AN APPROXIMATE CALCULATION OF HYDRODYNAMIC PRESSURE ON THE MIDSHIP SECTION CONTOUR OF A SHIP HEAVING AND PITCHING IN REGULAR HEAD WAVES

TASAI, Fukuzo
Research Institute for Applied Mechanics, Kyushu University

https://doi.org/10.5109/7165076

出版情報：Reports of Research Institute for Applied Mechanics. 14 (48), pp.55-65, 1966. 九州大学応用力学研究所

Reports of Research Institute for Applied Mechanics
Vol. XIV, No. 48, 1966

AN APPROXIMATE CALCULATION OF HYDRODYNAMIC PRESSURE ON THE MIDSHIP SECTION CONTOUR OF A SHIP HEAVING AND PITCHING IN REGULAR HEAD WAVES

By Fukuzō TASAI

Summary

An approximate calculation of the hydrodynamic pressure acting on the surface of the midship section of a ship going in regular waves under the head sea condition, neglecting the effect of pitching motion, is carried out. Next, it is discussed on the more precise calculation method which considers both heaving and pitching motions.

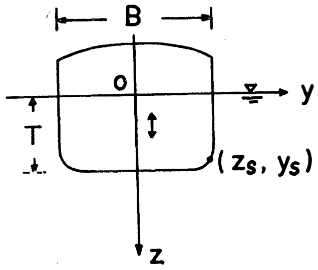
1. Introduction

A topic in the calculation of a ship's transverse strength is now laid on how to consider the hydrodynamic pressure distribution on the ship's hull travelling in waves.

Here is described how the hydrodynamic pressure on the midship section contour changes with frequency and how to calculate the ratio of its magnitude to hydrostatic pressure. Furthermore, an outline of pressure distribution is given with some examples of numerical calculations.

2. Hydrodynamic pressure on the two-dimensional body heaving on the still water.

When the section of a two-dimensional body is in Lewis form, the hydrodynamic pressure acting upon its contour will be given by the following equation from Tasai's [1]*.



$$P=-\rho\frac{\partial\phi}{\partial t}=\frac{\rho g\eta_0}{\pi}\Big[(\Phi_c+S)\sin\omega t-(\Phi_s+E)\cos\omega t\Big] \qquad \cdots\cdots(1)$$

In the acove equation η_0 represents the amplitude of progressive wave generated by heaving. Now, letting Z_0 be the heaving amplitude and using $\bar{A}=\eta_0/Z_0$, equation (1) becomes as follows:

Note: * Numbers in the brackets designate References at the end of this paper.

$$P=\frac{\rho g Z_0 \bar{A}}{\pi}\Big[(\Phi_c+S)\sin\omega t-(\Phi_s+E)\cos\omega t\Big] \qquad \cdots\cdots(2)$$

where. Φ_c, Φ_s are obtained from periodic source potentials at origin 0. S and E will be also given by the following equations:

$$\left.\begin{aligned}S&=\sum_{m=1}^{\infty}P_{2m}(\xi_B)\Big[\cos 2m\theta+\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{\cos(2m-1)\theta}{2m-1}+\frac{a_1\cos(2m+1)\theta}{2m+1}-\frac{3a_3\cos(2m+3)\theta}{2m+3}\Big\}\Big]\\ E&=\sum_{m=1}^{\infty}q_{2m}(\xi_B)\Big[\quad〃\quad〃\quad\Big\{\quad〃\quad〃\quad〃\quad\Big\}\Big]\end{aligned}\right\} \qquad \cdots\cdots(3)$$

provided that $\xi_B=\frac{\omega^2 B}{2g}$

Suppose now that the two-dimensional body oscillates vertically with a small displacement $Z=Z_0\cos\omega t$ ……(4)

In this case, force F that acts on the body in $-Z$ direction (upward) will be expressed as follows.

$$F=m\ddot{Z}+\rho g A_w Z+c_1\ddot{Z}+c_2\dot{Z} \qquad \cdots\cdots(5)$$

In the above equation,

m=mass of body, A_w=water plane area

c_1=added mass, c_2=damping coefficient

The first term in the right side is inertia force, the second term is buoyancy. $c_1\ddot{Z}$ and $c_2\dot{Z}$ are hydrodynamic forces.

