EFFECTS OF SHEAR DEFLECTION AND ROTATORY INERTIA ON THE DAMPING OF THE FLEXURAL VIBRATION OF A SHIP HULL

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EFFECTS OF SHEAR DEFLECTION AND ROTATORY INERTIA ON THE DAMPING OF THE FLEXURAL VIBRATION OF A SHIP HULL*

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Abstract The relation between the logarithmic decrement of the damping and the natural frequencies of a free-free beam which is in higher modes of flexural vibration is theoretically investigated here, taking into account as parameters the shear deflection and rotatory inertia factors as well as the inner damping force due to the normal and tangential viscosities of the material or structure of the beam. The external damping force is ignored. It is concluded that the shear deflection and the tangential viscosity of the beam have more conspicuous effect than the rotatory inertia and normal viscosity. In regard to the rotatory inertia, the logarithmic decrement increases with the increase of rotatory inertia, though the natural frequency decreases. The results of the investigation will be applied to the response calculation in the higher mode of the ship hull vibrations excited by the propeller blade force.

Introduction

The inner damping force of the vibration of a ship's hull, which is assumed to consist of the normal and tangential viscosities of the hull structure, will affect the logarithmic decrement of the vibration of the ship, if the shear deflection and the rotatory inertia factors are taken into account. The author has previously deduced the fundamental equation of the damped vibration of the uniform prismatic ship hull girder, the result of which was put forward as an empirical expression for estimating the damping coefficient of the higher mode of the ship hull vibration [2].

During the last five years, numerous results of logarithmic decrements of ship hull vibrations have been presented and compared with the empirical results, see refs. [3], [4], [5] and [6]. However, there is considerable scatter in the expression for the relation between the logarithmic decrements and the corresponding natural frequencies. Thus, further investigation into this problem is required.

In this article the relation is taken up for consideration between the logarithmic

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decrement of the damping and the natural frequencies up to the ten-node mode of the flexural vibration of the free-free beam, where the shear deflection and tangential viscosities are taken into account as parameters. The external damping force depending on the velocity of the vibrating beam has been neglected.

Theoretical Calculation

The Timoshenko equation [1] in which the inner damping force due to the normal and the tangential viscosities of the material or structure of the beam are considered, where the external damping force, however, is ignored, has been deduced by the author as follows [2]:

$$EI\left(1+\mu\frac{\partial}{\partial t}\right)\left(1+\nu\frac{\partial}{\partial t}\right)\frac{\partial^4 y}{\partial x^4}-\sigma\left\{\frac{EI}{k'GA}\left(1+\mu\frac{\partial}{\partial t}\right)+r^2\left(1+\nu\frac{\partial}{\partial t}\right)\right\}\frac{\partial^4 y}{\partial x^2\partial t^2}$$

$$+\sigma\left(1+\nu\frac{\partial}{\partial t}\right)\frac{\partial^2 y}{\partial t^2}+\frac{\sigma^2 r^2}{k'GA}\frac{\partial^4 y}{\partial t^4}=0, \tag{1}$$

provided that

$$\text{bending moment}=-EI\left(1+\mu\frac{\partial}{\partial t}\right)\frac{\partial\varphi}{\partial x},$$

$$\text{shearing force}=k'GA\left(1+\nu\frac{\partial}{\partial t}\right)\left(\frac{\partial y}{\partial x}-\varphi\right),$$

where μ = normal viscosity
ν = tangential viscosity
φ = slope due to bending moment
y = deflection of the beam
EI = flexural rigidity of the beam
σ = mass per unit length of the beam
$k'GA$ = shear rigidity of the beam
r = the radius of gyration of the mass moment of inertia of the section the beam

The normal function of the solution of (1) in the mode of a free-free beam will as an approach be taken as $\cos\lambda x/L$, so that the solution of (1) may be written:

$$y=\cos\lambda x/L.\ \exp(i\omega-\varepsilon)t, \tag{2}$$

where: ω = natural circular frequency
ε = damping coefficient
L = length of the beam with the origin at the left end

