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NOTE

Some Notes on the Virtual Mass Associated with Vertical Vibration of Ships**

By Toyoji KUMAI*

Abstract. The present notes summarize the author's recent investigation made in quest of the solutions for the disagreement lying between theory and experiments on the virtual inertia coefficient in connection with ship vibration proposed by F. H. Todd [3], P. Kaplan [5] and I. S. S. C. report 1961 [6]. The first part of this paper gives a definition to the virtual inertia coefficient in ship vibrations, tracing the cause of disagreement mentioned above. In the latter part, the longitudinal reduction factor of the virtual inertia coefficient in the vibration of a cylinder of quasi finite length is abstracted. A method for obtaining the local reduction factor for three dimensional virtual mass in the vertical vibration of ships availing the measurements of hydrodynamical pressures at the special points of the surface of the hull of a ship-shaped model vibrating in water is recommended.

1. Virtual inertia coefficient

The apparent displacement, Δ_1, used for estimating the natural frequency of the ship vibration is conveniently calculated as follows [1], [3],

$$\Delta_1=\Delta(1+\tau_n), \tag{1}$$

where, Δ is the displacement of ship and τ_n denotes the vrtual inertia coefficient in the n-noded mode of the vibration of the ship.

In the case, $n=0$, the ship motion is the transrational oscillation. With regard to the case of the nodal vibration of the ship hull, τ_n should be clearly different from τ_0 admitting that the longitudinal reduction factor for n-noded mode is considered, namely $\tau_0 \cdot J_n$ does not show τ_n in n-noded vibration of ships.

The inertia coefficient, τ_n, for n-noded vibration is defined by the ratio of the total kinetic energy of the added mass to that of the ship hull both in the n-noded mode, namely,

$$\tau_n=\frac{\int_0^L m_e y^2\,dz}{\int_0^L my^2\,dz}\cdot J_n, \tag{2}$$

where, m_e and m are the virtual added mass and mass of the ship both in unit length respectively, y_n is the n-noded mode or the velocity amplitude of the hull vibration, L denotes the ship length and J_n the longitudinal reduction factor for n-noded mode of the

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ship vibration.

If the distributive function of m_e is to be identical with that of m, a square of the velocity amplitude $y_n{}^2$ vanishes in (2) and τ_n is written by

$$\tau_n=\frac{\int_0^L m_e\, dz}{\int_0^L mdz}\cdot J_n. \tag{3}$$

Kaplan defined the above expression when he calculated inertia coefficient [5]. However, since the distributive function of m_e is not to be identical with that of m in ordinary ship, the coefficient τ_n should be generally calculated by the expression (2). It should be noted that the expression (3) is applicable only to the flexural vibration of the cylindrical ship [7].

As was shown in the expression (2), if the distribution of m varies along ship length in the same displaced ship, there will occur the corresponding variation in the inertia coefficient, τ_n. The effect of longitudinal distribution of load upon τ_n-value is clearly calculated from the expression (2) by the strip method in actual ship, and the experimental verification can easily be carried out by the use of ship-shaped model. Table I shows the results of τ_2 obtained by calculations and experiments in the two node vertical vibration of the wooden model of a cargo ship in the two types of load distributions both in the light draught as an example [7].

Table I. τ_2 from both calculations and experiments of cargo ship model

type of loading	loaded at the part near the nodal points	loaded at the part near the loop and both end of ship	reduction factor, by Taylor [2]
τ_{2cal}	0.97	0.56	J_2=0.74
τ_{2exp}	0.94	0.50	——
τ_{0cal}	1.21	1.21	J_2=0.74

As will be seen in Table I, while τ_2 from calculation agree with that from experiments, τ_0 from the expression (3) gives a value much higher than that from experiments. In addition, no effect of distribution of load upon τ_2 is explained from (3). In the above calculation of m_e, the well-established C-value derived by Prof. F. M. Leiws [1] and that proposed as a chart by Prof. C. W. Prohaska [4] was conveniently used and Taylor's J-value [2] was also applied for the reduction factor.

In regard to the inertia coefficient in the higher mode of the hull vibration, it is to be noted that the expression (2) is also applicable for estimating the τ_n-value in n-noded mode other than that in the two node using the corresponding J_n-value.

