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ON THE VIRTUAL INERTIA COEFFICIENT IN THE VERTICAL VIBRATION OF AN ELLIPTICAL CYLINDER OF FINITE LENGTH*

By Toyoji KUMAI**

Abstract. An investigation is made into the three-dimensional correction factor of virtual inertia coefficient in the vertical vibration of an elliptical cylinder of quasi finite length in water. The experimental verifications are given including higher modes of the vibration of the elliptical cylinders.

Introduction

A theorectical investigation was carried out on the three-dimensional correction factor of the virtual inertia coefficient in the vertical vibration of an infinitely long circular cylinder floated partly immersed in water by J. Lockwood Taylor [1] in 1930, and the same problem has been recently studied by O. Grim [2] and Leibowitz-Kennard [3]. In addition, Joosen-Sparenberg [9] presented a theoretical study on the longitudinal reduction factor for the added mass in the vibration of infinitely long rectangular cylinder. With regard to the model experimental results, the above-mentioned correction factor in experiments [4], [5] is considerably smaller than that obtained by the theoretical calculation.

To explain the cause for the discrepancy between the results obtained by the existing theory and the experiments, an approximate calculation of the three-dimensional correction factor of the virtual inertia coefficient in the vertical vibration of the circular and elliptical cylinders of quasi finite length is given in the present paper. Furthermore, the effect of the sectional form on the three-dimensional coefficient is shown dealing with various elliptical sections of the cylinder of finite length.

1. Hydrodynamical inertia coefficient for vertical vibration of a ship

The inertia coefficient for the vibration of a ship is computed as the ratio of the kinetic energy of the vibration of the water surrounding the hull to that of the vibration of the ship hull itself. Since the two-dimensional calculation availing strip method is used for computing the inertia coefficient, the three-dimensional correction factor should be taken into account. The inertia coefficient is obtained

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as follows;

$$\tau = \frac{\int_0^L m_e y^2 dz}{\int_0^L m y^2 dz} \cdot J , \tag{1}$$

where,

τ virtual inertia coefficient
m_e virtual mass per unit length of the ship
m mass of the ship per unit length
y velocity amplitude of vibration of the ship
z length co-ordinate of the ship
L length of the ship
J three-dimensional correction factor defined by Lewis

For evaluating the above expression (1), the integrands $m_e y^2$ and my^2 are calculated by means of strip method for ship hull vibration. As to the three-dimensional correction factor, the well established Taylor's J-value may be used for such a fine ship as formed a half of an ellipsoid of revolution under waterline. The above J-value is not to be applicable for a ship as full as tanker. When we consider the three-dimensional correction factor for the inertia coefficient of the vibration of a full-formed ship, the cylinder of finite length will be referred to. While experimental results from J-value of the cylinders variously formed were shown in the author's previous paper [4], tests were recently made by Burrill, Robson and Townsin [5]. To the best of the author's knowledge, there is no theoretical investigation made so far on the J-value in the vertical vibration of a cylinder except that by Taylor [1], Grim [2], and Leibowitz-Kennard [3] who used a circular cylinder of infinite length. These theoretical results, however, show a great difference from those of experiments. On the other hand, the three dimensional inertia coefficients of the vibrations of the ellipsoid were adopted from mathematical treatise by Kruppa [6]. This result will be more useful for considering the vibration of the fine ship than that of ellipsoid of revolution.

2. Three-dimensional correction factor, *J*, in the vibration of a circular cylinder of finite length

As a special case, when the distributive function of m_e equals to that of m as it occurs to a homogeneous cylinder in water, the expression (1) will lead is the following equation:

$$\tau = c \cdot J , \tag{2}$$

where, c is constant for the sectional form of the cylinder under water line. The c takes unity in the case of the vibration of a circular cylinder immersed partly in water, $c = B/2d$ in elliptical cross section.

