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ON THE MODE FACTOR IN CALCULATING THE RESPONSE OF SHIP VIBRATION AT A HIGHER FREQUENCY

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Abstract

The mode factor in calculating the response of ship vioration at a higher frequency has been taken up for further investigation here. Considering the great influence of the magnitude of acceleration response of resonant vibration upon the load distribution of a ship, hull response was calculated for various weight distributions with discontinuity of ship load. As a result, it has been clarified that the cause of irregularity of the mode factor for the number of nodes is mainly in the discontinuity of ship weight in its distribution along her length.

Introduction

The theoretical approach on the respone calculation of the ship vibration in the higher mode was already reported by one of the authors in the previous paper [4]. The present paper gives the results of investigations made according to the previous theory, where with varying some parameters in the weight distributions of ship were changed variously to investigate the effect of weight distribution upon the magnitude of acceleration response of the resonant vibration of a ship hull.

The coefficient of acceleration response at resonance is defined by

$$\frac{a}{F} = C_n \cdot \frac{g\pi}{\Delta_1 \delta_n} \tag{1}$$

in which,

$$C_n = \frac{\Delta_1 \eta(u) \cdot \eta(v)}{L \int_0^1 w\eta^2 d\xi} \tag{2}$$

where, C_n mode factor
a acceleration of hull vibration at $\xi = u$
F force excited at $\xi = v$
η amplitude of vibration
δ_n log-decrement in n-node mode vibration

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w weight of ship per unit length
Δ_1 effective displacement

Estimations of hull response in the lower mode of the vibrations were investigated in several ways by J. Lockwood Toylor [1], McGoldrick [2] and Ramsay [3]. The estimate formula have been recently proposed also by R. Tsunoda [5], T. Tomita [6] and A. J. Johnson and P. W. Ayling [7] based on the measurements of hull vibrations on board ships, where the mode factors, C_n, up to four-node modes are practically used.

Since the response of the hull vibration up to ten node modes is required for the excitation of blade frequency, the calculation of the mode factor C_n, in the modes so high that they are excited by propeller blades are shown in the present paper.

1. Theoretical calculations

The solutions to be considered here involve the shearing vibration of the beam of variable cross section like a ship. In regard to the distributions of weight and shear rigidity, the curves are tapered to ends with m-th and r-th order parabolas respectively from each end to $\xi_1 (0 \leqq \xi_1 \leqq 1/2)$ in length and from ξ_1 to 1/2, where both curves are assumed to be uniform. The above distributions are considered to be symmetrical to midship. These are shown by,

$$\left.\begin{array}{ll} \left.\begin{array}{l} k'GA = k'GA_0 (\xi/\xi_1)^r \\ w = w_1 (\xi/\xi_1)^m \end{array}\right\} & 0 \leqq \xi \leqq \xi_1 \\ \left.\begin{array}{l} k'GA = k'GA_0 \\ w = w_2 \end{array}\right\} & \xi_1 \leqq \xi \leqq 1/2 \end{array}\right\} \tag{3}$$

where, $k'GA$=shear rigidity, w=weight per unit length. The solutions of the fundamental equation of the shear vibration deduced in the previous paper [4] with the above distributions of the rigidity and the weight are written by

$$\eta = B\xi^{\mu} J_{-\nu}\left(\frac{\nu}{\mu}\lambda\xi^{\mu/\nu}\right) + C\xi^{\mu} J_{\nu}\left(\frac{\nu}{\mu}\lambda\xi^{\mu/\nu}\right) \tag{4}$$

where,

$$\left.\begin{array}{l} \mu = \dfrac{1-r}{2} \\ \nu = \dfrac{1-r}{2+m-r} \end{array}\right\} \tag{5}$$

λ eigenvalue
$J_{\pm\nu}$ Bessel function of ν-th order including fractional order
B, C integration constants

