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KUMAI, Toyoji
Research Institute for Applied Mechanics, Kyushu University : Professor

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THE EFFECT OF DISTRIBUTION OF LOAD UPON THE VIRTUAL INERTIA COEFFICIENT IN THE VERTICAL VIBRATION OF A SHIP*

By Toyoji KUMAI**

Abstract. Here is investigated the effect of distribution of load on the virtual inertia coefficient in the vertical vibration of a ship. The author emphasized that, since the inertia coefficient is defined as the ratio of kinetic energy of the vibrating water surrounding the ship hull to that of the ship itself, the velocity amplitude of the hull vibration should be taken into account in the calculation of the inertia coefficient under consideration. Some numerical examples on the effects of load distribution and the number of nodes of vibration upon the inertia coefficient are shown and the results are confirmed by the model experiments.

Introduction

Since the virtual inertia coefficient in the flexural hull vibration of a ship is defined as the ratio of the kinetic energy of the added mass of the surrounding water to that of the ship itself, the velocity amplitude of the ship vibration should be taken into account in computing the maximum kinetic energies mentioned above. The energy of the ship hull in the nodal vibration considerably depends on the load distribution along ship length, whereas the energy of the vibration of the added mass of the water is influenced only slightly by the load distribution due to a little change of the velocity amplitude. Accordingly, the inertia coefficient of water in the nodal hull vibration is considerably affected by the distribution of ship load along her length. Present paper shows some numerical examples on the problem mentioned above with the aid of an experiment using a model of cargo ship.

1. Expression of inertia coefficient in the vibration of a ship

The natural frequency of the vertical vibration of a ship in which shear and rotary inertia effects are ignored is computed by

$$f_n^2=\left(\frac{1}{2\pi}\right)^2\frac{\int_0^L EI\left(\frac{d^2y_w}{dx^2}\right)^2dx}{\int_0^L my_w^2dx(1+\tau_n)}\ , \tag{a}$$

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** Professor of Kyushu University, the member of the Research Institute for Applied Mechanics, Kyushu University, Fukuoka, Japan.

where,

$$\tau_n = \frac{\int_0^L m_e\, y_w^2\, dx}{\int_0^L m\, y_w^2\, dx} \cdot J_n \tag{1}$$

The m_e and m denote the added mass of water and mass of the ship per unit length respectively, and y_w is the n-noded mode of vibration of the ship hull in water. J_n presents the three-dimensional correction factor defined by Lewis [1] and Taylor [2], and τ_n the virtual inertia coefficient of water under consideration. From the above relation (a), the well-established formula is derived as follows,

$$f_n = \beta \sqrt{\frac{EI_0}{L^3 \Delta (1+\tau_n)}}\,, \tag{b}$$

where, Δ denotes displacement and β the eigenvalue which includes the integral value. In the expression (b), τ_n is the inertia coefficient defined by equation (1).

As was seen in equation (1), the τ-value is defined as the ratio of the kinetic energy of the vibration of the entrained surrounding water to that of the ship itself, but it is the ratio of added mass of water to displacement only in the following two special cases: In the first place, the distributive functions of m_e equals to that of m, such as cylindrical ship form with uniform load. Next, in the case of translational oscillation of a hull or o-node vibration such as heaving of the ship. In these two cases, the inertia coefficient yield

$$\tau_n = \frac{\int_0^L m_e dx}{\Delta} \cdot J_n. \tag{2}$$

The above expression of τ is the well known Lewis's method. In general cases, the distributive function of m_e in the ship vibration differs from that of m, and o node mode vibration is beside the question. In calculating the virtual inertia coefficient of the water in ship vibration, therefore, the equation (1) should be generally used instead of the Lewis's method. Lewis confirmed his calculation by Nicholls's experiments in three cases. This experiment was carried out using uniform bar, instead of ship model. The formula (2) is therefore available as uniform distributions for both of m_e and m.

