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ON THE SIMPLIFIED CALCULATION OF CHARACTERISTIC EQUATIONS FOR UNSTEADY FLOW IN OPEN CHANNELS*

By Toshihiko UEDA**

Abstracts

The method of characteristics in the calculation of unsteady flow is a excellent graphic solution induced by Massau through an ingenious mathemati cal process. This method, however, requires troublesome trial calculations, for there are too many equations to be satisfied in the process of evaluating each point on the characteristics of $x \sim t$ plane.

In the present report, the author proposed a method to calculate without trial the points other than those on t axis of $x \sim t$ plane to which boundary conditions are given. In the calculation, there are taken simultaneously the two known points of up and down streams at which two characteristics start and there U (function of water depth)$\sim t$ graph and V (mean velocity)$\sim t$ graph are utilized. And he simplified the graphical solution depending on the characteristics of unsteady flow and furthermore, applied the method of calculation to the flood experiment of the Shintakase River, Kyoto City and showed that the calculated results agree nearly with the experiment.

1. Introduction

There is no perfect solution reported so far for the equations of motion and continuity in the calculation of unsteady flow, nor is it an easy task to carry out direct numerical integration. The method of characteristics[1] belongs to this calculation of unsteady flow, which is a graphic solution induced by Massau through an ingenious numerical process. Even with this method, however, there is involved complicated trial. For it has two many equations to be satisfied, if started on a wrong assumption. Later on, the calculation was considerably simplified by Pin-Nam Lin[2] who proposed a method to calculate with Δt as constant in 1952. Still it seems to take much time for calculation, since its trial part retains.

In the present report, the author considered that he could simplify the calculation considerably by letting Δt be constant and attempted to draw a simplified graphic calculation of unsteady flow. That is, the author proposed a method

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of calculation without trial, except for the points on t axis of x-t plane to which boundary conditions are given (this axis will be hereafter expressed as t axis). Furthermore, he applied this method to the result of the flood experiment of the Shintakase River in Kyoto, Japan.

2. Fundamental equation

If mean velocity is assumed as V, water depth as h, cross-sectional area as A, discharge as Q, acceleration of gravity as g, bottom slope as S_0, so-called friction slope as S_f, distance along the stream as x, and time as t, the equation of motion and continuity of unsteady flow will be

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial h}{\partial x}=g(S_0-S_f) \qquad \cdots\cdots\cdots(1)$$

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \qquad \cdots\cdots\cdots(2)$$

Putting now as $A=Wh^m$ and assuming coefficient W as function of x and index number m as constant, we will get from the equation of continuity (2)

$$m\frac{\partial h}{\partial t}+h\frac{\partial V}{\partial x}+mV\frac{\partial h}{\partial x}+\frac{Vh}{W}\frac{dW}{dx}=0 \qquad \cdots\cdots\cdots(3)$$

Multiplying equation (3) by $\sqrt{g/mh}$ and taking the sum and the balance of the resultant and equation (1), we will have

$$\left\{\frac{\partial}{\partial t}+(V\pm\sqrt{gh/m})\frac{\partial}{\partial x}\right\}(V\pm 2m\sqrt{gh/m})=g(S_0-S_f)\mp\frac{V\sqrt{gh/m}}{W}\frac{dW}{dx} \qquad \cdots\cdots\cdots(4)$$

Putting as $\sqrt{gh/m}=U$ and representing the above equation with the equation of characteristics,

$$C_f : dx/dt=V+U \qquad \cdots\cdots\cdots(5)$$

$$\frac{d}{dt}(V+2mU)=g(S_0-S_f)-\frac{VU}{W}\frac{dW}{dx} \qquad \text{(along a curve } C_f)\cdots(6)$$

$$C_b : dx/dt=V-U \qquad \cdots\cdots\cdots\cdots(7)$$

$$\frac{d}{dt}(V-2mU)=g(S_0-S_f)+\frac{VU}{W}\frac{dW}{dx} \qquad \text{(along a curve } C_b)\cdots(8)$$

where, according to Manning formula, S_f will be $S_f=n^2V^2/R^{4/3}=n^2V^2/f(U)\cdots(9)$ and C_f and C_b are forward and backward characteristics respectively. Thesolution will be obtained by evaluating U, V at the crossing point of the characteristics of equations (5) and (7) on $x\sim t$ plane. Consideration will be forwarded here for the case of ordinary flow, that is $V<U$ and the case of equivalent section throughout upstream and downstream, that is, $dW/dx=0$.

