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MEASUREMENTS OF STRESS CONCENTRATION AND BREAKING STRENGTH OF POLYMETHYL METHACRYLATE BY STRAIN BIREFRINGENCE*

By Masakazu HIGUCHI**

Strain birefringence of polymethyl methacrylate subjected to monotonically increasing tensile stress is investigated at straining rates $10^{-5} \sim 10^{-2}$ per second, stresses and strains at notches of several kinds of forms of the material have been evaluated photo-visco-elastically, and the relation of the stress concentration factors to the rate of straining is presented.

The strain optical coefficient is independent of the magnitude of strain but depends on the straining rate. The rate dependence has a transition below which it is hardly discernible.

1. Introduction

The present paper is concerned with stresses concentrating at notches in polymethyl methacrylate plates. Though rheological behaviors of synthetized materials have long been dealt with, we can not describe the distribution of stresses yet when they are not uniform in such nonlinear-viscoelastic materials as polymethyl methacrylate.

We measured, prior to mathematical treatments, the factors of stress concentration produced at notches of Neuber type and of sharp U-type, and at artificially made cracks.

The factors depend upon the rate of increase in tensile force as expected. They were determined in the range from a slow rate in which the time to break specimens was about 80 minutes to the rate as high as possible for our testing machine.

The stresses at notch bottoms and crack tips (in the strict sense of the words we should say, at the parts as near to bottoms or to tips as possible) were obtained photo-visco-elastically. For the purpose, we measured the sensitivity of strain birefringence in relation to strain rates, and at strains amounting to $4 \sim 5\%$, at which our specimens broke.

Such measurements have already been done for several materials when strained a little amount. One of those measurements shows that the sensitiveness of birefringence does not vary, or can not be distinguished from the errors of measurements even in an impulsive rate of straining. Some on the other hand indicate the rate dependence for other materials.

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Many investigations have been done for the purpose of stress measurement of photoelasticity and in almost all of them the magnitude of stresses to which the materials were subjected are small.

The investigation on Urethan rubber (Hysol 8705) is an exception[1]; strains in measurement are several hundreds. Its stress optical coefficient is constant over a broad range of straining rates, though different from the value obtained from the statical measurement. There must be, between both measurements, a range of transition from static behavior to dynamic[2] where the coefficient is to vary. There is seemingly no measurement only for the rubber but also for other materials in the transitional range of straining. Our request is for this range; in which the static sensitiveness changes to the dynamic one[2]. The stresses were evaluated by calibrating the fringes by the use of the data. The strain rate characteristics thus obtained differ from the other already given results in which one or other linear-viscoelastic behavior had been assumed. Linearity could not be allowed to apply to our material of polymethyl methacrylate.

One of researches indicated experimentally that fringe orders varied with linear sums of principal stress difference and such strain difference for a nonlinear viscoelastic material. This formula also could not be used in our case.

2. Specimens

A 5 mm thick plate of polymethyl methacrylate, Acrylite of Mitsubishi Rayon, Inc., was used. The degree of polymerization was about 9,000. The specimens were taken from the plate as their directions of lengths were parallel. Specimen lengths were 230 mm, the breadths were 15 mm at the middle parts and 30 mm at the ends. On the surfaces of plate specimens prepared for determining stress optical coefficients were 100 mm distant gage lines for extension measurement.

The specimens for measuring the stress concentraion were shaped with bits prepared particularly. The tip radii of the bits were of four kinds, 2.5, 1.2, 0.6 and 0.175 mm and a set of roughing and finishing bits was used for each kind of the radius. With these bits Neuber type notches (hyperbola type contour) could be shaped.

Specimens were annealed after shaping in order to remove residual strains. That the strains were not existing were ascertained under polarized light; the notch dimensions were precisely measured with a projector after annealing. Annealing was done in a liquid paraphin bath of 120°C. For raising the temperature up to 120°C we allowed an hour or more, and held the specimens at the temperature 120 ± 1°C for an hour or more, and after it the specimens were cooled at the rate of $0.1\sim0.5$°C/min. The slowest rate of cooling naturally was obtained both in the beginning of temperature lowering and in the end of cooling. To ensure uniformity of temperature distribution, the specimens were soaked in a double-walled case of copper plate.

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1) Durelli, A. J., Riley, W. F., Carey, J. J., Symposium on photoelasticity (ed. M. M. Frocht), 251 (1963)

2) Dally, J. W., Riley, W. F., Durelli, A. J., Trans. ASME, 26, 613 (1959)

By annealing, the specimens increased in thickness and decreased in width as given in Table 1, from which values we found no volume change of the material.

Table. 1. Changes in thickness and breadth of specimens

	Thickness: After annealing, mm / Before annealing, mm	%	Breadth: After annealing, mm / Before annealing, mm	%
E-6	$\frac{5.52}{5.08}$	+2.7	$\frac{14.92}{15.16}$	−1.6
E-7	$\frac{5.17}{5.01}$	+3.1	$\frac{14.94}{15.15}$	−1.4
E-8	$\frac{5.17}{5.00}$	+3.4	$\frac{14.84}{15.08}$	−1.6
Mean		+3.07		−1.53

Changes in length are probably equal to those in breadth.