Let us consider a circular cylinder of length L, floated on water with draught a, where a is the radius of its section. The cylinder is now in vertical vibration and that with the natural frequency. The mode of vibration is assumed as an approach to be of a cosine mode, namely, the velocity distribution is shown as follows;

$$v = U \cos \frac{n\pi}{L} z \cdot e^{i\omega t}, \tag{3}$$

where, U unit velocity
n number of nodes of vertical vibration of the cylinder

The velocity v is also expressed by means of Fourier expansion as follows;

$$v = \frac{4U}{\pi} \sum_{m}^{\infty} \left(\frac{m}{m^2 - n^2} \right) \sin k_m z \cdot e^{i\omega t}, \tag{4}$$

where, $k_m = \frac{m\pi}{L}$

$m = 1, 3, 5, \cdots\cdots$ for $n = 0, 2, 4, \cdots\cdots$

$m = 2, 4, 6, \cdots\cdots$ for $n = 1, 3, 5, \cdots\cdots$

On the other hand, the Laplace equation with respect to the potential function for the three-dimensional flow in the cylindrical co-odinate is written by

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \tag{5}$$

The velocity potential under consideration should satisfy the following boundary conditions:

At the boundary of the free surface,

$$\phi = 0, \qquad \text{for } \theta = 0 \text{ and } \pi,$$

the water parted infinitely distant from the cylinder,

$$\phi = 0, \qquad \text{for } r \to \infty$$

for the end condition of the quasi finite length as approximate consideration,

$$\phi = 0, \qquad \text{for } z = 0 \text{ and } L$$

The solutions of equation (5) satisfying the above conditions are written as follows;

$$\phi = \sum_{m}^{\infty} A_m K_1(k_m r) \sin k_m z \cdot \sin \theta \cdot e^{i\omega t}, \tag{6}$$

where, A_m constants
$K_1(k_m r)$ K-Bessel function or Hankel function with imaginary argument

As the last condition, the normal velocity of the wetted surface of the cylinder should identify that of the surrounding water, namely,

$$\left(\frac{\partial\phi}{\partial r}\right)_{r=a} = v\cdot\sin\theta. \tag{7}$$

Substituting (4) and (6) into (7), constants A_m are determined as follows;

$$A_m = \frac{4mU}{\pi(m^2-n^2)K_1'(k_m a)}\ , \tag{8}$$

where, dash denotes the derivative of $K_1(k_m r)$ with respect to r.

The velocity potential function satisfying above conditions will be written as

$$\phi = \frac{4U}{\pi}\sum_m^{\infty}\frac{mK_1(k_m r)}{(m^2-n^2)K_1'(k_m a)}\sin k_m z\cdot\sin\theta\cdot e^{i\omega t}. \tag{9}$$

A comparison of the distributions of the velocity amplitude v and the potential function $\phi_{r=a,\theta=\pi/2}$ along z-axis which considered by Taylor with that investigated by the present author is shown in figure a. As is seen in the figure, the major difference between the above two assumptions will be found in the distribution of the potential function near from both ends of cylinder. In both cases, no boundary conditions at the plane of flat ends of a cutting out cylinder are satisfied.

The kinetic energy due to vibration of the water surrounding the hull is computed from the above potential function and its derivative, as follows;

$$2T = -\rho\int_0^L\int_0^\pi\left(\phi\frac{\partial\phi}{\partial r}\right)_{r=0} a\cdot d\theta\cdot dz$$

$$= \frac{-\rho\pi U^2a^2}{2}\cdot\frac{L}{2}\sum_m^{\infty}\frac{16}{\pi^2}\frac{m^2}{(m^2-n^2)^2}\cdot\frac{K_1(k_m a)}{aK_1'(k_m a)}\ . \tag{10}$$

On the other hand, the two-dimensional calculation of the energy of vibrating water surrounding the same cylinder as mentioned above is easily shown by