In Fig. 1, a part of the characteristics of $x\sim t$ plane is given. P_u denotes the point on the upstream side and P_d that on the downstream side, and it is assumed that the characteristics C_f and C_b that started P_u, P_d cross each other at P.

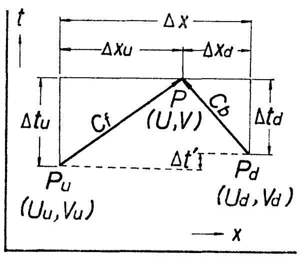

Fig. 1. Characteristics on x-t plane

If equation (6) is expressed by a difference equation in accordance with this figure and with respect to the case of $dW/dx=0$,

$$(V+2mU)-(V_u+2m\,U_u)=gS_0\Delta t_u-g\int_t^{t+\Delta tu} S_f dt \fallingdotseq gS_0\Delta t_u-g\frac{S_{fu}+S_f}{2}\Delta t_u$$

$$=K_u\Delta t_u+K\Delta t_u \cdots\cdots\cdots(10)$$

Here, $$\left.\begin{array}{l} K_u=(S_0-S_{fu})g/2=(S_0-n^2V_u^2/R_u^{4/3})g/2, \\ K=(S_0-S_f)g/2=(S_0-n^2V^2/R^{4/3})g/2 \end{array}\right\}\cdots\cdots\cdots(11)$$

and n=roughness coefficient of Manning formula.

Expressing similarly equations (5)$\sim$(8) with difference equations,

$$\Delta x_u=(V_u+U_u+V+U)\Delta t_u/2 \qquad \cdots\cdots\cdots(12)$$

$$V+2mU-K\Delta t_u=V_u+2mU_u+K_u\Delta t_u \qquad \cdots\cdots\cdots(13)$$

$$\Delta x_d=(V_d-U_d+V-U)\Delta t_d/2 \qquad \cdots\cdots\cdots(14)$$

$$V-2mU-K\Delta t_d=V_d-2mU_d+K_d\Delta t_d \qquad \cdots\cdots\cdots(15)$$

Here K_d conforms to equation (11).

Furthermore, since the position of P_u, P_d on $x\sim t$ plane is given, Δx and $\Delta t'$ represented by

$$\Delta x=\Delta x_u-\Delta x_d \qquad \cdots\cdots\cdots(16)$$

$$\Delta t'=\Delta t_u-\Delta t_d \qquad \cdots\cdots\cdots(17)$$

are already known. The method of characteristics is to evaluate U, V of P and its position on $x\sim t$ plane, that is, Δt_u, Δt_d, Δx_u and Δx_d by giving U, V of P_u, P_d and their positions from equations (12)$\sim$(17).

For that, we generally adopt a method to assume gradients of characteristics C_f and C_b, evaluate Δx_u, Δx_d, Δt_u, Δt_d from the graph of the characteristics on $x\sim t$ plane, calculate U, V of P from equations (13), (15) by them and check lastly the gradients of C_f, C_b from equations (12), (14).
This is however troublesome trial, since the equations to be satisfied are many.

3. Method proposed by the author

(i). Method to calculate U, V at the point other than t-axis.

In Fig. 2, we assume that characteristic C is known, that is, U, V of points

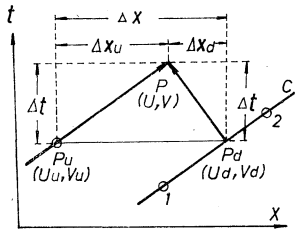

Fig. 2. Calculation of point P on x-t plane.

1, 2 are evaluated before, and $U\sim t$, $V\sim t$ curves for characteristic C are evaluated and assume that U, V of P_u, that is, U_u, V_u and its position on $x\sim t$ plane are evaluated.

Now, drawing $\overline{P_u\ P_d}$ from P_u in parallel to abscissa and putting crossing point with C line as P_d, U, V of P_d, that is, U_d, V_d are known from $U\sim t$, $V\sim t$ curves for characteristic C.

