.880	0.823	0.718

H_0=0.8

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.952	0.904	0.812	0.725	0.569
0.9	1.0	0.956	0.910	0.824	0.742	0.592
0.8	1.0	0.958	0.917	0.836	0.759	0.617
0.7	1.0	0.962	0.923	0.848	0.776	0.641
0.6	1.0	0.965	0.929	0.861	0.794	0.667
0.5	1.0	0.969	0.937	0.875	0.813	0.695

H_0=1.0

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.951	0.901	0.803	0.709	0.531
0.9	1.0	0.953	0.908	0.817	0.728	0.560
0.8	1.0	0.957	0.915	0.830	0.746	0.586
0.7	1.0	0.961	0.921	0.843	0.764	0.613
0.6	1.0	0.964	0.929	0.854	0.782	0.640
0.5	1.0	0.967	0.934	0.869	0.801	0.668

H_0=1.2

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.949	0.898	0.793	0.691	0.492
0.9	1.0	0.953	0.906	0.808	0.711	0.521
0.8	1.0	0.957	0.912	0.822	0.730	0.551
0.7	1.0	0.961	0.920	0.836	0.750	0.579
0.6	1.0	0.963	0.925	0.847	0.768	0.608
0.5	1.0	0.967	0.932	0.861	0.788	0.640

H_0=1.4

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.949	0.895	0.783	0.668	0.447
0.9	1.0	0.953	0.903	0.798	0.690	0.478
0.8	1.0	0.957	0.910	0.813	0.712	0.508
0.7	1.0	0.960	0.917	0.827	0.733	0.540
0.6	1.0	0.963	0.924	0.840	0.753	0.573
0.5	1.0	0.966	0.930	0.854	0.774	0.609

H_0=1.6

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.949	0.892	0.771	0.642	0.398
0.9	1.0	0.952	0.900	0.787	0.668	0.430
0.8	1.0	0.956	0.908	0.803	0.691	0.463
0.7	1.0	0.959	0.915	0.818	0.714	0.498
0.6	1.0	0.962	0.921	0.833	0.736	0.535
0.5	1.0	0.965	0.927	0.846	0.759	0.576

H_0=2.0

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.947	0.884	0.742	0.588	0.296
0.9	1.0	0.951	0.892	0.760	0.615	0.328
0.8	1.0	0.954	0.901	0.778	0.642	0.365
0.7	1.0	0.958	0.909	0.796	0.669	0.406
0.6	1.0	0.961	0.916	0.813	0.697	0.452
0.5	1.0	0.963	0.922	0.829	0.726	0.504

H_0=3.0

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.938	0.855	0.647	0.418	0.042
0.9	1.0	0.943	0.866	0.672	0.452	0.074
0.8	1.0	0.948	0.878	0.694	0.490	0.115
0.7	1.0	0.953	0.888	0.722	0.530	0.166
0.6	1.0	0.956	0.898	0.750	0.576	0.233
0.5	1.0	0.959	0.908	0.780	0.630	0.322

H_0=4.0

σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	1.0	0.927	0.818	0.528	0.230	−0.114
0.9	1.0	0.934	0.831	0.561	0.269	−0.092
0.8	1.0	0.940	0.846	0.592	0.315	−0.059
0.7	1.0	0.945	0.860	0.629	0.369	−0.012
0.6	1.0	0.950	0.875	0.671	0.435	0.056
0.5	1.0	0.955	0.894	0.720	0.520	0.155

Table II-1 *P–R* (χ=90°)

	H_0=0.2						H_0=0.4					
σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0	0	0.1	0.2	0.4	0.6	1.0
1.0	−0.4720	−0.4433	−0.4165	−0.3677	−0.3254	−0.2560	−0.4367	−0.4109	−0.3869	−0.3428	−0.3036	−0.2379
0.9	−0.4445	−0.4179	−0.3937	−0.3489	−0.3099	−0.2457	−0.4010	−0.3777	−0.3559	−0.3161	−0.2807	−0.2215
0.8	−0.4128	−0.3895	−0.3672	−0.3269	−0.2915	−0.2334	−0.3629	−0.3427	−0.3227	−0.2870	−0.2555	−0.2029
0.7	−0.3760	−0.3556	−0.3363	−0.3012	−0.2703	−0.2187	−0.3196	−0.3014	−0.2845	−0.2537	−0.2263	−0.1806
0.6	−0.3303	−0.3132	−0.2979	−0.2689	−0.2433	−0.2001	−0.2684	−0.2533	−0.2396	−0.2138	−0.1912	−0.1532
0.5	−0.2715	−0.2596	−0.2490	−0.2272	−0.2082	−0.1757	−0.2039	−0.1924	−0.1826	−0.1637	−0.1462	−0.1174