Substituting (2) for (1) and ignoring the higher order of the small quantities, the natural frequency, the damping coefficient, and the logarithmic decrement will, respectively be obtained after some calculations as:

$$\omega=\lambda^2\sqrt{\frac{EI}{\sigma L^4\{1+(\alpha+\beta)\lambda^2\}}}, \tag{3}$$

$$\varepsilon=\frac{\{\mu+(\alpha\nu+\beta\mu)\lambda^2\}\omega^2}{2\{1+(\alpha+\beta)\lambda^2\}}, \tag{4}$$

$$\delta=\frac{2\pi\varepsilon}{\omega}=\frac{\pi\omega\{\mu+(\alpha\nu+\beta\mu)\lambda^2\}}{1+(\alpha+\beta)\lambda^2} \tag{5}$$

where : λ = eigenvalue = $n\pi$
$\alpha = EI/k'GAL^2$ = shear deflection factor
$\beta = r^2/L^2$ = rotatory inertia factor
δ = logarithmic decrement

From the relations between ω and $(\alpha+\beta)$ in (3) and also those between δ and α, β, μ and ν in (5), the effects of shear deflection and rotatory inertia as well as the normal and tangential viscosities of the material or the structure of the beam on the logarithmic decrement of the vibration will be investigated for given node of the natural vibration or natural frequency of the beam.

Numerical Example

A box beam like prismatic ship hull of 174 meters in length vibrating in water was considered, as in the example of the previous paper [2]. The relations between logarithmic decrements and frequencies with the parameters α, β, μ and ν of the free-free bar in the symmetric mode are plotted for the number of nodes up to the tenth-node in Figs. 1 to 4, respectively. Some of the results of the

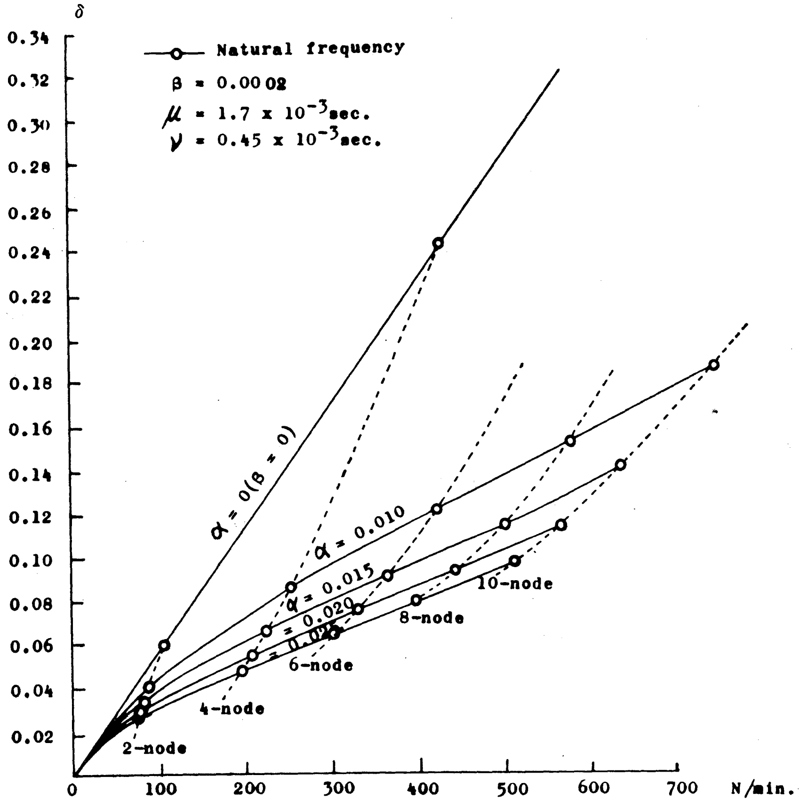


Fig. 1 The effect of shear deflection on the logarithmic decrement.

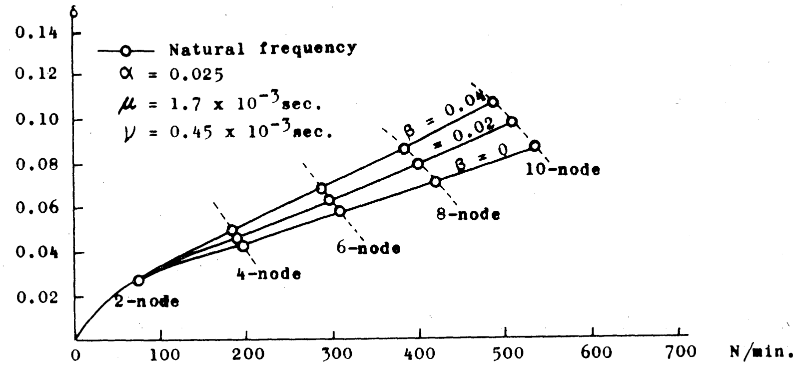


Fig. 2 The effect of rotatory inertia on the logarithmic decrement.

