

WAVE HEIGHT AT THE SIDE OF TWO-DIMENSIONAL BODY OSCILLATING ON THE SURFACE OF A FLUID

TASAI, Fukuzo
Research Institute for Applied Mechanics, Kyushu University

<https://doi.org/10.5109/7164828>

出版情報 : Reports of Research Institute for Applied Mechanics. 9 (35), pp.121-142, 1961. 九州
大学応用力学研究所
バージョン :
権利関係 :



WAVE HEIGHT AT THE SIDE OF TWO-DIMENSIONAL BODY OSCILLATING ON THE SURFACE OF A FLUID

By Fukuzo TASAI

Summary

The wave height at the side of two-dimensional body with Lewis-form section oscillating on the surface of a fluid was calculated.

It is defined that ζ_h is the ratio of the wave amplitude at the side of a cylinder produced by heaving to the amplitude of heaving and ζ_s is the one in case of swaying.

ζ_R is the ratio of the wave amplitude to $\frac{B\theta_0}{2}$, where B is the breadth of a section and θ_0 is the amplitude of rolling about the origin in the free surface.

ζ_h , ζ_s and ζ_R are given in many figures.

The phase difference between the motion and the wave at the side of a cylinder, that is, ε_h , ε_s and ε_R were also obtained.

Introduction

When a ship sails over in a bad weather, owing to the violent motion and the shipping water, it happens that the ship weakens the stability and suffers the damage on the upper or bridge deck, so that the speed of the ship is reduced purposely. With regard to the phenomenon of shipping water several researches have been done, for example, J. L. Kent [1], R. N. Newton [2], and R. Tasaki [3] etc..

R. Tasaki [3] carried out experiments on the shipping water during experiments of the propulsive performance of tanker and liner models in regular head waves. He measured the amount of shipping water per one cycle. Then he gave a formula for estimating the amount of shipping water.

Through the experiments of forced pitching Tasaki also obtained an experimental formula to evaluate the height of the dynamical swell-up of water surface due to relative vertical velocity of the bow and waves.

In this paper, in order to analyse the dynamical swell-up of the water surface due to ship motion, calculations of the wave height at the side of two-dimensional body oscillating (heaving, swaying and rolling) on the surface of fluid were carried out for cylinders with Lewis-form section. In using the results mentioned above for the evaluation of the shipping water of a ship, we must take into consideration a considerable amount of the effect of the three-dimensional motion of fluid. These two-dimensional results, however, will be used as a means of the analysis as regards the shipping water.

I. Calculating formula of the wave amplitude

1.1. Heaving oscillation

When a cylinder with Lewis-form section, which was dealt in [4], performs heaving oscillation two kinds of waves are created, namely, the standing wave and progressive one.

Both of them contribute to the wave height at the side of a cylinder. The two-dimensional wave height at the side of a cylinder can be calculated using the velocity potential given in the author's paper [5].

Suppose now that an infinitely long cylinder oscillates vertically with a small amplitude $y_h = h \cos(\omega t + \varepsilon)$ (See. Fig. 1).

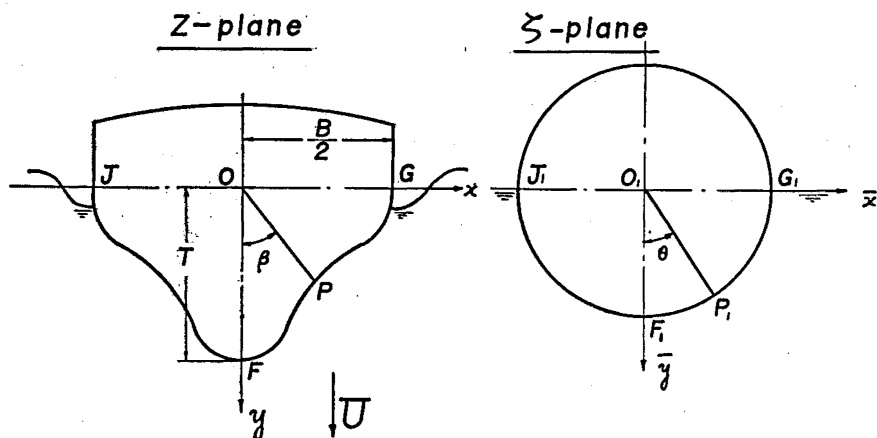


Fig. 1.

The fluid motion is two-dimensional. It is assumed that the depth of water is infinite, the fluid is inviscid, incompressible and the fluid motion irrotational.

In case of the heaving the velocity potential of the fluid is expressed as follows :

$$\begin{aligned}
 \phi = & \left(\frac{g\eta}{\pi\omega} \right) \left(\Phi_c(\xi_B, \alpha, \theta) \cos \omega t + \Phi_s(\xi_B, \alpha, \theta) \sin \omega t \right. \\
 & + \cos \omega t \sum_{m=1}^{\infty} p_{2m} \left[e^{-2m\alpha} \cos 2m\theta + \frac{\xi_B}{1+a_1+a_3} \left\{ \frac{e^{-(2m-1)\alpha}}{2m-1} \cdot \cos(2m-1)\theta \right. \right. \\
 & \quad \left. \left. + \frac{a_1}{2m+1} e^{-(2m+1)\alpha} \cos(2m+1)\theta - \frac{3a_3 e^{-(2m+3)\alpha}}{2m+3} \cdot \cos(2m+3)\theta \right\} \right] \\
 & \left. + \sin \omega t \sum_{m=1}^{\infty} q_{2m} \left[e^{-2m\alpha} \cos 2m\theta + \frac{\xi_B}{1+a_1+a_3} \left\{ \frac{e^{-(2m-1)\alpha}}{2m-1} \cdot \cos(2m-1)\theta \right. \right. \right. \\
 & \quad \left. \left. + \frac{a_1 e^{-(2m+1)\alpha}}{2m+1} \cdot \cos(2m+1)\theta - \frac{3a_3 e^{-(2m+3)\alpha}}{2m+3} \cdot \cos(2m+3)\theta \right\} \right] \Bigg) \quad (1)
 \end{aligned}$$

where η is the amplitude of progressive wave and $\xi_B = \frac{\omega^2}{g} \cdot \frac{B}{2}$.

Φ_c and Φ_s are the potentials by the two-dimensional source at origin 0.

Letting ζ denote the amplitude of the surface wave (taking positive downwards), we have the following equation in the linearised form;

$$\zeta = \frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right)_{y=0} \quad (2)$$

Then, putting $\alpha=0$ the wave amplitude at the side of a cylinder can be obtained.

Writing ζ_0 the amplitude of wave at the side of a cylinder we get

$$\begin{aligned} \zeta_0 = & \frac{\eta}{\pi} \left(-\Phi_{c0} \left(\xi_B, \frac{\pi}{2} \right) \sin \omega t + \Phi_{s0} \left(\xi_B, \frac{\pi}{2} \right) \cos \omega t \right. \\ & - \sin \omega t \sum_{m=1}^{\infty} p_{2m} \left[\cos 2m\theta + \frac{\xi_B}{1+a_1+a_3} \left\{ \frac{\cos(2m-1)\theta}{2m-1} \right. \right. \\ & \quad \left. \left. + \frac{a_1 \cos(2m+1)\theta}{2m+1} - \frac{3a_3 \cos(2m+3)\theta}{2m+3} \right\} \right]_{\theta=\pi/2} \\ & + \cos \omega t \sum_{m=1}^{\infty} q_{2m} \left[\cos 2m\theta + \frac{\xi_B}{1+a_1+a_3} \left\{ \frac{\cos(2m-1)\theta}{2m-1} \right. \right. \\ & \quad \left. \left. + \frac{a_1 \cos(2m+1)\theta}{2m+1} - \frac{3a_3 \cos(2m+3)\theta}{2m+3} \right\} \right]_{\theta=\pi/2} \Bigg). \end{aligned}$$

By virtue of

$$\cos 2m \left(\frac{\pi}{2} \right) = (-1)^m$$

$$\cos(2m-1) \frac{\pi}{2} = 0$$

we obtain

$$\zeta_0 = \frac{\eta}{\pi} \left[-\Phi_{c0} \left(\xi_B, \frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m p_{2m} \right] \sin \omega t + \left\{ \Phi_{s0} \left(\xi_B, \frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m q_{2m} \right\} \cos \omega t. \quad (3)$$

Write now

$$\zeta_0 = \eta \theta_0 \cos(\omega t + \epsilon'),$$

where θ_0 and the phase difference ϵ' become as follows:

$$\theta_0 = \frac{1}{\pi} \sqrt{\left\{ \Phi_{s0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m q_{2m} \right\}^2 + \left\{ \Phi_{c0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m p_{2m} \right\}^2} \quad (4)$$

$$\tan \epsilon' = \frac{\Phi_{c0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m p_{2m}}{\Phi_{s0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m q_{2m}}. \quad (5)$$

Then we put

$$\zeta_0 = \eta \theta_0 \cos(\omega t + \epsilon') = \eta \theta_0 \cos(\omega t + \epsilon + \epsilon_h), \quad (6)$$

where ϵ_h is the phase difference between the motion of the heaving and ζ_0 .