We will consider the case to evaluae, U, V and position of crossing point P of the two characteristics starting P_u, P_d.

In this case, there is $\Delta t_u=\Delta t_d=\Delta t$ in equation (12)$\sim$(15) and Δx is measured on $x\sim t$ plane.

Then from (13)-(15) $\quad U=\{V_u-V_d+2m(U_u+U_d)+(K_u-K_d)\Delta t\}/4m$ (18)

from (12)-(14) $\quad \Delta x=\Delta x_u-\Delta x_d=(2U+V_u-V_d+U_u+U_d)\Delta t/2$

or $\quad \Delta t=2\Delta x/(2U+V_u-V_d+U_u+U_d)$ ·········(19)

Substituting the above into equation (18) and crossing out Δt, we can obtain the following equation as the calculation equation of U.

$$U=-\{(2m-1)/8m\}(V_u-V_d)$$
$$+\sqrt{[\{(2m+1)/8m\}(V_u-V_d)+(U_u+U_d)/2]^2-(\Delta x/8m)(M_uV_u^2-M_dV_d^2)} \quad (20)$$

here, $M=gn^2/R^{4/3}$ accordingly $M_u=gn^2/R_u^{4/3}$, $M_d=gn^2/R_d^{4/3}$ ······(21)

Δt is, therefore, calculated from equation (19).

In the next place, we get from equation (13) or (15) the following equation as the calculation equation of V.

$$V=[-1+\sqrt{1+2M\Delta t\{V_u+2m(U_u-U)+(gS_0-M_uV_u^2/2)\Delta t\}}]/M\Delta t \quad \cdots(22)$$

or $$V=[-1+\sqrt{1+2M\Delta t\{V_d+2m(U-U_d)+(gS_0-M_dV_d^2/2)\Delta t\}}]/M\Delta t \quad \cdots(23)$$

Δx_u is, therefore, calculated from equation (12).

The position of P on $x\sim t$ plane will be determined by Δx_u and Δt.

(ii) Method to calculate V or U on t axis.

This case is that in Fig. 3, characteristic C is known, V or U of P on t axis is given and we evaluate unknown value V or U of P. In this case, as $\overline{PP_d}$ is backward characteristics, equation (14) and (15) must be satisfied and the method of calculation will become trial. The author proposes the following method though miscellaneous trial methods are considered.

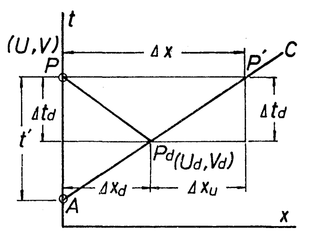

Fig. 3. Calculation of point P on t axis.

Now, we will assume that the position of P on t axis is given and the unknown value of P is V. First in place, referencing gradient of backward characteristic evaluated before and drawing assumed backward characteristic $\overline{PP_d}$ from P, we will get P_d as the crossing point with curve C. Measuring Δx_d, Δt_d on $x \sim t$ plane and knowing U_d, V_d of P_d from $U \sim t$, $V \sim t$ curves for characteristic C, we will get V of P from equation (23), calculate Δx_d from equation (14) and check whether it agrees with Δx_d measured before.

If it is not satisfied, varing assumption of Δt_d, that is, position of P_d, we will repeat the above calculation.

Accidentally, when U, V on the characteristic C is known and constant, trial will be unnecessary as follow. Drawing $\overline{PP'}$ from P in parallel to abscissa and putting crossing point with C line as P', we will measure $\Delta x = \overline{PP'}$ on $x \sim t$ plane and take t', Δx_u as shown in Fig. 3.
Assuming that $\overline{AP'}$ is straight line approximately, we will get

$$\Delta x_u / \Delta t_d = \Delta x / t' = \beta \qquad \cdots\cdots\cdots(24)$$

Accordingly, from equations (14) and (24)

$$\Delta x_d = \Delta x_u - \Delta x = (V_d - U_d + V - U)\Delta t_d/2$$

$$\text{or} \quad \frac{\Delta x_u}{\Delta t_d} - \frac{\Delta x}{\Delta t_d} = (V_d - U_d + V - U)/2$$

$$\therefore \ \Delta t_d = 2\Delta x/(2\beta + U + U_d - V - V_d) \qquad \cdots\cdots\cdots(25)$$