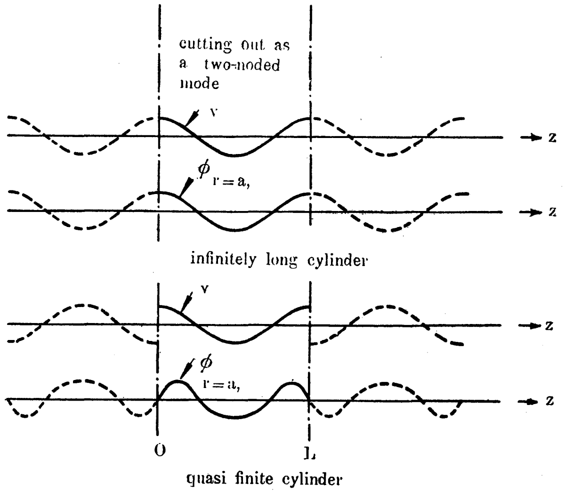

Fig. a. Distributions of v and ϕ of two node vibration of cylinder cutting out from two types of infinite cylinders.

$$2T_{II}=\frac{\rho\pi U^2a^2}{2}\cdot\frac{L}{2}\ . \tag{11}$$

Taking the ratio of (10) to (11), the three-dimensional correction factor in n-noded vertical vibration of the circular cylinder is obtained as follows,

$$J=\frac{T}{T_{II}}=\frac{16}{\pi^2}\sum_m^{\infty}\frac{\left(\frac{m}{m^2-n^2}\right)^2}{1+k_m a\ \frac{K_0(k_m a)}{K_1(k_m a)}}\ , \tag{12}$$

where,

$$k_m a=\frac{m\pi}{2}\cdot\frac{B}{L}$$

As was seen in the above formula, the three-dimensional correction factor of the circular cylinder of the quasi finite length in two-node vertical vibration, for an example, is obtained with L/B as variable, as shown in Fig. 1. The figure shows J-value comparing with the existing results of calculations and of the experiments. The results of the present calculation made on the number of nodes of vibration in the parameter are shown in Fig. 2(b).

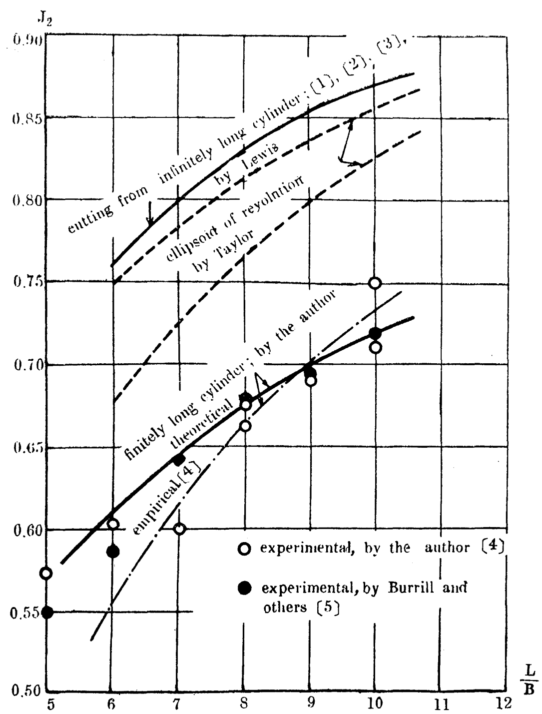

Fig. 1. J-values of 2-node vertical vibration of infinitely and finitely long cylinders and of ellipsoid of revolution.