On the other hand, in order to determine the inertia coefficient by the experiment, the natural frequency and the mode of the vibration of model ship should be measured in air and in water. The natural frequency of the ship in air is expressed by

$$f_a^2 = \left(\frac{1}{2\pi}\right)^2 \frac{\int_0^L EI \left(\frac{d^2 y_a}{dx^2}\right)^2 dx}{\int_0^L m\, y_a^2\, dx}\,, \tag{3}$$

where, y_a; mode of vibration of the ship model in air. Taking the ratio of (3) to (a), the inertia coefficient τ is easily obtained from the following relation,

$$\tau=\left(\frac{f_a}{f_w}\right)^2 \cdot \nu-1\ , \tag{4}$$

$$\nu=\frac{\int_0^L I\left(\frac{d^2y_w}{dx^2}\right)^2 dx}{\int_0^L m\, y_w{}^2 dx} \cdot \frac{\int_0^L m\, y_a{}^2 dx}{\int_0^L I\left(\frac{d^2y_a}{dx^2}\right)^2 dx}\ . \tag{5}$$

If we assume y_a as equaling to y_w, ν takes unity. As an approach, ν-value may be assumed to be unity in practice.

As was seen in equation (1), the kinetic energy of the mass of the ship will considerably change for the distribution of the ship load, whereas the distribution of the kinetic energy of the vibration of added mass of water is almost constant. Accordingly, it is necessary to consider the effect of the distribution of the ship load on the inertia coefficient in the vibration of a ship hull.

2. Numerical examples and experimental verification

Since an analytical calculation of the energies in (1) is difficult to make even in a model ship, the numerical examples should be illustrated first by the use of the results of vibration measurements given on a model ship in the present section.

A wooden model with 1/100 scale of a cargo ship was used for the numerical calculations and measurtements of virtual inertia coefficients of water in the vertical vibration of the ship hull with three types of load condition.

The principal dimensions and particulars are shown in table I, and also the body plan of the model ship is given in Fig. 1. The vibration tests were carried out in a small tank with the ratio of depth of water to draught about 10.

Table I. Particulars of the cargo ship model used in strip calculation and experiment.

L	145 (cm)
B	19.6 〃
D	12.5 〃
d	6.13 〃
Δ	10.04(kg)
hull wt.	6.40 〃
load	3.64 〃

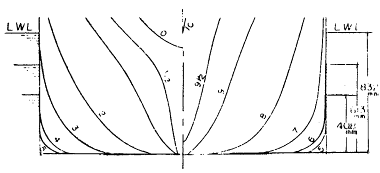
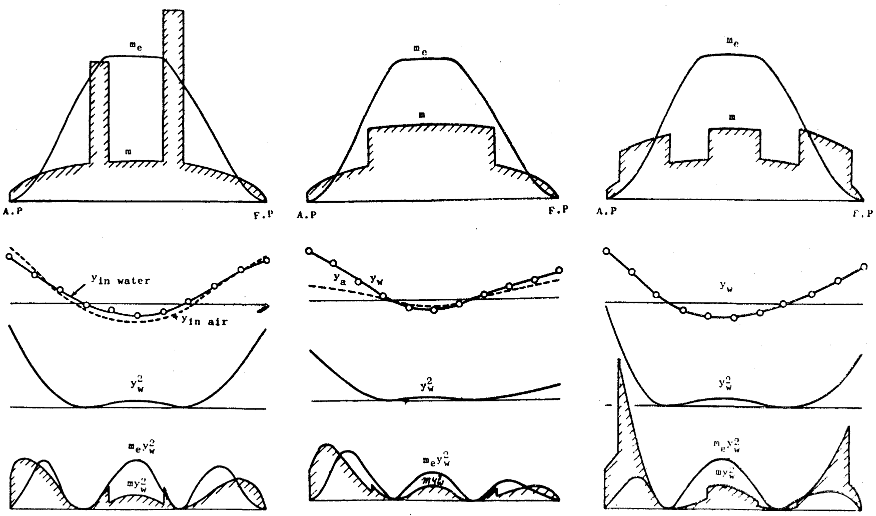


Fig. 1. Body plan of the cargo ship used for strip calculation and experiment.

