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KUMAI, Toyoji
Research Institute for Applied Mechanics, Kyushu University : Professor

OCHI, Yoshio
Hull Design Section, Kure Shipbuilding Co. Ltd.

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ON THE VIBRATION OF SHIPS IN WAVE

Toyoji KUMAI* and Yoshio OCHI**

Abstract

An investigation has been made into the vibration of ships in wave theoretically and through some experiments. Since the instantaneous virtual mass of water in a vibrating ship slightly changes when she floats among waves, frequency modulation should be accounted in the natural vibration of ship hull. Since modulation factor is easily calculated based on the wave profile, the modulated frequency at the free vibration can be estimated from the stable solution of Mathieu equation. Some calculations have been carried out on the forced vibration of the system mentioned above in the present paper. A particular solution was obtained as the frequency modulated vibration with modulated amplitude. The maximum amplitude of the forced vibration depends on the ratio of the natural frequency of ship hull and that of the wave and on her damping coefficient.

Introduction

The resonance of the hull vibration of a ship afloat in wave which is one of the problems in ship vibration, has relative importance in practice. Since the virtual mass of water surrounding ship hull slightly varies in accordance with the wave profile, the natural frequency of the ship hull ought to vary periodically. The solution of the problem in the free vibration is approximately obtained as the frequency modulated vibration.

In the present paper the forced vibration of the system mentioned above is considered as a single degree of freedom. The relation between the modulation ratio of the natural frequency of the hull to the wave encountered and the maximum amplitude of the forced vibration was obtained. The results of theoretical approach were confirmed by experimental studies made on the model ship in the standing wave.

1. Theoretical calculation

For the sake of simplicity, the vibration system is assumed to be of the single degree of freedom in the theoretical calculation. The fundamental equation of forced vibration is therefore written by

$$m\left(\frac{d^2y}{dt^2}+2\lambda\frac{dy}{dt}\right)+ky=F_1\sin pt \tag{1}$$

* Professor of Kyushu University, Research Institute for Applied Mechanics, Fukuoka,
** Hull Design Section, Kure Shipbuilding Co. Ltd., Kure.

where, y amplitude of vibration
λ damping factor
ky force of restitution
p frequency of exciting force
F_1 exciting force
m mass of the system

In the above expression, the mass corresponding to the displacement of ship slightly changes with the frequency of wave encountered due to the variation of virtual inertia of the water. This is approximately expressed by

$$m = m_0(1+\varepsilon\cos\omega t) \tag{2}$$

where, $m_0 = m_s + m_e$

$$\varepsilon = \frac{\Delta m_e}{m_s + m_e}$$

ω frequency of wave encountered to ship

The variation of mass Δm_e is assumed to be small compared with m_0 and changes $+\Delta m_e$ for sagging and $-\Delta m_e$ for hogging conditions.

Substituting from (2) into (1), the equation becomes

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + n^2(1-\varepsilon\cos\omega t)y = F(1-\varepsilon\cos\omega t)\sin pt \tag{3}$$

where, $n = \sqrt{k/m_0}$

$F = F_1/m_0$

The equation of the free vibration of the above system becomes

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + n^2(1-\varepsilon\cos\omega t)y = 0 \tag{4}$$

The solutions of the above equation is obtained as that of the Mathieu equation in the stable range providing $\varepsilon \ll 1$ as follows [1],

$$\left.\begin{aligned} y_1 &= Ae^{-\lambda t}\cos(\sigma t - h\sin\omega t) \\ y_2 &= Be^{-\lambda t}\sin(\sigma t - h\sin\omega t) \end{aligned}\right\} \tag{5}$$

where, $h = \dfrac{\varepsilon\sigma}{2\omega}$

$\sigma = \sqrt{n^2 - \lambda^2}$

The above solution means that the vibration appears as that of the frequency modulation with the modulated frequency $\sigma(t) = \sigma\left(1 - \frac{\varepsilon}{2}\cos\omega t\right)$.

As will be seen in the above solution, there is no amplitude modulation in the free vibration of the system provided that ε is small compared with unity. This fact is also applicable to the case of free vibration of a ship afloat in wave.