To calculate τ from the measurements of natural frequencies in water (f_w) and that in air (f_a), the relation is written by

$$\tau=\left(\frac{f_a}{f_w}\right)^2\cdot \nu-1. \tag{4}$$

In the above equation, ν is the coefficient depending not only on the modes of vibrations in air and water, but on the distributions of mass and rigidity of ship hull. If both modes in air and water are equivalent, ν takes unity [5]. Kaplan proposed the above formula on an assumption that ν always represents unity.

As an attempt to verify experimentally the above mentioned inertia coefficient, the vibration tests on a 200 ton tugboat in air and in water were carried out by the author and his collaborators [9]. The results of experiments and some calculations are abstracted in Table II.

Table II. Values τ_2, J_2 and ν_2 obtained by tests on board 200 ton tugboat in air and water

τ_{2IIcal}	0.834	two-dimensional calculation
τ_{2exp}	0.524	experimental
J_{2exp}	0.628	experimental
J_{2cal}	0.574	by Kruppa [8]
J_{2cal}	0.490	by Taylor [2]
J_{2emp}	0.564	by Prohaska [*], ($J_2=0.38\sqrt[3]{L/B}$)
ν_2	0.982	

[*] Lodrette Skibssvingninger med To Knuder. 1941, p. 46

As a result, we may safely assume that the virtual inertia coefficient in the vertical vibration of ships calculated by (2) gives correct value, since some results computed by (2) were defintely confirmed by experiments on the ship-shaped models and an actual ship. For rapidly estimating the τ_2-value, the following formulae may be used for the ship form and cylinder of adequate ratio of L/B in the two node vertical vibration in water [7],

$$\text{for a ship;} \qquad \tau_2=0.24\frac{B}{d}, \tag{5}$$

$$\text{for a cylinder;} \qquad \tau_2=0.35\frac{B}{d}, \tag{6}$$

where, B and d denote beam and draught of the ship or the cylinder respectively. It should be noted that the empirical factors in the above formulae depend on the longitudinal reduction factor as well as beam-draught ratio.

2. Three dimensional reduction factor of a cylinder

As an existing practice, Lewis's or Taylor's J-value of ellipsoid of revolution [1],[2] has been used for the three dimensional reduction factor for a virtual inertia coefficient in the flexural vibration of ships. In the case of a full ship, the above reduction factor in a cylinder which was deduced first by Taylor [2] may be conveniently used in practice. The same result of J-value in a circular cylinder was obtained by some researchers [10], [11]. By the experimental study, however, the above theoretical results greatly differ from those of experiments [12], [13] of a finite cylinder under the same consideration. The author ascribed the cause of discrepancy between the theory and experiment to the existence of considerable difference of pressure distribution at the ends of finite and infinite cylinders vibrating in water. Accordingly, a theoretical investigation was attempted under the correct assumption of the end conditions of circular and elliptical cylinders of quasi finite length with various ratio of breadth to depth [14].

The expressions of the three dimensional potential functions, ϕ, obtained by Taylor and by the present author in the same circular cylinder of radius a, are compared as follows,

$$\phi = U\frac{K_1(kr)}{K_1'(ka)}\cos kz.\sin\theta, \qquad \text{by Taylor [2]}, \tag{7}$$

$$\phi = \frac{4U}{\pi}\sum_{m}^{\infty}\frac{m\quad K_1(k_m r)}{(m^2-n^2)K_1'(k_m a)}\sin k_m z.\sin\theta, \qquad \text{by the author [14]}, \tag{8}$$

where z is the length coordinate with the origin at the left end of the cylinder and L the length of the cylinder. The mode of vibration is presented by a cosine wave in both calculations. n denotes the number of node of vibration, m's take odd (even) numbers when n is even (odd) number. Table III shows some results of calculations of J_2-values obtained by Taylor and the author [14].

Table III. J_2-value of a circular cylinder with various length beam ratio

$\frac{L}{B}$	J_2	
	by Taylor [2]	by the author [14]
5	0.720	0.570
6	0.760	0.610
7	0.800	0.645
8	0.827	0.675
9	0.855	0.700
10	0.872	0.720

The reduction factor obtained by the above mentioned theory were comfirmed by model experiments including higher modes of vertical vibration of elliptical cylinders with various ratio of breadth to depth in water [14].

On the other hand, the distribution of potential along the keel of an elliptical cylinder obtained by the theory was also comfirmed by the measurements of hydrodynamical pressure along the keel in the two-three- and four-noded vertical vibrations of wooden model of an elliptical cylinder in water [15].

