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ON THE APPARENT MASS OF CARGO OIL IN VIBRATIONS OF A TANKER*

By Toyoji KUMAI**

Abstract The present paper shows some hydrodynamic coefficients for correcting the mass of cargo oil or the liquid load in a tank with free surface in estimation of the natural frequency of ship vibrations. The apparent mass of liquid cargo in a ship vibration will be calculated as virtual mass of the liquid performed inside cargo tank under consideration. So that, it will be expected that the apparent mass depends on the cross section of cargo tank and the direction of vibration.

Introduction

What is notable about the vibration of ships is that the inertia force due to liquid cargo is different from that due to general cargo as heavy as the liquid cargo. In estimating the natural frequencies of hull vibrations of a tanker vibrating in respective directions, the so-called apparent mass or mass moment of inertia of the liquid in cargo tank should be taken into consideration.

The hydrodynamic pressure in the vibration of dam due to earthquake was early discussed by Westergaard[1)], while the dynamical pressure of the tank vibrating at the top of a tower has been recently calculated by Senda and Nakagawa[2)], and the impulsive pressure due to surface wave in a rectangular tank analyzed by Hagiwara and Yamamoto[3)]. From these calculations, the apparent mass of the liquid in tank will be deduced by some additional calculations. For estimating the natural frequencies of a tanker, however, the convenient apparent mass ratio for given directions of vibrations are required in the stage of initial design of ship hull, so that some calculations on the ratio of apparent mass with its frozen mass in tank should be required as an information for estimating the natural frequency and calculating the responce of the hull vibrations.

The present paper considers the two dimensional calculation of the apparent mass of perfect fluid which is partly contained in a tank, and the ratio of apparent mass or mass momet of inertia of fluid in a tank so arranged that the cross section of a tank is assumed to be of an ellipse in this calculation.

In the vertical vibration, the effect of the bottom vibration of a cuboid tank on the apparent mass in vibration is also shown in the latter part of this paper.

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1. Apparent mass of the liquid cargo in tank of elliptical cross section

The ratio of apparent mass moment of inertia of the ideal fluid with its frozen state a half which is contained in a tank of elliptical section, is considered. The location of the centre of rotation is assumed to be arbitrary at $R(-x_o, -y_o)$. The apparent mass ratios of the fluid in tank due to vertical and horizontal vibration are obtained from the results of the above rotational vibration as two extreme cases $x_o \to \infty$, and $y_o \to \infty$ respectively.

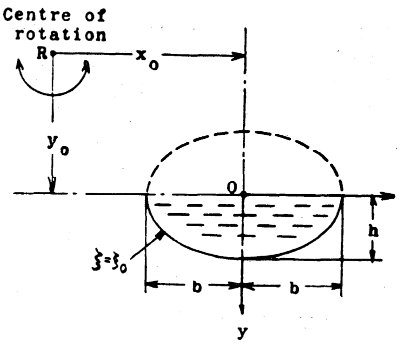


Fig. 1 Co-ordinate system of vibrating tank of elliptical section.

The elliptical section in the cartesian coordinate is presented by the use of elliptical coordinates ξ, θ as follows,

$$\left.\begin{aligned} x&=c\cosh\xi\cos\theta, \\ y&=c\sinh\xi\sin\theta, \end{aligned}\right\} \quad (1)$$

where, c is a half length of focal distance. The cross section of the tank is presented by $\xi=\xi_o$ and the free surface of the fluid is $\theta=0$ and π as shown in Figure 1. Denoting b and h, a half length of breadth of the tank at the free surface and the depth of fluid at the centre of the tank respectively, thus b, h, and ξ_o are represented by

$$\left.\begin{aligned} b&=c\cosh\xi_0 \\ h&=c\sinh\xi_0 \\ \xi_0&=\tanh^{-1}\lambda \\ \lambda&=h/b \end{aligned}\right\} \quad (2)$$