In regard to the forced vibration of the above system, forcing of the vibration is the amplitude modulation type, as was seen in equation (3). It is a well known fact that such an exciting force system has upper and lower sidebands at the frequencies $\sigma \pm \omega$ respectively with the amplitude $\varepsilon/2$ [2]. For the sake of simplicity, however, the effect of these sidebands on the response are ignored in the present problem. Consequently, the equation (3) is considered to be like the following simplified form,

$$\frac{d^2y}{dt^2}+2\lambda\frac{dy}{dt}+n^2(1-\varepsilon\cos\omega t)y=F\sin pt \tag{6}$$

The particular solution of the above forced vibration is easily obtained as follows

$$y=\frac{F}{C}\left[y_2\int y_1\sin pt\cdot dt-y_1\int y_2\sin pt\cdot dt\right] \tag{7}$$

where, $C=y_1\dot{y}_2-\dot{y}_1y_2=\sigma(1-\varepsilon/2)$.

The modulated waves y_1 and y_2 are conveniently expanded by Jacobi's expansion [3] that in a Fourier series in which the coefficients are Bessel functions of the first kind with integer suffix, viz.

$$\left.\begin{aligned}\sin(\sigma t-h\sin\omega t)&=\sum_{r=-\infty}^{\infty}J_r(h)\sin(\sigma-r\omega)t\\ \cos(\sigma t-h\sin\omega t)&=\sum_{r=-\infty}^{\infty}J_r(h)\cos(\sigma-r\omega)t\end{aligned}\right\} \tag{8}$$

Substituting (8) into (7), the forced vibration will be written by

$$y=\frac{F}{2C}\sum_{r=-\infty}^{\infty}J_r(h)\left[\frac{-\cos\{(p-r\omega)t+h\sin\omega t-\alpha_r\}}{\sqrt{(\sigma-r\omega+p)^2+\lambda^2}}+\frac{\cos\{(p+r\omega)t-h\sin\omega t-\beta_r\}}{\sqrt{(\sigma-r\omega-p)^2+\lambda^2}}\right] \tag{9}$$

For the resonance of the system, p equals to σ, thus leading to the solution as follows,

$$y=\frac{F}{p^2\left(1-\frac{\varepsilon}{2}\right)}\sum_{-\infty}^{\infty}J_r(h)\frac{\cos\{(p+r\omega)t-h\sin\omega t-\beta_r\}}{\sqrt{4\left(r\frac{\omega}{p}\right)^2+\frac{4\lambda^2}{p^2}}} \tag{10}$$

where, $\beta_r=\tan^{-1}\left(-\frac{r\omega}{\lambda}\right)$

The above expression becomes a form more convenient for calculation of y by the use of the relation $\beta_r=-\beta_{-r}$ as follows;

$$y=\frac{F}{p^2\left(1-\frac{\varepsilon}{2}\right)}\sqrt{\left\{A_0+2\sum_{r=0}^{\infty}A_{2r}\cos(2r\omega t+\beta_{2r})\right\}^2+\left\{2\sum_{r=1}^{\infty}A_{2r-1}\sin(\overline{2r-1}\omega t+\beta_{2r-1})\right\}^2}$$

$$\times\cos(pt-h\sin\omega t+\theta) \tag{11}$$

where, $A_0=J_0(h)/\delta/\pi$

$$A_{2r}=J_{2r}(h)\Big/\sqrt{\left(\frac{\delta}{\pi}\right)^2+4\left(2r\frac{\omega}{p}\right)^2}$$

$$A_{2r-1}=J_{2r-1}(h)\Big/\sqrt{\left(\frac{\delta}{\pi}\right)^2+4\left(\overline{2r-1}\,\frac{\omega}{p}\right)^2}$$

δ=logarithmic decrement, θ=phase angle

As a result, the forced vibration becomes the type of frequency modulated wave with modulated amplitude as will be seen in the above expression.

