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MEASUREMENTS OF TENSILE STRENGTHS OF POLYMETHYL METHACRYLATE SPECIMENS PATCHED WITH GLASS CLOTH REINFORCED PLASTIC LAMINATES RELATED TO THE LOAD RATE*

By Masakazu HIGUCHI**

Partial reinforcement of plastic plates by patching sometimes gives rise to the decrease in strength for other stresses concurrently acting on the plates. Measurements were made for such strengths of the specimens of polymethyl methacrylate to which patches of glass cloth reinforced polymethyl methacrylate laminate were stuck with adhesive. There were taken into account stress concentration owing to the step of the patch end, residual stresses caused by adhesion of the patch, and dependence of the strength on the stress rate.

The strength of patched specimens here used were nearly half of the strength of plate specimens of polymethyl methacrylate at the moderate rate of loading. Unexpectedly, the strengths were almost independent of the rate in the range of 10^{-3}~10kg/mm²sec in spite of the rate sensitive strength property of polymethyl methacrylate.

The insensitivity is ascribable to the early growth of crazes on the surface of specimens where stress concentrates; the crazes have been dormant there on account of a rather long time elpsed after annealing.

On the other hand, annealing was found to bring forth the undesirable result that it produces residual stresses as well as high temperature curing does. The residual stresses were below 0·5 kg/mm² in the direction of the external force of test thanks to the effective aslant cut of the patch edges, whereas they amount to 2 kg/mm² in the cross direction.

No residual stress was observed when the adhesive bond had been cured at the room temperature.

1. Introduction

It is a common practice partly to strengthen a plastic plate by the aid of an adhering patch of glass cloth reinforced plastic laminate, whereas the step formed with the patch end may act as a stress raiser and weaken the plate for another stress concurrently produced in the plate, especially when the material of the patch is rigid. If materials obey the Hooke's law, the factor of stress concentration can be calculated one way or other in the case of the most complicated condition of boundaries. Polymethyl methacrylate is, however, a visco-elastic

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material and that of nonlinear type when strained to some percent.

We made direct measurements of breaking strengths for the specimens which have patches stuck on their faces with adhesive, subject to tension along their length. We used two kinds of materials for patches, glass cloth laminate and polymethyl methacrylate; the latter for a patch material was used for comparison. Some specimens had the patches adhered on one face and some had on both faces. The others were shaped with a bit to the form of the specimen having patches on both faces. The patches were beveled at the ends near the specimen center in order to minimize concentration of stress.

It had been presupposed that one-face patched specimens would be apparently weak because of offset loading owing to unsymmetry of the form; there would be residual stress which would do harm and the bad effect would be reduced by annealing; the breaking strength would depend on the rate of loading. We further needed the measurements of distribution of stresses under tension, of residual stresses, of the warp of specimens due to adhesion, its effect on the condition of loading and the change in warp by annealing. It proved that some presuppositions did not come true.

2. Specimens and Test Procedures

We used Acrylite, cast polymethyl methacrylate plate, the degree of polymerization of which was about 9,000. Laminates were stuck to the base plates of specimens with adhesive. The laminates were glass cloth reinforced polymethyl methacrylate. Their textile base was of plain weave, the code of warp and weft

Table 1 Kinds and Numbers of Specimens

Kinds of specimens		Stress rate ranges, kg/mm^2sec			
		$10^{-3}\sim10^{-2}$	$10^{-2}\sim10^{-1}$	$10^{-1}\sim1$	$1\sim10$
# 1	Both face step cut	5	7	5	10
# 2	Both face 4mm thick Acrylite patched	1	5	5	6
# 3	Both face 2.5mm thick Acrylite patched	—	5	5	6
# 4	Both face 8-ply laminate patched	1	5	4	6
# 5	Both face 5-ply laminate patched	—	7	—	5
# 6	Both face 3-ply laminate patched	1	5	2	5
# 7	One face 8-ply laminate patched	1	5	4	5
# 8	One face 5-ply laminate patched	—	5	5	5
# 9	One face 3-ply laminate patched	1	5	—	6
#10	2mm thick Acrylite plate	—	—	2	4
	2mm thick laminated plate	4	5	7	5
#11	4mm thick Acrylite plate	—	—	1	6
	4mm thick laminated plate	—	12	2	9
#12	6mm thick Acrylite plate	—	—	2	6
	6mm thick laminated plate	5	—	8	10
#13	10mm thick Acrylite plate	—	—	1	7
	10mm thick laminated plate	—	6	4	13

yarns 45–2/5 (JIS–R3401)[1], and the degree of twists 4.4 S. The number of warp threads per cm was 11 and that of weft threads 6. The cloth was 432 g/m^2 in weight and 0.37～0.48 mm thick. The glass cloths were laminated with polymethyl methacrylate adhesive. The laminates were adhered to the base plates of specimens with the adhesive of which the principal ingredient was polymethyl methacrylate again. They were pressed with paper clips at room temperature till the adhesive fastened and cured enough.

The specimens which needed the job of adhesion (Specimen Nos. 2～9) were prepared at a designated workshop. Other specimens of flat plate (Nos. 10 ～13) and those with shaped steps (No. 1) were worked at the workshop of our Institute of Kyushu University. The forms of the specimens are illustrated in Fig. 1

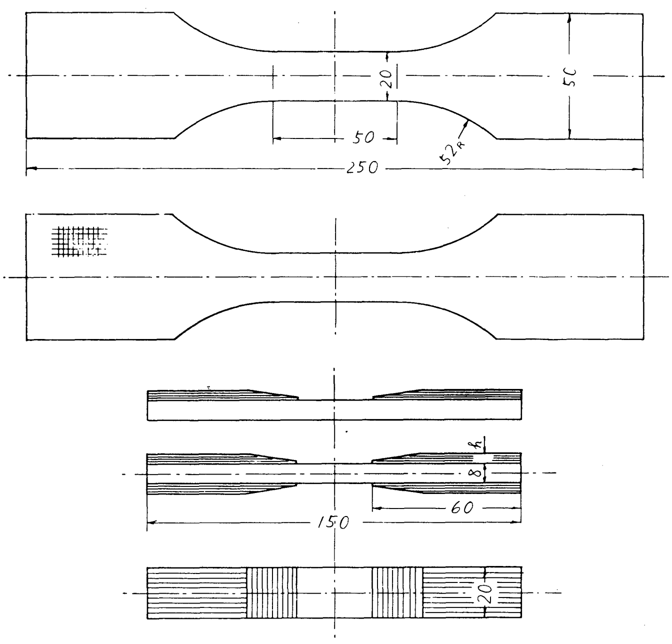


Fig. 1. Specimens.

and the nominal dimensions are written. The actual dimensions were measured accurately in the room of 23±1°C and 50±2% RH. Particularly, the sectional areas of parallel parts of the specimens had been measured at three sections and the value of the area nearest to the broken section was adopted.

Breaking test were made after the specimens had been left in the room of the condition, 23±1°C and 50±2% RH, during 24 hours. In making the tests at higher rates of straining, we used three chucks of different types to set the specimens to the testing machine (Universal type, Shimadzu RH 30) ; the chucks had been specially made respectively for the stepped specimens and for the flat plate specimens of Acrylite and of laminate. When tested at the slow rates of straining

1) Japanese Industrial Standard.

below 1 kg/mm²sec, however, the specimens were mostly gripped directly with the testing machine chucks, not universally jointed, after the way of the commercial test.