Finally we obtain

$$\theta_0 = \frac{1}{\pi} \sqrt{E_1^2 + E_2^2} \quad (7)$$

provided that

$$E_1 = \phi_{s_0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m \cdot q_{2m}, \quad E_2 = \phi_{c_0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m \cdot p_{2m}. \quad (8)$$

Putting $E_2/E_1 = k$ (9)

we get

$$\epsilon_h = \tan^{-1} \left(\frac{kB_0 - A_0}{kA_0 + B_0} \right) \quad (10)$$

where A_0 and B_0 are defined by the equation (31) in [5] and $A_0/B_0 = \tan \epsilon_h$. (11)

Accordingly, the ratio of the wave amplitude at the side of the cylinder to the amplitude of the heaving is given by the following equation;

$$\bar{\zeta}_h = \frac{|\zeta_0|}{h} = \frac{\eta \theta_0}{h} = \bar{A}_h \cdot \theta_0 \quad (12)$$

where $\bar{A}_h$ is the ratio of the progressive wave amplitude produced by the heaving oscillation to the heaving amplitude.

The $\bar{\zeta}_h$ is given as a function of $\epsilon_B = \frac{\omega^2}{g} \cdot \frac{B}{2}$ generally.

When ϵ_B tends to zero $\bar{\zeta}_h$ and ϵ_h tend to zero.

Therefore, in the case of $\epsilon_B \rightarrow 0$, when a cylinder plunges into its lowest position the water surface at the side of the cylinder comes down its lowest position, and when the cylinder heaves up to its highest position, the swell-up of the water surface will become maximum.

1.2. Swaying and rolling oscillation

For a cylinder with Lewis-form section swaying with a small displacement $x_s = S \cos(\omega t + r)$ (Fig. 2), its velocity potential of the fluid motion is given in

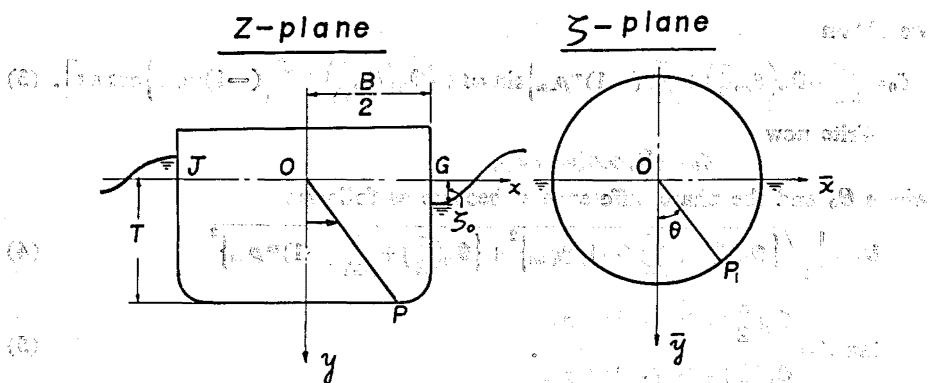


Fig. 2.

[6] or [7].

That is,

$$\phi = \frac{g\eta}{\pi\omega} \left\{ \bar{\phi}_c(\xi_B, \alpha, \theta) \cos \omega t + \bar{\phi}_s(\xi_B, \alpha, \theta) \sin \omega t \right\}$$

$$\begin{aligned}
 & + \cos \omega t \sum_{m=1}^{\infty} P_{2m}(\xi_B) \left[e^{-(2m+1)\alpha} \sin(2m+1)\theta + \frac{\xi_B}{1+a_1+a_3} \left\{ \frac{e^{-2m\alpha}}{2m} \sin 2m\theta \right. \right. \\
 & \quad \left. \left. + \frac{a_1 e^{-(2m+2)\alpha}}{2m+2} \sin(2m+2)\theta - \frac{3a_3 e^{-(2m+4)\alpha}}{2m+4} \cdot \sin(2m+4)\theta \right\} \right] \\
 & + \sin \omega t \sum_{m=1}^{\infty} Q_{2m}(\xi_B) \left[\begin{array}{cccc} " & " & " & " \end{array} \right] \quad (13)
 \end{aligned}$$

where $\bar{\theta}_s$ and $\bar{\theta}_s$ are the potentials by the two-dimensional horizontal doublet at the origin 0.

In Fig. 2, ζ_0 denotes the amplitude of the wave at right-hand side of the cylinder, that is, the one at G and is positive downwards. Since the wave at J , the left side of the cylinder, has the same magnitude and adverse phase to ζ_0 , it is sufficient to take into consideration ζ_0 only. Using the potential (13) and putting $\alpha=0$ in the equation (2) we can obtain the ζ_0 .

Writing $\zeta_0 = \eta \theta_0 \cos(\omega t + \tau + \epsilon_s)$ by the same reduction as done in case of the heaving, we get

$$\left. \begin{aligned} \theta_0 &= \frac{1}{\pi} \sqrt{E_1^2 + E_2^2} \\ \epsilon_s &= \tan^{-1} \left(\frac{k Q_0 - P_0}{Q_0 + k P_0} \right) \end{aligned} \right\}, \quad (14)$$

where P_0 and Q_0 are defined in [6] and $\frac{-P_0}{-Q_0} = \tan \tau$, and

$$\left. \begin{aligned} E_1 &= \bar{\theta}_{s_0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m Q_{2m} \\ E_2 &= \bar{\theta}_{c_0} \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m P_{2m} \\ k &= E_2 / E_1 \end{aligned} \right\}. \quad (15)$$

Then the amplitude ratio $\bar{\zeta}_s$ is given by the following equation

$$\bar{\zeta}_s = \frac{|\zeta_0|}{S} = \bar{A}_s \cdot \theta_0, \quad (16)$$

where $\bar{A}_s$ is the ratio of the progressive wave-amplitude produced by swaying oscillation to the swaying amplitude.

These $\bar{\zeta}_s$ and ϵ_s should be the function of $\xi_B = \frac{\omega^2}{g} \cdot \frac{B}{2}$ generally.

When ξ_B tends to zero, $\bar{\zeta}_s$ and ϵ_s tend to zero. That is to say, in the case of a slow swaying oscillation, when a cylinder sways to the extreme position in the right-hand direction the surface wave at G falls in and swells up at J , and next, in the extreme negative position of the cylinder the water surface is swelled up at G and falls in at J .

In the case of rolling oscillation having a small angular displacement $\theta = \theta_0 \cos(\omega t + \delta)$ about the origin 0 in the clockwise direction as is shown in Fig. 3, we can dealt with the problem according to the same manner with the swaying oscillation.