Now, assuming unknown value of P is V and crossing out Δt_d from equations (15) and (25), we will get the equation of second order for V as follow,

$$[M\Delta x - 1]V^2 + \{2\beta + (2m+1)U - (2m-1)U_d\}V$$
$$-\{2m(U - U_d) + V_d\}(2\beta + U + U_d - V_d) + \Delta x(M_d V_d^2 - 2gs_0) = 0 \quad (26)$$

From the above equation, we will get V except trial. When Q is given and V and U are unknown on t axis, assuming U of P and calculating V of P by the above process, we will have only to check the result by the following equation,

$$Q = (m/g)^m W U^{2m} V \qquad \cdots\cdots\cdots(27)$$

(iii) Process of calculation.

Here will be given an account on the method of calculation of unsteady flow depending on the characteristics by using $x \sim t$ plane, $U \sim t$ plane, $V \sim t$ plane and the above-mentioned (i), (ii). Suppose flood-water flows in from the end of upstream in a waterway under the condition of uniform flow (mean velocity is V_0, water depth h_0 or U_0) and with the same upstream and downstream section and assume that U on t axis is given.

Accordingly the points of t_1, t_2, $\cdots$ on Fig. 4(b) are plotted. At first, the characteristics C_0 of Fig. 4(a) will be drawn by equation (5). On this straight line and inside of it, all are V_0, U_0. C_0 line of Fig. 4(b) and (c), therefore, is expressed by lines of $U = U_0$, $V = V_0$.

In the next place, in evaluating the characteristics C_1 starting t_1, we can evaluate V of t_1 by the method in which no trial is done in (ii) since U, V of P_d are U_0, V_0. This V will be plotted as t_1 point of Fig. 4(c). Then, in Fig. 4(a),

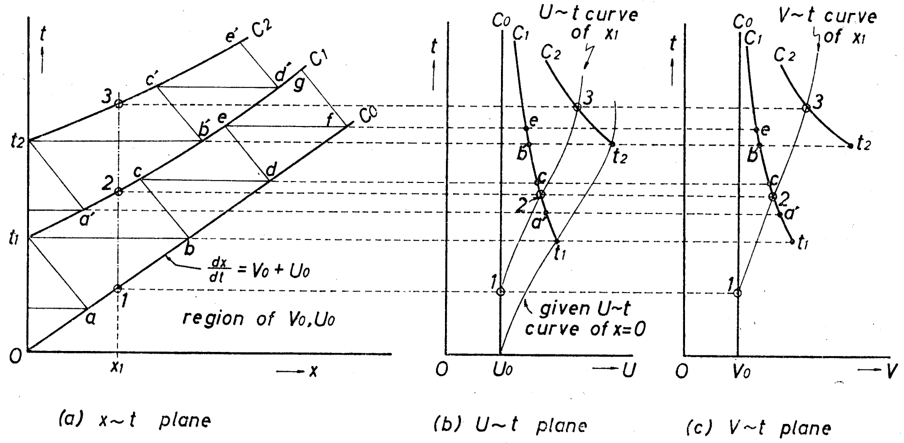

Fig. 4. Calculation method of characteristic equations.

a line is drawn from t_1 in parallel to abscissa, and its crossing point b with C_0 line is taken. Getting Δx by measuring $\overline{t_1 b}$ in the figure, we will evaluate U, V at c point and its position (Δx_u, Δt) using U, V of b, that is, U_0, V_0 and depending on the method of (i). Then c point in Fig. 4(a)$\sim$(c) will be plotted.

Next, e will be evaluated similarly by drawing $\overline{cd}$ in Fig. 4(a), in parallel to abscissa from c. Thus the points on the characteristics C_1 can be evaluated in turn and C_1 curve of Fig. 4(b), (c) is drawn.

When we evaluate characteristic C_2, the process is same as the above except the case that t_2 point becomes trial and a', b' d' $\cdots$ of Fig. 4(a) are evaluated from C_1 curve of Fig. 4(b), (c) drawn above.