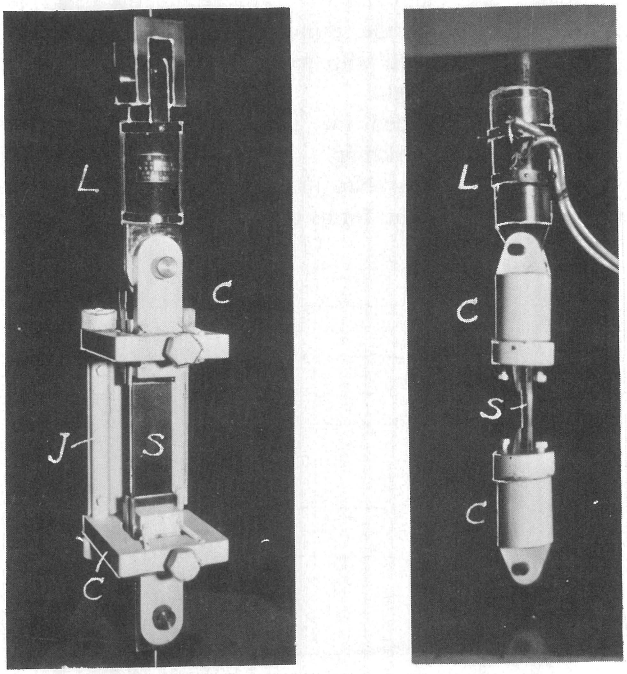


Photo 1. Chucks and load cells.
L load cells C chucks S specimens
J jigs to be removed on testing
Left for flat plate specimens
Right for step specimens.

The unparallelism of the pair of chucks of the machine was detected by the strains caused on the surfaces of the flat test pieces, which were made of mild steel plate and of Acrylite. After we, gripping one of such test pieces with the machine chucks under zero load, measured its surface strains, it was removed and regripped foreside back; the surface strains were measured again. The plate was then regripped upside down. Repeating these procedures and changing the specimen for another, we could exclude the effect owing to the slight warp of the test plate itself and the unparallelism of the machine chucks was determined with the bending strain on the surface of a plate piece as 0.027~0.030%.

Both breaking loads and mean rates of loading were read from oscillograph records of the load cell, which were statically calibrated with a loop dynamometer; the vibrators of H-type were used there.

Our highest rate of loading was the limit of the machine speed, at which speed the specimens broke in about one second. Before loading we had decided a position for the oil pressure valve to be stopped (9, 10, or full open) and turned the valve so far at a dash. Naturally there was, on getting started, left a foot on the record of load. But it was not such that reading the duration of time to break was perplexing. It is impossible to read the load on the dial of the machine; the pointer can not follow the higher rate of loading, but it may happen even that the red maximum load indicator is sent flying and goes up too much after break of specimens. Our data are free from such errors.

3. Test Results

(3.1) Specimens with steps :-

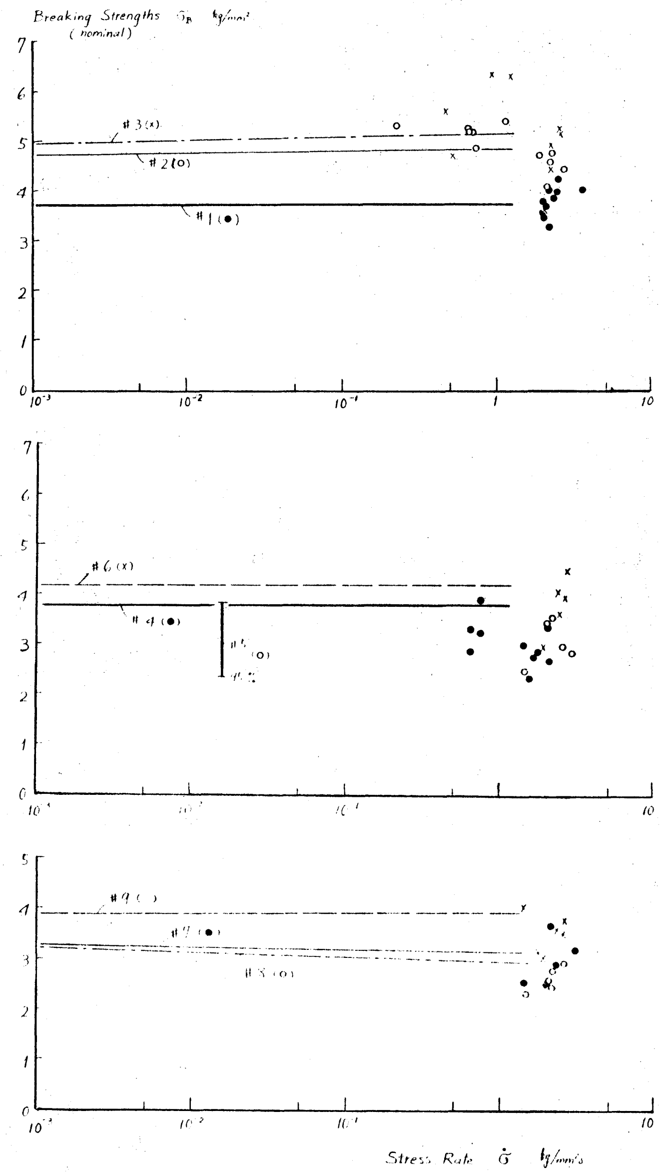
All the specimens with steps started to break at one of the steps along a slightly curved plane, not normal to the direction of loading, as would be expected when the direction of principal stresses was considered.

As shown in Figs. 2~4, the tensile breaking strengths were nearly independent of the rate of loading in the range of the measurements. It goes against our expectation. The straight lines drawn in the figures are fitted to the measurements in the range of the stress rate below 0.3 kg/mm^2 sec and not referred to those at the higher rates. We can see the same insensitivity hold beyond the rate of 0.3 kg/mm^2 sec at least in our rates of measurements. This excites some interest; and demands the explanation.

Comparing the three kinds of specimens shown in Fig. 2, we find the strengths of those with cut steps (No. 1) obviously below the other two patched, the patches of which were 4 mm and 2.5 mm thick respectively and made of the same material, polymethyl methacrylate, as the base pieces. Of the two, the specimens having higher steps (No. 2) were weaker. Accepting the significance of tha difference, we reasonably look upon the thicker patch of laminate as the heavier stress raiser and the step of cut type as the sharper notch. The notches built up by adhering the patches are not so sharp because of the excess of adhesive forced out there in the process of adhering. There must be also some effect of local attack on the adherend by the solvent of the adhesive; it will reduce the rigidity and keep off stress raising.