The fluid potential ϕ in the rolling motion of a two-dimensional body with

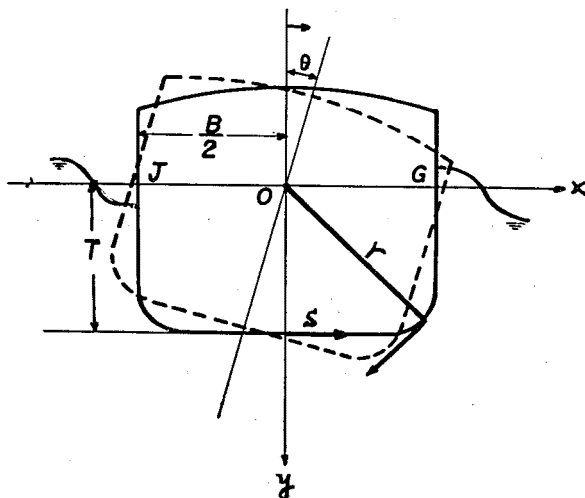


Fig. 3.

the Lewis-form section is given in [7].

Letting ζ_0 denote the amplitude of the wave at the right-hand side of a cylinder taking positive downward, $\bar{\zeta}_R$ the ratio of ζ_0 to $\frac{B\theta_0}{2}$, is given by the following equation

$$\bar{\zeta}_R = \frac{|\zeta_0|}{\frac{B\theta_0}{2}} = \bar{A}_R \cdot \theta_0, \quad (17)$$

where $\bar{A}_R$ is the ratio of the progressive wave-amplitude produced by the rolling oscillation to $\frac{B\theta_0}{2}$.

With regard to θ_0 the same equation with (14) and (15) hold.

That is,

$$\left. \begin{aligned} \theta_0 &= \frac{1}{\pi} \sqrt{E_1^2 + E_2^2} \\ E_1 &= \bar{\theta}_0 \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m \cdot Q_{2m} \\ E_2 &= \bar{\theta}_0 \left(\frac{\pi}{2} \right) + \sum_{m=1}^{\infty} (-1)^m \cdot P_{2m} \end{aligned} \right\}. \quad (18)$$

Using the calculated results given in [7] we can evaluate E_1 , E_2 and accordingly θ_0 for the cylinder with Lewis-form section.

Writing ε_R the phase difference between the rolling motion and ζ_0 , namely,

$$\zeta_0 = \eta \theta_0 \cos(\omega t + \delta + \varepsilon_R)$$

we have

$$\varepsilon_R = \tan^{-1} \left(\frac{kQ_0 - P_0}{Q_0 + kP_0} \right), \quad (19)$$

where P_0 and Q_0 are defined in [7] and $\tan \delta = \frac{P_0}{Q_0}$ and $k = E_2/E_1$.

Finally we can obtain $\bar{\zeta}_R$ and ε_R as a function of $\xi_B = \frac{\omega^2}{g} \cdot \frac{B}{2}$ or $\xi_d = \frac{\omega^2}{g} \cdot T$ for a cylinder with Lewis-form section.

when ξ_B tends to zero, $\bar{\zeta}_R$ tends to zero and ε_R to π . That is to say, with a small ξ_B when a cylinder rotates to the extreme position in the clockwise direction, the water surface at the right-hand side of the cylinder G swells up to the maximum, while in the adverse position of the cylinder the water surface at G falls in the lowest position.

The phase difference ε_R decreases with the increase of ξ_B .

II. Numerical calculation and the results

Using the results given in [5] and [7], the numerical calculations were carried out for several cylinders with Lewis-form section. Fig. 4 shows $\bar{\zeta}_h$ for the several elliptical sections as a function of $\xi_B = \frac{\omega^2}{g} \cdot \frac{B}{2}$. These $\bar{\zeta}_h$ curves have a similar tendency to the curve of $\bar{A}_h$.

Fig. 5 shows the ε_h as a function of ξ_B .

Suppose now that the ratio of the half breadth of a ship at the midship section to the draught T , that is, $H_0^* = \frac{B^*}{2T}$ equals 1.25 and $\xi_B^* = \frac{\omega^2}{g} \cdot \frac{B^*}{2}$ at the natural heaving or pitching period is equal to about 0.9. And the draught T is assumed to be constant throughout the length of a ship.

Then ξ_B of an arbitrary section of a ship is given by the following equation:

$$\xi_B = \left(\frac{H_0}{H_0^*} \right) \xi_B^*,$$

where H_0 is the ratio of the half breadth of an arbitrary section to the draught T . Then, ξ_B of the ship-sections at $H_0=0.2$ and $H_0=2/3$ becomes 0.144 and 0.48 respectively.

Reading the ε_h for the sections $H_0=0.2$, $2/3$ and 1.25 from Fig. 5, we have

H_0	0.2	2/3	1.25
ξ_B	0.144	0.48	0.9
ε_h	-96°	-93°	-92°

It will be seen that all ε_h are nearly the same.

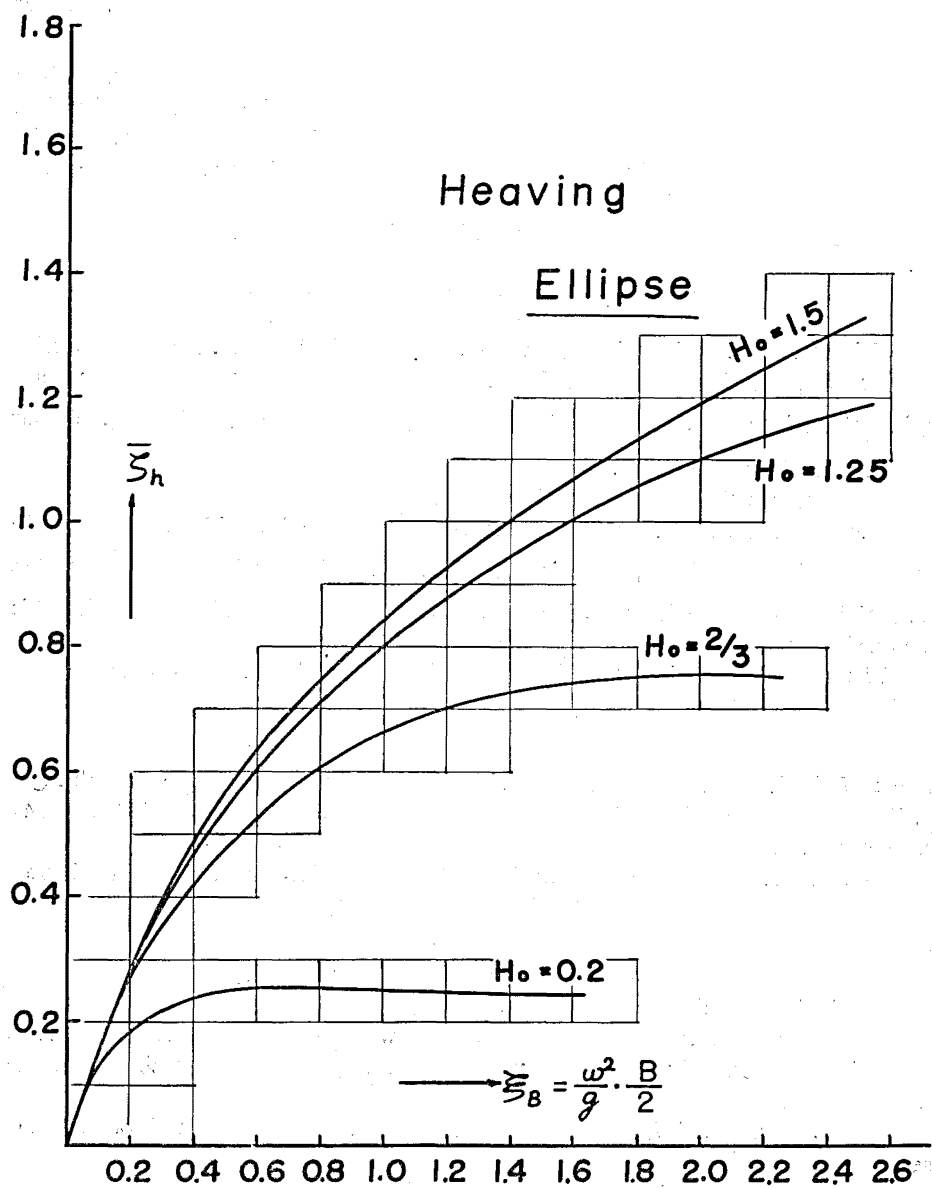
ε_h for the sections of fore part of a ship will suffer the effect of the three dimensional motion of the fluid, they will, therefore, differ considerably from the above values.