When Q on t axis is given, U, V of t axis is evaluated by as the method mentioned in (ii) and U, V of the points other than those on t axis are evaluated samely above. Thus the points other than those on t axis can be calculated without trial. Incidentally, in calculating M, M_u and M_d of equation (21), it is convenient to prepare beforehand diagrams of $M\sim U$. In the next place, in order to evaluate $U\sim t$, $V\sim t$ curves of a given distance x_1, we have only to read the time by the points 1, 2, 3, $\cdots$ on each characteristic curve in x_1 of Fig. 4(a) and to connect points 1, 2, 3, $\cdots$ of Fig. 4(b), (c) corresponding in the curve and the time to the preceding points. Again in evaluating $Q\sim t$ curves, we can do it from the above-mentioned U, V by using equation (27).

What has been considered so far is confined to the case where there is no condition at the end of downstream, but the same method will be applicable also to the case when such condition as water level-time curve is given there.

4. Examples of application

Having pursued the flood of the Shintakase River in Kyoto City depending on the present method, the author examined the adoptability of the results thereby to the observed values.

i) Flood experiment of the Shintakase River

The Shintakase river is, as shown in Fig. 5, a straight waterway of about 2.3 km, which flows through Fushimi-ku, Kyoto City, and its cross–section presents such a compound section as shown in Fig. 6. The upper width of low waterway is about 16 m and low water depth is about 0.6 m and its surface width is about 12 m, presenting almost regular form of section over the whole length of upstream and downstream. Again, the gradient of water surface at low water level at the interval of 2.3 km total length is, as shown in Fig. 7, nearly 0.00085~0.0010 in the interval of 1.4 km in upstream and nearly 0.00011~0.00013 in the interval of 0.9 km in downstream.

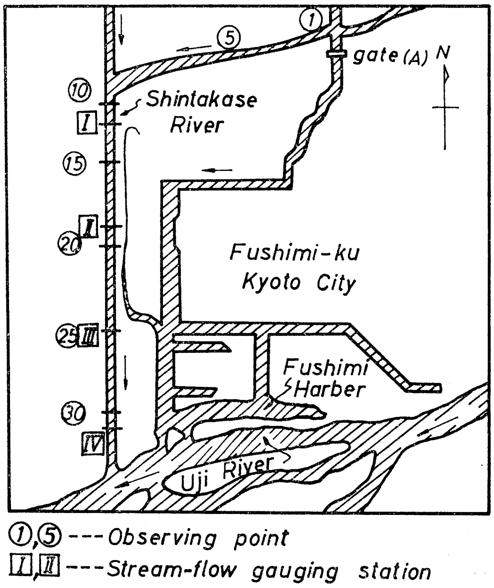

Fig. 5. Plan of the Shintakase River.

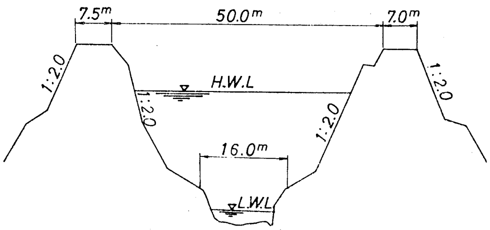

Fig. 6. Cross-section of the Shintakase River.

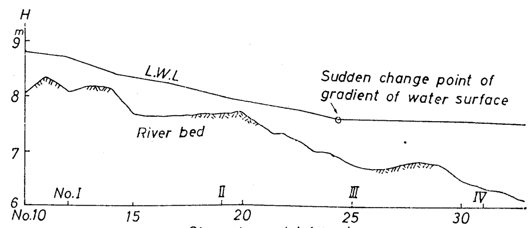

Fig. 7. Profile of the Shintakase River.

The experiment which was undertaken three times by Dr. Masafumi Yoneda[3] availing this low waterway was carried out by moving the regulating gate (A) of Fig. 5 and creating an artificial flood in the Shintakase River. Water level was observed every two minutes at the observing points Nos. 10~32 (interval of about 100 m) as shown in Fig. 5, while velocity was observed at observing points of No. 12, No. 19 (0.7 km downstream from No. 12), No. 25 (1.3 km downstream from No. 12), No. 31 (1.9 km downstream from No. 12)(hereafter represented as stations No. I, No. II, No. III, No. IV) and dividing one section into three parts, he observed for 30~60 seconds every two minutes with a Price current-meter. The sectional diagrams of the four stations are shown in Fig. 8. The application of the present calculation was