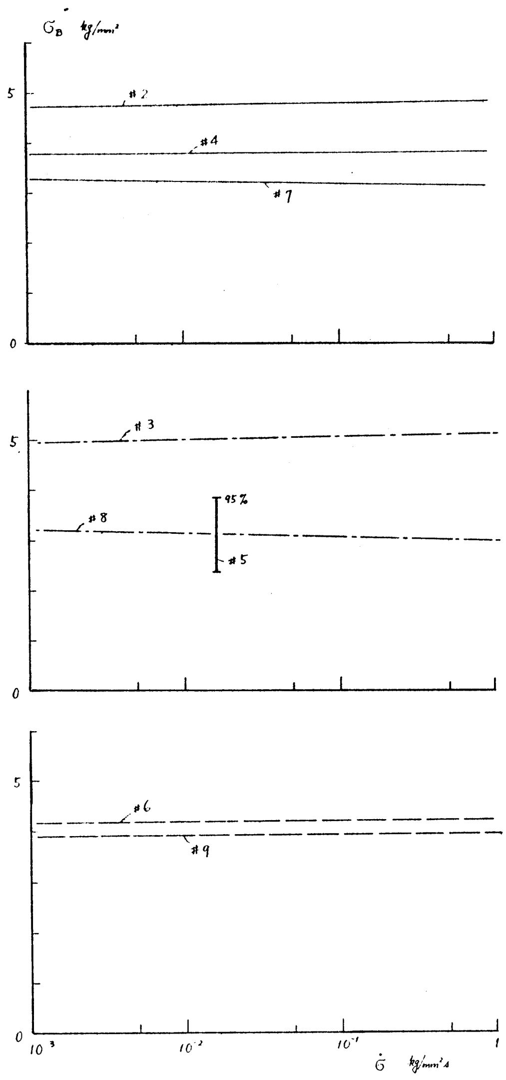
The fact that the thinnest laminate patched specimens are the strongest among those of the same type in Fig. 3 owes possibly to the low degree of stress concentration in them. Contradictory to it, the specimens No. 5 are inferior to the specimens No. 4 as seen in the same figure, though the data for the former are not enough and lacking in the range of the load rate. But the reversal of the same kind, slight as it is, is also found in the specimens with one side patches. The author is inclined, regarding the reversal as such, to think it attributable to such other reasons as residual stress due to adhesion, difference of the working condition of adhering due to different thicknesses of the patches, unsimilarity of the shapes of patches because of the same beveled edge angle and so on. At any rate, it has been unable to know its definite reason as for the both face patched specimens. Nevertheless, the warp owing to the residual stress produced by adhesion is supposed to be a valid cause of the strength reversal of one face patched specimens, for the warp curvature also tends to decrease with the thickness of patches after its reaching a maximum value (cf. Sects. 4 and 7).

Let us now consider the effect of the unsymmetrical form of the specimens



Figs. 2-4. Breaking strengths.

undergoing a load of tension. For convenience of comparison, we give Figs. 5~7 which are the rearrangement of the data presented in the foregoing figures. The strength of the specimens No. 5 is not different from that of No. 8; the strength of No. 9 is too close to that of the specimens No. 6 to be stated that the former is inferior to the latter (the difference between the strengths of them has been roughly estimated 0.89~−0.41 kg/mm² at the 5 percent level). But we can say that the both face patched specimens are stronger than the one face patched when the patches are thick considerably. Simply, the offset loading due to the unsymmetrical form of the specimens is responsible for it, and it works little on the specimens with thinner patches, not only because of the little offset but because of the cancelling of the effect with the warp existing in the specimens from the outset owing to the adhesion of patches; the warp has been found by measuring most remarkable for the specimens of medium patch thickness. For the same reason, the fact is comprehensible to us that there is seen a difference between No. 6 and No. 9 in Fig. 7 though not very affirmative as mentioned above.



Figs. 5-7. Breaking strengths

An affirmative difference is seen between the specimens with laminate patches and those with polymethyl methacrylate patches. The latter exceed the other in strength, irrespective of the thickness.(So as not to be misunderstood, the author adds that the excellence mentioned here is not in the strength of the plates which has been primarily intended to enhance with the patches.) The excellence owes almost certainly to the fact that the concentration of tensile stress at the step and the residual stress caused by adhesion are less in the specimens with the patches of the same material; the more stress concentrates at the step when the Young's modulus of the patch is higher, and the more harmfull initial stress is caused by the wider difference of coefficients of thermal expansion of the paired adherends.

(*3. 2*) *Flat plate specimens* :–

The measurements of strengths presented so far are collectively represented again with common marks ○ in Fig. 8 and compared with the strengths of flat stepless plate specimens of polymethyl methacrylate. Each of the four points denoted with + is the mean of the measurements for nine specimens of 1.5, 3.2, 4.7, and 6.3 mm thick respectively, whereas many other points are for so many specimens, of which those marked *g*, being below the others, represent the strengths of the specimens to which resistance strain gages had been adhered for control of the straining rate. From the statistical point of view, by the way, the adhesion of the gages (large deformation-type paper gages here) makes the tensile strength fall.

A straight line has been drawn on reference to the current data in the figure to indicate the tendency of the increase in strength with the stress rate. Comparing it with that of the stepped specimens, we recognize that they will not remain insensitive to the stress rate by far at the slow rate outside the scope of our measurements, at which rate both the straight lines cross. The stress of concentration ought to be regarded completely release there. At the higher rates, on the other hand, the strengths of the plate specimens mostly remain on the same level, irrespective of materials for the present case. When the plate specimens of polymethyl methacrylate are tested after annealing, the increase in strength further continues as seen in another paper of the author[2]. He considers that the increase was prevented with the dormant crazes in the material having been kept a long time in a stock. The crazes appeared in such materials even at the higher rates of straining, whereas they were missing when tested immediately after annealing.

We can interpret the situation on the basis of the conception of the crack nuclei, *i. e.* in the term of, slightly modifying, the Griffith-Orowan theory. According to the quadric nature of the relation between the critical stress for a crack to start to propagate and the extent of the initial crack nucleus, there is pronounced dependence of the critical stress on the crack size when the size is very small and *vice versa*. The load rate insensitivity is expected thus in the circumstances that the crack nucleus is ready to widen in the early stage of loading, as in the

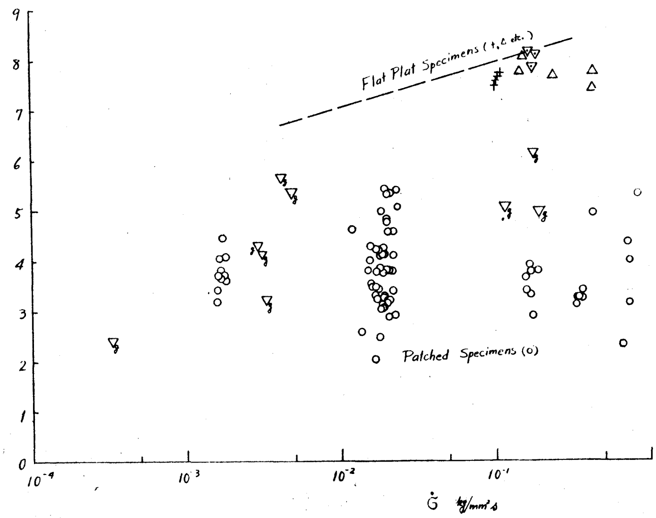


Fig. 8. Comparison of strengths of patched specimens with flat plate specimens.

2) Read at the Meating of Japanese Society for Applied Physics, Oct. 3, 1963.

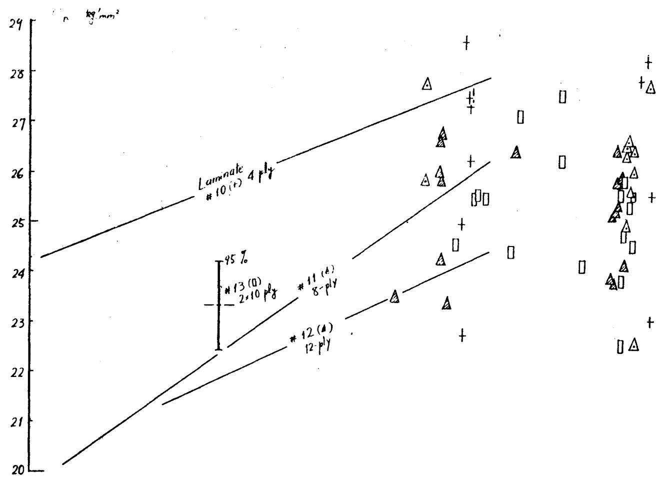


Fig. 9. Strengths of plate specimens of laminate.

specimens having stress raisers, having been kept a long time in stock, or affected by the solvent of adhesive.