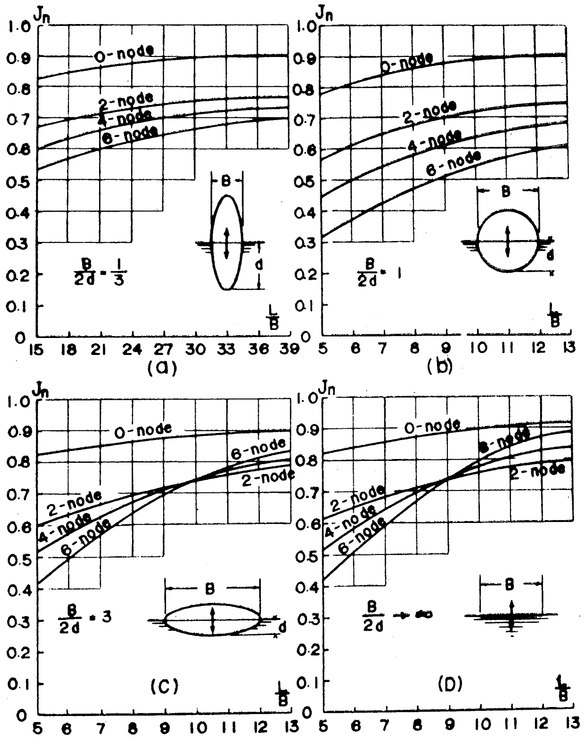

Fig. 2. J-value versus L/B for $B/2d=1/3$, 1, 3 and ∞, number of nodes, 0, 2, 4 and 6 as parameter.

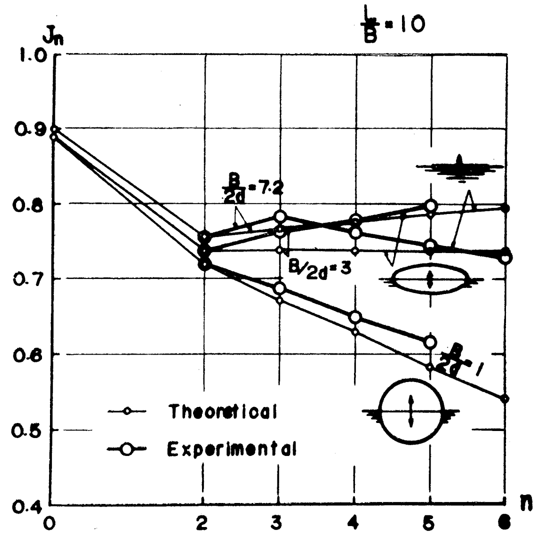

Fig. 3. Experimental verification on J-value versus n, in cylinders of various cross sections.

The experimental verification of the theoretical results in the higher modes of the vibration of the circular cylinder under consideration was carried out by the use of a wooden model of $L/B=10$. The result is shown in Fig. 3. As will be seen in the figure, while the J-value decreases slightly by the increase in the number of nodes of vibration in the circular cylinder, the tendency in the theory agrees with that in the experiment.

3. J-value in the vertical vibration of an elliptical cylinder

For evaluating the effect of the sectional form of the cylinder of finite length on the three-dimensional correction factor for the virtual inertia coefficient in the vertical vibration of the cylinder in water, the calculation of the three-dimensional virtual inertia coefficient of an elliptical cylinder of the finite length vibrating in water will be shown as follows.

The Laplace equation for the three-dimensional potential flow in the cartesian co-ordinates is

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0. \tag{13}$$

When the potential function ϕ is presented by $\phi' \sin k_m z$, provided that the mode of vibration is assumed to be of the trigonometric function, the following equation is obtained,

$$\frac{\partial^2\phi'}{\partial x^2}+\frac{\partial^2\phi'}{\partial y^2}-k_m{}^2\phi'=0. \tag{14}$$

By the use of the following elliptical co-ordinates ξ, η,

$$x=h\cosh\xi\cos\eta\ ,$$
$$y=h\sinh\xi\sin\eta\ ,$$

$2h$: focal distance,

and also putting $\phi'=\phi_1\cdot\phi_2$, the following two equations are obtained,

$$\left.\begin{aligned}&\frac{d^2\phi_1}{d\xi^2}-(a-2q\cosh 2\xi)\phi_1=0,\\&\frac{d^2\phi_2}{d\eta^2}+(a+2q\cos 2\eta)\phi_2=0,\end{aligned}\right\} \tag{15}$$

where, a: separation constant

$q=\kappa^2$

$\kappa=k\cdot h/2$

The solutions of the above two types of Matheu equations are easily obtained [7] and the ϕ becomes,

$$\phi=\sum_m^{\infty}\{C_m\mathrm{Gek}_1(\xi,-q_m)\mathrm{se}_1(\eta,-q_m)+D_m\mathrm{Fek}_1(\xi,-q_m)\mathrm{ce}_1(\eta,-q_m)\}\sin k_m z\cdot e^{i\omega t}. \tag{16}$$