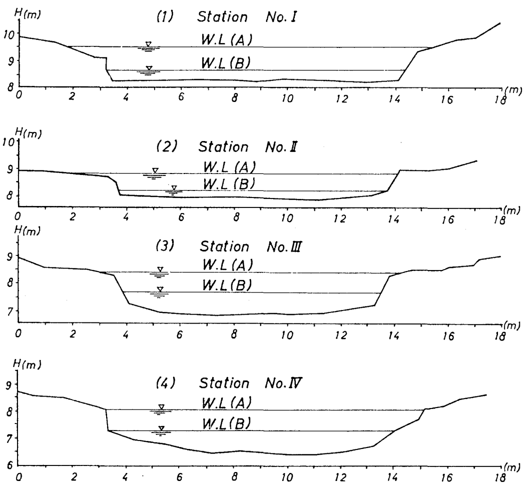

Fig. 8. Cross-sections of low waterway of streamflow gauging stations of the Shintakase River.
W.L.(A)…highest water level in flood experiment.
W.L.(B)…lowest water level in flood experiment.

carried out on the first experiment.

ii) Decision of various constants

In the application of the present method, it will be assumed that gradient and section are both constant and waterway is infinitely long.

Bottom slope $S_0=8.5\times10^{-4}$ will be assumed with reference to the foregoing value. Then, Manning's n will be approximately calculated from Manning formula $n = R^{2/3}I^{1/2}/V$ by gradient of water surface I, hydraulic mean depth R and mean velocity V at the time of the highest water level in the flood experiment by Dr. Yoneda. We have the results by the above-calculation as shown in Table 1. Accordingly we will use $n=0.03$ (m, s unit) as the mean value for these.

Then, $A\sim h$ curves evaluated from each section of Fig. 8 will be shown in Fig. 9 and Table 2. In this case we will use $W=9.3$ (m unit), $m=1.2$ (m unit) as

Table 1. roughness coefficient n of each section

Station	Experiments	H_{max} (m)	A (m²)	R (m)	I	V (m/s)	n (m, s unit)	mean. n
I	1st	9.29	11.716	0.811	9.14×10^{-4}	0.97	0.0271	
	2nd	9.49	14.340	0.995	11.85	1.25	0.0274	
	3rd	9.04	8.847	0.708	10.33	0.74	0.0344	0.0296
II	1st	8.58	7.394	0.654	10.25	0.98	0.0246	
	2nd	8.78	9.440	0.710	9.45	1.25	0.0195	
	3rd	8.35	5.027	0.468	6.70	0.67	0.0233	0.0225
III	1st	8.09	11.738	1.041	4.33	0.53	0.0403	
	2nd	8.37	14.375	1.130	4.55	0.78	0.0296	
	3rd	7.90	9.873	0.908	2.70	0.44	0.0350	0.0350
IV	1st	7.88	13.538	1.065	2.20	0.44	0.0351	
	2nd	8.06	15.800	1.201	5.55	0.54	0.0494	
	3rd	7.74	11.926	0.959	2.45	0.37	0.0412	0.0419
							mean of n.	0.0323

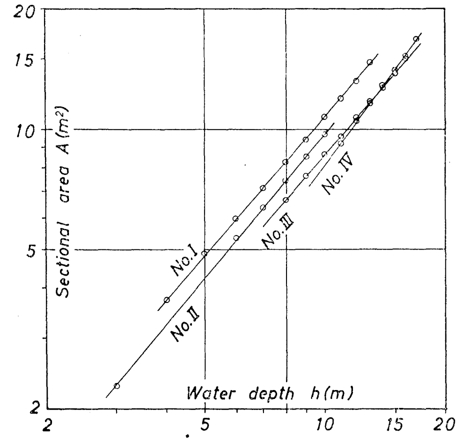

Fig. 9. $A \sim h$ diagrams of each section.

Table 2. W, m of each section (m unit)

Station	m	W
I	1.15	10.60
II	1.19	9.71
III	1.15	8.57
IV	1.38	8.15
mean	1.22	9.26

the mean value for these.