Considering similarly as in equation (5), we will divide the hydrodynamic pressure into two parts P_a and P_d. The former is in phase with $\ddot{Z}$ and the latter with $\dot{Z}$.

By putting as

$$\left.\begin{aligned}P_a&=\rho g P_a'\ddot{Z}=\rho g Z_0 P_a''\cos\omega t\\ P_d&=\rho g P_d'\dot{Z}=\rho g Z_0 P_d''\sin\omega t\end{aligned}\right\} \qquad \cdots\cdots(6)$$

P_a'', P_d'' will be obtained respectively by the following equations:

$$P_a''=-\xi_B\left(\frac{P_sB_0+P_cA_0}{A_0^2+B_0^2}\right),\quad P_d''=-\xi_B\left(\frac{P_sA_0-P_cB_0}{A_0^2+B_0^2}\right) \qquad \cdots\cdots(7)$$

It is provided that $P_s=\Phi_s+E$ and $P_c=\Phi_c+S$.

And E, S, A_0, B_0 have also various values according to the form of section and ξ_B, as given in [1]. Again, P_a', P_d' are given by the following equations:

$$P_a'=-\frac{B}{2g}\frac{P_a''}{\xi_B},\quad P_d'=-\sqrt{\frac{B}{2g}}\frac{P_d''}{\sqrt{\xi_B}} \qquad \cdots\cdots(7)'$$

Next, what corresponds to the second term in the right side of equation (5) is,

$$P_b = \rho g Z_0 \cos \omega t \qquad \cdots\cdots(8)$$

Hence the total pressure fluctuation at a point on the surface of body will be

$$P' = \rho g Z_0 \{(1+P_a'')\cos \omega t + P_d'' \sin \omega t\} \qquad \cdots\cdots(9)$$

Putting $P'/\rho g Z_0 = \overline{P'} \cos(\omega t - \nu_0)$

$$\left.\begin{aligned} \overline{P'} &= \{(1+P_a'')^2 + P_d''^2\}^{1/2} \\ \nu_0 &= \tan^{-1}\{P_d''/(1+P_a'')\} \end{aligned}\right\} \qquad \cdots\cdots(10)$$

P_a'', P_d'' that have been calculated on the Lewis form section of which $H_0 = B/2T = 1.25$, σ (area coefficient) $=0.9420$, as shown in Fig. 1, are given in Figs. 2 and 3, while $\overline{P'}$ of equation (10) is shown in Fig. 4.

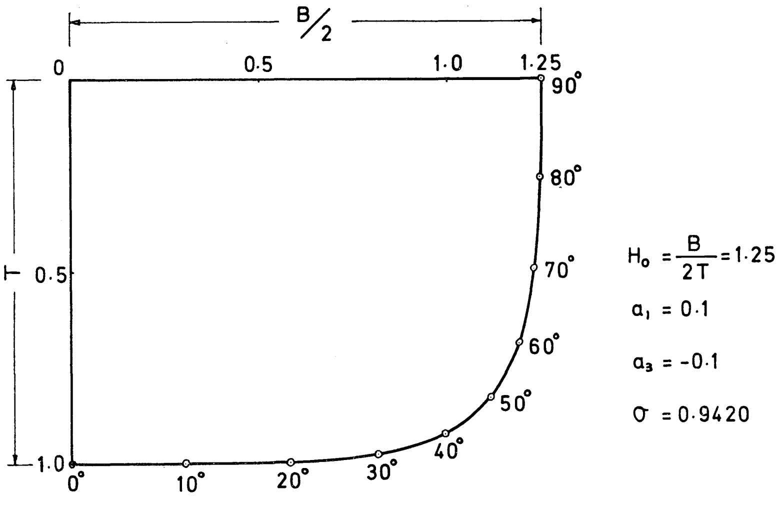


Fig. 1.

As seen from Fig. 4, in the case of $\xi_B < 1.0$, $\overline{P'}$ is larger at the ship's side than at the bottom.