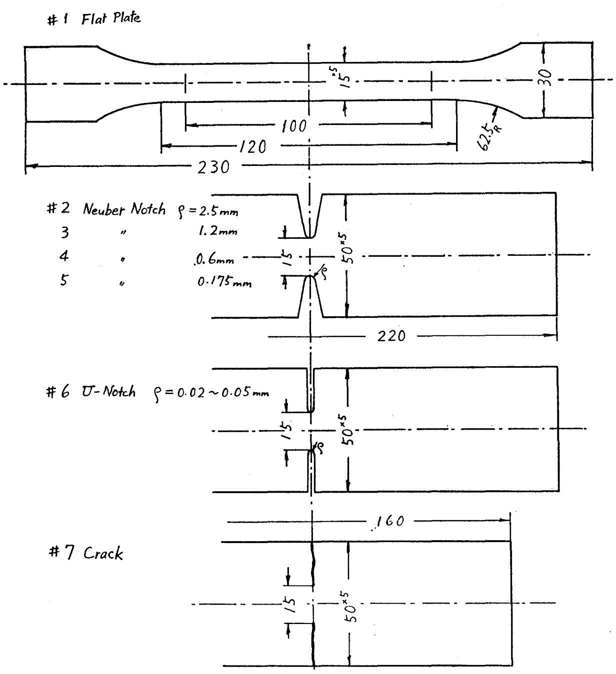

Fig. 1. Specimens.

The notches of U-type were 1 mm wide grooves made by sawing halfway from both sides and finished with sand papers and then emery papers. The notch radii, difficult to finish identically, were in the range of $\rho=0.02\sim0.05$ mm.

The sharpest notches were cracks. The cracks were made by pushing a knife into the sides of specimens under an electric lamp. Owing to the heat of the lamp the environment was at 50∼60°C. The tip distances between both side cracks were about 15 mm.

3. Test Procedures

The tests were made under the conditions of 23±1°C and 50±3% RH. A load cell was inserted between the specimen and the upper chuck of the testing machine. The cell was statically calibrated at each test with a loop dynamometer. (The maximum of used loading speeds did not exceed 500 kg/sec.) Loads were recorded on oscillogram papers; strains were obtained from photographs, on which the changes of the distance of 100 mm apart gage lines could be measured. On one of the line the foot of a pointer was clamped. The movement of the pointer on the scale was continuously photographed with a motor driven 35 mm camera.

The scale here used was made by taking a photograph of the image of a Micromat on a sensitive film. We read the scale down to 0.05 mm. Our sensitivity then is 1 % for the extension of 4∼5 mm at break. But the accuracy decreases to about 3 % at the strain of medium extension. Namely, there is an ambiguity of 0.0005 at its highest in the measurements of strains; when referred to stress it is about ±0.05 kg/mm^2.

The rates of straining were not regulated with strain gages as in a previous investigation, but the oil valve of the testing machine was set at an adequate position at each test. Thus we got satisfactorily constant rates of straining as we could assure ourselves of the fact from the oscillograms with one exception. However, it was unavoidable for the strain rate more or less to increase on the approach of the break of specimens (beyond about a strain of 0.04). The stress-strain diagrams must have had the final parts with gentler slopes provided that the constant rates of straining could have been controlled exactly to the last extremity.

In investigating stress optical properties, measuring apparatus for photoelasticity was arranged before and behind the specimen which was mounted on the testing machine. A measuring head, a photocell, of a light meter for microphotographs was applied to measurement of the field brightness; the incident light was circularly polarized. The brightness was expressed with the amplitude of alternate current of 120 cycle/sec because the mercury lamp flickers on account of the frequency of electric sourse.

We could use an amplifier for audio frequency. In addition to this, oscillograms recorded the load, the times when the photographs for strain measurement were taken, and time marks.

Strains in the vicinity of notch bottoms were measured for some of notched specimens by taking magnified photographs of the displacements of tiny gage points, which were faint scratch marks on 1/100 inch distant points of sputtered silver.

Most measurements of strains as well as stresses were indirectly made by successively photographing the fringes of specimens subjected to increasing load. Curves of calibration, fringe *vs* stress and fringe *vs* strain attended with a parameter of strain rates, had been prepared beforehand by the aid of flat parallel plate specimens.

We can not know the true rates of straining, the parameter, in the first place at the part of stress concentration, for we are now going to know true strains produced there. We resorted to an iterative method; at first, determining mean rates of strain from the rates of loading by referring to stress-strain diagrams and taking them as approximate parameters, we read stresses and strains corresponding to the measured fringe numbers from the curves of calibration; thus obtained strains were used to determine the second approximate values of strain rates at the notch bottom, the more accurate values of the parameter. Again consulting the curves of calibration, the stress- and strain-fringe reduction scales, we evaluated stresses and strains at the notch bottom. Twice repeating of the procedures was enough to obtain the values of strains and stresses; they converged practically.