In Fig. 9, we have given the rate dependence of the strengths of plate specimens of laminate. It is interesting that the inclinations of the straight lines by estimation are roughly equal to that of the polymethyl methacrylate plate specimens. There may be some reason for the fact that such is the case in spite of the discrepancy of the magnitudes of strengths of both kinds of specimens.

As already known, the specimens had the strengths dependent upon their thickness. Speaking of the four-ply specimens denominated 4 mm thick with marks +, the strength per unit thickness of the thin specimens is slightly higher than that of the thick ones (in effect, of much resin content).

4. Warp Owing to Adhesion

It is valid as stated in the foregoing section, as far as the specimen form concerns, that the thicker the patch, the more the stress concentrates at the step, and the apparent strength the more decreases, as seen when the strengths of the specimens No. 3 and No. 9 are compared with the others, except for the relations of the specimens No. 4 to No. 5 and of No. 7 to No. 8; though the difference in the latter case is too small to recognize. The reversal can not, however, determinately be elucidated, for which the author has only mentioned some possible reasons. On the other hand, we could know the effect of the form of specimens, one face- or both face-patched. We think at first that the specimens with one face patches will be apparently weak on which a tensile load acts eccentrically to produce the stress of bending, and surely it is, as seen in Fig. 5. When we refer to Figs. 6 and 7, however, we find it fail and we will ascribe it mainly to the warp of specimens owing to unsymmtrical patching, as has been mentioned already.

Unsymmetrical patching curves a specimen as the patched face is convexed.

Table 2 Warping of One-Side Patched Specimens

Specimens	Curvatures measured along whole lengths mm^{-1}	Curvatures measured partly at left 60mm mm^{-1}	Curvatures measured partly at right 60mm mm^{-1}	$\frac{\text{Laminate thickness}}{\text{Acrylite thickness}}$
#9-11	1.85×10^{-4}	3.33×10^{-4}	2.78×10^{-4}	$\frac{1.63}{7.62}$
9- 6	1.83	1.67	4 33	$\frac{1.61}{7.77}$
9- 2	1.11	1.89	2.78	$\frac{1.66}{7.83}$
#8-16	2.72	4.00	4.44	$\frac{2.62}{7.61}$
8-11	2.46	3.78	3.56	$\frac{2.66}{7.72}$
8- 6	2.67	3.67	3.67	$\frac{2.67}{7.79}$
8- 3	1.22	2.78	2.78	$\frac{2.67}{7.86}$
#7-16	2.33	4.00	4.33	$\frac{4.10}{8.27}$
7-14	2.38	4.00	4.11	$\frac{4.14}{8.32}$
7- 9	2.17	3.00	3.11	$\frac{4.24}{8.40}$
7- 6	2.11	3.55	3.22	$\frac{4.30}{8.41}$
7- 2	2.15	4.89	4.67	$\frac{4.23}{8.37}$

The warp acts against working of an offset load. The curvatures of our specimens are shown in Table 2. The mean values of them are plotted on the ratios of thicknesses of patch laminates and base materials (Fig.10). As a matter of course, it increases at first with the patch depth. But, reaching a maximum, it is supposed to tend to zero again asymptotically if the patch becomes absolutely rigid because

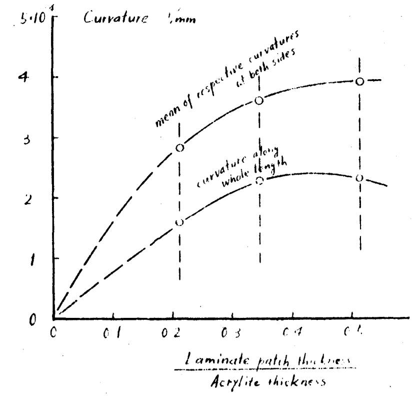


Fig. 10. Warps differing with thickness ratios.

of its being infinitely thick. The tendency seems to appear in our depth ratios.

The warped specimens are set in the bent state on the testing machine when gripped without universal joints. The set stress thus induced cancels the bending stress caused by the offset load. As an example, Fig. 11 is shown, in which each of the strains of both faces is shown for a one face-patched specimen No. 7 and compared with the strains of a both face-patched specimen No. 6 and of a plate specimen TS–2. The strain of the plate specimen was measured with resistance wire strain gages for large strain (Kyowa Musen LE-type). The others were measured with paper gages of usual type, so the measusements were confined in the small range of strains.

Here we see the specimen already strained at the zero load. The strains amount to $\pm 0.194\%$ respectively on both faces of the parallel part. For other one face-patched specimens with various thicknesses, strains are listed together with the values calculated from their curvatures in the fourth and the third columns in Table 3 (the latter do not include the strain component produced by the unparallelism of the machine chucks). The difference of the strains of both faces, diminishing with the increase in load, vanishes when the mean stress of the specimen reaches about 0.6 kg/mm^2 and then increases conversely with the reversed sign. But for the initial warp, the straight lines in Figs. 5, 6 and 7 ought to come a little lower, 0.6 kg/mm^2 or thereabouts, for No. 7 and No. 8, and least for No. 9. For all the above considerations and the measurements of bending strains at break induced by offset loading (see the fifth column in Table 3), we can not understand that the specimens of the type No. 5 are rather weak.

We think after all that the specimens might not have been prepared under such similar conditions as to stand the quantitative comparison beyond this.

Table 3 Strains of Specimen Surfaces due to Initial Warp at Zero Load

Specimens	Half-breadth at mid-length mm	ε_0 calculated %	ε_0 measured %	Bending strain at break %
TS 7- 2	4.19	-0.200	-0.206	0.745
- 6	4.21	-0.143	-0.187	0.830
- 9	4.20	-0.123	-0.194	0.785
-14	4.16	-0.169	-0.180	0.769
-16	4.14	-0.173	-0.181	0.661
8- 3	3.93	-0.109	-0.123	0.647
-11	3.86	-0.142	-0.163	0.458
-16	3.80	-0.160	-0.186	0.504
9- 6	3.88	-0.117	-0.109	0.439

ε_0 calculated=(halfbreadth)$\times$(curvature).

ε_0 measured=obtained from strain gages of mid-length surfaces of specimens at the zero load.

Bending strain=measured with large deformation type strain gages.

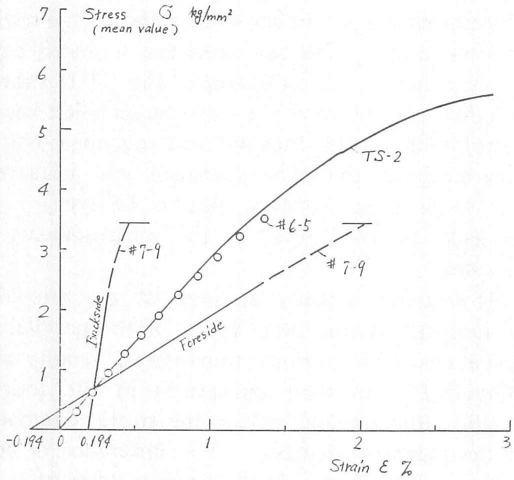


Fig. 11. Offset strain due to a warp.