The integration constants C_m and D_m in the above solutions are determined by

$$D_m=0, \quad \text{for} \quad \eta=0\sim\pi,$$

$$C_m=0, \quad \text{for} \quad \eta=\frac{\pi}{2}\sim\frac{3\pi}{2},$$

the Matheu functions $se_1(\eta, -q_m)$ and $ce_1(\eta, -q_m)$ for small value of q_m are apploximately written by

$$se_1(\eta, -q_m) \doteqdot \sin\eta,$$
$$ce_1(\eta, -q_m) \doteqdot \cos\eta.$$

From the foregoing solutions, the three-dimensional correction factor for elliptical cylinder of the section, $\xi=\xi_0$, is computed by using the same method as that in the previous circular cylinder and the results are obtained as follows;

$$J=\frac{16}{\pi^2}\sum_m^\infty\left(\frac{m}{m^2-n^2}\right)^2\frac{\mathrm{Gek}_1(\xi_0, -q_m)}{-\mathrm{Gek}_1'(\xi_0, -q_m)}, \quad 0\leqq\eta\leqq\pi,$$
$$J=\frac{16}{\pi^2}\sum_m^\infty\left(\frac{m}{m^2-n^2}\right)^2\frac{\mathrm{Fek}_1(\xi_0, -q_m)}{-\mathrm{Fek}_1'(\xi_0, -q_m)}, \quad \pi/2\leqq\eta\leqq3\pi/2, \tag{18}$$

where, the dash denotes the derivative of respective function with respect to ξ.

The modified Matheu functions in the above formulae are approximately represented by the products of modified Bessel functions as shown by following expressions.

$$\frac{\mathrm{Gek}_1(\xi_0, -q_m)}{-\mathrm{Gek}_1'(\xi_0, -q_m)}=\frac{1+\dfrac{I_1(u_m)K_0(v_m)}{I_0(u_m)K_1(v_m)}}{1-\dfrac{I_1(u_m)K_0(v_m)}{I_0(u_m)K_1(v_m)}+(u_m+v_m)\left\{\dfrac{K_0(v_m)}{K_1(v_m)}+\dfrac{I_1(u_m)}{I_0(u_m)}\right\}},$$
$$\frac{\mathrm{Fek}_1(\xi_0, -q_m)}{-\mathrm{Fek}_1'(\xi_0, -q_m)}=\frac{1-\dfrac{I_1(u_m)K_0(v_m)}{I_0(u_m)K_1(v_m)}}{1+\dfrac{I_1(u_m)K_0(v_m)}{I_0(u_m)K_1(v_m)}-(u_m-v_m)\left\{\dfrac{K_0(v_m)}{K_1(v_m)}-\dfrac{I_1(u_m)}{I_0(u_m)}\right\}}, \tag{19}$$

where, $u_m=\kappa_m e^{-\xi_0}$, $v_m=\kappa_m e^{\xi_0}$, $\kappa_m=k_m h/2$.

The numerical values of the modified Bessel functions in the above equations are obtained by Shibagaki's table [8].

When the sectional form of the elliptical cylinder approaches to circular one, ξ_0 becomes large and then the values presented by (18) approach to that of (12). On the contrary, when ξ_0 approaches zero, the sectional form presents the flat plate and the values are calculable by the use of (18). The computed results of the J-values on the cylinders of typical elliptical sections including higher modes of the nodal vibrations are shown in Figs. 2(a), (b), (c) and (d). As will be seen in these figures, it is to be noted that the coefficient in the high mode of the vibration shows a higher value in the flat shaped section than that in the circular section, whereas the effect of the sectional form on the J-value in the lower mode does not change so considerably as is shown in Fig. 4. As an example, Fig. 3 shows the J-values versus number of nodes of the vibration of the cylinders of three kinds of the sectional form which were calculated and tested. The J-values versus $B/2d$ of the elliptical section with $L/B=10$ are shown in Fig. 4 with the number of nodes of the vibration in the parameter. There seems to be a little discrepancy between the theory and the experiments as is seen in Fig. 3. However, the tendency of the effect of the sectional form upon the J-value of the cylinder in the high mode of the vibration in water is shown fairy well.