For the sake of simplicity, r is taken equal to m. Consequently, μ becomes equal to ν. By the end conditions at $\xi = 0$; $\dfrac{d\eta}{d\xi} = 0$, and from the above assumption η becomes

$$\left.\begin{array}{l}\eta=B\xi^{\nu}J_{-\nu}(\alpha\lambda\xi):\ \ 0\leqq\xi\leqq\xi_1\\ \eta=D\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\left(\frac{1}{2}-\xi\right)\lambda:\ \ \xi_1\leqq\xi\leqq1/2\end{array}\right\} \tag{6}$$

$$C=0$$

where,

$$\left.\begin{array}{l}\nu=\dfrac{1-r}{2}\\ \alpha=\sqrt{\dfrac{w_1}{w_2}}\\ \lambda=\omega\sqrt{\dfrac{L^2w_2}{gk'GA_0}}\end{array}\right\} \tag{7}$$

D integration constant for the solution in the parallel part

In the above expression, cos (sin) presents the symmetric (antisymmetric) mode of the vibration. From the boundary conditions at $\xi=\xi_1$, the frequency equation becomes,

$$\frac{\alpha J_{-\nu+1}(\alpha\lambda\xi_1)}{J_{-\nu}(\alpha\lambda\xi_1)}=\begin{cases}-\tan\ (1/2-\xi_1)\ , & \text{symmetric mode}\\ \ \ \cot\ (1/2-\xi_1)\ , & \text{antisymmetric mode}\end{cases} \tag{8}$$

in addition,

$$\frac{D}{B}=\frac{\xi_1{}^{\nu}J_{-\nu}(\alpha\lambda\xi_1)}{\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\left(\frac{1}{2}-\xi_1\right)\lambda} \tag{9}$$

Evaluating integration following to the expression (2), C_n is obtained as follows,

$$C_n=\frac{2\left\{\alpha^2\dfrac{\xi_1}{r+1}+\left(\dfrac{1}{2}-\xi_1\right)\right\}\Big/\left\{\left(\dfrac{1}{2}\alpha\lambda\right)^{\nu}\cdot\Gamma(1-\nu)\right\}^2}{\alpha^2\xi^{2(1-\nu)}\left\{J^2_{-\nu}+J^2_{-\nu+1}+\dfrac{2\nu}{\alpha\lambda\xi_1}J_{-\nu}\cdot J_{-\nu+1}\right\}+\dfrac{D^2}{B^2}\left\{\left(\dfrac{1}{2}-\xi_1\right)\pm\dfrac{1}{2\lambda}\sin 2\left(\dfrac{1}{2}-\xi_1\right)\lambda\right\}} \tag{10}$$

where, $\Gamma(\ )$ is Gamma function, the argument of Bessel functions J is represented by $(\alpha\lambda\xi_1)$ and $\pm$ corresponds to cos or sin respectively in (9).

The magnitude of C_n will be computed for given values m, r and α etc. As an extreme case, for $\xi_1=0$, there should be uniform beam so that the value of C_n may become

$$C_n=2,\ \ (\text{independent of } n) \tag{11}$$

As the second extreme case, put $\xi_1=1/2$, C_n is written by

$$C_n=\frac{2+m-r}{1+r+m}\frac{\eta^2{}_{\xi=0}}{\eta^2{}_{\xi=1/2}} \tag{12}$$

It is interesting that the C_n is in proportion with the square of the ratio of amplitude at the end to that at midship in this case.

2. Numerical examples

(1) *The mode curves and C_n in the beams with continuous distribution of weight*

From the above-mentioned analytical solutions in which $\xi_1=1/2$ or $\xi_1=0$ with the parameter m and r, some of the vibration modes are shown in Figure 1. It is to be noted that the increase of the amplitude at the end of the hull which is accounted for the increase of the values m and r is considerably more than that in $m=r=0$, as is seen in Figure 2. Therefore, it is easily seen that the response at the ends of beam increases more conspicuously in sharply tapered beams than that in uniform beams.