Putting ϕ the velocity potential of the fluid in tank vibrating about the centre of rotation $R(-x_o, -y_o)$, the velocity of the fluid which is normal to the inner boundary of the tank is expressed by

$$\left(\frac{\partial\phi}{\partial\xi}\right)_{\xi=\xi_0}=\omega\left\{(x+x_0)\frac{\partial y}{\partial\xi}-(y+y_0)\frac{\partial x}{\partial\xi}\right\}\cos\omega t. \quad (3)$$

Denoting $x_0/b=\alpha$, $y_0/h=\beta$, and

$$\cos\theta=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{2n}{4n^2-1}\sin 2n\theta,$$

the equation (3) becomes

$$\left(\frac{\partial\phi}{\partial\xi}\right)_{\xi=\xi_0}=\omega\cdot b^2\Big\{\alpha\sin\theta+\left(\frac{1-\lambda^2}{2}-\frac{8\lambda^2\beta}{3\pi}\right)\sin2\theta$$
$$-\frac{4\lambda^2\beta}{\pi}\sum_{n=2}^{\infty}\left(\frac{2n}{4n^2-1}\right)\sin2n\theta\Big\}\cos\omega t. \tag{4}$$

The velocity potential ϕ should satisfy the above condition at $\xi=\xi_0$ and also

$$\phi=0 \text{ at } \theta=0 \text{ and } \pi \tag{5}$$

and

$$\frac{\partial^2\phi}{\partial\xi^2}+\frac{\partial^2\phi}{\partial\theta^2}=0 \tag{6}$$

General expression of ϕ, which satisfies the above boundary conditions and Laplace equation, is written by

$$\phi=\omega\cdot b^2\Big\{A\ \sinh\xi\cdot\sin\theta+\sum_{n=1}^{\infty}A_{2n}\sinh2n\xi\cdot\sin2n\theta\Big\}\ \cos\omega t \tag{7}$$

The unknown coefficients in (7) are determined by (4), thus the required velocity potential is obtained.

The maximum kinetic energy of the vibration of fluid in tank is computed by

$$2T=\rho\int_0^{\pi}\left(\phi\frac{\partial\phi}{\partial\xi}\right)_{\xi=\xi_0}d\theta, \tag{8}$$

and that of the frozen fluid is easily obtained as follows,

$$2T_0=\rho I_p\omega^2 \tag{9}$$

where, ρ is the density of fluid and I_p is the mass moment of inertia of the frozen fluid about the given centre of rotation. The ratio of the apparent mass moment of inertia denoted by η is thus computed by

$$\eta=\frac{T}{T_0}.$$

Using the velocity potential mentioned above, η is calculated by

$$\eta=\frac{\alpha^2+\left(\dfrac{1+\lambda^2}{2}-\dfrac{8\lambda^2\beta}{3\pi}\right)^2\dfrac{1}{1+\lambda^2}+\dfrac{16\lambda^3\beta^2}{\pi^2}\displaystyle\sum_{n=2}^{\infty}\frac{2n}{(4n^2-1)^2}\tanh2n\xi_0}{\alpha^2+\dfrac{1+\lambda^2}{4}+\left(\beta^2+\dfrac{8\beta}{3\pi}\right)\lambda^2}, \tag{10}$$

From the above relation, the three extreme cases of vertical, horizontal and torsional vibrations about the centre of the tank are obtained.

a) Vertical vibration:

Assuming that α approaches infinity, we can presume the direction of the vibration of fluid as vertical, and therefore the ratio of apparent mass, η becomes,

$$\eta_{\text{vert.}}=1. \tag{11}$$

b) Horizontal vibration:

Providing that the β approaches infinity, the apparent mass ratio in the horizontal direction is obtained as follows;

$$\eta_{\text{horiz.}} = \frac{16\lambda}{\pi^2}\sum_{n=1}^{\infty}\frac{2n}{(4n^2-1)^2}\tanh 2n\xi_0. \tag{12}$$

c) Torsional vibration with centre at origin:

If we put $\alpha=\beta=0$ in (10), the ratio of apparent mass moment of inertia of the torsional or rotational vibration centred at origin is obtained from (10) and is written by

$$\eta_{\text{tor.}} = \frac{(1-\lambda^2)^2}{(1+\lambda^2)^2}. \tag{13}$$

The above formulae, (11) (12) (13) and (10) are conveniently used for calculating the ratio of apparent mass and mass moment of inertia of the fluid in elliptical tank for the vertical, horizontal and torsional vibrations respectively. The η value of the fluid in horizontal vibration in the elliptical tank is slightly less than that in the tank of rectangular cross section, as will be seen in Figure 2.