5. Measurements of Stress Concentration

We wished for reference to know the concentration of stress at the inside corner of the step where the specimen starts to break. It could not be expected, however, to use the specimens prepared for measuring breaking strengths, for it was difficult to estimate the stresses from the fringe patterns of polymethyl methacrylate, a material of low stress optical coefficient, which included the residual stresses induced by adhesion besides the stresses to be measured, especially at the

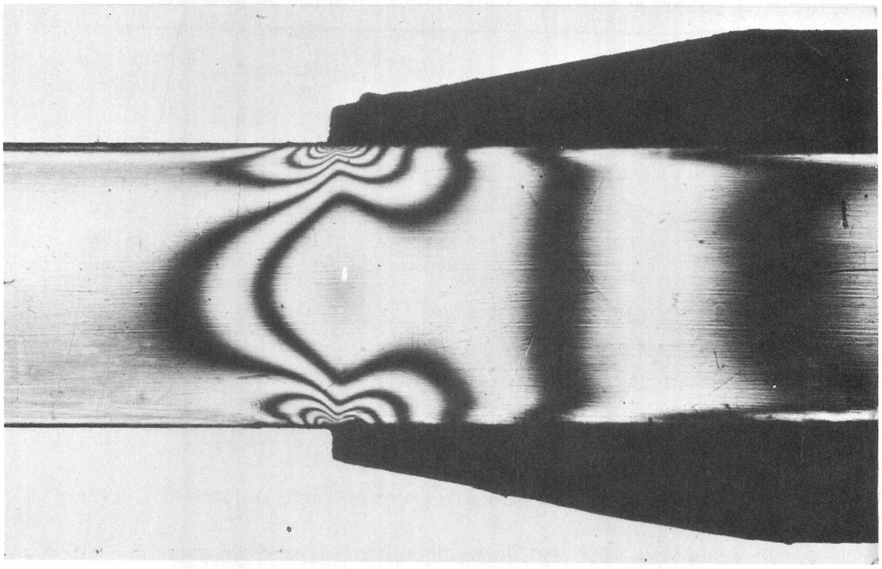


Photo 2. Isochromatic lines in a steel-patched epoxy resin specimen.

area of complicated distribution of stresses.

So they were obtained from a specimen of photoelastically more sensitive epoxy resin of the similar shape, the circumstances being not altogether identical, and the measurements were taken in the state of low stresses having little effect of creep. Hence the state of stresses at measurement was elastic.

The epoxy resin of Kakenlite was used. Steel pieces were adhered with Araldite No. 121 and the hardener 951 at the room temperature to the specimen (as will later be described, residual stresses arise in the course of cooling succeeding high temperature adhesion); any initial strain in the specimen had been removed beforehand by sufficient annealing. Naturally minor stresses soon appeared on free surfaces, but were not found along the adhering face.

Photo 2 shows a fringe pattern for the specimen when pulled in its longitudinal direction. In Fig. 12 are shown the principal stress lines. As usual, it

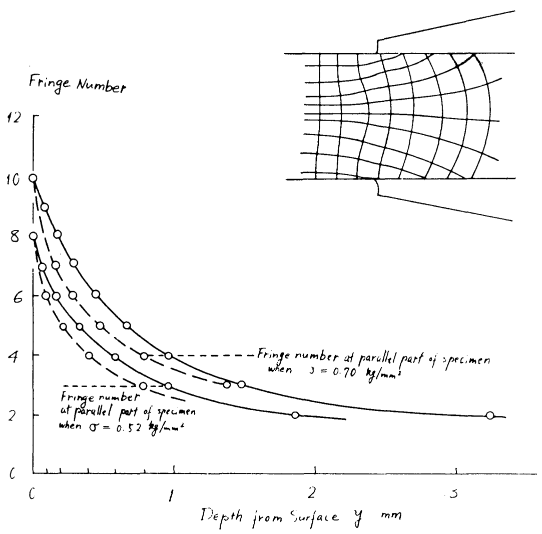


Fig. 12. Numbers of fringes at the patch end section in an epoxy resin specimen with steel patches and principal stress lines.

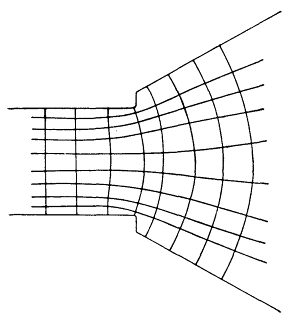


Fig. 13. Principal stress lines in an epoxy resin specimen with cut steps.

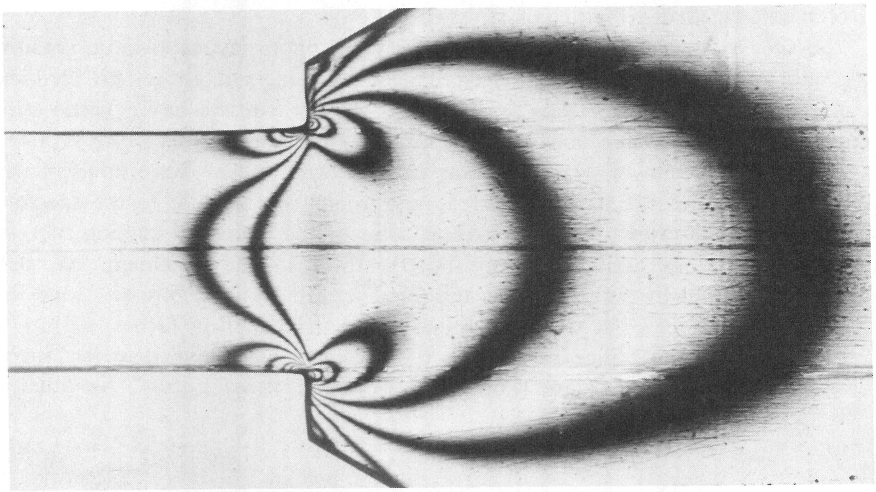


Photo 3. Isochromatic lines in a cut step epoxy resin specimen.

is difficult to determine the fringe number at the inside corner of the step. So counting the numbers of the fringes and plotting them as in Fig. 12 the number at the notch bottom was read by producing the curve connecting the plots to the left. It was done for two different loads. When the mean tensile stress was 0.52 kg/mm^2 on the minimum section, it was 8 and when 0.70 kg/mm^2, 10. One of the principal stresses on any free surface is the stress normal to the surface and is zero. The state is supposed to hold in the region close enough to the notch bottom. Since the degrees of fringes are 3 and 4 at the center of the parallel part for the respective mean stresses, the ratios 8 to 3, and 10 to 4, or 2.7 and 2.5 give the factors of stress concentration.

The values of factors thus obtained must be obviously larger compared with that of laminate patched specimens. On the other hand, if we determine the value for the specimen with cut type steps, it will give a factor of less magnitude, because the Young's modulous of polymethyl methacrylate is about 300 kg/mm^2 and is small compared with that of about 1,400 kg/mm^2 for the laminate. According to this measurement, the factor was 2.0 for both the specimens with the steps shaped by a bit 4 mm and 2 mm high (Fig. 13, Photo 3), which case the radii of the fillets could be defined and were 0.7 mm after annealing.

Summing up the above, the factor of stress concentration at the step of both face laminate patched specimens under tension is supposed to be a little more than 2 at the moderate rate of loading.

6. Residual Stresses

When we were considering breaking strengths, the question was left untouched that some residual stress might have been produced in the process of adhering and has an injurious effect on the strength. We measured such stresses for eight specimens. For the most part, the specimens prepared for the breaking strength tests were used before the tests and so they were not sliced, 20 mm thick. Most of the measurements were done only in the vicinity of the inner corner of the step. The distribution of the stresses covering a wide area was obtained for a one face- and for a both face-laminate patched specimen of polymethyl methacrylate, and for a both face-duralmin plate patched specimen of Kakenlite. The last was especially prepared for this measurement. Duralmin has the Young's modulous approximate to that of the laminate and Kakenlite to that of polymethyl methacrylate and is sensitive to stress birefringence. With this we could clarify the ambiguity of the fringe patterns of polymethyl methacrylate of low sensitivity.