In the Figs. 6 and 7, the effect of the area coefficient σ are shown for the section with $H_0=2/3$.

From these figures it will be found that the fuller the section is the smaller the $\bar{\zeta}_h$ is, however, on the contrary, the ε_h becomes larger as the section is fuller.

In the next place, in Figs. 8 and 9, $\bar{\zeta}_s$ and ε_s by the swaying oscillation are given as a function of ξ_B for five elliptical sections.

In the Figs. 10 and 11, $\bar{\zeta}_s$ and ε_s are given as the function of $\xi_d = \frac{\omega^2}{g} T$.



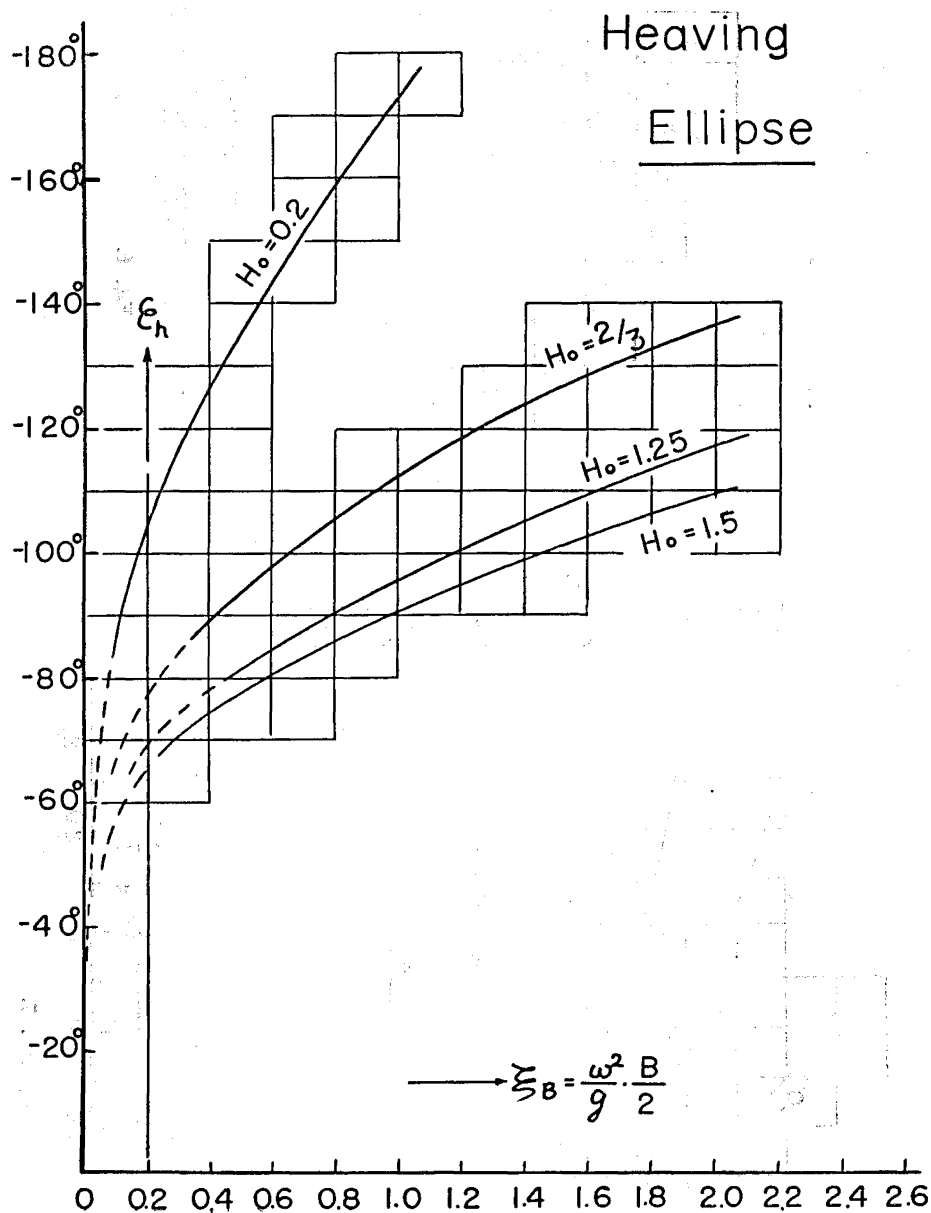


Fig. 5.

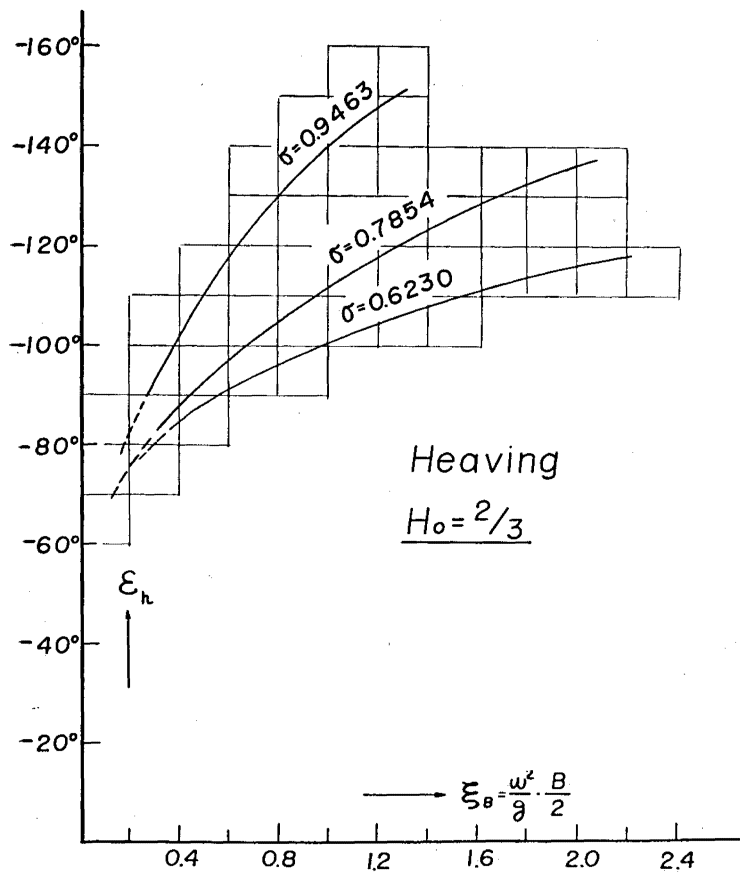


Fig. 7.

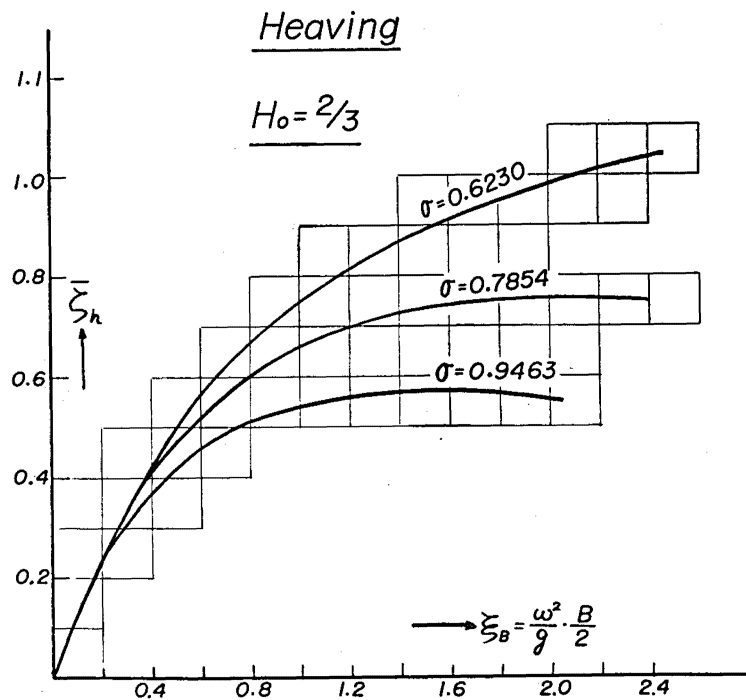


Fig. 6.