3. Local reduction factor of a cylinder and a ship-shaped model

Kaplan has recommended the experimental verification of the distribution of local reduction factor in the vibration of an ellipsoid of revolution in water [5]. However, the author attempted direct estimation of the local reduction factor in the vertical vibration of a ship-shaped model by means of pressure measurements as follows [15].

The two-dimensional potential functions of fluid surrounding a Lewis-formed cylinder in the vertical vibration is completely calculated by Lewis [1] and the results were comfirmed by Prohaska [4]. In the present study, the three dimensional potential function on the surface of the hull under water line represented by Lewis form is assumed in reference to that of two-dimensional one as follows,

$$\Phi_{\alpha=0} = (q_1 \sin\beta + q_3 \sin 3\beta)e^{i\omega t}, \tag{9}$$

where, Φ is the three dimensional potential function, q_1, and q_3 are unknown functions of length coordinate, z, and β is angular coordinate of Lewis form, in which the function of β is assumed to be the same as that in two-dimensional potential as an approach. If the hydrodynamical pressures at $\beta=\pi/2$ (keel line) and at $\beta=\pi/3$ along $\alpha=0$, in each cross section are measured by the vibration test of ship-shaped model of Lewis-formed cross section, q_1 and q_3 are obtained for given cross section, namely,

$$q_1=\frac{g}{\rho\omega}\cdot\frac{p_3}{0.866}$$
$$q_3=\frac{g}{\rho\omega}\left(\frac{p_3}{0.866}-p_1\right), \tag{10}$$

where, p_1, p_3 are the pressure at $\beta=\pi/2$ and $\beta=\pi/3$ of a given cross section respectively. The three dimensional potential at $\alpha=0$ is therefore obtained through experiments. The normal velocity at $\alpha=0$ in the three dimensional potential is identical with that in the two dimensional one. The corresponding kinetic energies in the two and three dimensions will be therefore calculated. The local reduction factor, j, is defined as the ratio of three- to two-dimensional kinetic energies at the given station, z, of the ship hull that

$$j=\frac{(1+a_1)q_1+3a_3q_3}{U_z\ b_0\{(1+a_1)^2+3a_3^2\}}, \tag{11}$$

where, a_1, a_3 and b_0 are coefficients for Lewis form computed from values of area coefficient and breadth-depth ratio of given section [16], U_z the velocity amplitude measured.

The reduced virtual inertia coefficient was therefore calculated as follows,

$$\tau=\frac{\int_0^L j\ m_e U_z^2 dz}{\int_0^L m U_z^2 dz}, \tag{12}$$

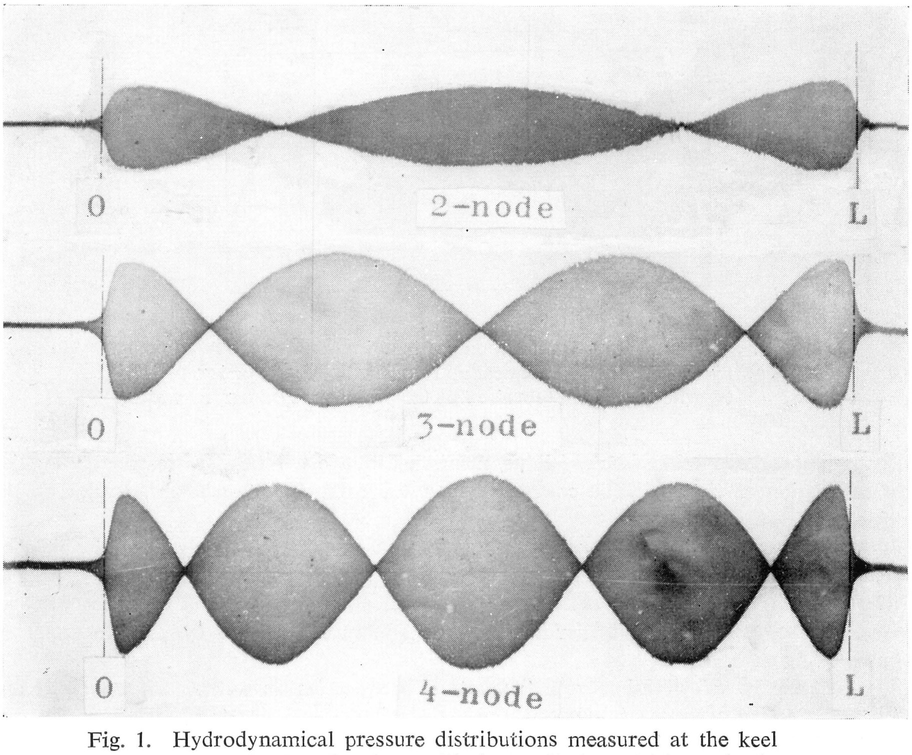

Fig. 1. Hydrodynamical pressure distributions measured at the keel along the length of an elliptical cylinder which vertically vibrating with 2-3 and 4-nodes in water