The experiments on the hydrodynamic pressure was carried out by Porter [2]. He measured the hydrodynamic pressure on the surface of the body by submitting a circular cylinder model to forced heaving oscillation. He proved, furthermore, that such a value from theoretical calculations as proposed in the present paper shows good agreement with his measured data. On the other hand, Paulling [3] took up a two-dimentional model with a section of ship form and confirmed the fact mentioned above by comparing the measured and calculated values.

Then Hou-Wen Huang [4] measured the pressure distribution on a model ship at its nine sections. From the result it has been proved that, while the sections at fore and aft of ship give the pressure distribution more or less varied from the two-dimensional calculation because of the three-dimensional effect, at midship section an extremely good agreement was found between the above result and the two-dimensional calculation.

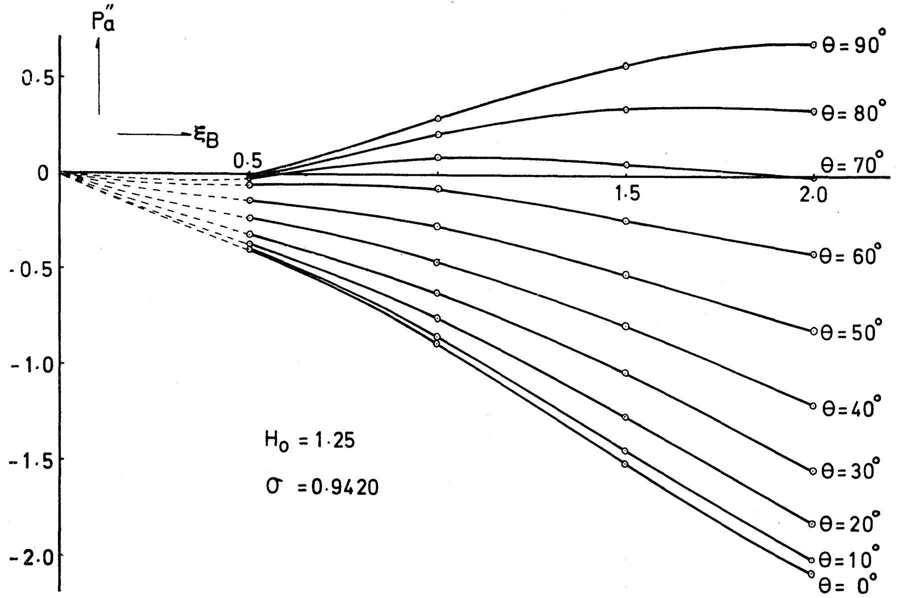


Fig. 2.

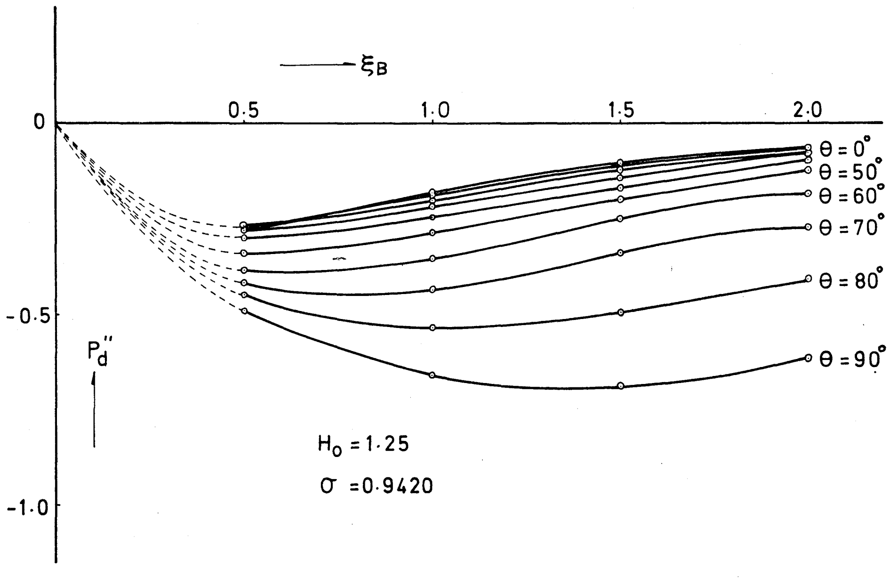


Fig. 3.