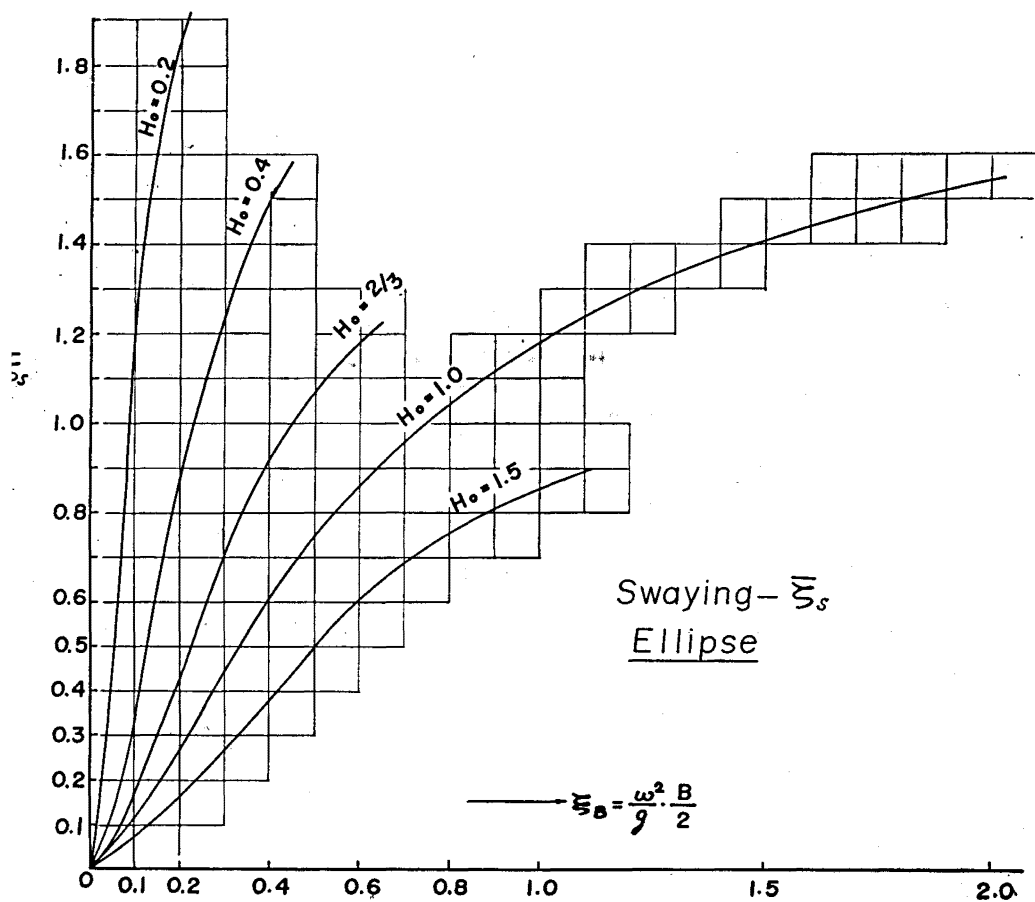


Fig. 8.

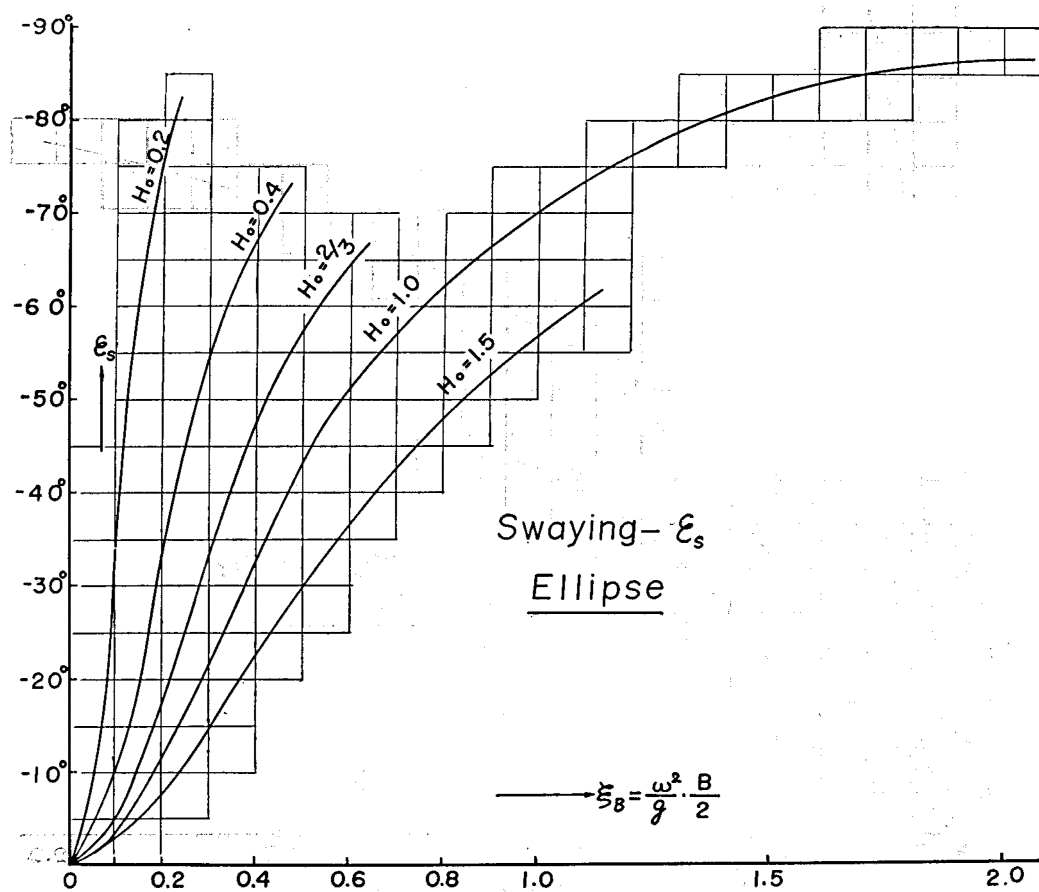


Fig. 9.

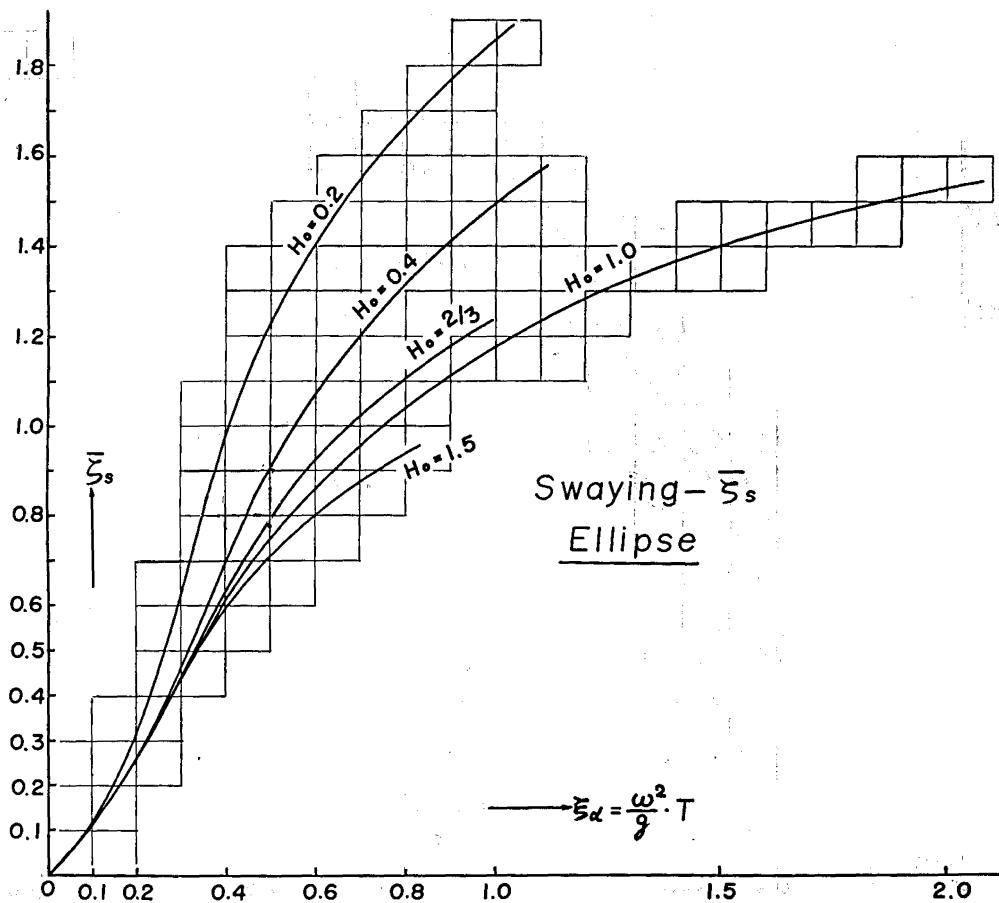


Fig. 10.