The former two of the specimens were sliced from 20 mm thick specimens and were a central slice of thickness 4 mm. The Babinet's compensator was used to investigate them. Another measurement was made of a specimen with an adhered patch of congenial polymethyl methacrylate for comparison. Naturally, the residual stresses are not uniformly distributed along the breadth (z-axis) of the specimen of finite breadth, which are recognized from the stresses shown in Fig. 14. There must be the rotation of the axes of principal stresses. Accordingly, the measurements depend on thickness (see Fig. 24. It is a matter of course that the measurements are converted into specific values per unit thickness; they also are to be independent of normal and tangential stresses in the direction of the axis of z if they have uniform magnitudes). We are now not concerned about measuring the three dimensional distribution of the residual stresses in the 20 mm thick specimens, though it might be necessary to find the residual stresses in the state of plane strain, that is in the case of far broad plates, from the data of measurements with the sliced 4 mm thick pieces.

The stress optical coefficient of our polymethyl methacrylate was -0.076 mm/kg when measured statically. The typical features of the distribution are

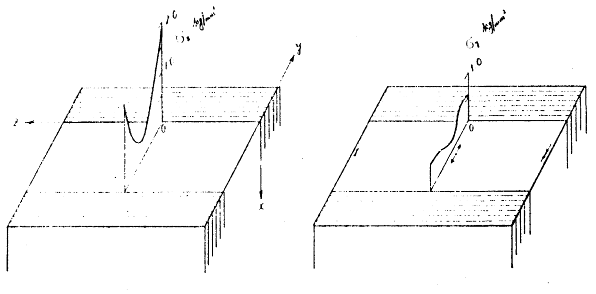


Fig. 14. Residual stresses acting perpendicularly to lengthwise directions.

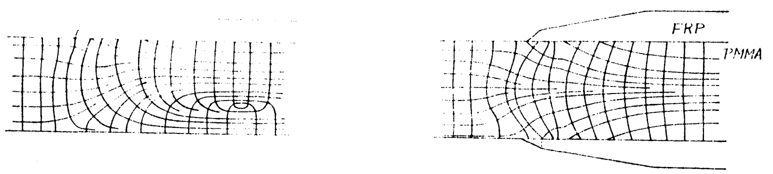
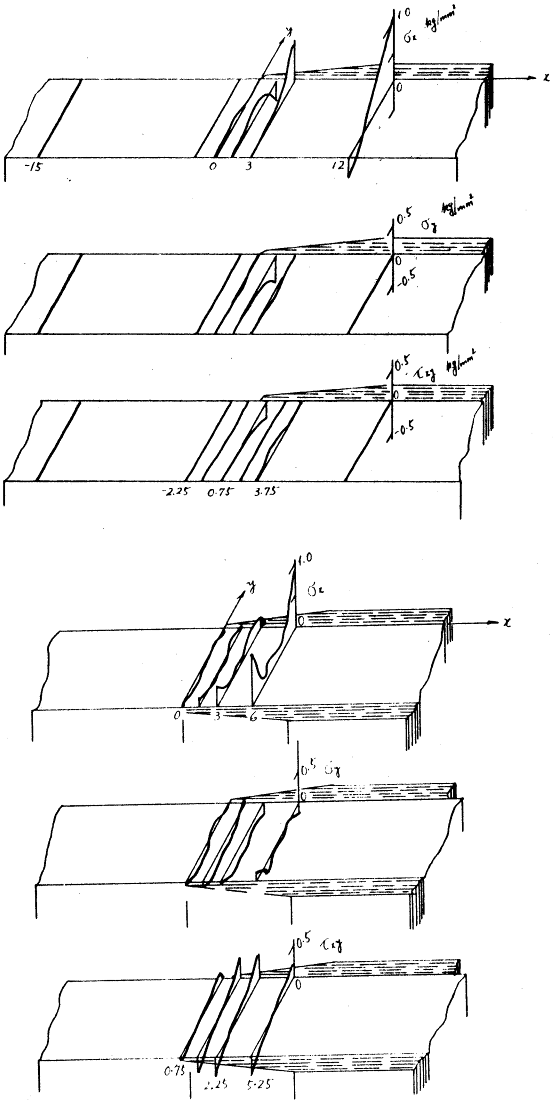


Fig. 15. Principal stress in 4mm sliced specimens of laminate-patched Acrylite.



Figs. 16 and 17. Residual stress distributions.

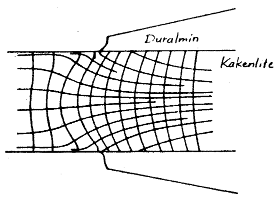


Fig. 18. Principal stress lines in the specimen shown in Photo 6.

shown in Figs. 15∼20 and in Photos 4∼6. On account of the rugged surface of the laminate, there appeared wavy fringes along adhering faces as seen in Photos 4 and 5, a chain of surplus stresses of the alternating sign. The effect was however discarded ; by prolonging the distribution of stresses in the inner area, we could obtain the stresses, which were regarded as the average.

Figs. 18 and 19 summarize the results of careful investigation of the Kakenlite specimen with a duralmin patch. The principal features agree with our expectation and with the data of the foregoing specimens, too Also it does not seem odd that both of the components σ_x and σ_y reverse their signs near the end of the adhering face.

The residual stress of interest, which had been produced at the part where break would possibly initiate, was surely below 0.5 kg/mm^2 ; it is far little compared with the breaking strength. But the largest value of the residual stresses occasionally reaches 2 kg/mm^2 at other part (see Fig. 14). This is not a little proportion to the breaking strength, though not of primary importance for our investigation at present.

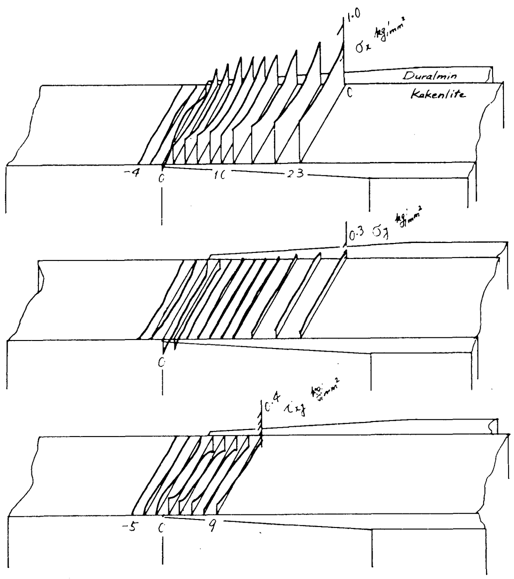


Fig. 19. Residual stress distributions in the specimen shown in Photo 6.