Since the aim of the present paper is laid in investigating the pressure distribution on the midship section, it is evident from [4] that two-dimensional calculation is appropriate and sufficient.

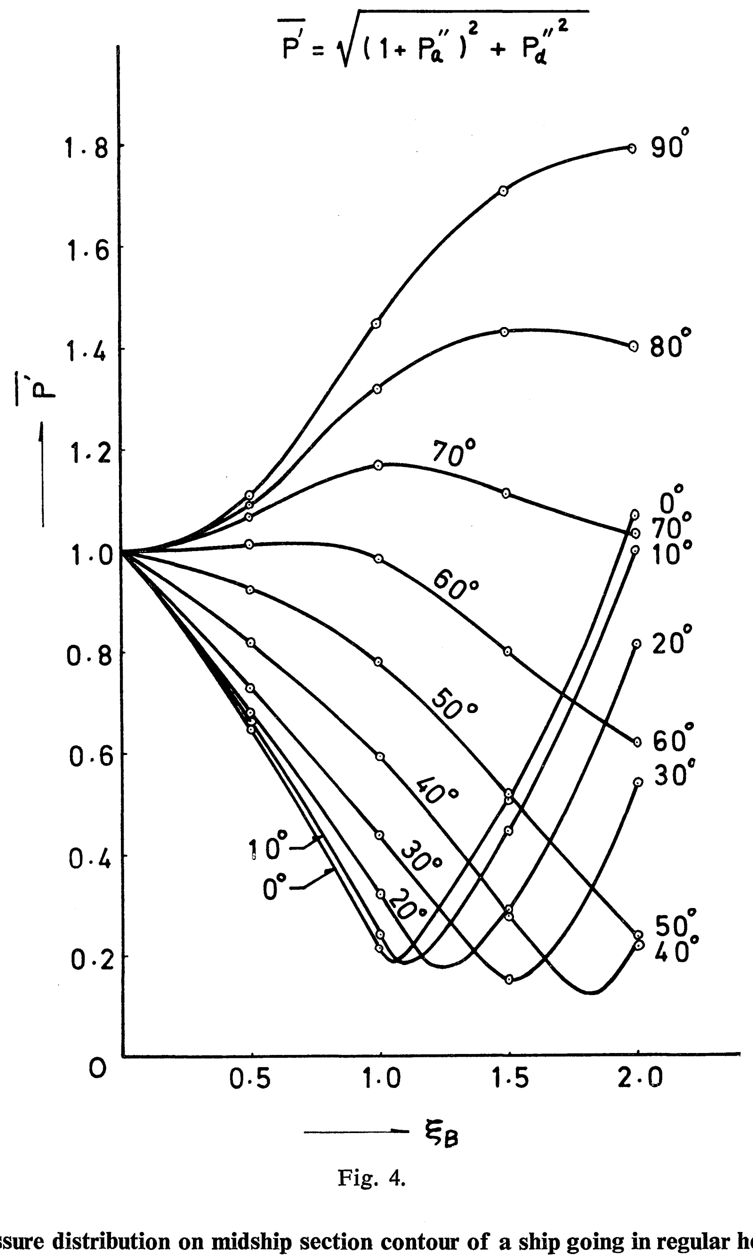
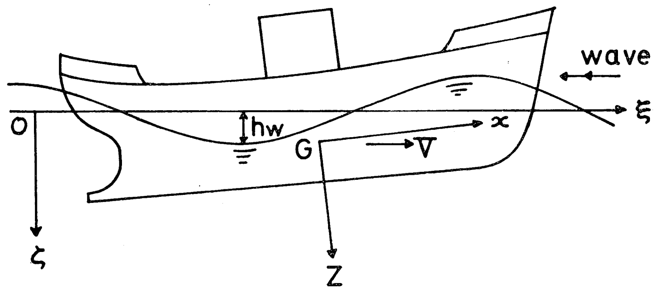


Fig. 4.