The numerical calculations were carried out with p/ω in variable with given value of ε and two values of δ. Figure 1 shows the modulated amplitude in $p/\omega=10$, 20 and 80 with given values of $\varepsilon=0.2$ and $\delta=0.0314$. Figure 2 shows also the modulated amplitude with different value of $\delta=0.314$. Figure 3 shows the maximum amplitudes for various values of p/ω in the cases $\delta=0.0314$ and $\delta=0.314$. The range of p/ω will be accounted from 5 to 50 in actual ship vibrations. It is interesting that the maximum amplitude of the forced vibration greatly varies for the change of the ratio p/ω from 5 to 50. We may safely assume that the ef-

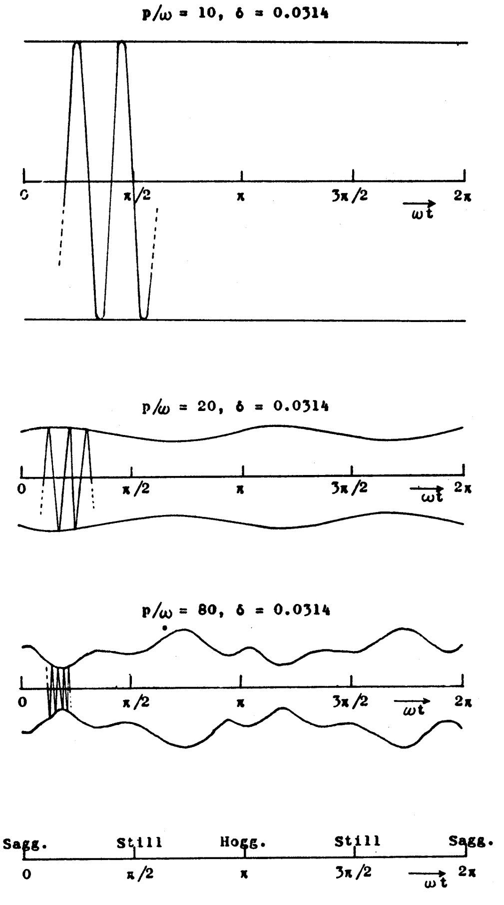

Fig. 1. Amplitude modulation at $p/\omega=10$, 20 and 80 in $\delta=0.0314$, $\varepsilon=0.2$.

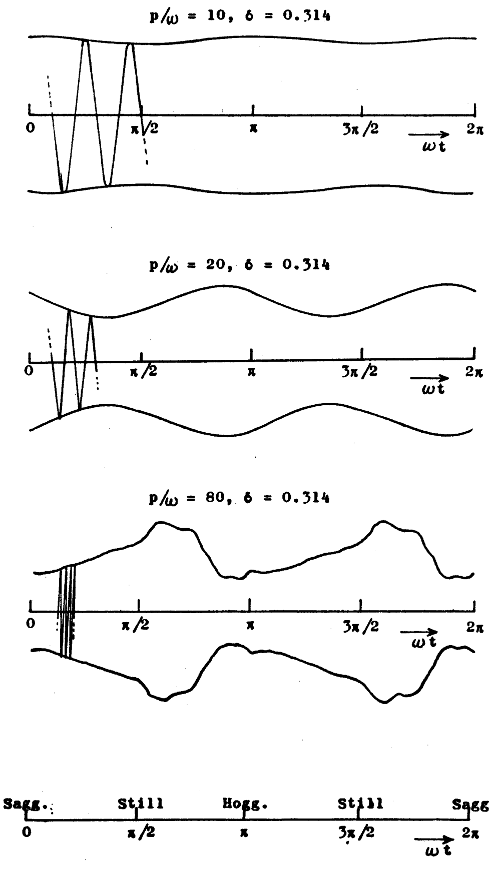

Fig. 2. Amplitude modulation at $p/\omega=10$, 20 and 80 in $\delta=0.314$, $\varepsilon=0.2$.