The vertical vibration was excited by a vibrator of a moving coil type speaker connected to thin rod at the end of the model, and the crystal pickup was used for measuring the natural frequency and its mode of the vibration of the model ship in air and in water. The inertia coefficient is computed from the formulae (4) and (5), provided that the measured results of the mode y_a and y_w, and the frequencies in air f_a and in water f_w are used.

On the other hand, the inertia coefficient is calculated from the formula (1) by means of the strip method, in which the Lewis's C-value of each section of the ship hull is obtained using the convenient chart proposed by Prohaska [3]. The modes of the nodal vibration measured in experiments were adopted in the present strip calculation. Fig. 2–(a), –(b), –(c) show the computed results of distribution of added mass of water and that of mass of the ship in which the displacement in half load takes unity with three kinds of load distribution cases I, II and III respectively in top view. The measured amplitude of the natural vibration of the model ship in air and on water, and square of y_w are shown in next view, and the distributions of energies of the vibration of added mass of water



(a) (b) (c)

Fig. 2. Results of calculations of m_e,m and $m_e y^2$, my^2 on the model ship in three loading conditions.

Table II. Inertia coefficients obtained by calculations and experiments.

	Case I	Case II	Case III	J_n
τ_{II0}	1.64	1.64	1.64	
τ_0	1.54	1.54	1.54	0.94 (Lamb)
τ_2(Lewis)	**1.21**	**1.21**	**1.21**	0.74 (Taylor)
τ_{II2}	1.31	1.06	0.755	
τ_2	**0.97**	**0.78**	**0.56**	0.74 (Taylor)
τ_{2exp}	**0.94**	**0.70**	**0.50**	—
f_a cps	310	242	215	—
f_w cps	217	190	181	—

and of the ship in water are presented in bottom view of Fig. 2. The two-dimensional inertia coefficients under consideration are computed by integration of the energy curves obtained as above. The three dimensional or longitudinal coefficients J_n-value in the vertical vibration of a ship obtained by Taylor [2] was used in the present calculation of τ. The numerical results are shown in table II.

As will be seen in the results, the inertia coefficient in the nodal ship vibration is different from that of the translational oscillation of the ship or that obtained by Lewis's method in the same draught of a ship. In addition, the effect of the load distribution on the inertia coefficient under consideration may be clearly seen in the present illustrations of the calculations and experiments as shown in the figure and table.

3. Estimation of the inertia coefficient in the vertical vibration of a ship

It is necessary to estimate the tendency of increase or decrease of the τ-value for various load conditions of a ship. So that some analytical evaluations of inertia coefficients should be carried out under the appropriate assumptions of the distributions of added mass, ship's mass and the mode of the nodal vibration of the ship.

i. Effect of load and added mass distributions on the inertia coefficient in the fundamental mode of the vertical vibration of ships

As the first example, the ship load as well as the added weight of water are assumed to be symmetrical about midship, and the function of both distributions of m_e and m which are close to the end of the ship are assumed as square of a quarter length of sine wave and the length of run or entrance in both curves as parameters respectively. In the mode of two-node vertical vibration, a cosine curve with corrected base line is assumed as an approach in the mode of the nodal vibration. The above mentioned three curves are written as follows, and also shown in figure 3–(a).

$$\left.\begin{aligned}
m_e &= m_0 \sin^2 \frac{\pi\xi}{2\xi_e} &&; \quad 0 \leqq \xi \leqq \xi_e \\
&= m_0 &&; \quad \xi_e \leqq \xi \leqq 1/2\ , \\
m &= \frac{1}{1-\xi_m} \sin^2 \frac{\pi\xi}{2\xi_m} &&; \quad 0 \leqq \xi \leqq \xi_m \\
&= \frac{1}{1-\xi_m} &&; \quad \xi_m \leqq \xi \leqq 1/2 \\
y &= \cos 2\pi\xi + \alpha .
\end{aligned}\right\} \qquad (6)$$