It is of some interest to compare the η values mentioned above with the ratio of virtual mass of water surrounding a half immersed elliptical cylinder of infinite length to its displacement. Table I shows the comparison of the η values of in-

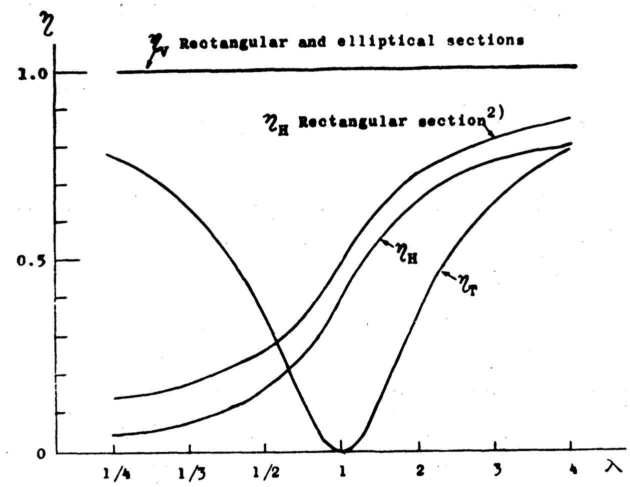


Fig. 2 Apparent mass ratios of the vibrating tanks for various ratios of depth of water in tank and half breadth.

Table I Comparison of the ratios of inside apparent mass and outside virtual mass.

	Inside	Outside
$\eta_{\text{vert.}}$	1	$1/\lambda$
$\eta_{\text{horiz.}}$	$\frac{16\lambda}{\pi^2}\sum_{n=1}^{\infty}\frac{2n}{(4n^2-1)^2}\tanh 2n\xi_0$	$4\lambda/\pi^2$
$\eta_{\text{tor.}}$	$\frac{(1-\lambda^2)^2}{(1+\lambda^2)^2}$	$\frac{(1-\lambda^2)^2}{2\lambda(1+\lambda)^2}$

side tank with that of outside virtual mass[4] in the vertical, horizontal and torsional directions of the vibrations of elliptical tank. It will be resulted that the η values differ from that of outside virtual mass except that of the circular cross section.

d) Numerical examples on the torsional vibration of a tanker

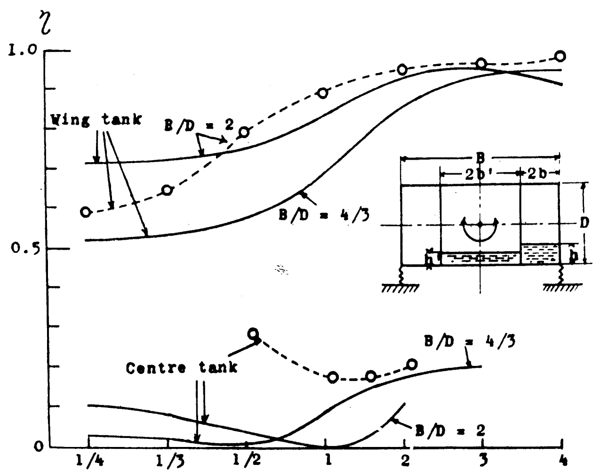
The ratio of apparent mass moment of inertia of the fluid partly filled in tanker was calculated by the use of the formulae (10) as an example. Both of the sections of the central tank and the wing tank are assumed to be of elliptical cross sections, and also the centre of torsional vibration is assumed to be at the figure centre of the cross section of the tanker. The apparent mass moment of inertias of the two types of the cross sections of the tankers were calculated with respect to the wing tank and the centre tank in each type. The numerical results of the ratio of apparent mass moment of inertia are shown in Figure 3.