(2) *C_n in the beam with discontinuous distribution of weight*

When the rigidity and weight curves with $r=m=1/2$ have discontinuity at $\xi=\xi_1$ with the ratio $w_2/w_1=2$, as shown in Figure 3 (a) for an example, the relation between the eigenvalue λ_n and the number of nodes becomes almost linear,

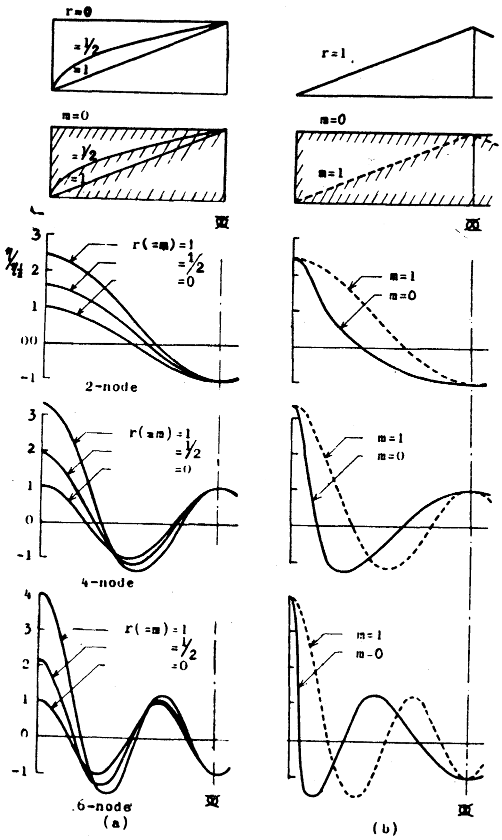


Fig. 1. Mode curves of the vibrations of beams with continuous distributions of rigidity and weight.

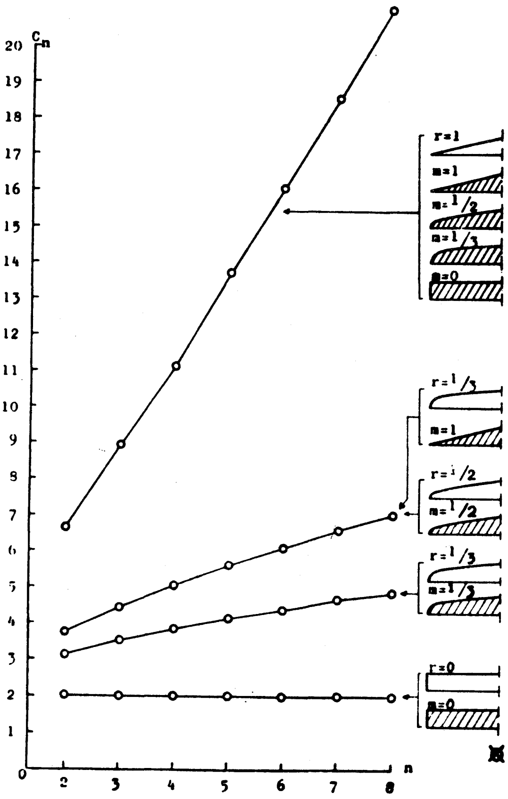


Fig. 2. Mode factor C_n versus number of nodes n of monotonously tapered beam.

as seen in Figure 3 (b). Nevertheless, the relations between C_n and the number of nodes with the parameter ξ_1 obtained are irregular except the curves in $\xi_1=0$ and $\xi_1=1/2$, as seen in Figure 4. Figure 5 shows C_n for ξ_1 *i. e.*, the location of the point of discontinuity with the number of nodes in parameter in the above example. The relations between C_n and w_2/w_1 for given value of $\xi_1=0.4$ are shown

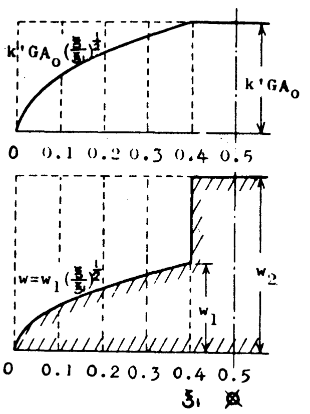


Fig. 3 (a). Typical distributions of shear rigidity and weight with discontinuity.