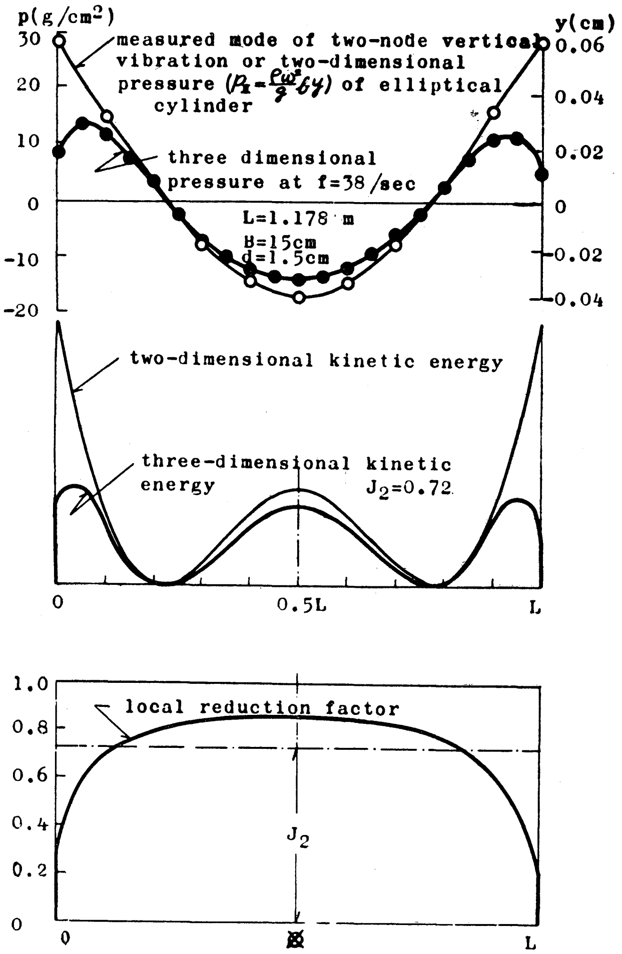

Fig. 2. Two-and three-dimensional distributions of hydrodynamical pressure, kinetic energies and local reduction factor in the two node vertical vibration of the elliptical cylinder in water

The local reduction factor, j-value, mentioned above will be more reasonably applied for calculation of virtual inertia coefficient than the total reduction factor, J-value, in the ellipsoid of revolution.

As a verification of the present calculation, an experimental study was attempted by the use of wooden model of an elliptical cylinder of 1.178 meter in length with $L/B=7.85$ and $B/d=10$. The pressure distributions along keel were measured by travelling pressure gauge in the two- three-and four-noded vertical vibrations of the cylinder in water as shown in figure 1.

The modes of vibrations and the natural frequencies as well as amplitudes at one end of the models were measured with respect to each mode of vibration, and the calculated results of distributions of the pressures and kinetic energies from the experiments in two- and three-dimensional ones and the local reduction factors in two node vertical vibration were obtained, as will be seen in the figure 2. The results of the total reduction factor,

J_2, obtained from measurements of natural frequencies in air and water agreed well with a mean value of local reduction factor, j, obtained from pressure measurement mentioned above.

Conclusions

In view of some recent investigations into the virtual inertia coefficient in the vibration of ship, it is to be noted that the square of velocity amplitude or that of the mode of the vibration should be taken into account in calculating kinetic energies of the virtual mass and ship mass per unit length. Such consideration will make it easier to understand the effects of the load distribution and of the number of nodes of the vibration upon the virtual inertia coefficient. The computed result of τ gives correct value provided that the Lewis's C-value aud Taylor's J-value are used.

In regard to the three dimensional correction factor for the virtual mass in the vertical vibration of a cylinder, it should be noted that there are significant reduction of hydrodynamical pressure at both ends of the cylinder, so that the distribution of local reduction factor greatly differs from that of the total reduction factor in the vibration of an infinite cylinder in water. Further experimental study into the local reduction factor for ship-shaped model is being carried out.

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