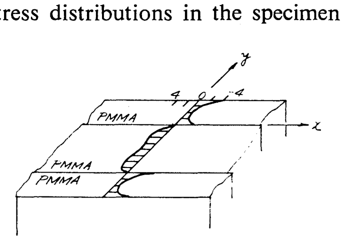
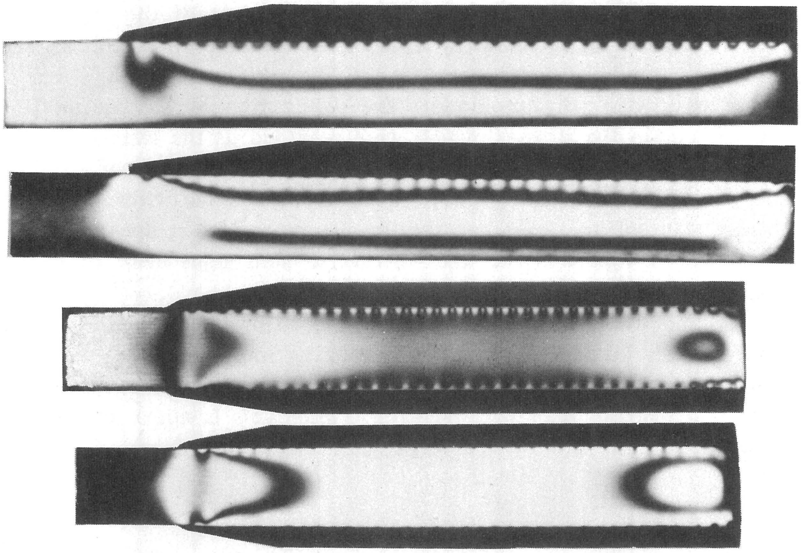


Fig. 20. Residual stress distributions in Acrylite adhesion as expressed with fringe shifts of a Babinet's compensator Tension is indicated with leftward shift.



Photos 4 and 5. Isochromatic lines in laminate-patched specimens 20mm thick.
Upper two bright field and dark field of a one side-patched specimen
Lower two a both side-patched specimen

The residual stresses produced on adhesion of the same materials were insignificant as commonly supposed. However they surely appeared and had a distribution as shown with the fringe shifts of a Babinet's compensator. The shifts here, having been measured in the region of small τ_{xy} and σ_y, namely at the part remote from the end of the adhering face, were to be nearly proportional to the stress σ_x. That there occurs the stress with the pair of materials of the same thermal expansion coefficient, will be ascribable to the condition that the outside temperature quickly lowers in advance in the early stage of cooling and so the adhering faces shift a little on each other in the middle stage. Contraction of adhesive, capable of occurring, can not be responsible for the distribution of residual stresses here observed, for the stresses accompanying to the contraction are to be distributed symmetrically at both sides of the plane of adhesive.

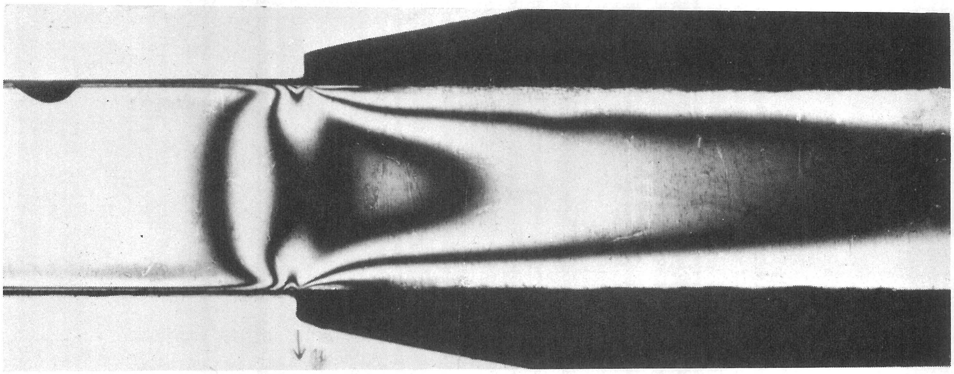
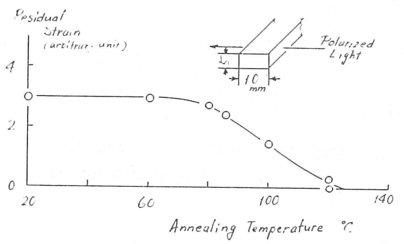


Photo 6. Isochromatic lines in a duralmin-patched Kakenlite 6 mm thick by 10 mm wide.

7. Thermal Stresses Frozen In

Whereas residual stresses in a polymethyl methacrylate plate vanish when the plate is annealed sufficiently, as known from the results of measurement shown in Fig. 21, the specimens with one face laminate patches become still more warped by annealing as seen in Photo 7. Accordingly the residual stresses must increase. We measured the coefficients of thermal expansion of the polymethyl methacrylate plate and the laminated plate of glass cloth reinforced polymethyl methacrylate. Fig. 22 shows the temperature dependent values of the coefficients.

The more increasing in warp by



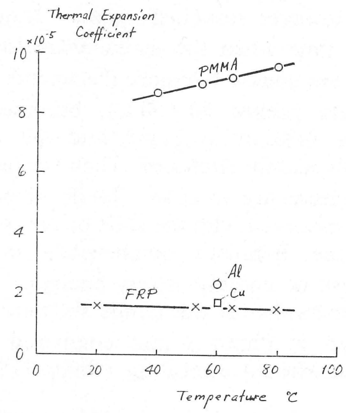


Fig. 21. (Left) Relation of residual strains to annealing temperatures. Residual strains are represented with fringe shifts.

Fig. 22. (Right) Thermal expansion coefficients.

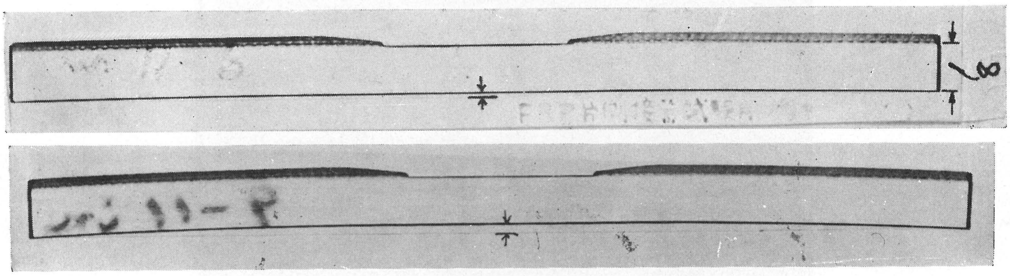


Photo 7. Warps of one side-patched specimens.

Top as received

Bottom after annealed at 115°C and cooled slowly

annealing is due possibly to the difference of the coefficients. The schematic explanation is given in Fig. 23. The adhesive bonds between the polymethyl methacrylate plate and the laminate, having been once fastened at the room temperature, will

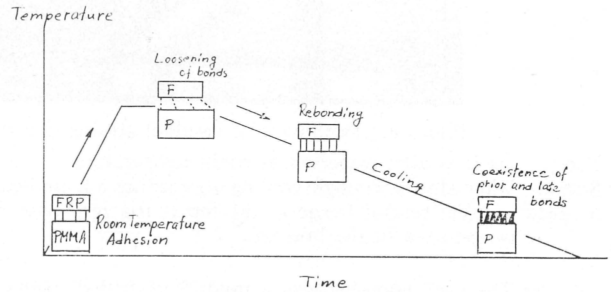


Fig. 23. Occurence of residual strains.

be stretched out as the temperature rises on account of the difference of their thermal expansion coefficients and at the annealing temperature will almost break (for local peeling is observed when one fails to press them). Then, some bonds will be made again in that state. Such bonds are however stretched again. It occurs this time when the temperature lowers. But the bonds, the more the temperature lowers (below 80~90°C), become the more difficult to break and the more bonds remain stretched. Then the residual stresses are to arise. Its development was observed with the shift of the fringes of the Babinet's compensator in the course of cooling slowly enough. The photographs for the fringe shift are produced in Photo 8 and compared with those without annealing (Photo 9)[3].