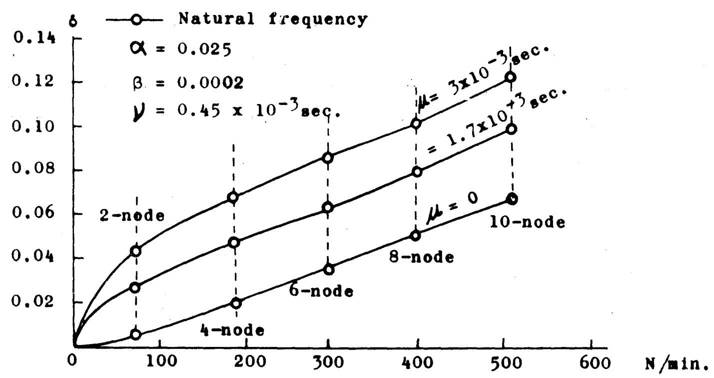


Fig. 3 The effect of normal viscosity on the logarithmic decrement.

theoretical calculations for the shear deflection factor α in typical cargo ship have already been presented by Dr. Y. Watanabe in 1934, [7].

For estimating the logarithmic decrement of the higher mode of the vibration of such a beam as a ship hull, the logarithmic decrement will be conveniently presented from (5) as the non-dimensional expression:

$$\frac{\delta_n}{\pi\mu\omega_n}=\frac{1+\left(\frac{\nu}{\mu}\alpha+\beta\right)n^2\pi^2}{1+(\alpha+\beta)n^2\pi^2}\equiv k_n, \tag{6}$$

k_n is the specific logarithmic decrement for the number of nodes, n, of the natural vibration of the beam provided that $\lambda_n=n\pi$. The theoretical calculations of k_n will be carried out from the right hand side of (6) for given values of α, β and ν/μ with respect to n. On the other hand, δ_n and λ_n are obtained from the resonances curves of the measurements of ship vibrations. μ and ν are also determined from the test results of flexural and torsional vibration [2], respectively. Consequently the specific logarithmic decrement k_n mentioned above, will be obtained by applying the left hand side of (6) on the vibration measurements.

Fig. 5 shows, as an example, a curve of k_n for various numbers of nodes for

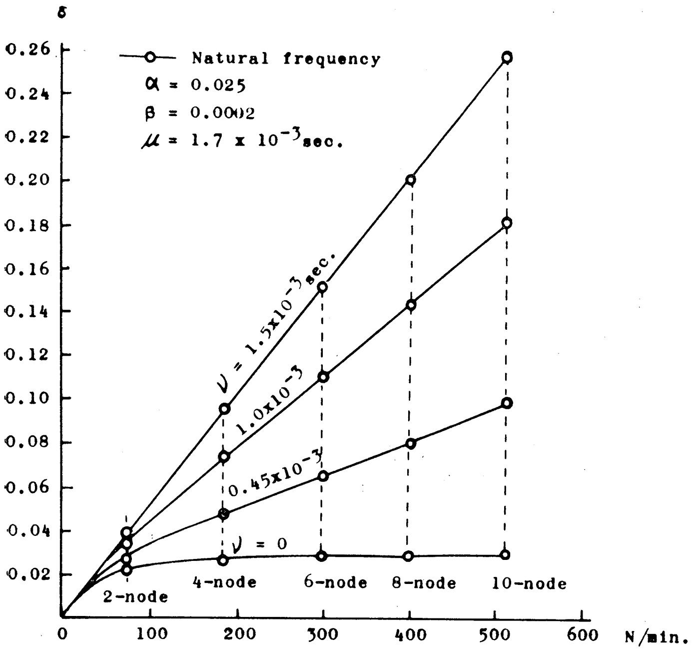
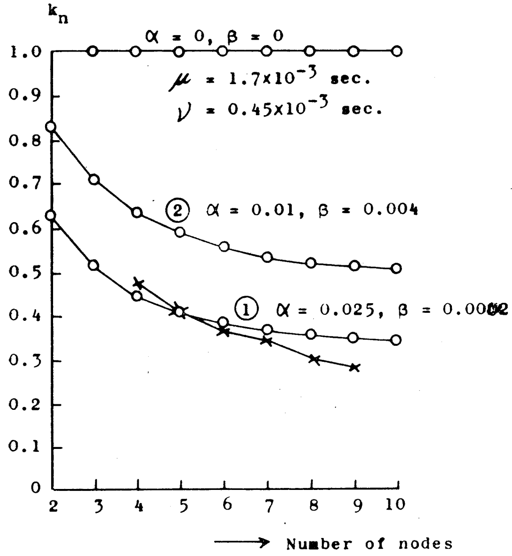