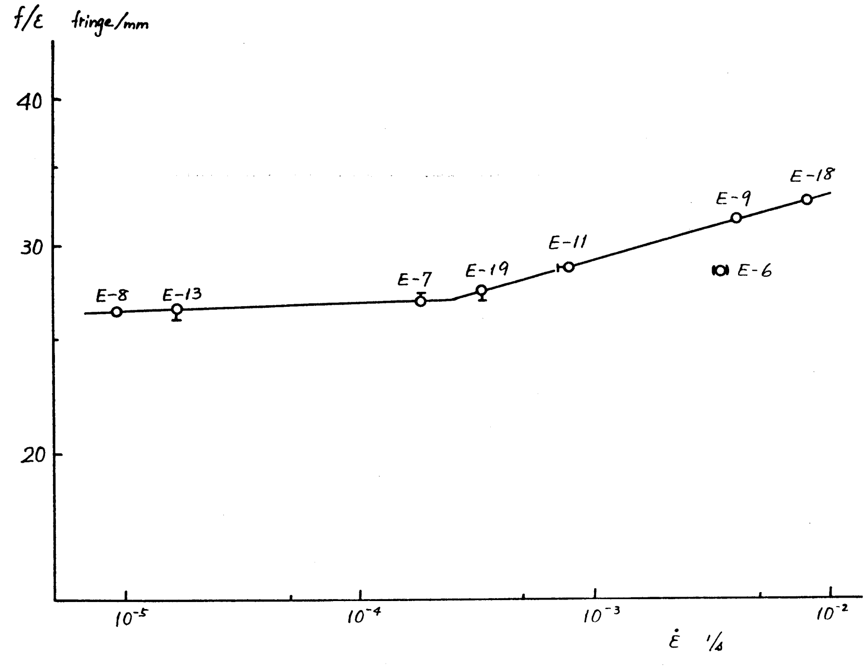

Fig. 2. Fringes per strain per thickness related to strain rates.

There is, however, another difficulty. As will be experienced, it is not easy to read the numbers of fringes at the bottoms of sharp notches. The number of fringes just at the bottom of a notch was determined as shown in Fig. 10, by extrapolating the readings obtained along the contour of the notch as well as along the minimum section; sufficiently agreeing values were obtained from both directions.

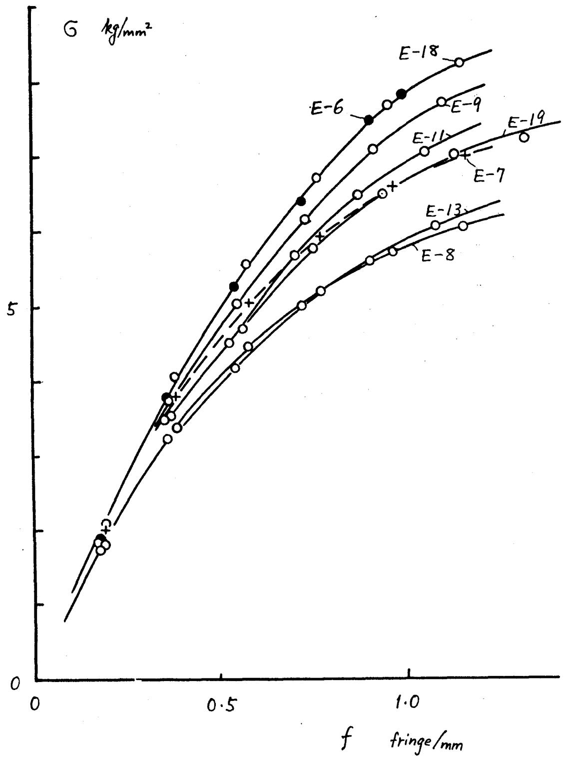

Fig. 3. Stress dependence.

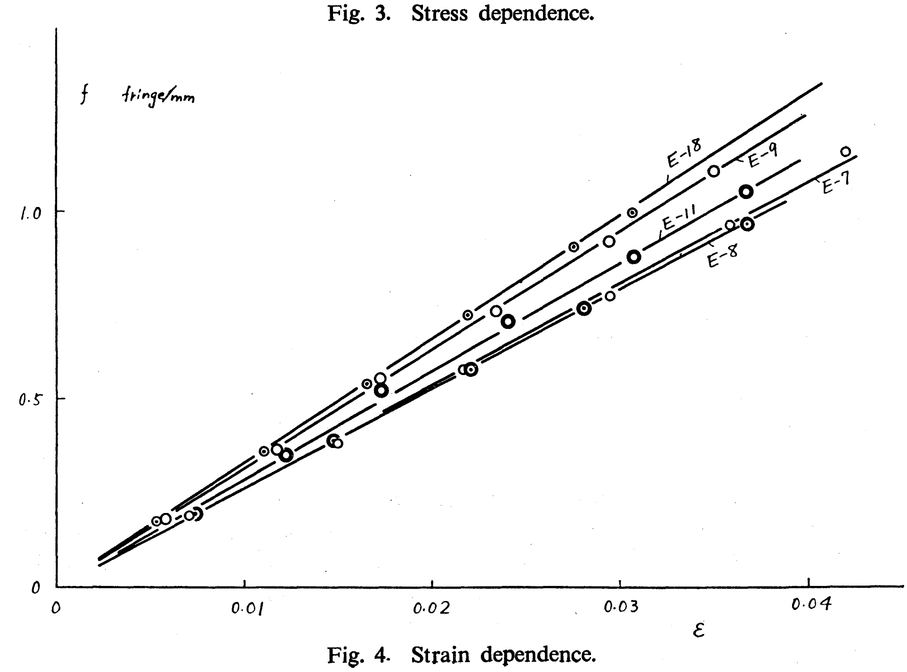

Fig. 4. Strain dependence.