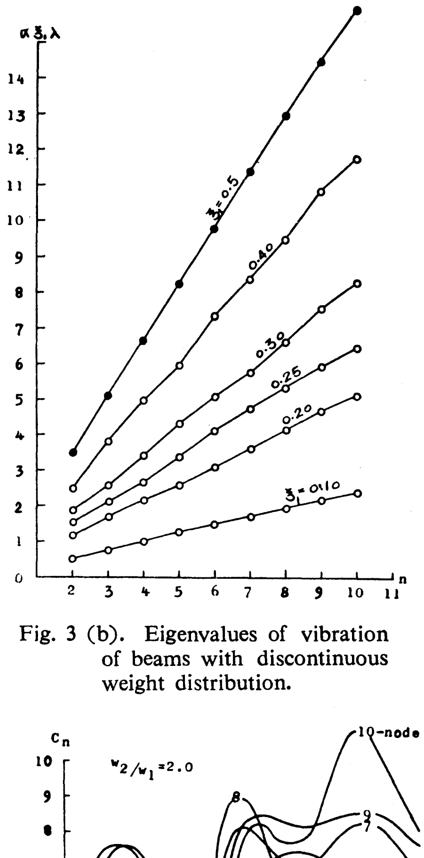


Fig. 3 (b). Eigenvalues of vibration of beams with discontinuous weight distribution.

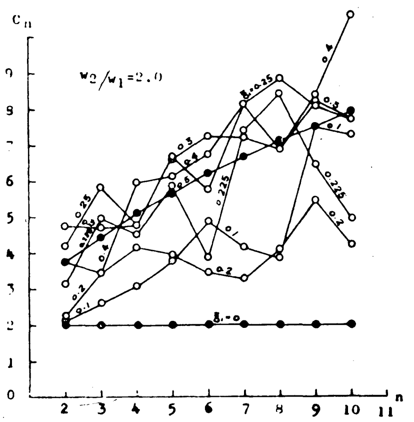


Fig. 4. Mode factor C_n versus n of the beams with discontinuous weight distribution.

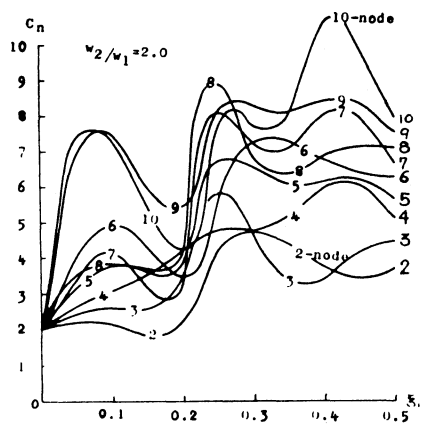


Fig. 5. Curves of C_n versus ξ_1 of the beam with $w_2/w_1=2$.

in Figure 6. We may safely assume, as shown it was seen above Figures 4, 5 and 6, that the cause of irregular variation of C_n for the number of nodes of vibration is due to the discontinuity of the weight distribution along ship length.

As another example of the effect of w_2/w_1 on the magnitude of C_n, the C_n values for number of nodes of the vibration of a tanker with symmetrical load and rigidity with $r=m=1$, $\xi_1=0.25$ as shown in Figure 8 were calculated. Figure 7 shows eigenvalues of them with various values of w_2/w_1. As will be seen in Figure 8, C_n varies conspicuously at the 4– and 5–nodes and at the modes above 7–node for high value of w_2/w_1. It is of some interest that C_n values in 5– and 6–node modes are comparatively lower than those in 4– and 7–node modes for $w_2/w_1=2$. Figure 9 shows C_n value for w_2/w_1 with the number of nodes in paremeter in the same example. The example mentioned above will characterize the mode factor in the vertical vibration of a tanker with various load conditions.

3. Model experiments and measurements on board tanker

A ship shaped 1/100 scale model of 40,000 *t. d. w.* tanker was made for verification of C_n value. The model ship 2.15 meter in length is entirely made of brass sheets with 0.2–0.5 mm thickness, and the structure of main model ship

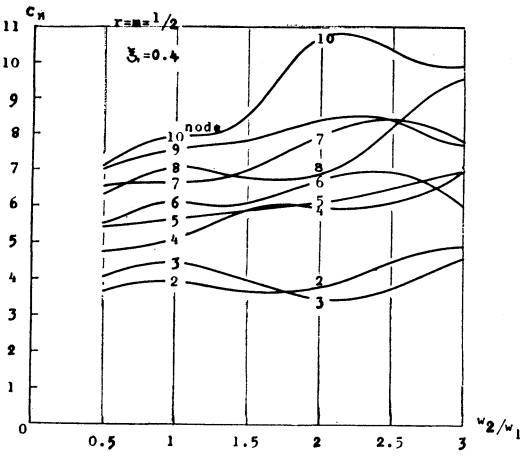


Fig. 6. Curves of C_n versus w_2/w_1 of the beam with $\xi_1=0.4$.