e) Model experiments

The prismatic model of a tanker of the rectangular cross section with both the centre tank and the wing tank with mass-spring system of rotational vibration were made by organic glass. Comparing the natural frequencies of the rotational vibration of the empty tank and those with water, the author obtained the ratio of the apparent mass of water for given depth of water by the following relations;

$$(2\pi f_0)^2 = \frac{g\cdot k\cdot l^2}{I_0}$$

$$(2\pi f)^2 = \frac{g\cdot k\cdot l^2}{I_0+\eta I}$$



——— Theoretical (Elliptical section)
○- - - -○ Experimental (Rectangular section, B/D=2)

Fig. 3 Comparison of η-values obtained from experiments and calculations.

f_0 natural frequency of empty tank
f natural frequency of tank with liquid load
k spring constant of the system at the distance l from the centre of rotation
I_0 mass moment of inertia of the empty tank about the centre of rotation
I mass moment of inertia of liquid load in frozen state
η ratio of apparent mass moment of inertia of the liquid load

From the above two equations, the ratio of apparent mass moment of inertia is obtained by the experiment as follows:

$$\eta = \frac{I_0}{I}\left(\frac{f_0^2}{f^2} - 1\right)$$

The results of the above experiments are shown in Figure 3. As will be seen in the figure, both results of calculations and measurements show fairly good agreement. It is to be noted that the apparent mass moment of inertia of the liquid cargo in centre tank is considerably less than that of wing tank in the torsional vibration of a tanker.

2. Apparent mass of the cargo in cuboid tank associated with the vibration of bottom panel in the vertical vibration

It is a well known fact the bottom vibration of the cargo hold or cargo tank associates with the vertical vibration of a ship hull. We can easily understand that the effect of flexibility of the bottom panel is replaced by the effective mass of the cargo in the hold in ship hull vibration. The above phenomenon occurs not only in tankers but in general dry cargo ships.

a) General cargo

Let us consider vertical vibration of a cuboid hold $2l$ in length and 2b in breadth, carrying general cargo that associates with the bottom vibration, where the deflection of side wall is neglected, but δ is assumed to denote the ratio of amplitude of bottom vibration at the centre to that of the vertical vibration of the side wall, as shown in Figure 4. The resultant velocity of the vibration of the tank with general cargo is expressed by

$$\begin{aligned} v &= U\{\cos(\omega t-\varphi_1)+\delta f(xy)\cos(\omega t-\varphi_2)\} \\ &= U\{1+\delta^2 f^2(xy)+2\delta f(xy)\cos(\varphi_1-\varphi_2)\}^{1/2}\cos(\omega t-\varphi_0) \end{aligned} \tag{14}$$

where, $f(xy)$ mode of vibration of bottom panel

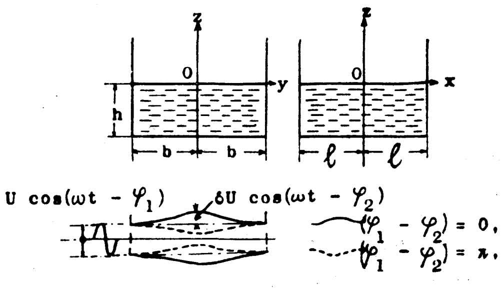


Fig. 4. Co-ordinate system of the vertical vibration of a cuboid tank with bottom vibration.