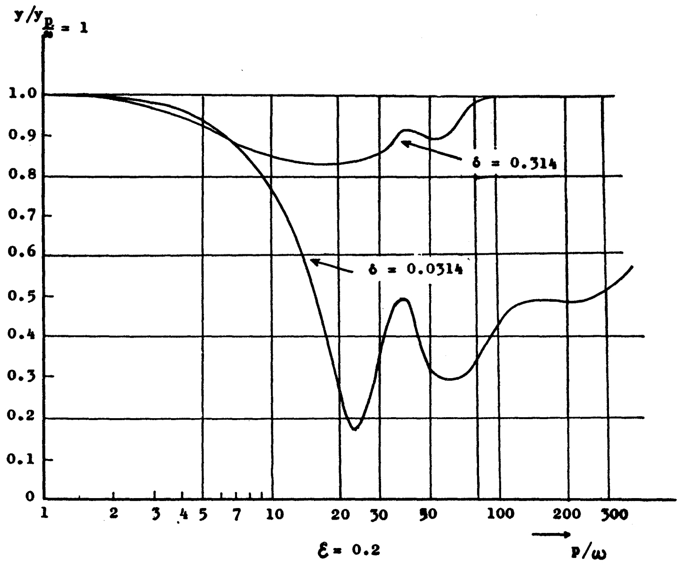

Fig. 3. Maximum values of modulated amplitude for p/ω in $\delta=$ 0.0314 and $\delta=0.314$.

fect of wave on the amplitude modulation of the forced vibration of ships is less in the low value of p/ω, but there appears complex modulation in the higher value of p/ω while there small amplitudes as will be seen in Figures 1 and 2. It is to be noted that the effect of wave on the maximum amplitude greatly depends on the damping coefficient, as shown for examples in Figure 3.

2. Experimental study by the use of model ship ($p/\omega \doteqdot 150$)

The modulation of the vibration of a wooden model ship of 145 cm length was measured in the standing wave with 4-node mode of the water, $L/24$ in wave height, in a 3 meter long tank. The profile of the testing apparatus of the model

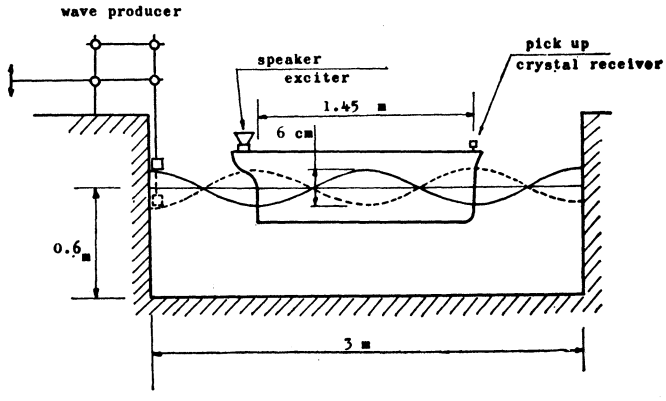

Fig. 4. Measuring apparatus.

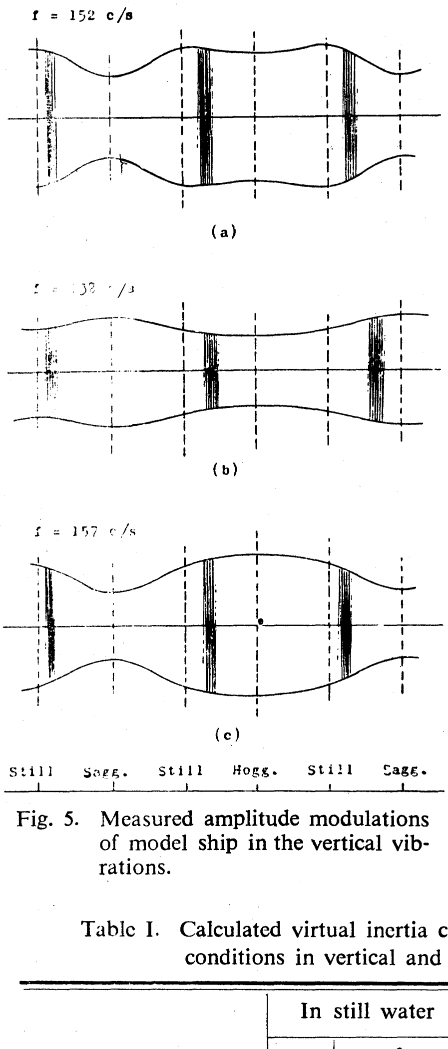

Fig. 5. Measured amplitude modulations of model ship in the vertical vibrations.