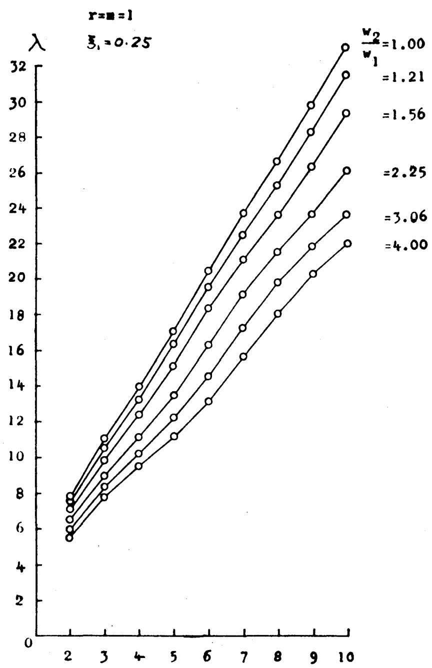


Fig. 7. Eigenvalues of the vibration of a tanker with various loading condition.

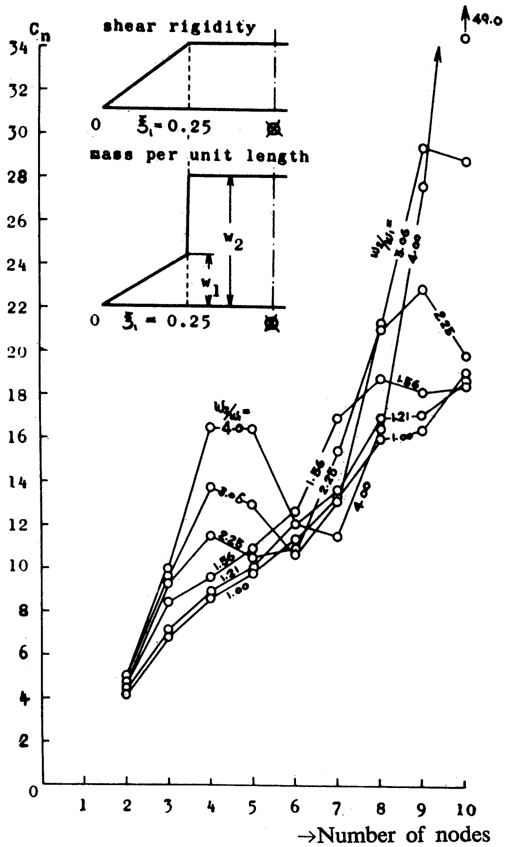


Fig. 8. C_n-values of the vibration of a tanker with various loading conditions.

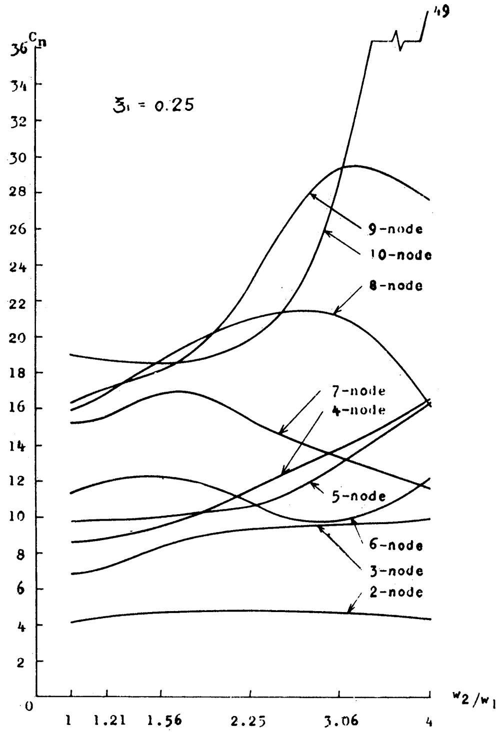


Fig. 9. C_n-values versus w_2/w_1 of a tanker with ξ_1=0.25.