	H_0=0.6						H_0=0.8					
σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0	0	0.1	0.2	0.4	0.6	1.0
1.0	−0.3738	−0.3539	−0.3349	−0.2987	−0.2654	−0.2070	−0.2832	−0.2713	−0.2575	−0.2356	−0.2111	−0.1645
0.9	−0.3277	−0.3099	−0.2928	−0.2610	−0.2322	−0.1821	−0.2246	−0.2142	−0.2043	−0.1842	−0.1650	−0.1293
0.8	−0.2795	−0.2633	−0.2485	−0.2208	−0.1979	−0.1536	−0.1626	−0.1535	−0.1460	−0.1288	−0.1139	−0.0879
0.7	−0.2247	−0.2111	−0.1981	−0.1745	−0.1535	−0.1189	−0.0918	−0.0843	−0.0766	−0.0644	−0.0537	−0.0368
0.6	−0.1619	−0.1506	−0.1396	−0.1204	−0.1037	−0.0767	−0.0078	−0.0012	0.0043	0.0131	0.0199	0.0278
0.5	−0.0803	−0.0727	−0.0639	−0.0495	−0.0378	−0.0196	0.0987	0.1024	0.1093	0.1126	0.1151	0.1134

	H_0=1.0						H_0=1.2					
σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0	0	0.1	0.2	0.4	0.6	1.0
1.0	−0.1650	−0.1640	−0.1618	−0.1536	−0.1416	−0.1119	−0.0193	−0.0347	−0.0419	−0.0538	−0.0585	−0.0519
0.9	−0.0923	−0.0917	−0.0910	−0.0869	−0.0810	−0.0661	0.0704	0.0586	0.0482	0.0316	0.0195	0.0052
0.8	−0.0124	−0.0123	−0.0121	−0.0118	−0.0110	−0.0093	0.1716	0.1607	0.1499	0.1297	0.1111	0.0792
0.7	0.0779	0.0776	0.0770	0.0746	0.0707	0.0605	0.2882	0.2781	0.2675	0.2446	0.2194	0.1776
0.6	0.1905	0.1900	0.1888	0.1834	0.1744	0.1529	0.4330	0.4238	0.4138	0.3890	0.3599	0.2927
0.5	0.3338	0.3334	0.3309	0.3240	0.3109	0.2761	0.6224	0.6140	0.6050	0.5789	0.5447	0.4618

Table II-2 P-R (χ=90)

	H_0=1.4						H_0=1.6					
σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0	0	0.1	0.2	0.4	0.6	1.0
1.0	0.1522	0.1240	0.1000	0.0618	0.0368	0.0123	0.3460	0.2988	0.2586	0.1910	0.1400	0.0769
0.9	0.2611	0.2356	0.2106	0.1681	0.1330	0.0812	0.4849	0.4419	0.4006	0.3256	0.2607	0.1588
0.8	0.3880	0.3640	0.3399	0.2936	0.2496	0.1712	0.6588	0.6169	0.5752	0.4930	0.4138	0.2711
0.7	0.5361	0.5143	0.4907	0.4422	0.3907	0.2866	0.8221	0.7861	0.7478	0.6656	0.5783	0.4016
0.6	0.7208	0.7010	0.6799	0.6292	0.5708	0.4400	1.0530	1.0215	0.9858	0.9013	0.8036	0.5865
0.5	0.9652	0.9492	0.9288	0.8780	0.8123	0.6518	1.3618	1.3360	1.3030	1.2175	1.1097	0.8456

	H_0=2.0						H_0=3.0					
σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0	0	0.1	0.2	0.4	0.6	1.0
1.0	0.8295	0.7379	0.6525	0.5023	0.3783	0.2037	2.4948	2.2391	1.9768	1.4647	1.0073	0.3541
0.9	1.0176	0.9324	0.8484	0.6875	0.5404	0.2998	2.8705	2.6289	2.3667	1.8137	1.2674	0.4333
0.8	1.2386	1.1604	1.0783	0.9094	0.7400	0.4298	3.3232	3.0983	2.8365	2.2122	1.6182	0.5511
0.7	1.5079	1.4366	1.3584	1.1827	0.9914	0.6051	3.8904	3.6830	3.4235	2.7833	2.0627	0.7238
0.6	1.8516	1.7898	1.7150	1.5335	1.3189	0.8455	4.6289	4.4449	4.1874	3.4976	2.6578	0.9779
0.5	2.3170	2.2800	2.2090	2.0330	1.7787	1.1956	5.6435	5.4864	5.2329	4.4799	3.4906	1.3621

	H_0=4.0					
σ \ ξ_d	0	0.1	0.2	0.4	0.6	1.0
1.0	4.8272	4.3148	3.7436	2.5422	1.4656	0.1640
0.9	5.4651	4.9770	4.3882	3.0665	1.7920	0.1378
0.8	6.2433	5.7802	5.1774	3.7209	2.2166	0.1211
0.7	7.2239	6.7897	6.1716	4.5597	2.7801	0.1196
0.6	8.5158	8.1150	7.4789	5.6738	3.5482	0.1429
0.5	10.3068	9.9486	9.2993	7.2237	4.6397	0.2100