4. Strain Birefringence and Rates of Straining

The relations between the numbers of fringes per unit thickness of the specimens, f, for the light of wave length 5461 A, and the stresses, σ, or the strains, ε, are shown in Fig. 3 or Fig. 4 respectively; the parameter is the rate of straining.

It is remarkable that the strain optical coefficients, f/ε, are independent of the magnitude of strains, but vary a little with the rates of straining as

$$f/\varepsilon = k\dot{\varepsilon}^{n} \tag{1}$$

and the constants, independent of ε, are

$$\left.\begin{array}{l} k_l=28.6,\ n_l=0.0067,\ \text{for } \dot{\varepsilon}<2.5\times10^{-4} \\ k_h=42.9,\ n_h=0.056,\ \text{for } \dot{\varepsilon}>2.5\times10^{-4} \end{array}\right\} \tag{2}$$

where suffixes l and h are used for the ranges of the lower and the higher rates of straining respectively. Here ε is the first principal strain in the state of plane stress, a ratio of the increase of the gage length to its initial, which is however nearly equal to the logarithmic strain in our case. $\dot{\varepsilon}$ is the increase in ε per sec. As the constants (2) show, the strain rate sensitiveness of birefringence of the material is apparent at higher rates of straining than 2.5×10^{-4} per sec, below which it is recognized only slightly.

The transition of the dependence on the rate is represented with the bends of the dotted lines, iso-fringe order curves written on the stress-strain diagrams (Figs. 5 and 6). Namely, the dotted lines are nearly vertical at lower rates (they would be exactly vertical if the strain optical coefficient should not depend on the rate), whereas the lines bend leftward in the range of our higher rates.

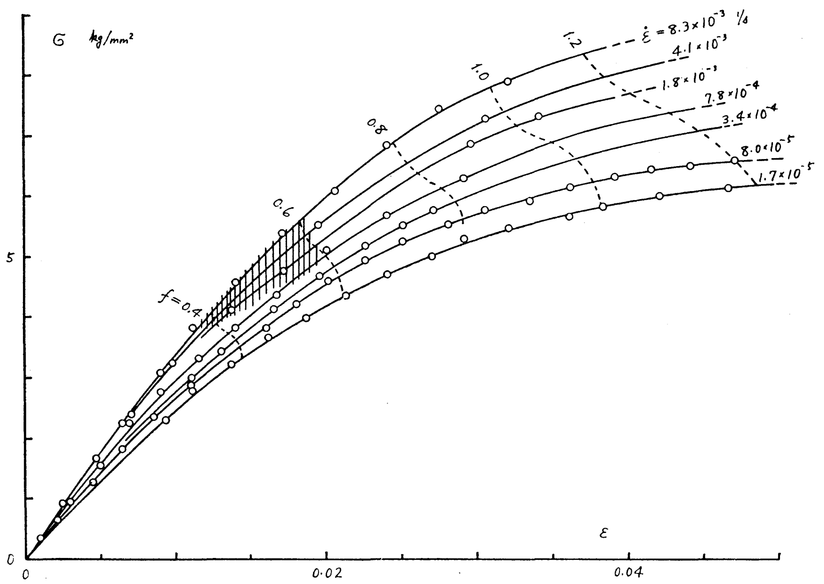

Fig. 5. Stress-strain diagrams.

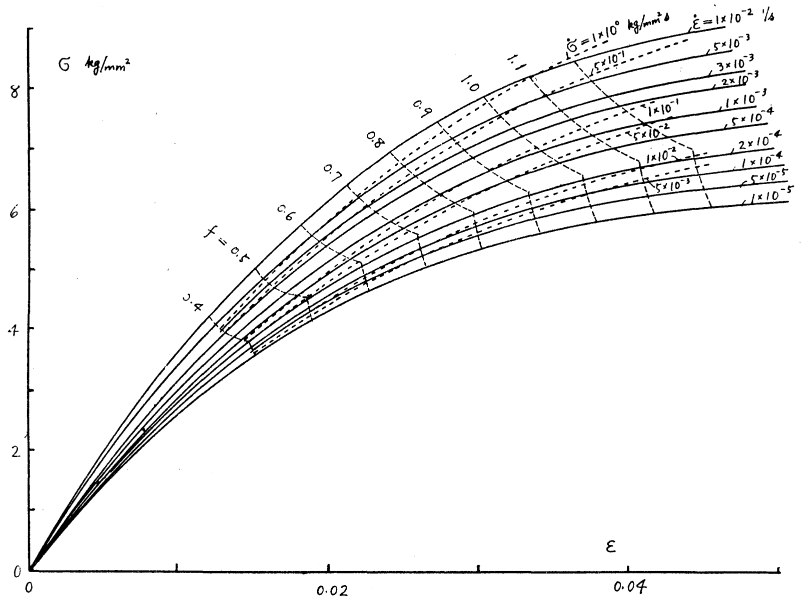

Fig. 6. Stress-strain diagrams arranged for reference.