Table I. Particulars of the model ship

scale	1/100
L	2.148 M
B	30.18 cm
D	15.95 cm
d_L mean	6.9 cm
d_F mean	11.4 cm
Δ_L	31.5 kg
Δ_F	58.0 kg
f_{2L}	86.0 c/sec.
f_{2F}	75.6 c/sec.

was made almost similar to that of an actual ship except in its local structures. Most of the connections of plates and girders were riveted and the rest soldered. The logarithmic decrement in the fundamental mode of the vertical vibration of the model ship in water was measured about 0.01 in ballast condition. The par-

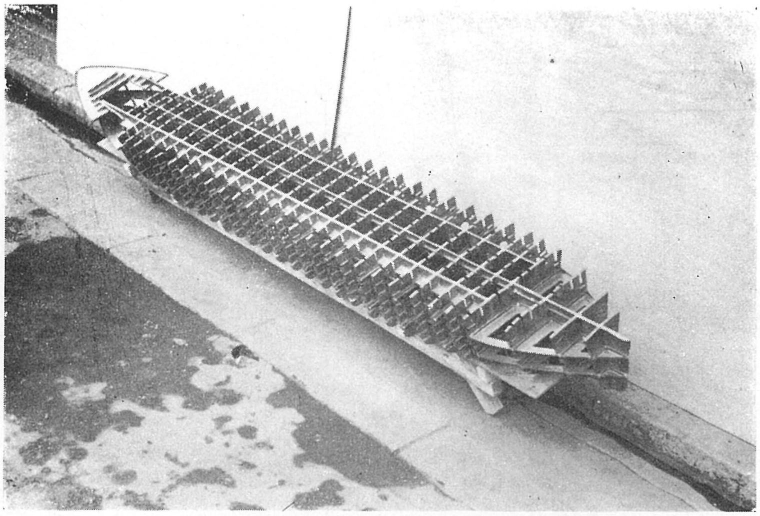


Photo. 1. Structural view of the model of tanker.

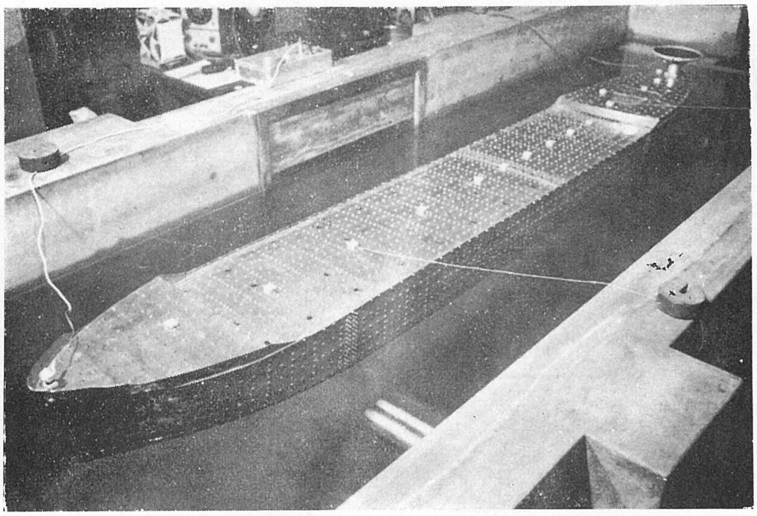
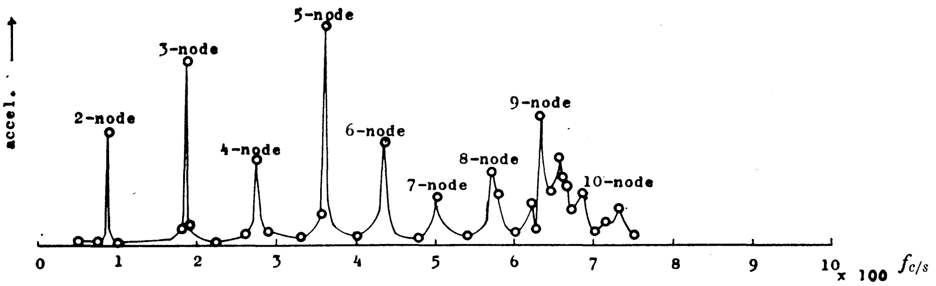