3. Pressure distribution on midship section contour of a ship going in regular head seas

In the fiigure next page, 0-ξ, ζ are space fixed coordinates, G-x, z hull coordinates. It will be assumed here for simplicity that the center of gravity G is in midship section, but when there is no wave, it is on the surface of still water 0-ξ.

The equation of regular wave progressing in $-\xi$ direction will be given by



the following equation when expressed approximately with hull coordinates x, z.

$$\zeta_w \doteqdot h_w \cdot e^{-KZ} \cos(Kx + \omega_e t) \qquad \cdots\cdots(11)$$

In the above equation h_w is wave amplitude, and putting λ for wave length $k = 2\pi/\lambda = \omega^2/g$. ω is the circular frequency of wave and ω_e the circular frequency of encounter, and V is ship's speed.

Here we put as $\quad \omega_e = \omega(1 + \omega V/g) \equiv \omega\kappa \qquad \cdots\cdots(12)$

Now that midship section is under consideration, by putting $x=0$ in equation (11) we obtain

$$\zeta_w = h_w \cdot e^{-KZ} \cos \omega_e t \qquad \cdots\cdots(13)$$

Orbital velocity and acceleration will be obtained as follows

$$\left.\begin{aligned} \dot{\zeta}_w &= -h_w \cdot \omega e^{-KZ} \sin \omega_e t \\ \ddot{\zeta}_w &= -h_w \cdot \omega^2 e^{-KZ} \cos \omega_e t \end{aligned}\right\} \qquad \cdots\cdots(13)'$$

The hydrodynamic pressure on the underwater surface of a floating body is consisted of the following parts,

(1) P_1=hydrodynamic pressure based on the movement in still water

(2) P_2=hydrodynamic pressure due to reflection of waves from the body under restrained condition

(3) P_3=pressure obtained from regular wave potential

In the calculation of hydrodynamic pressure on the midship section contour, assuming that the effect of pitching motion is very small, only the heaving motion is considered. (The pressure attributed to pitching motion will be discussed later)

The heaving motion of a ship will be assumed as follows.

$$Z = Z_0 \cos(\omega_e t - \mu) \qquad \cdots\cdots(14)$$

In this case, the pressure varies with ω_e.

P_1 is to be given by the following equation from eqeations (9) and (10) which were discussed in the preceeding section.

$$P_1 = \rho g Z_0 \{(1 + P''_{aw})\cos(\omega_e t - \mu) + P''_{dw} \sin(\omega_e t - \mu)\} \qquad \cdots\cdots(15)$$

P''_{aw}, P''_{dw} in equation (15) are P_a'', P_d'' coresponding to $\xi_{Be} = \omega_e^2 B/2g$. Namely,

P''_{aw}, P''_{dw} can be obtained by putting abscissa as ξ_{Be} in Figs. 2 and 3.

Next, P_2 will be expressed by using $\kappa=\omega_e/\omega$ of equation (12),

$$P_2=-\rho g h_w e^{-KZs}\left\{\frac{P''_{aw}}{\kappa^2}\cos\omega_e t+\frac{P''_{dw}}{\kappa}\sin\omega_e t\right\} \quad \cdots\cdots(16)$$

Lastly,

$$P_3=-\rho g h_u e^{-KZs}\cos\omega_e t \quad \cdots\cdots(17)$$

Z_s in equation (16) and (17) is vertical coordinate of section contour. The total pressure fluctuation in longitudinal waves, therefore, will be

$$\begin{aligned}P_H=P_1+P_2+P_3&=\rho g Z_0[G\cos\omega_e t+H\sin\omega_e t]\\&=\rho g Z_0 J\cos(\omega_e t-\nu)\end{aligned} \quad \cdots\cdots(18)$$