A. Kuske[3], having given his general views on stress birefringence, did not touch upon such as the above. He measured both changes of strains and fringes of a material subjected to various constant stresses together with the time elapsed. Such measurements will give a like figure to Fig. 5 when the strains and stresses are plotted taking the elapsed times as a parameter. By connecting equal fringe order points on the chart, he got a leftward leaning straight line with polymethyl methacrylate and a rightward leaning one with epoxy resin. These may correspond to the above described which have the rates of straining as a parameter.

His views, not yet definite on the physical basis, state that two or more phases contribute to birefringence; and do not differ from the generally conceived conception of a time sensitive model to explain phenomenologically photoelasticity sensitiveness. His views deserve special notice in that the iso-fringe order lines turn to the left for the material in which the contributions of the two phases are of the same sign and otherwise for the material with different signs.

His explanation gives a suggestion to interpret the characteristics shown in Fig. 6. Since we naturally regard the glass state behavior to dominate at higher rates[4], distortional biregringence is expected to contribute more to an equivalent strain of the material than orientational briefringence, the contribution of which will be large at slower rates; and there may be no sensitiveness to the rate of straining or, if any, it may be of the different sign to bring forth the rightward leaning.

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3) Kuske, A., Exper. Mech., 2, 278 (1962)

4) Stein, R. S., Physik der Hochpolymeren, Bd. IV (hrsgb. H. A. Stuart), Kap. I (1956)

There needs an explanation of some specimens which were dispensed with measurement of strains. For such specimens, we used the values of strains corresponding to measured stresses on the stress-strain diagrams of specimens having had nearly equal times to break with them. A specimen, E–6 (Fig. 2), was obtained there out of a common character of the material. The deviation was found attributable to preloading; only this specimen had slipped at one side of the chucks when subjected to load; we tried firmer grips further four times to succeed to break it. Meanwhile, the specimen had twice experienced the stresses of nearly two thirds of the breaking load. Though the time to break for the last course of loading approached that of E–9, the relation of the stresses and the fringes (which were directly measured) does not resemble to that of E–9 but of E–18 which were tested at the higher rate of straining (Fig. 3). It seems that preloading lowers the strain sensitivity of birefringence. Naturally, judging from the above we think also that we might not obtain the same data for a material having been stored for a long period as well as for a preloaded material.

5. Straining Rates and Stressing Rates

We obtained the relation between the rate of stressing, $\dot{\sigma}$, and that of straining,

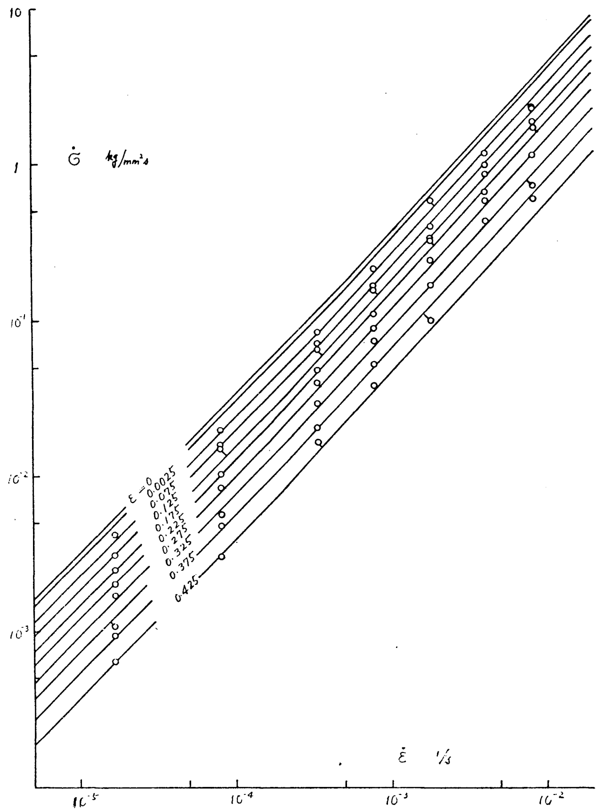

Fig. 7. Stress rates *vs* strain rates under monotonically increasing loads

$\dot{\varepsilon}$. Taking $\dot{\varepsilon}$ as a parameter at first, we get the relation

$$\dot{\sigma}=\frac{\delta\sigma}{\delta t}=\frac{\delta\sigma}{\delta\varepsilon}\cdot\frac{\delta\varepsilon}{\delta t}=\frac{\delta\sigma}{\delta\varepsilon}\cdot\dot{\varepsilon} \tag{3}$$

Then, reading stress increments $\delta\sigma$ for strain increments $\delta\varepsilon$ ($=0.005$, say) in various rates of straining $\dot{\varepsilon}$ we can determine the rates of stress $\dot{\sigma}$. The numerical values thus obtained were plotted on a logarithmic scale (Fig. 7). The relation was found to have astonishing linearity and parallelism, that is,

$$\dot{\sigma}=\exp[B(\varepsilon)]\cdot\dot{\varepsilon}^{m}. \tag{4}$$

Since the relation has been obtained from the measurements under the condition of each being at constant strain rates, it applies only to cases of monotonic increase in extension and of negligible acceleration if any.