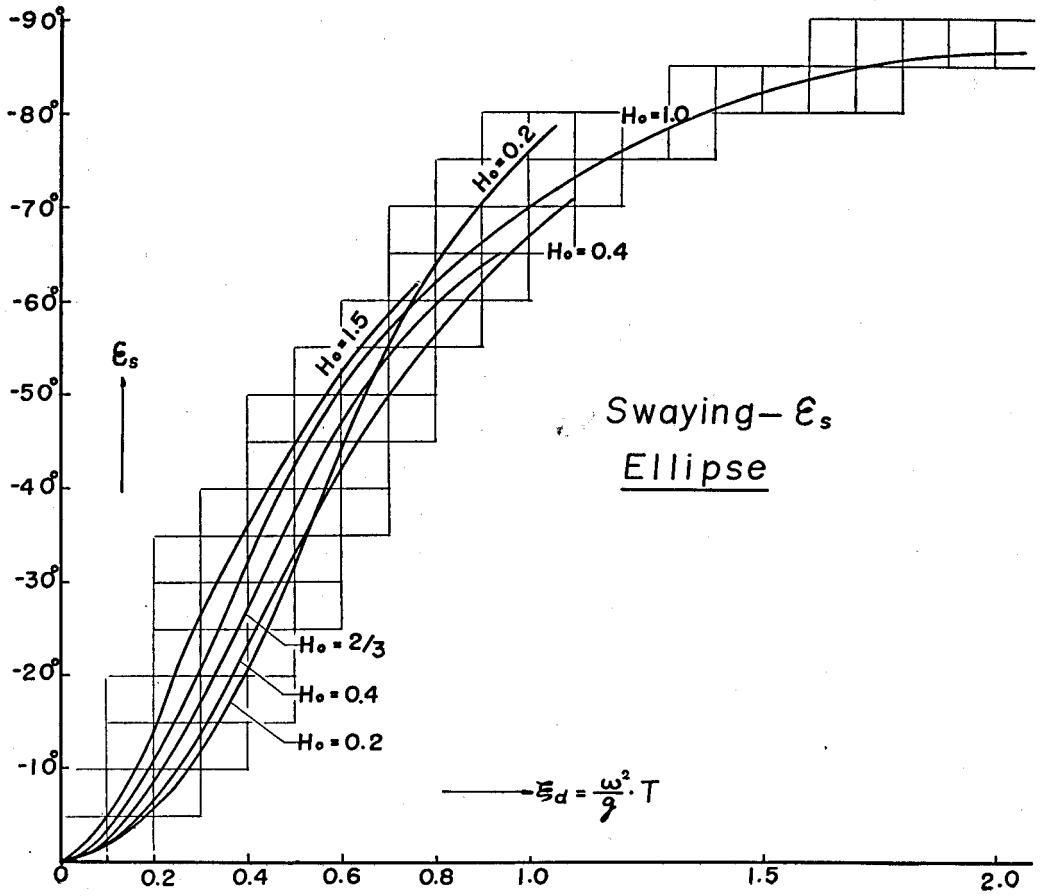


Fig. 11.

As seen in the Figs. 8 and 10, $\bar{\zeta}_s$ becomes larger as H_0 is smaller contrary to the heaving oscillation.

When we discuss the shipping water in bad weather, $\bar{\zeta}_s$ is so large on the section with small H_0 that we should take into consideration ζ_s by all means.

Comparing Fig. 4 with Fig. 8, it is found that $\bar{\zeta}_s$ is larger than $\bar{\zeta}_h$ generally, however, for the section with large H_0 , $\bar{\zeta}_h$ is larger than $\bar{\zeta}_s$ in the case of small ξ_B .

Figs. 12 and 13 show the effect of the area coefficient σ on the $\bar{\zeta}_s$ and ε_s . The fuller the section is the larger the $\bar{\zeta}_s$, ε_s are and the tendency of $\bar{\zeta}_s$ is opposite to the case of the heaving.

In the Figs. 14 to 16, $\bar{\zeta}_R$ are shown as a function of ξ_d .

In Fig. 14, $\bar{\zeta}_R$ becomes larger as the section is fuller.

$\bar{\zeta}$ has the same tendency in the case of $H_0=2/3$ and 1.0 also. However, at $H_0=1.5$, on the contrary, we have large $\bar{\zeta}_R$ with the finer section.

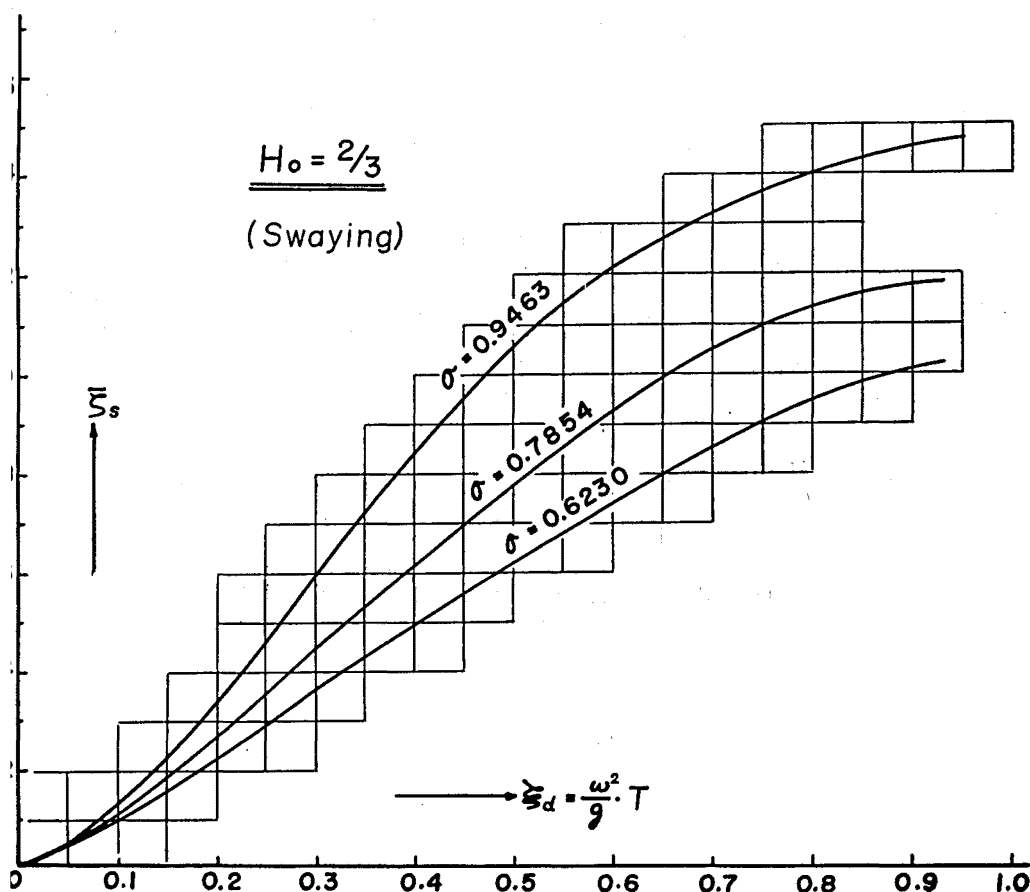


Fig. 12.