Fig. 4 The effect of shear viscosity on the logarithmic decrement



—○— Theoretical values

—×— Experimental results on board supertanker (vertical vibration)

① Vertical vibration of a supertanker

② Horizontal vibration of a supertanker

Fig. 5 k_n-n curves for given values of α, β, μ and ν in the vertical and horizontal vibrations of a supertanker.

the vertical vibration of a supertanker obtained by (6). Agreement between estimation and measurements of k will be fairly good as is seen in the figure. The value of k_n presented in Fig. 5 is more reasonably used than that obtained from the existing expression, as the natural frequency is lacking as a parameter in expression (6). Further numerous ship data are required to confirm the k_n-n curves in various kinds of ships.

Conclusions

The notable results qualitatively obtained are as follows:

1. The shear deflection factor and the coefficient of the tangential viscosity affect the logarithmic decrement in the higher mode of vibration of the beam to a higher degree than do the rotatory inertia factor and the normal viscosity.

2. A reasonable expression for the logarithmic decrement of the ship vibration was put forward as $k_n = \delta_n / \mu\omega\pi$. The curve of k_n versus n will be more reasonably used than existing expressions for estimating the logarithmic decrements in ship vibrations.

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Appendix

On the Distinction between Inner Damping and External Damping in the Vibration of a Beam

The inner damping force of the material or the structure is defined by the force proportional to the time derivative of the curvature in the pure bending vibration and of the slope in the shearing vibration of a beam. Whereas the external damping force is that proportional to the time derivative of the amplitude of the vibration of the beam in the pure bending and that in the shearing vibration.

1. Pure bending vibration:

The equation of the vibration of a beam considered internal and external damping forces is presented by,

$$\frac{\partial^2}{\partial x^2}EI\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 y}{\partial x^2}+m\left(\frac{\partial^2 y}{\partial t^2}+\gamma\frac{\partial y}{\partial t}\right)=0 \tag{1}$$

where, αEI: normal viscosity of the structure or material which defined as inner damping force

γm: external damping force

Assuming the normal function $y(x)$, the solution of the equation (1) becomes

$$y=y(x)\cdot e^{(ip-q)t} \tag{2}$$

where p and q is natural frequency and the damping coefficient of the vibration of the beam. Putting $y(x)=\sin\frac{\lambda x}{L}$, where L is the length of a beam, and substitute (2) into (1), the relation between p and q is easily obtained as follows;

$$p=\sqrt{p_o^2-q^2} \qquad \text{where } p_o^2=\frac{\lambda^4 EI}{mL^2}$$

$$q=\frac{1}{2}(\alpha p_0^2+\gamma)$$

The logarithmic decrement is approximately presented by

$$\delta=\frac{2\pi q}{p_0}=\pi\left(\alpha p_0+\frac{\gamma}{p_0}\right), \tag{3}$$

provided that the damping force is so small. As was seen in the above result, the logarithmic damping due to inner damping force is proportional to the natural frequency of the vibration of the beam, whereas, that due to the external damping force is inversely proportional to the natural frequency of the vibration. We can easily assume that the inner damping force is more essential than the external damping force in the calculation of the logarithmic decrement of the ship hull vibration in the higher mode.

2. Shearing vibration:

The equation of the shearing vibration considering the inner and external damping forces is expressed by

$$\frac{\partial}{\partial x}k'GA\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial y}{\partial x}=m\left(\frac{\partial^2 y}{\partial t^2}+\gamma\frac{\partial y}{\partial t}\right) \tag{4}$$

where, $\beta k'GA$: inner damping force due to shear viscosity of the beam or the material

γm : external damping force

By the same calculation as mentioned above, the logarithmic decrement δ becomes,

$$\delta=\pi\left(\beta p_0+\frac{\gamma}{p_0}\right), \tag{5}$$

The logarithmic decrement in the shearing vibration of the beam is also obtained similar form like that in pure bending vibration.

The calculation of the relations between the inner and external damping forces in the vibration of Timoshenko beam is more complicate. The fundamental equation and the solution in this case were already shown in the author's previous paper and represented in this paper.

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