provided that

$$J=|P_H|/\rho g Z_0=\{G^2+H^2\}^{1/2} \quad \cdots\cdots(19)$$

$$\nu=\tan^{-1}(H/G) \quad \cdots\cdots(20)$$

Also,

$$\left.\begin{aligned}G&=(1+P''_{aw})\cos\mu-P''_{dw}\sin\mu-\frac{e^{-KZs}}{\overline{Z}}(1+P''_{aw}/\kappa^2)\\H&=(1+P''_{aw})\sin\mu+P''_{dw}\cos\mu-e^{-KZs}P''_{dw}/\overline{Z}\kappa\end{aligned}\right\} \quad \cdots\cdots(21)$$

where $\overline{Z}=Z_0/h_w$ $\quad \cdots\cdots(22)$

4. Examples of numerical calculation

The next step is to carry out the calculation of equation (21) by using $\overline{Z}$, κ and others obtained by solving the coupled equations of pitch and heave in regular wave, but here we will tentatively carry out an approximate calculation by assuming as follows just to make a rough estimation of the magnitude and distribution of P_H.

1) Midship section is the one given in Fig. 1.

2) Now, letting T_z be the natural heaving period, Z_0 generally becomes max at $T_z=T_e=2\pi/\omega_e$. We will therefore confine our calculation only to the condition of $\omega_e=\omega_z=2\pi/T_z$.

3) According to Gerritsma's experiment [5], ξ_{Be} at the resonance becomes as follows with respect to Todd's 60 series.

C_B	0.60	0.70	0.80
ξ_{Be}	0.813	0.748	0.738

As a result, it will be assumed here as $\xi_{Be}=0.75$.

4) Phase lag μ is smaller than 90° at the resonant point in general. Here calculation will be made on $\mu=60°$ and 90°.

5) It will be assumed as $\overline{Z}=Z_0/h_w\doteqdot1.35$.

6) Also, $F_n=V/\sqrt{Lg}=0.2$, $\lambda/L=\sigma_0\doteqdot 1.25$.

In other words, the calculation is to be made on the case when a ship navigates in the regular waves of $\lambda/L=1.25$ at $F_n=0.2$ under head sea condition and in the condition of heave resonance.

Now, κ will be rewritten as follows,

$$\kappa=1+F_N\sqrt{2\pi/\sigma_0} \qquad \cdots\cdots(23)$$

In this case we have $\kappa=1.448$.

Because of $\xi_B=\xi_{Te}/\kappa^2$, we obtain $\xi_B=0.357$.

Furthermore, we have the following equation.

$$kZs=\omega^2 Zs/g=\xi_B Zs/H_0T \qquad \cdots\cdots(24)$$

Evaluating now various values for the case of a ship of $L=200$ m, we can obtain the results as given in the table below.

Since	$\lambda/L=1.25$,	it results	$\lambda=250$ m,	$T_w=13$ seconds
Since	$\pi B/\lambda=0.357$,		$B=28.4$ m,	$L/B=7.0$
Since	$B/2T=1.25$,		$T=11.36$ m,	$L/T=17.6$
Since	$V/\sqrt{Lg}=0.20$,		$V=17.2$ kt	

$T_e=T_z=8.5$ seconds.

The dimensions given above all change if the ship's length L changes, but J doesn't since it is non-dimensional.

T_z for a ship of $L=200$ m will become about 7.5 sec and $T_z=8.5$ is too large, which is due to the assumption of $T_e=T_z$.

In the present example, therefore, we may consider it as the case of $\overline{Z}=1.35$ at $T_e=8.5$.

The results of calculation of J, ν are given in Fig. 5.

Putting now as $\overline{P_H}'=P_H/\rho g h_w=\overline{Z}\cdot J\cdot\cos(\omega_e t-\nu)$ ……(25)

$|\overline{P_H}'|$ is to be obtained simply by multiplying J of Fig. 5 by $\overline{Z}$.

Since we have put as $\overline{Z}=1.35$ in the present paper, $|\overline{P_H}'|$ takes tne order of 0.5～0.8 at ship's bottom and about 1.5 near the water surface of ships side.