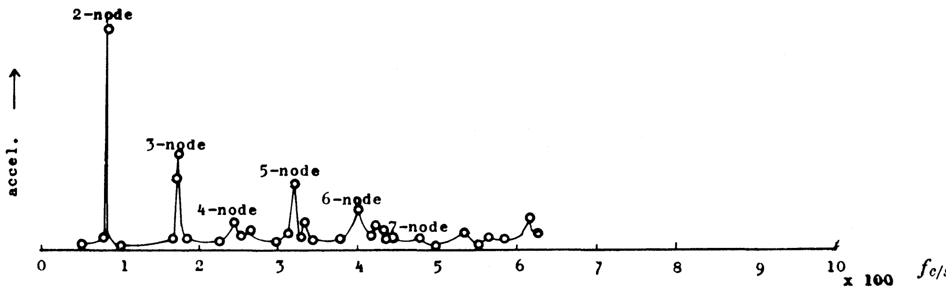
Photo. 2. General view of the model of tanker.

ticulars of the above model ship are shown in Table I. The photographs 1 and 2 show the inner and general views of the model ship respectively. Figures 10 (a) and (b) show the measured results of response of vertical vibration of model ship in ballast and full load conditions respectively.

The weight distributions assumed based on foregoing calculations are compared with those of model ship in ballast and full load conditions, as shown in Figures 11 (a) and (b). Figures also show some of the calculated and measured modes of vertical vibrations in water. Both the theoretical and experimental modes



(a) Light load



(b) Full load

Fig. 10. Response curves of the vertical vibrations of the model ship of tanker.

show good agreement except lower mode in two loaded conditions. As the results, C_n values in the calculation above 5–node modes in both conditions fairly agree with those measured in model ship as shown in Figure 12.

Figure 13 shows the comparison of C_n value in the calculation with that measured on board two tankers in ballast condition [8] based on an assumption that logarithmic decrement is $\delta_2 = 0.024$. There is disagreement of C_n value in both results, though the tendency of increase of C_n for the number of nodes of vibration in the theory almost agree with measurements. The value of C_n from measurement mentioned above is written by

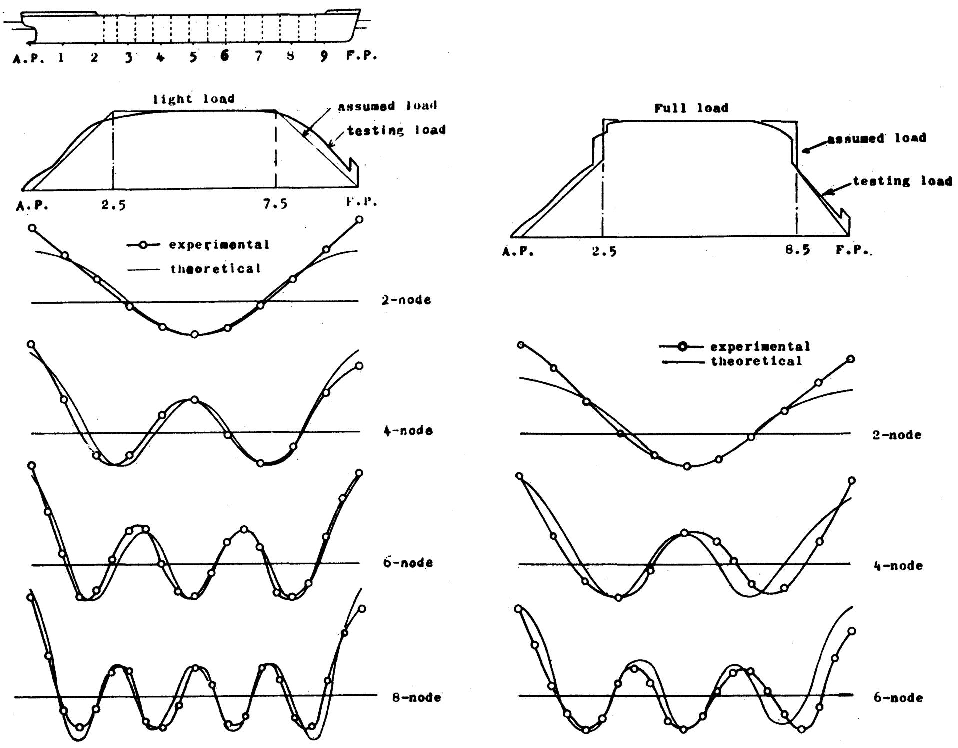
$$C_n = \left(\frac{a}{F}\right)_{\text{exp.}} \frac{\varDelta_1 \delta_n}{g\pi} \tag{13}$$