In the next place, since the cross sections of channel are wide as shown in Fig. 8, we will take $R=h/m=U^2/g$ approximately. Accordingly $M \sim U$ diagrams are drawn.

iii) Result of calculation

Since the initial discharge of the flood experimented in the Shintakase River to be applied is $Q_0=0.85$ m³/s, V_0, U_0 under the uniform flow of initial condition will be as follows.

In Manning formula

$$V_0=(h_0/m)^{2/3}S_0^{1/2}/n,\ h_0=(nm^{2/3}Q_0/WS_0^{1/2})^{1/(m+2/3)}\ \text{and}\ U_0=\sqrt{gh_0/m}$$

Substituting into these $S_0=8.5\times10^{-4}$, $n=0.03$, $W=9.3$ and $m=1.2$, we will get $U_0=1.565\ m/s$, $V_0=0.386\ m/s$. After that we have only to give observed curve of the discharge in $x=0$, that is, No. I and evaluate U, V at each point on $x \sim t$ plane by the afore-mentioned method. The process of calculation is shown in Fig. 10(a)~(c) and Table 3 and the results are shown in Fig. 11. Incidentally, in the case of $n=002$ (m, s unit) also, the results are shown in Fig. 11 for comparison. From this figures it is seen that the observed values of No. II and No. III are between

Table 3. calculation of points on $x \sim t$ plane in Fig. 10.

(a) points on t axis.

Point in Fig. 10	U_d (m/s)	V_d (m/s)	U assumed (m/s)	V equ. (26) (m/s)	Q equ. (27) (m³/s)	Q given (m³/s)	Remark
$_0C_1$	1.565	0.386	1.730	0.525	1.46	1.40	$\beta=\Delta x/t'$
	〃	〃	1.713	0.510	1.39	〃	$=468/(4\times60)$
	〃	〃	1.716	0.512	1.40	〃 o.k	$=1.950$

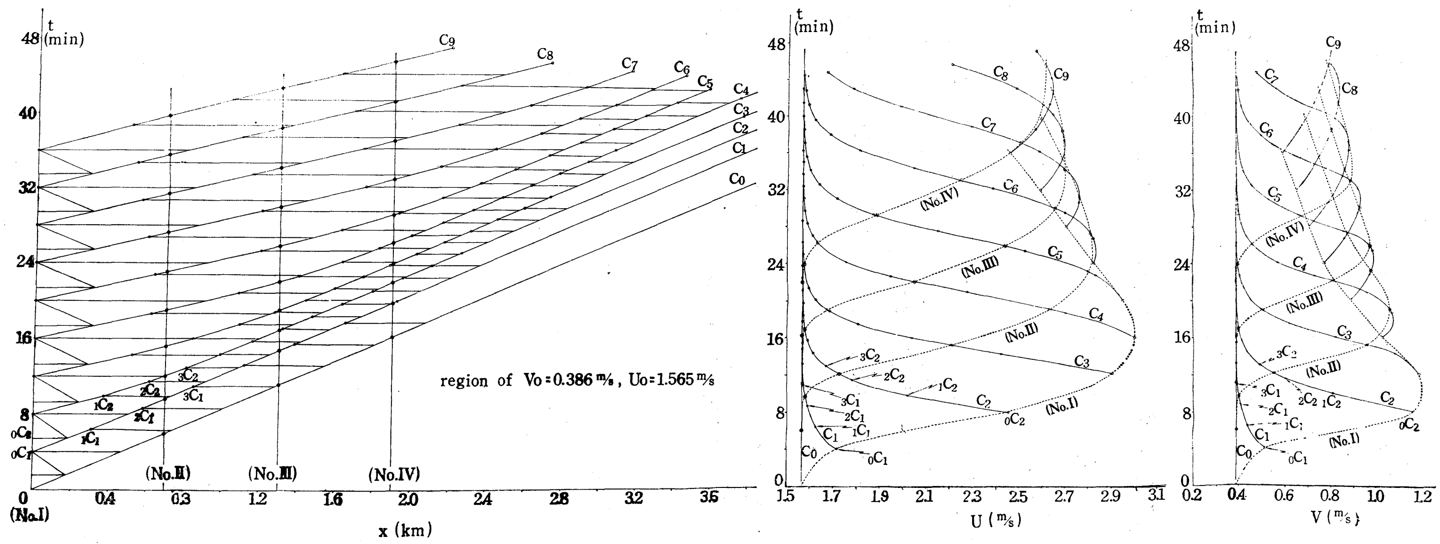

Fig. 10. Calculation diagrams of flood flow in the Shintakase River.