Next, if the hydrostatic pressure is put as P_s, total pressure P_0 will be

$$P_0=P_s+P_H, \quad \text{where} \quad P_s=\rho g Z_s=\rho g T\cdot Z_s/T \quad \text{and}$$

$$P_H=\rho g T J Z_0/T\cos(\omega_e t-\nu).$$

Then we obtain

$$\overline{P_0}=P_0/\rho g T=\overline{P_s}+\overline{P_H}\cos(\omega_e t-\nu)$$
$$=Z/T+JZ_0/T\cos(\omega_e t-\nu) \qquad \cdots\cdots(26)$$

The second term in the right side of equation (26) is the fluctuating pressure. On the other hand, since we can put as

$$Z_0/T=h_w/T\cdot Z_0/h_w=\overline{Z} \quad h_w/T=\frac{h_w}{\lambda}\cdot\sigma_0\cdot\frac{L}{T}\overline{Z} \qquad \cdots\cdots(27)$$

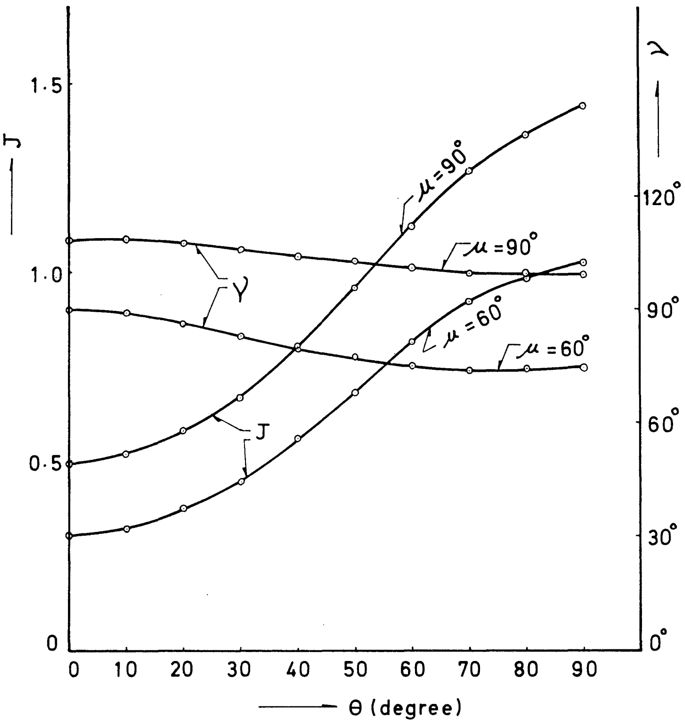

1.5
1.0
0.5
0
J
120°
90°
60°
30°
0°
λ
μ=90°
μ=90°
μ=60°
μ=60°
γ
J
0 10 20 30 40 50 60 70 80 90
θ (degree)

Fig. 5.

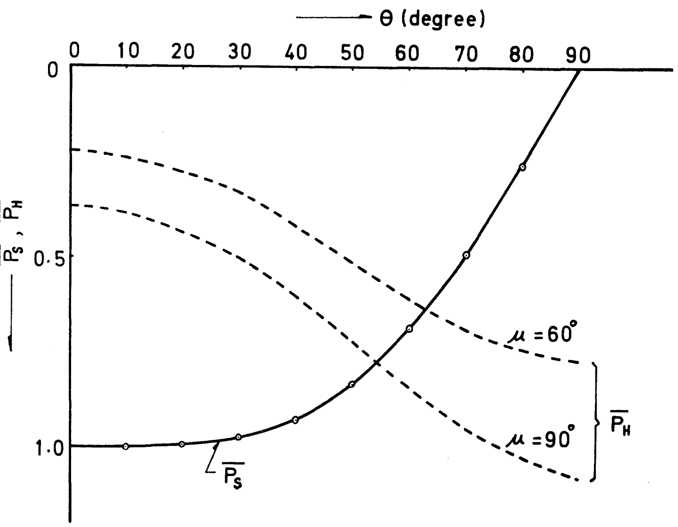

θ (degree)
0 10 20 30 40 50 60 70 80 90
0
0.5
1.0
$\overline{P_S}$, $\overline{P_H}$
μ=60°
μ=90°
$\overline{P_H}$
$\overline{P_S}$

Fig. 6.