In the above expressions, ξ_e and ξ_m are coordinates of the ends of parallel parts of the curves m_e and m respectively. The m_0 is written by

$$m_0 = \frac{\pi}{8} \frac{C_{1/2}}{C_b} \frac{B}{d}\ ,$$

provided that,

$$\frac{1}{\Delta}\int_0^L m dx = 1 \;.$$

where,

$C_{1/2}$ Lewis's C-value at midship

C_b block coefficient

B/d breadth-draught ratio

By substitution of the above expressions into equation (1), the τ-values are analytically evaluated, and the parameter, α, in the above quation is determined by means of the minimum energy principle as follows,

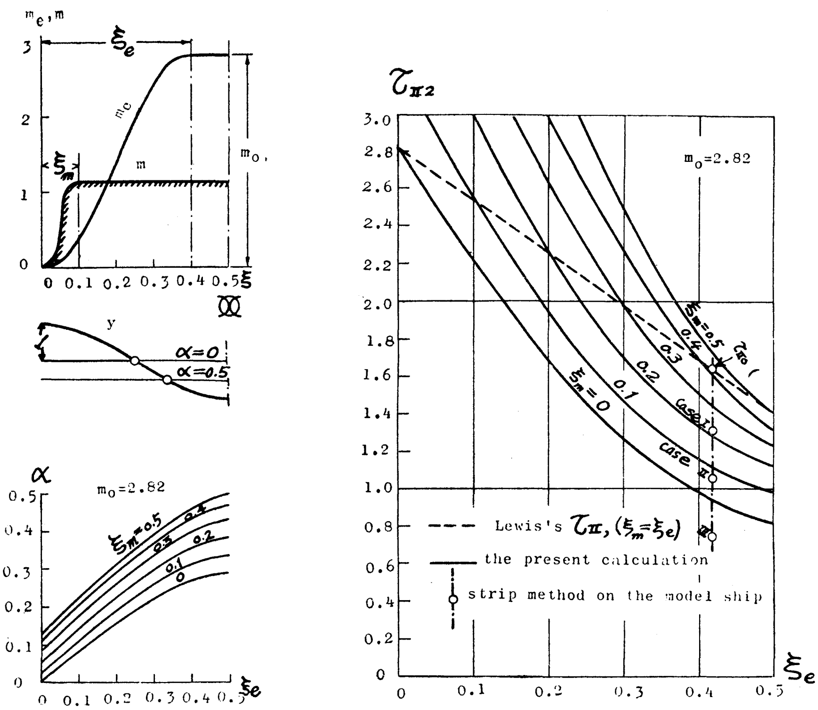
$$\left.\begin{aligned}\int_0^{1/2} m_e y^2 d\xi &= \frac{m_0}{2}\left[\left(\frac{1}{2}+\alpha^2\right)(1-\xi_e)-\frac{1}{\pi}\left\{\alpha\frac{1}{1-4\xi_e^2}\sin 2\pi\xi_e\right.\right.\\ &\qquad\left.\left.+\frac{1}{8}\cdot\frac{1}{1-16\xi_e^2}\sin 4\pi\xi_e\right\}\right],\\ \int_0^{1/2} m y^2 d\xi &= \frac{1}{2(1-\xi_m)}\left[\left(\frac{1}{2}+\alpha^2\right)(1-\xi_m)-\frac{1}{\pi}\left\{\alpha\frac{1}{1-4\xi_m^2}\sin 2\pi\xi_m\right.\right.\\ &\qquad\left.\left.+\frac{1}{8}\cdot\frac{1}{1-16\xi_m^2}\sin 4\pi\xi_m\right\}\right],\end{aligned}\right\} \tag{7}$$

from $$\frac{\partial\int_0^{1/2}(m_e+m)y^2 d\xi}{\partial\alpha}=0,$$

$$\alpha=\frac{m_0\dfrac{1}{1-4\xi_e^2}\sin 2\pi\xi_e+\dfrac{1}{1-\xi_m}\cdot\dfrac{1}{1-4\xi_m^2}\sin 2\pi\xi_m}{2\pi\{1+m_0(1-\xi_e)\}}\;. \tag{8}$$