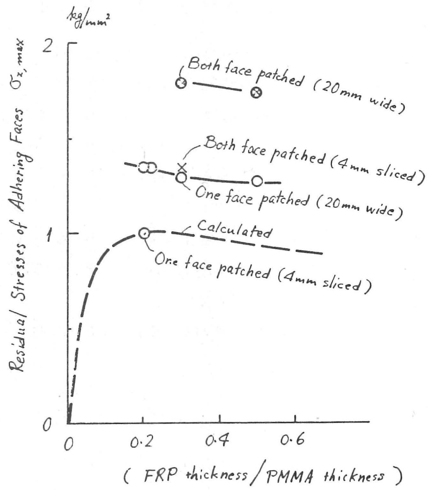


Fig. 24. Residual stresses vs adherend thickness ratios.

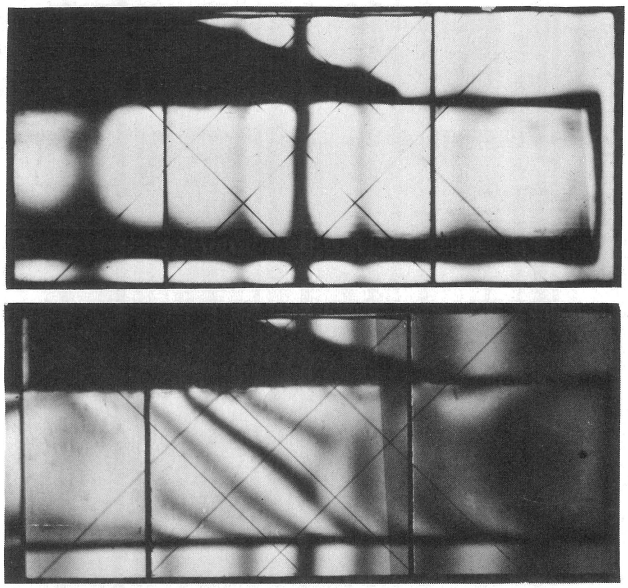


Photo 8. Occurence of residual strains due to heat treatment

Top as it is after adhering at room temperature
Bottom after slowly enough cooling succeeding 8 hour heat treatment at 100°C
A thick vertical central fringe in the top is the zero base of the Babinet's compensator; it is distorted in the bottom

3) The used adhesive was a mixture of methyl methacrylate monomer and methylene chrolide together with benzoyl peroxide as a hardener.

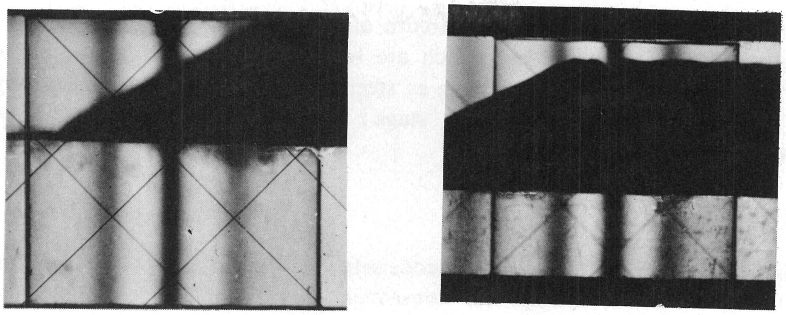


Photo 9. Not heat treated specimens.
Top after adhering at room temperature
Bottom after 5 days at room temperature

Since there is no variation of the stress in the horizontal direction in this part of the specimen, the shifts of the fringes are proportionnal to the stress. The specimen with a one face-laimnate patch being likened to a bi-metal, the residual stress in the face of adhesion was calculated and is shown with a broken line in Fig. 24, where the temperature was taken to change from the room temperature to 85°C and again to 23°C and the difference of the thermal expansion coefficients was supposed to be equal to the value 75×10^{-6} per °C at the mean temperature, 54°C, throughout the range. With these reasonable values, the calculation agrees with the measurement for the one face-patched specimen of 4 mm thickness. The residual stresses we got are thus said one of the photoelastic measurements of thermal stresses of composite beams[4].

Lastly, we need to consider the creep behavior of the material in the aspect of the residual stress as much as our measurements. According to observation, the change in the fringe n, compared to the initial value n_0, is as shown in Fig. 25 under simple tension of 0.8 kg/mm^2. The change, amounting to a few percent in early several hours, is retarded rapidly. The change in strain is much the same and not more than one percent. The data are about the same as Marin and Pao had obtained[5].

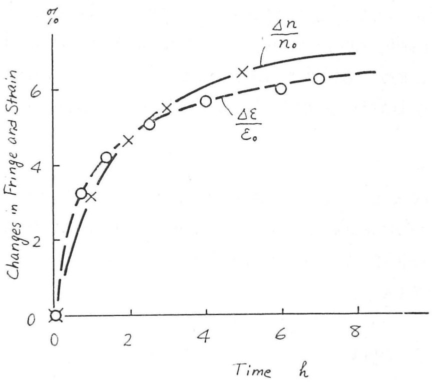


Fig. 25. Increase in fringes and creep.

Thus it is thought that the residual strains produced by adhesion will increase more or less as time goes on after the work has finished; and so the residual stresses will decrease together.

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4) For example, T. Kawamoto, Proc. 8th Japan Natl. Congr. Appl. Mech., 225 (1950); H. Shimada, Tech. Rep. Tohoku Univ., 25, 2 (1957); A. J. Durelli and C. H. Tsao, J. Appl. Mech., 22 (1955).

5) J. Marin and Y. H. Pao, Trans. ASME, 1, 232 (1952).

But the quantities are within one hundredth of the breaking strength after all. Not small proportion of the stresses, which are to occur in the neighborhood of the step of the pair of such a rigid plate as steel and a plate of plastic, are supposed to fall off with a rush in the early stage; after it the stresses remain nearly constant.

8. Conclusions

Since the annexed figures tell concisely the main part of the conclusions, briefly to summarize some of the consequences will suffice to supplement the figures.

Partial reinforcement of polymethyl methacrylate plates with adhered patches of laminate has a counterbalancing disadvantage in strength decrease of the plates at the ends of the patches. The decrease is due mainly to the concentration of stress; the factor of stress concentration is about 2 at the moderate rate of loading.

Whereas the strength of polymethyl methacrylate plates increases with the stress rate, that of the patched plates left uncovered in the room is nearly independent of the rate in the range of stress rates $10^{-3} \sim 10\ \mathrm{kg/mm^2\ sec}$. It is because the crack nucleus is ready to widen in the early stage of loading for such materials[2)].

Thicker patches are not necessarily advantageous when all sorts of stresses are considered.

It makes the problem complicated to use one face-patched specimens for strength testing. It is not only because of offset loading due to the unsymmetry of the specimen form, but also because of the warp of the specimens due to adhesion.

The residual stresses caused by adhesion were below $0.5\ \mathrm{kg/mm^2}$ at the part of our concern, thanks to the effective aslant cut of the patch edges. But they occasionally amount to $2\ \mathrm{kg/mm^2}$ in unfavorable conditions. They are produced by the difference of thermal expansion coefficients of the adherends in the course of cooling. No stress is frozen in when the bond is cured at the room temperature.

Acknowledgements. The specimen materials used in this work were provided by the Research Laboratories of the Mitsubishi Rayon, Inc. The author thanks the staffs, particularly the former Director R. Iuchi and Dr. N. Kishi, for their aid and interest in the course of this work.

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