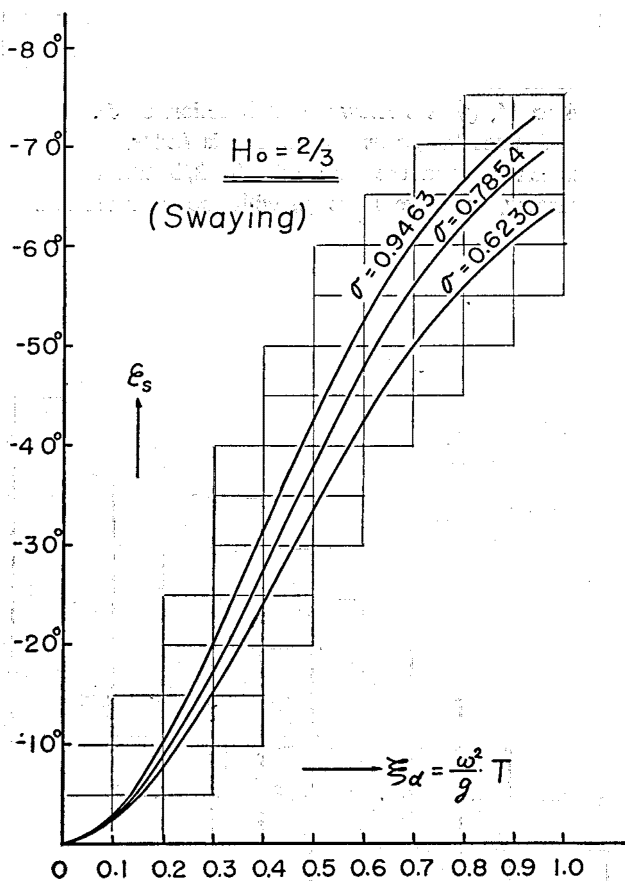


Fig. 13.

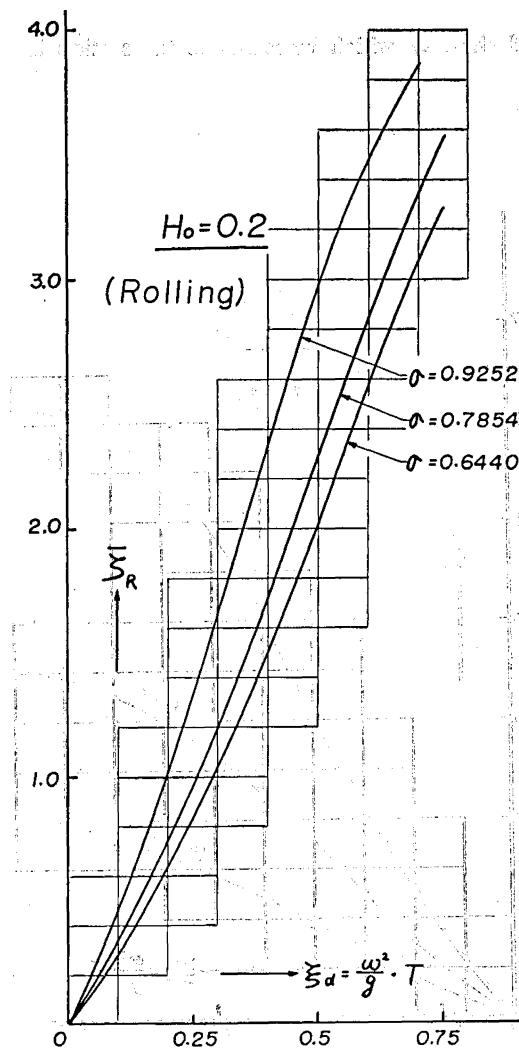


Fig. 14.

$\bar{\zeta}_R$ in the case of $H_0=0.2$ is so large and H_0 at the fore part of a ship is so small that $\bar{\zeta}_R$ in the neighbourhood of the stem will become large. But in the stem, as the $\frac{B\theta_0}{2}$ is generally small the effect of the yawing will be larger than the one of rolling with respect to the problem of shipping water at the fore part of a ship.

Figs. 17 and 18 show ϵ_R which increases as the section is finer.

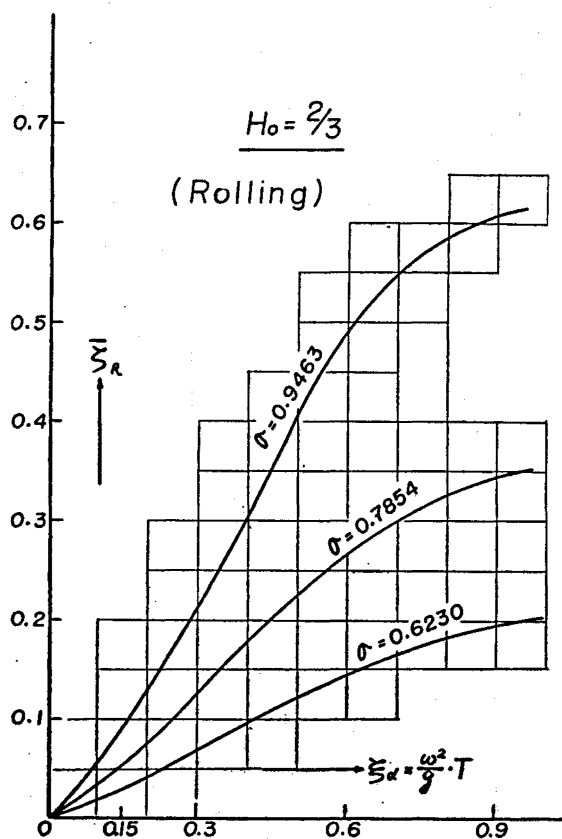


Fig. 15.

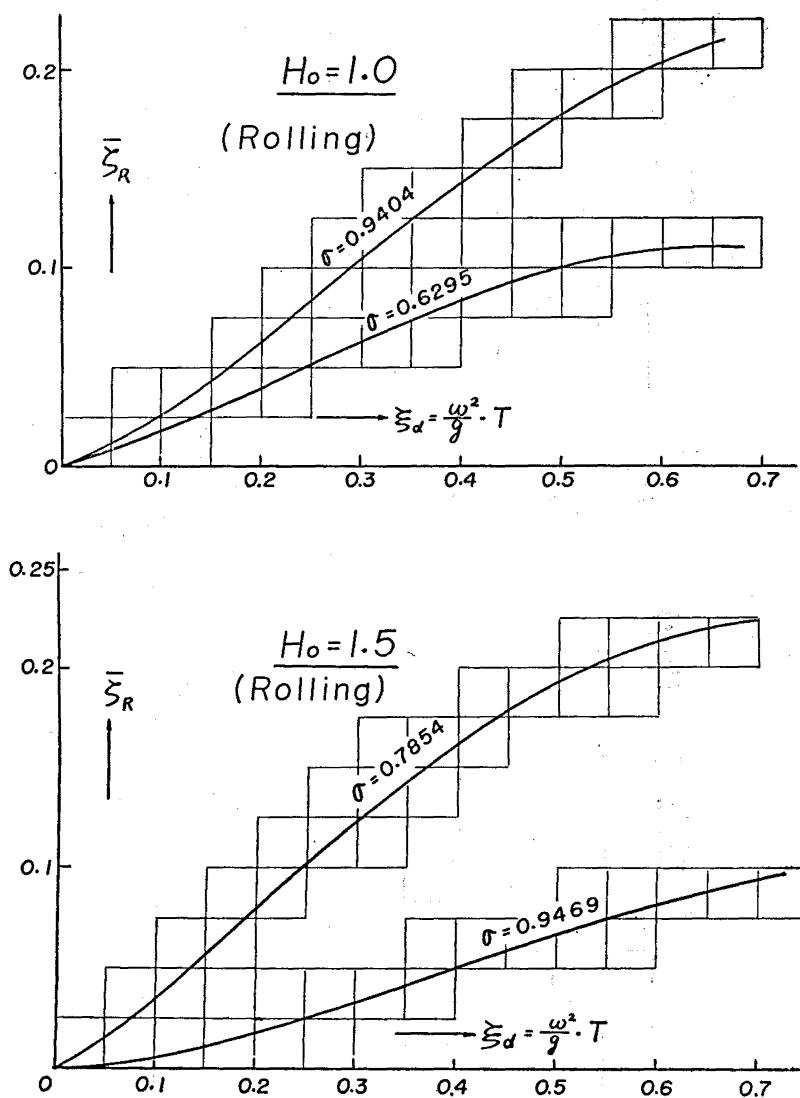


Fig. 16.