ω frequency of the tank wall

φ_1, φ_2 phases of vibrations of wall and bottom panel respectively

Now if we put the n the natural frequency of the bottom panel, the relations between the phase lag and frequencies will be as follows:

$$\omega<n: \quad \varphi_1-\varphi_2 \doteqdot 0: \quad \text{in phase,} \tag{15}$$

$$\omega>n: \quad \varphi_1-\varphi_2 \doteqdot \pi: \quad \text{out-of-phase,} \tag{16}$$

The resultant velocities in the above two states are expressed respectively by

$$v=U\{1+\delta f(xy)\}\cos(\omega t-\varphi_0), \text{ in phase,} \tag{17}$$

$$v=U\{1-\delta f(xy)\}\cos(\omega t-\varphi_0), \text{ out-of-phase,} \tag{18}$$

Denoting m the mass per unit area of general cargo, we can express the maximum kinetic energy by

$$2T=4\int_0^l\int_0^b mv^2\,dxdy, \tag{19}$$

when the bottom plate does not associate with the vibration of tank,

$$2T_0=4\int_0^l\int_0^b mU^2\,dxdy, \tag{20}$$

Now, if we assume the mode of vibration of the bottom panel as a type of a rectangular plate with its four edges clamped, the approximate mode will be presented by

$$f(xy)=\cos^2\frac{\pi x}{l}\cdot\cos^2\frac{\pi y}{b} \tag{21}$$

Using the above mode of the vibration of the bottom panel and taking the ratio of T and T_0, η will become

$$\left.\begin{aligned} &\eta=1+\frac{\delta}{2}+\frac{9}{64}\delta^2 \quad \text{for inphase} \\ &\eta=1-\frac{\delta}{2}+\frac{9}{64}\delta^2 \quad \text{for out-of-phase} \end{aligned}\right\} \tag{22}$$

As will be seen in the above equations, η is larger than the unity in the inphase vibration of bottom and side wall, whereas this is less than the unity in the case out-of-phase.

b) Liquid cargo

The apparent mass ratio η is computed by the use of potential function in the vibration of the tank with liquid cargo. The boundary conditions of the liquid cargo in cuboid for the velocity potential ϕ are written by,

$$\left.\begin{aligned} &z=0: \quad \phi=0 \\ &x=\pm l: \quad \frac{d\phi}{dx}=0 \\ &y=\pm b: \quad \frac{d\phi}{dy}=0 \\ &z=-h: \quad \frac{d\phi}{dz}=-U\left[\cos(\omega t-\varphi_1)+\delta\cos^2\frac{\pi x}{l}\cos^2\frac{\pi y}{b}\cos(\omega t-\varphi_2)\right] \end{aligned}\right\} \tag{23}$$

It is, however, provided that the mode of the bottom vibration is the same as the previous example. The potential function that satisfies not only the above boundary conditions but also the Laplas equation is shown by

$$\phi=-U\Big[z\cos(\omega t-\varphi_1)+\frac{\delta}{4}\Big\{z+\frac{\sinh\alpha z\cdot\cos\alpha x}{\alpha\cosh\alpha h}+\frac{\sinh\beta z\cdot\cos\beta y}{\beta\cosh\beta h}$$
$$+\frac{\sinh\gamma z\cdot\cos\alpha x\cdot\cos\beta y}{\gamma\cosh\gamma h}\Big\}\cos(\omega t-\varphi_2)\Big] \tag{24}$$

where, $\alpha=\pi/l$, $\beta=\pi/b$, $\gamma=\sqrt{\alpha^2+\beta^2}$

Using the above potential function, the apparent mass ratio is computed by the same means of the previous example as follows;

$$\eta=1\pm\frac{\delta}{2}+\frac{\delta^2}{16}\Big\{1+\frac{\tanh\alpha h}{2\alpha h}+\frac{\tanh\beta h}{2\beta h}+\frac{\tanh\gamma h}{2\gamma h}\Big\} \tag{25}$$

In the above equation, plus sign is used for inphase and minus one for the out-of-phase. Figure 5 shows the η values for various δ in parameter for general cargo (dotted line) and liquid cargo (full line). The η value in the case of liquid cargo is much less than that of general cargo in the vibration with the out-of-phase.