The above results are illustrated in Fig. 3-(b), letting ξ_e and ξ_m be variable and parameter respectively. As a special case, if we put $\xi_e=\xi_m$, the τ-value will be presented by dotted straight line, as seen in the figure. This line just shows Lewis's expression (2). As an example, when a fine ship (assume $\xi_e=0.5$) is fully loaded along ship length (assume $\xi_m=0.15$), the τ-value becomes a half of that obtained by Lewis's method. As an extreme case, taking $\xi_e=\xi_m=0$, this value will show that of the vibration of a uniform bar. On the contrary, if the load is concentrated almost at the midship ($\xi_m=0.5$) on the above ship form ($\xi_e=0.5$), the τ becomes maximum, it corresponds to the Lewis's result. The distribution of the ship load, however, shows no such concentrated load, because the hull weight is generally distributed along ship length. As mentioned in the above examples, the τ-value is seen to be smaller than that computed by Lewis's method in an ordinary type of ship.

ii. Inertia coefficient in the higher mode of the hull vibration



(a) (b)

Fig. 3. The results of calculations of τ-value with given m_e, m and y.

The distributions of m_e, m, and y are assumed in the higher mode of vibration as follows,

$$\left.\begin{aligned} m_e &= m_0 \sin^2\pi\xi, \\ m &= \frac{\pi}{\mu\pi+2(1-\mu)}\{\mu+(1-\mu)\sin\pi\xi\}, \\ y &= \cos n\pi\xi+\alpha\cos(n-2)\pi\xi,\quad n=2, 3, \cdots\cdots \end{aligned}\right\} \tag{9}$$

where μ is a parameter which gives the distribution of load. The n is the number of nodes of the vibration of the ship. Substituting m_e, m and y mentioned above into (1), the results will be written as

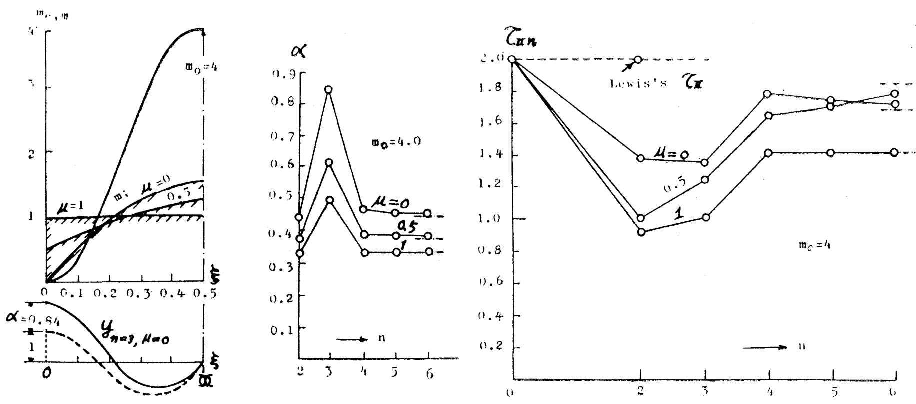
$$\int_0^1 m_e y^2 d\xi = \frac{m_0}{4}\left[(1+\alpha^2-\alpha)-\alpha(1-\alpha)\frac{\sin(2n-4)\pi}{(2n-4)\pi}-\frac{\alpha^2}{2}\frac{\sin(2n-6)\pi}{(2n-6)\pi}\right], \tag{10}$$

$$\int_0^1 my^2 d\xi = \frac{1}{2}(1+\alpha^2)+\frac{\mu\alpha^2}{2\{\mu\pi+2(1-\mu)\}}\frac{\sin(2n-4)\pi}{(2n-4)\pi}$$

$$-\frac{2(1-\mu)}{\mu\pi+2(1-\mu)}\left\{\frac{\alpha^2}{2(2n-3)(2n-5)}+\alpha\left(\frac{1}{3}+\frac{1}{(2n-1)(2n-3)}\right)+\frac{1}{2}\frac{1}{4n^2-1}\right\}. \tag{11}$$