The constants contained in (4) take different values between two ranges of the straining rates; using suffixes l for $\dot{\varepsilon}<2.5\times10^{-4}$ per second, we obtained the expressions

$$\left.\begin{aligned} m_l&=1.017\\ B_l(\varepsilon)&=2.303\{2.58-18.7\varepsilon-(16.5\varepsilon)^{5.8}\}\end{aligned}\right\} \tag{5}$$

and those for $\dot{\varepsilon}>2.5\times10^{-4}$ per second with suffixes h

$$\left.\begin{aligned} m_h&=1.097\\ B_h(\varepsilon)&=2.303\{2.83-16.5\varepsilon\} &&\text{for } \varepsilon<0.02\\ B_h(\varepsilon)&=2.303\{2.68-1.3\varepsilon-(19.5\varepsilon)^2\} &&\text{for } \varepsilon>0.02.\end{aligned}\right\} \tag{6}$$

It is not only for convenience of curve-fitting that the expression of $B(\varepsilon)$ in the latter case changes from of the first degree to of the second at a magnitude of strain 0.02. But the change is accompanied with another transition of an optical property of the material, which arises also in the vicinity of the same value of strain. For the former range no transition was found.

From (4), we get

$$\frac{d\sigma}{d\varepsilon}=\exp[B(\varepsilon)]\cdot\dot{\varepsilon}^{m-1} \tag{7}$$

with $B(\varepsilon)$ and m for each range of straining rates provided that strains increase monotonically and $\ddot{\varepsilon}\doteqdot 0$.

With the same qualifications

$$\frac{df}{d\sigma}=k\cdot\exp[-B(\varepsilon)]\cdot\dot{\varepsilon}^{n+1-m} \tag{8}$$

are derived from (1) and (7). This expresses that the more ε increases, the larger $df/d\sigma$ is, whereas it is the smaller for the larger value of $\dot{\varepsilon}$.

6. Breaking Strengths Related to Strain Rates

Strengths of plate specimens of polymethyl methacrylate depend upon the strain rates as already known. The results of our tests are shown in Fig. 8. The specimens were annealed just before each test. Some specimens were weaker because of pits, or rather dimples in their appearance, on the specimen surfaces.

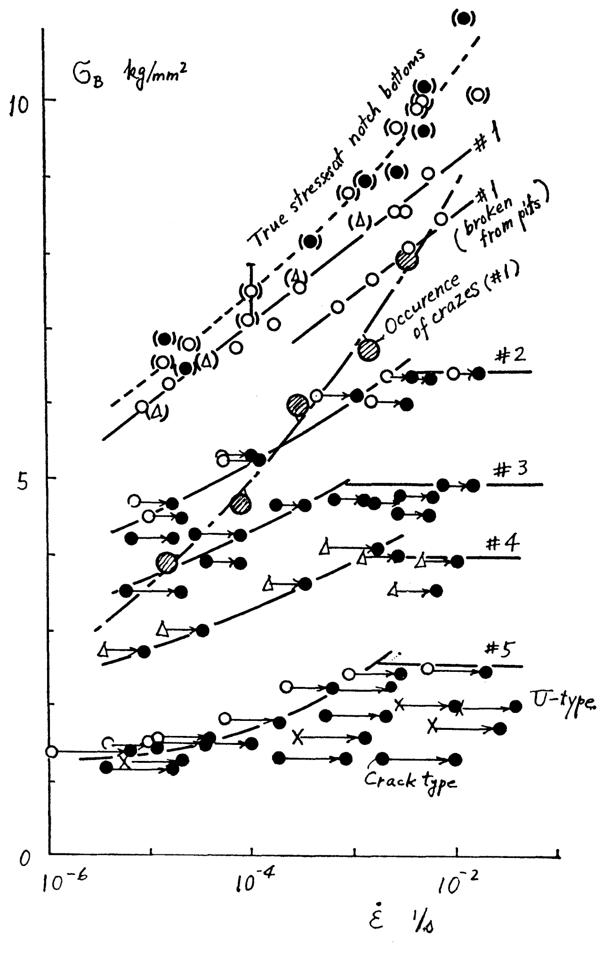

Fig. 8. Breaking strengths.

The dimples were neither those made by bad surface finish nor by small dusts sunk in the box of annealing. The depth of the dimples is nearly equal to the increase in thickness of the specimens owing to annealing; It appears as if the dimples are as many pinned points to the initial thickness.

Roughly speaking, the strength gains 4 kg/mm^2 with four decade increase in the strain rate. The sharper the notches, the weaker the specimens are when the strengths are expressed with mean stresses in their minimum sections, where the width of the specimens were all 15 mm and equal to that of the parallel part of flat plate specimens. Speaking of the true stresses of concentration at notch bottoms, the strengths are nearly the same as, or slightly higher than, those of flat plate specimens. They are distinguished with brackets in Fig. 8 and represented with a dotted line. They were iteratively evaluated by the aid of the fringe orders at notch bottoms as mentioned already. On plotting the nominal breaking strengths of notched specimens, the iteratively evaluated strain rates of the abscissae are indicated with arrow-heads; the points of tails indicate the strain rates of the first approximation obtained from the cross head speeds of the testing machine.