3. Application of apparent mass ratio for correcting the higher natural frequency of ship hull vibration

The rate of increase of the natural frequencies of the vertical vibration of a

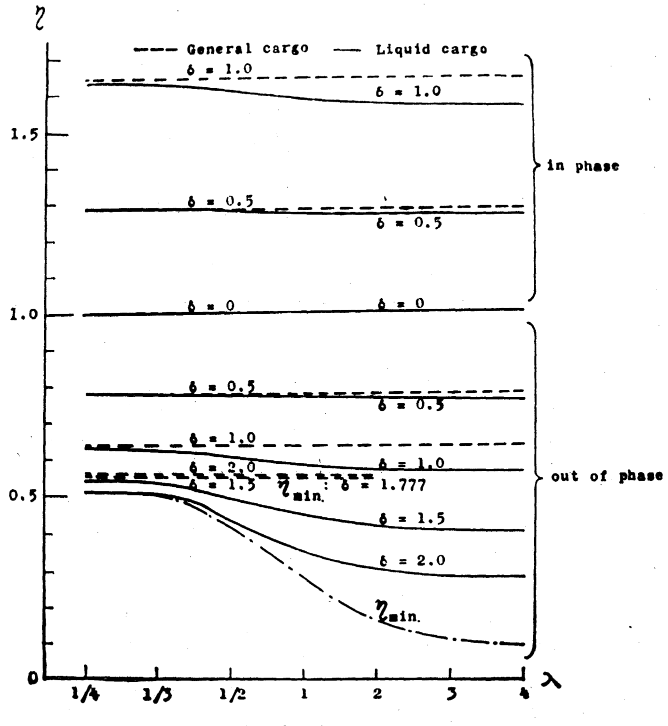


Fig. 5 Apparent mass ratios in the vertical vibration of cuboid tank with dry cargo and liquid.

ship hull for the corresponding number of nodes is computed almost linear in large tanker, whereas the results of measurements of the natural frequencies are generally plotted somewhat lower than that of calculations without consideration of the local vibrations of ship structure. It was proved that one of the causes for the discrepancy in the curves of the natural frequencies mentioned above is in the local vibration of the ship structure.[5] On the other hand, the most effective local vibration we can expect in the vertical hull vibration is bottom vibration in ship structures.[6] The natural frequency of the bottom panels of a large tanker is roughly estimated as the same or little less than the blade frequency in a large tanker. It will be assumed, therefore, that the bottom vibration is the state of inphase akin to the blade frequency, as was previously measured in an actual tanker. Following the present calculation, we find the apparent mass of the cargo oil to increase for its frozen mass in the state of the inphase. Consequently the natural frequencies of the hull vibration give lower value than that ignor the apparent mass of cargo oil.

The amplitude of the bottom vibration δ will gradually increase by the increase in the frequency of the hull vibration up to the natural frequency of the vibration of the bottom panel.

After the resonance of the bottom and hull vibrations, the phase of both vibrations change to the state out-of-phase. The apparent mass ratio therefore becomes less than the unity, as seen in the present calculation, and therefore, the natural frequency of the ship hull increases, and the discontinuous curve of frequency-nodes appears.[5] As a result, the ship hull vibration is assumed to be of such a vibration system as having apparent mass increasing due to local vibrations of bottom up to the natural frequency of the bottom panels. The correction factor of natural frequency of a ship hull for the local vibration of bottom panels will be conveniently made for practical use by the above investigation. Further study, however, will be required on this problem.

Conclusions

The conclusive note obtained are as follows:

As an example, the results were presented as the ratio of apparent mass of perfect fluid with its frozen mass which filled a half in the tank of an elliptical cross section in vertical, horizontal and torsional vibrations of ships hull. It is to be noted that the apparent mass ratio and mass moment of inertia ratio are obtained less than unity in the horizontal and torsional vibrations respectively, whereas the ratio is calculated as unity in the vertical vibration. The effects of the vibration of bottom plate of the tank of rectangular cross section upon the effective mass of general cargo and fluid in the tank which is vertically vibrating were also investigated. The apparent mass ratio in this case is greater than unity when the vibration of bottom plate is inphase, on the contrary the ratio is less than unity in the case of out-of-phase of bottom vibration. The apparent mass ratio of the liquid cargo is some more less than that of general cargo in the case of antiphase of bottom vibration.

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