The parameter α in the expression y should be determined according to the minimum energy principle as follows,

$$\alpha=\frac{\dfrac{2(1-\mu)}{\mu\pi+2(1-\mu)}\left\{\dfrac{1}{3}+\dfrac{1}{(2n-1)(2n-3)}\right\}+\dfrac{m_0}{4}\left\{1+\dfrac{\sin(2n-4)\pi}{(2n-4)\pi}\right\}}{1+\dfrac{1}{\mu\pi+2(1-\mu)}\left\{\dfrac{\mu\pi\sin(2n-4)\pi}{(2n-4)\pi}-\dfrac{2(1-\mu)}{(2n-3)(2n-5)}\right\}+\dfrac{m_0}{2}\left\{1+\dfrac{\sin(2n-4)\pi}{(2n-4)\pi}-\dfrac{1}{2}\dfrac{\sin(2n-6)\pi}{(2n-6)\pi}\right\}}\ . \tag{12}$$



(a) (b)

Fig. 4. Computed τ_n-values in the higher modes of vibration of ships

Fig. 4–(a) shows assumed distributions of m_e, m and y, and α determined for $m_0=4$. Fig. 4–(b) presents the results of calculations of τ_n. It will be interesting to note that, for given value of μ in the load distribution, it greatly influences to the τ_n as well as the number of nodes n, as seen in the expressions (10), (11), (12) and the figure. The illustration mentioned above will give us a clear understanding on the tendency of the relationship of the inertia coefficient and the number of nodes of vibration and the load distribution.

iii. Empirical formula

As was seen in the expression of (2) in the previous section, if the ship form is uniform, the existing Lewis's method of calculation and Todd's formula for inertia coefficient in the vertical hull vibration is available. However, in a general case, as the velocity amplitude of the vibration should be taken into account in a ship hull vibration, the empirical expression of the inertia coefficient of a cylinder considerably differs from that of a ship hull as will be seen in Fig. 5.

As a result, the formulae yield as follows,

$$\tau_2=c\,\frac{B}{d}\,, \tag{13}$$

where, $c=0.35$; for a cylinder, (14)

$$c=0.24; \quad \text{for a ship hull} \tag{15}$$

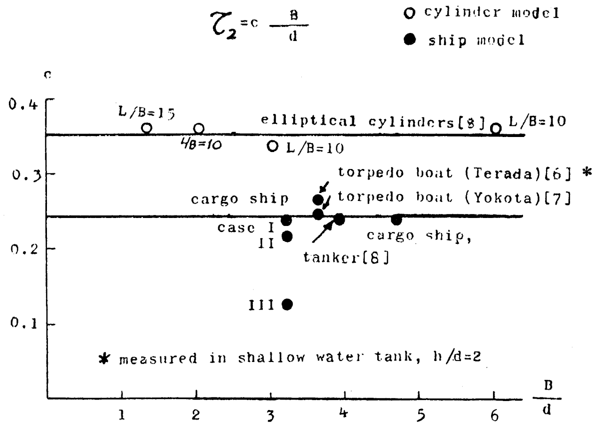


Fig. 5 c-values obtained by experiments on elliptical cylinders and ship-shaped models (see also [6], [7] and [8]).

Conclusions

In the present paper, the author emphasized that since the virtual inertia coefficient in a ship hull vibration is defined as the ratio of kinetic energy of the vibration of water entrained by ship hull to that of the ship itself, the velocity amplitude of the hull vibration should be taken into account in the calculation of the virtual inertia coefficient. As the results of the calculations of the coefficients on a model ship and the analytical evaluation of energies under appropriate assumptions, the inertia coefficient in a ship vibration is seen to be considerably smaller than that in the translational oscillation of a ship or the flexural vibration of a cylinder. In addition, it is to be noted that the load distribution as well as the number of nodes of the hull vibration considerably affects the inertia coefficient in the ship hull vibration. We may safely assume that one of the reasons of the discrepancy between the Lewis's τ-value and the experimental result on board ship, as suggested by McGoldrick and Russo [5], Dr. F.H. Todd [4] and P. Kaplan [9], is clearly explained by the present investigation. Further study into the three-dimensional correction in the above problem is required for determining the virtual inertia coefficient of ship hull vibration.

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