There is seemingly no more increase in the breaking strengths of the notched specimens at the higher rates than about 3×10^{-3} per sec. The stresses, having been acting when crazes were produced on the surfaces of the flat specimens, are shown with shaded circles for each rate of straining in the figure. They are, however, not so exact ones because coming about of the crazes was observed with the unaided eye and a push botton switch was used to insert the signals in oscillograms. Nuclei of cracks which bring the specimens to break are produced by widening of crazes, except that nuclei frequently were found to have grown in the inner area, not adjacent to but a little apart from surfaces, at the highest straining rates of us. It can not yet be explained whether not increasing in strength with rate increasing has any relation with coming about of crazes or not. But the specimens which had been left intact in a room for a month or two showed the tendency of not increasing in their breaking strengths at rather lower rates of straining and there came about crazes on the specimen surfaces in an early stage of increasing load.

The measurements of the strains of specimens at break were scattered, particularly for notched specimens as shown in Fig. 9. The strains of flat plate specimens at break may be said not to vary with the rates of straining, at least within the scope of rates of our investigation, though lower values were seen at both ends of the range of our rates.

In spite of the inferiority in the strength, the break strain of the specimens broken from their tiny surface flaws or dimples, do not differ.

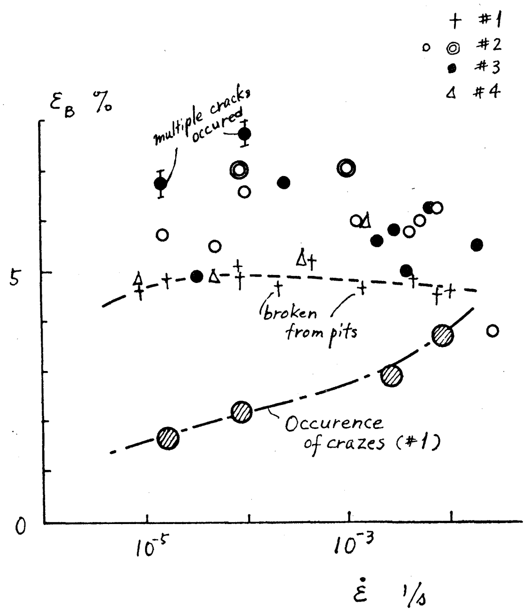

Fig. 9. Strains at break.

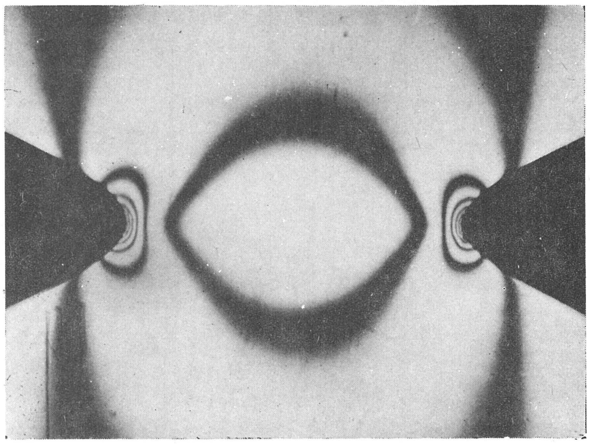

Photo 1. Occurence of wavy fringes due to crazes.

As seen in Photo 1, it was difficult to count the fringes of notch bottoms precisely on account of coming about of crazes. It can be said that the measurements of strains as well as stresses will give smaller values for flat plate specimens because we are scarcely lucky in measurement to meet that very parts at which the break of the specimens starts. Photomicroscopically the break strain may be a little more than 5 %.

7. Stress Concentraton Factors

We here do not consider the concentration of stress at the tip of a fracture nucleus, which grows previously to breaking, for its growing speed is quick enough (9∼17 mm/sec on an average according to our measurement) and the nucleus tip stress is to be studied separately together with propagating crack tip stresses. However, crazes widen slowly (2 μ/sec on an average) and they come about early prior to break. We can know their disturbance of stress distribution from Photo 1, particularly at slow rates of straining in which case they are produced earlier enough to be considered. Notwithstanding we did not take it into account. In such a not very strict sense, the stresses measured at notches are called true stresses. Nominal stresses are average values of stresses, tensile forces divided by the areas of minimum sections.

The principal stress differences in the neighborhood of a notch bottom are distributed as exemplified in Fig. 10, ones along the contour of the notch and others on the minimum section. The distributions along each line change with the magnitude of mean stresses, not in proportion to mean stresses. That is, the material behaves nonlinear-viscoelastically.

Just at the bottom of the notch, the stresses measured along both lines agree with each other satisfactorily. The calculated values from elasticity theory of the stress differences, $\sigma_y - \sigma_x$, according to Neuber's work[5] are given in the figure. When the nominal stresses are large, the discrepancy in the way of distribution is