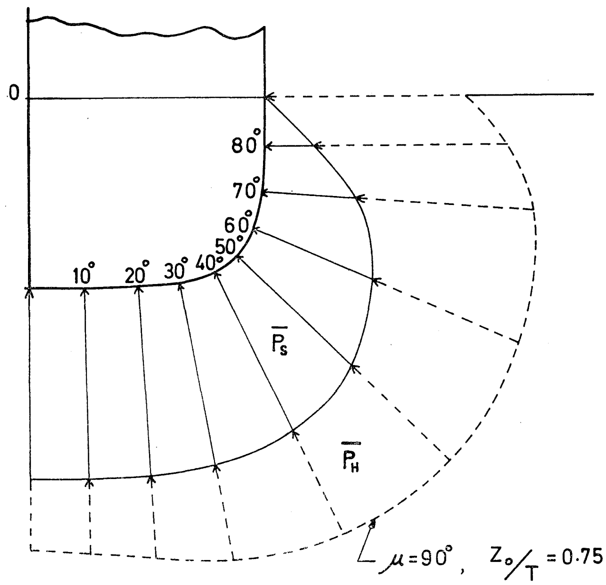


Fig. 7.

by assuming steepness as $2h_w/\lambda=1/20$, we obtain $Z_0/T=0.75$

by assuming steepness as 〃 $=1/30$, we obtain $Z_0/T=0.50$

Fig. 6 shows the calculation results of $\overline{P_s}$ and $\overline{P_H}$ by assuming as $2h_w/\lambda=1/20$. While $\overline{P_H}$ is about 20∼30% of $\overline{P_s}$ at the bottom, at the upper part of ship's side, it becomes very large.

In Fig. 7, the relative magnitude of hydrostatic and fluctuating pressure is shown with respect to the section of $H_0=1.25$.

5. Conclusion

In order to calculate hydrodynamic pressure distribution precisely, we have to consider not only the heaving motion but also the pitching motion.

The hpdrodynamic pressure attributed to the pitching motion will become $f_1P''_{aw}V\dot{\theta}$ and $f_2P''_{aw}V\theta$, in which f_1, f_2 are certain constant coefficients and pitching angle θ should be evaluated by solving the equation of motions.

The strip theory ([6], [7], etc.) has been practically used to solve the coupled equations of pitch and heave in longitudinal waves. Furthermore, the calcution results show a fairly good coincidence with the results obtained from the model experiment in the water tank.

It is possible to evaluate more precisely the distribution of hydrodynamic pressure by finding solutions of heaving and pitching motions through such a method as mentioned above and calculating in accordance with what is shown in this paper. Moreover, it will be indispensable to check the result of calculation with the result obtained from the model experiment.

Though the examples of numerical calculations given in this paper are those of approximation where only the heaving motion is considered, the author believes that what are shown in Figs. 6 and 7 represent a rough shape of the pressure distribution.

References

[1] F. Tasai: "On the Damping Force and Added Mass of Ships Heaving and Pitching" Report of the Research Institute for Applied Mechanics, Kyushu University, Vol. VII. No. 26, 1959 and Vol. VIII, No. 31, 1960

[2] W. R. Porter: "Pressure Distribution, Added Mass, and Damping Coefficient for Cylinders Oscillating in a Free Surface" University of California, I.E.R. Report, 82 July 1960

[3] J. R. Paulling and R. K. Richardson: "Measurements of Pressures, Forces, and Radiating Waves for Cylinders Oscillating in a Free Surface" University of California, I.E.R. Report, June 1962

[4] Hou-Wen Huang: "Measurement of Pressures and Hydrodynamic Forces on a Shiplike Model Oscillating in a Free Surface", I.E.R. Report 1965

[5] J. Gerritsma: "Shipmotions in Longitudinal Waves" T.N.O. Report, No. 35, Spring, 1960

[6] Y. Watanabe: "On the Theory of Heaving and Pitching Motions of a Ship" Technology Report of the Kyushu University, Vol. 31, No. 1, June, 1958

[7] J. Fukuda: "On the Midship Bending Moments of a Ship in Regular Waves" Journal of the Society of Naval Architects of Japan, No. 110, December, 1961

(Received Dec. 24 1966)