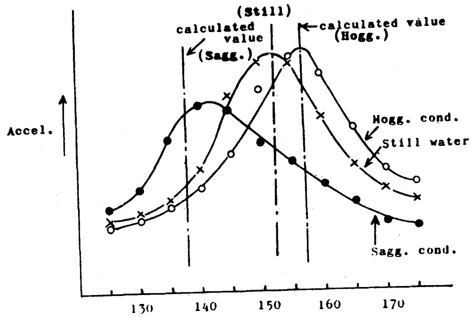

Fig. 6. Measured responses and calculated natural frequencies in the vertical vibration of model ship.

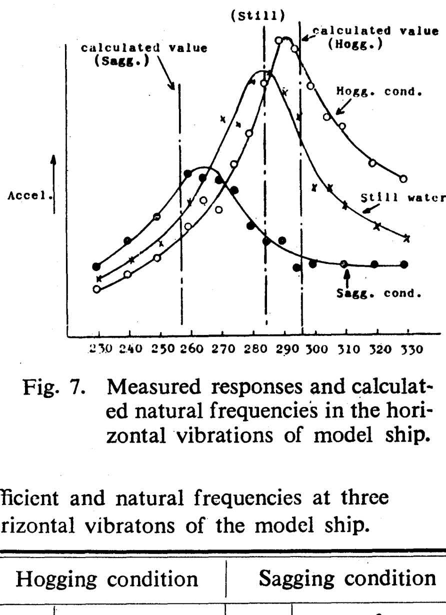

Fig. 7. Measured responses and calculated natural frequencies in the horizontal vibrations of model ship.

Table I. Calculated virtual inertia coefficient and natural frequencies at three conditions in vertical and horizontal vibratons of the model ship.

	In still water		Hogging condition		Sagging condition	
	τ_0	f_0	τ_H	f_H	τ_S	f_S
Vertical vibration	0.94	152 c/sec	0.802	$1.032\times f_0=157$ c/s	1.35	$0.908\times f_0=138$ c/s
Horizontal vibration	0.40	284 c/sec	0.290	$1.04\times f_0=296$ c/s	0.72	$0.904\times f_0=257$ c/s

ship is sketched in Figure 4. The natural frequency of the standing wave was rated about one cycle per second and the frequency ratio was estimated as $p/\omega\doteqdot 150$, and the logarithmic decrement measured is $\delta=0.314$ both in the fundamental mode of the vertical and horizontal vibrations of model ship.

The modulated amplitudes were measured on the three types of different resonant points, namely, respective frequency in still, hogging and sagging conditions as shown in Figures 5 and 6 in vertical vibrations as well as in the horizontal vibrations of such model ship in wave as shown in Figure 7 were obtained.

On the other hand, the virtual inertia coefficients in hogging, still and sagging conditions in both vertical and horizontal vibrations were calculated based on the measurement of fundamental modes of respective direction of vibrations. As a result, the instantenious natural frequencies in each conditions were estimated to be in steady state. The results are shown in Table I and Figures 6 and 7. As will be seen in the said table and figures, deviation of the natural frequencies in hogging and sagging from those in still water show different values, being about $+3\%$ and -10% respectively. The estimate of the frequencies in steady state fairly agrees with that of experiments.

With regard to the amplitude modulation, measured amplitude agrees with the foregoing calculation. There seems to be, however, a little difference between both results of amplitude modulation measured and calculated as are shown in Figure 2, $(p/\omega=80)$ and Figure 5 (a) $(p/\omega=150)$ both in the same damping coefficient $(\delta=0.314)$. The cause for the difference in the theory and the experiment will be ascribed to the different assumptions of modulation factors and the effect of difference of the mode curves in hogging and sagging conditions in the theory.

Conclusions

Brief conclusions will be drawn from the present study on the hull vibration of ship in wave, that,

a) The frequency modulation occurs in the free vibration of hull of ship in wave, for slight change of the virtual inertia of water surrounding ship hull is accounted for hogging and sagging conditions.

b) The amplitude modulation as well as the frequency modulation occurs in the forced vibration of hull. The maximum amplitude becomes smaller than that in still water and it depends on the damping factor and the ratio of the natural frequency of the hull to that of wave. Further study on the present problem in which ship going on wave is required.

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