where, $\varDelta_1$ is the displacement with virtual weight and δ_n is estimated by $\delta_n = \delta_2 \times (N_n/N_2)^{3/4}$ [9] as an approach.

Conclusions

Brief conclusions will be drawn from the present investigation into the mode factor C_n for the calculation of hull response as follows:

1. C_n value for the number of nodes of the hull vibration continuously increases in the case of ship hull with the weight distribution monotonously tapered,



(a) (b)

Fig. 11 (a)(b). Comparisons of the results measured and calculated in two load distributions and in the modes of vertical vibrations of model ship.

2. C_n value irregularly varies for the number of nodes in the ship hull with discontinuous weight distribution like an actual ship. Nevertheless the corresponding eigenvalues almost linearly increase for the number of nodes of the vibration.

Some of examples of the calculations based on the present theory were confirmed by experiments on a ship shaped model and ship on board, though further investigations into the effective displacement and the damping coefficient as well as mode factor is necessary for estimating the response coefficient in a higher mode of hull vibrations of ships.

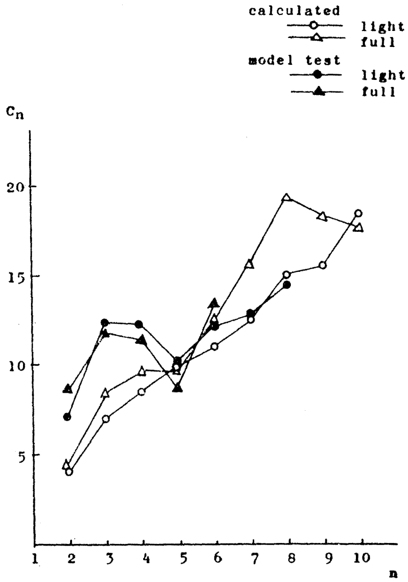


Fig. 12. C_n-n relations obtained by theory and experiments on the model ship.

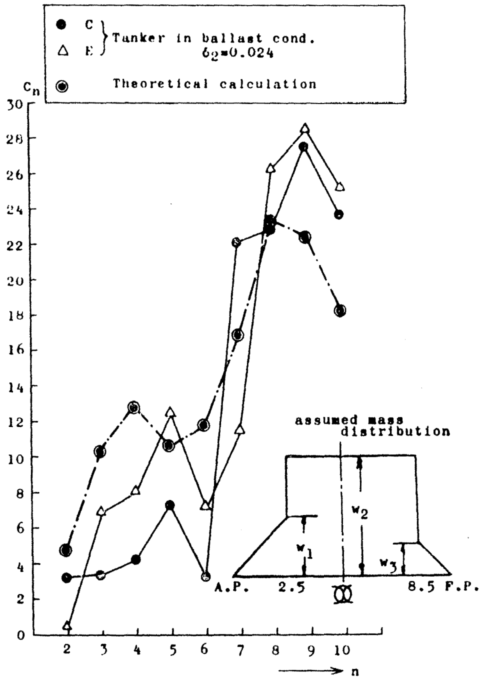


Fig. 13. Comparison of C_n in the theory with that in actual ships.

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