5) Neuber, H., Kerbspannungslehre, Nomogramm

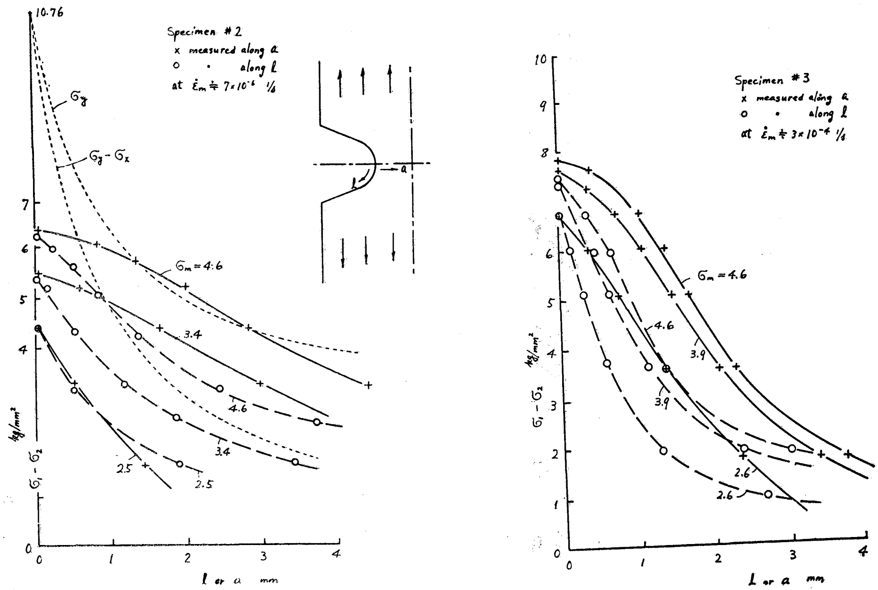

Fig. 10. Determination of fringe orders by extrapolation.

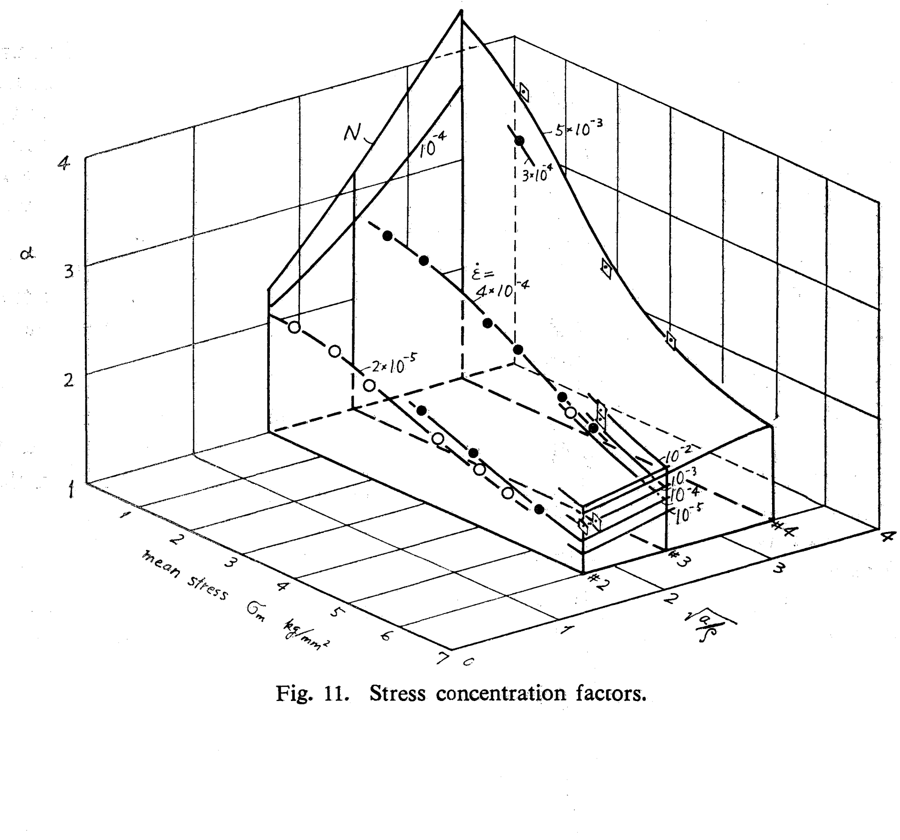

Fig. 11. Stress concentration factors.

marked.

The stress concentration factors are collectively given in Fig. 11. Large factors are obtained under small nominal stresses, but there arises no larger value than that obtained from the calculation of elasticity theory. We did not obtain any factor beyond about 10 even with artificially made cracks. As will be expected, the sharpness of notches will work on the concentration of stress to relax as well as enhance the concentration. The factors are small at slower rates of straining under otherwise same conditions.

8. Summary

Our aim has been to investigate what the unique material quantity to determine the break would be, while the strength of polymethyl methacrylate plate varies with strain rates, sharpness of notches, or other peculiarities. As seen from Fig. 8, the stresses which were acting at notch bottoms when specimens broke did not much differ from the strengths of flat plate specimens (though slightly larger in general). But the stresses depended on strain rates. On the other hand, the strains at break were not so varied (Fig. 9). But the measurements were rather scattering.

If the material behaves as linear-viscoelastic, stress concentration factors ought not to be related to strain rates. Polymethyl methacrylate is nonlinear-viscoelastic; stress concentration factors rely on strain rates as shown in Fig. 11.

The stresses and strains were obtained photoviscoelastically. Strain optical coefficients vary with strain rates as shown in Fig. 2, but not with strains. We could obtain the relations (1) and (2).

There was a transition at the rate of 2.5×10^{-4} per second.

We could further derive a relation between both rates of stresses and strains (4), though it should be applied only to the case of negligible strain acceleration.

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