Point in Fig. 10	Δt_d assumed (sec)	U_d (m/s)	V_d (m/s)	U assumed (m/s)	V equ.(23) (m/s)	Q equ.(27) (m³/s)	Q given (m³/s)	$-\Delta x_d$ equ.(14) (m)	$-\Delta x_d$ measured (m)
$_0C_2$	147.0	1.648	0.453	2.500	1.220	8.23	7.50	182	200
	151.0	1.651	0.455	2.400	1.105	6.76	〃	188	191
	152.0	1.651	0.456	2.450	1.156	7.43	〃	189	189
	152.0	1.651	0.456	2.454	1.160	7.49	〃	189	189
	151.9	1.651	0.456	2.455	1.161	7.50	〃	189	189 o.k

(b) points except those on t axis.

Point in Fig. 10	Δx measured (m)	U_u (m/s)	V_u (m/s)	U_d (m/s)	V_d (m/s)	U equ.(20) (m/s)	Δt equ.(19) (sec)	V equ.(22) (m/s)	Δx_u equ.(12) (m)
$_1C_1$	468	1.716	0.512	1.565	0.386	1.626	140.6	0.437	301.7
$_2C_1$	441	1.626	0.437	〃	〃	1.587	137.4	0.405	278.6
$_3C_1$	430	1.587	0.405	〃	〃	1.572	136.3	0.392	269.6
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$_1C_2$	502	2.455	1.161	1.596	0.412	2.021	113.6	0.811	366.2
$_2C_2$	361	2.021	0.811	1.577	0.396	1.786	95.1	0.602	248.2
$_3C_2$	301	1.786	0.602	1.570	0.390	1.669	87.2	0.491	198.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

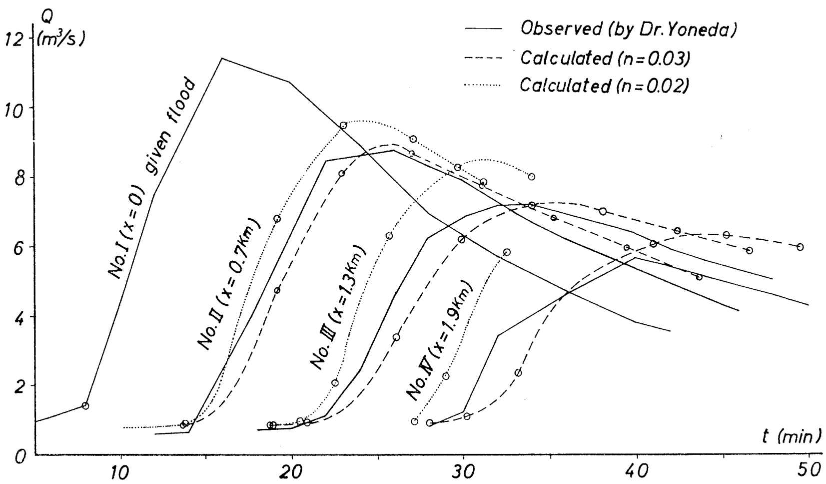

Fig. 11. Results of Calculation for a flood of the Shintakase River.

the calculated values of $n=0.03$ and $n=0.02$ and come nearer to the calculated values of $n=0.03$ than those of $n=0.02$. Anyway, on the whole, they seem to be in good agreement in No. II and No. III. In the case of No. IV ($x=1.9$ km), error is large, which will be ascribed to the circumstance in which bottom slope in this neighborhood has become considerably small value than 8.5×10^{-4} as shown in Fig. 7.

5. Conclusion

The author believes that the present method renders calculation considerably simpler than before by putting as $\Delta t_u=\Delta t_d$, utilizing $U\sim t$, $V\sim t$ curves for each characteristics and saving trial except the calculation on t axis. He also believes that it is an effective method to prepare Fig. 4(b), (c), for he can forward calculation while checking it and besides, the figures of $U\sim t$, $V\sim$t at a given point are shown in Fig. 4(b), (c) just as they are.

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