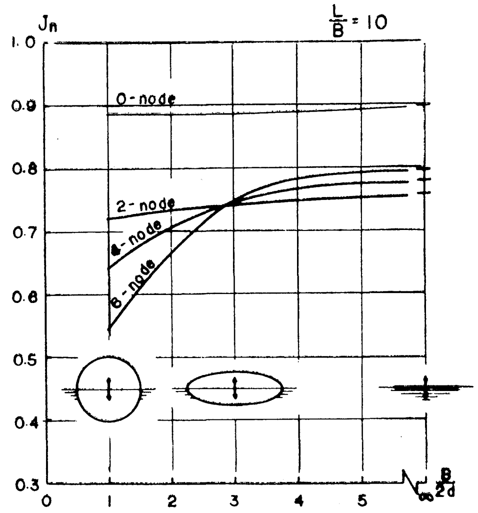

Fig. 4. J-value versus $B/2d$ with number of nodes as parameter in $L/B=10$.

4. Longitudinal distribution of the hydrodynamical pressure in the vertical vibration of the cylinder

The hydrodynamical pressure along keel in the vertical vibration of the cylinder under consideration as mentioned above is presented by

$$p=\frac{\rho}{g}\left(\frac{\partial\phi}{\partial t}\right)_{r=a,\ \theta=\pi/2} \tag{20}$$

The distribution of the above pressure is easily obtained from the previous calculations. As an example, Fig. 5 shows the distributions of the ratios of three- and two-dimensional pressures at the bottom centerline of the circular cylinders of three kinds of L/B in the vertical translational oscillation and the two node vibration in water with a half immersion. As will be seen in the figure, there is considerable difference between the two- and three-dimensional pressure distributions especially near the end of the cylinder. It will be supposed that the above difference in the ellipsoid is not so conspicuous compared with that of the cylinder as shown in these illustrations. The theoretical result will be comfirmed by the model experiment in near future.

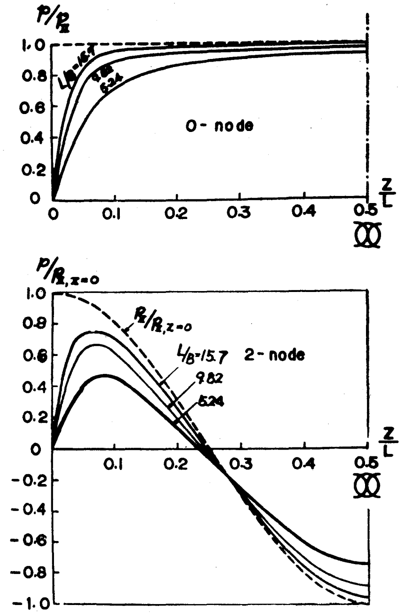

Fig. 5. Distributions of hydrodynamical pressure at the bottom centre line of the circular cylinder

Conclusions

As a contribution to the calculation of the virtual inertia coefficient of the vertical hull vibration of a ship, the three-dimensional correction factors in the vibration of a circular and elliptical cylinders of the quasi finite length were calculated under some appropriate assumptions and the results of the calculations were compared with that of some model experiments.

So far as the results of the present investigation are concerned, the effect of the end condition of the vibrating cylinder of finite length upon the three-dimensional correction factor in the vertical vibration seems considerably large compared with that of infinitely long cylinder, whereas there is little effect of the sectional form on this factor in lower mode of the vibration of the cylinder in water. In regard to the higher modes of the vibration, however, the J-value appears considerably high. The tendency of this effect was confirmed by the model experiments in the present study. We may assume that one of the causes of the difference between the inertia coefficients of a slender ship and that of a full-formed ship with flat bottom in the higher mode of the vertical vibration of the model ship was clarified by the present investigation. As was suggested by Joosen-Sparenberg [9], we should bear in mind that the three dimensional influence is more important for hull forms of which bottom has a larger distance to the surface of the water. Further study is required into the present problem in ship-shaped beam.

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