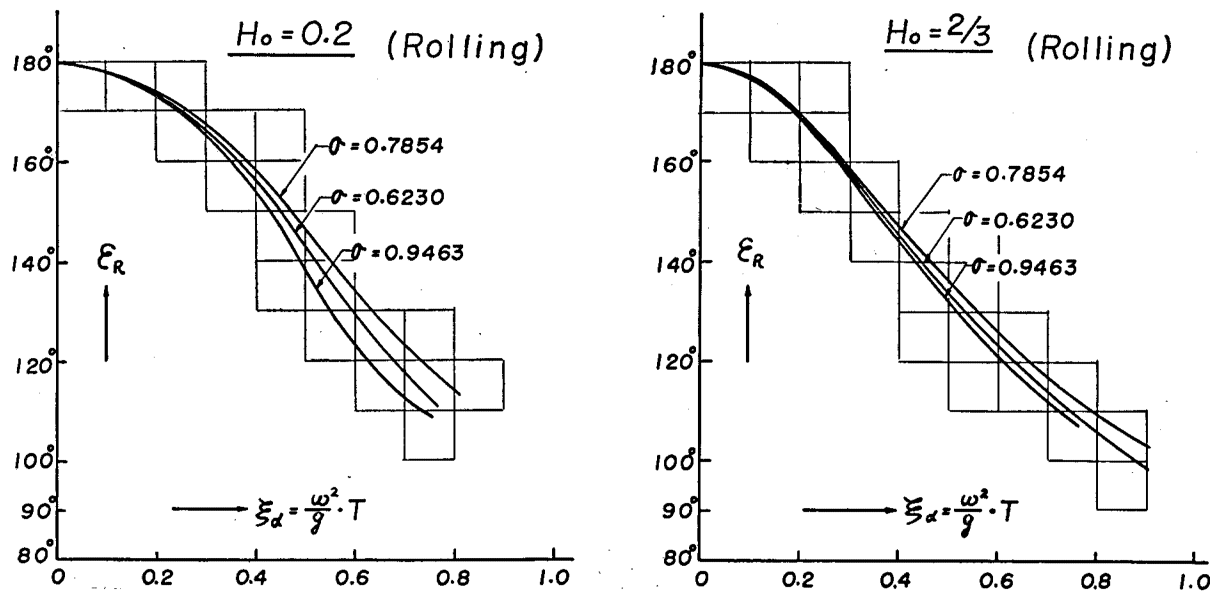


Fig. 17.

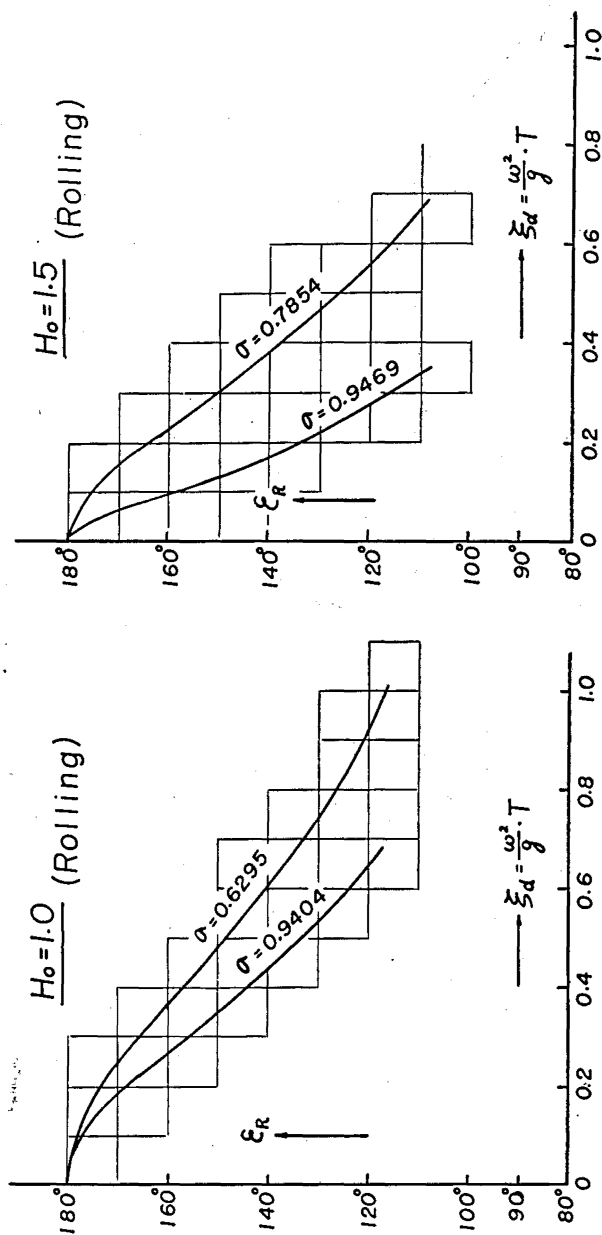


Fig. 19.

III. Conclusions

Calculations of the wave height at the side of two-dimensional body oscillating on the surface of water were carried out for the three cases of heaving, swaying and rolling on the Lewis-form cylinder. The summary is those that follows:

- (1) $\bar{\zeta}_s$ is generally larger than $\bar{\zeta}_h$ except for the case of large H_0 with small ξ_B .
- (2) Except for the $\bar{\zeta}_R$ of $H_0=1.5$, $\bar{\zeta}_s$ and $\bar{\zeta}_R$ become large as the section is fuller. While $\bar{\zeta}_h$ becomes small as the fuller the section is.

When we discuss the shipping water of a ship in bad weather we must take into consideration the swell-up of the water surface at the side of a ship due to the ship motion, namely, ζ_0 produced by the heaving, pitching, rolling, swaying and yawing. As known from the results calculated above the effect of $\bar{\zeta}_s$ is so large that we should not forget ζ_s due to the swaying and yawing motion of a ship.

Throughout these works, the writer is very grateful to Messers. Arakawa and Yamasaki for their help in numerical calculations as well as in presenting the manuscript.

References

- [1] J. L. Kent: "The Design of Seakindly Ships" North-East Coast Institute of Engineers and Shipbuilders in Scotland. 1938.
- [2] R. N. Newton: "Wetness related to Freeboard and Flare", T. I. N. A., Vol. 102, No. 1, 1960, pp. 49-81.
- [3] R. Tasaki: "On shipping water", Monthly Reports of Transportation Technical Research Institute Vol. 11, No. 8, Aug., 1961.
- [4] F. M. Lewis: "The inertia of the water surrounding a vibrating ship", S. N. A. M. E., 1929.
- [5] F. Tasai: "On the Damping Force and Added Mass of Ships Heaving and Pitching", Reports of Research Institute for Applied Mechanics, Kyushu University, in Japan, Vol. VII, No. 26, 1959.
- [6] F. Tasai: "Hydrodynamic Force and Moment Produced by Swaying Oscillation of Cylinders on the Surface of a Fluid", J. S. N. A. in Japan, Vol. 110.
- [7] F. Tasai: "Hydrodynamic Force and Moment Produced by Swaying and Rolling Oscillation of Cylinders on the free Surface", Reports of Research Institute for Applied Mechanics, Kyushu University in Japan, Vol. IX, No. 35, 1961.

(Received December 23, 1961)