HYDRODYNAMIC FORCE AND MOMENT PRODUCED BY SWAYING AND ROLLING OSCILLATION OF CYLINDERS ON THE FREE SURFACE

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HYDRODYNAMIC FORCE AND MOMENT PRODUCED BY SWAYING AND ROLLING OSCILLATION OF CYLINDERS ON THE FREE SURFACE

By Fukuzo TASAI

CONTENTS

Summary

Two-dimensional values of the hydrodynamic force and moment produced by swaying and rolling oscillation on the surface of a fluid were exactly calculated for cylinders with Lewis-form section.

Added mass coefficient K_x, progressive wave height ratio $\bar{A}$, the inertia coefficient of moment $K_{S\varphi}$ and the coefficient of the damping moment $\alpha_{S\varphi}$ of swaying oscillation, and also the coefficient of added moment of inertia $K_{\varphi T}$, wave height ratio $\bar{A}$, the coefficients of swaying force K_{RS}, α_{RS} of rolling oscillation are shown in several tables and figures.

Introduction

When a cylinder floating on the surface of a fluid oscillates horizontally (swaying oscillation) it suffers hydrodynamic force. This force is approximately resolved into two components: 1) the added inertia force in phase with the horizontal acceleration and 2) damping force in phase with the velocity. In the case of swaying oscillation of cylinders, since the motion of the fluid is not symmetrical the abovementioned force generally does not act upon the center of gravity of the cylinder, so that rolling moment will be produced. This moment becomes the coupled moment by swaying in the coupling motion between swaying and rolling.

These hydrodynamic force and moment are generally the function of frequency. O. Grim [1] calculated the force and moment two-dimensionally on the three elliptic cylinders. Then he showed graphically the added mass and progressive wave height ratio as the function of $\xi_B=\frac{\omega^2}{g}\frac{B}{2}$, where ω is the circular frequency of swaying oscillation and B is breadth of the cylinder at the free surface.

O. Grim [2] has dealt with the added mass, rolling moment and progressive wave height of cylinders with Lewis-form section in the neighbourhood of $\omega\to0$. On the other hand, making use of several model ships S. MOTORA [3] measured the added mass and added moment of inertia of swaying and yawing motion in case of $\omega\to\infty$. But the above-mentioned data will not be sufficient for calculating the added mass, added moment of inertia and damping force of swaying and yawing oscillation of a ship, by the strip method for example, as a function of frequency.

The author treated of the added mass and damping force of heaving oscillation of cylinders with Lewis-form section in [4].

In this paper, two-dimensional values of the added mass, progressive wave height ratio and rolling moment produced by swaying oscillation of cylinders with Lewis-form section were exactly calculated, applying above method.

The problem with respect to the hydrodynamic force and moment of a cylinder rolling about the origin 0 could be dealt with in the same manner as done for swaying, and then the two-dimensional values of added mass moment of inertia, wave-making damping and swaying force produced by rolling motion were also obtained.

Making use of the hydrodynamic force and moments created by swaying and rolling motion of the cylinder we are able to discuss the coupled oscillation between swaying and rolling motion of the cylinder.

I. Swaying oscillation

1.1. Progressive wave height

In Fig. 1 let us take Cartesian coordinates with its x-axis taken horizontally

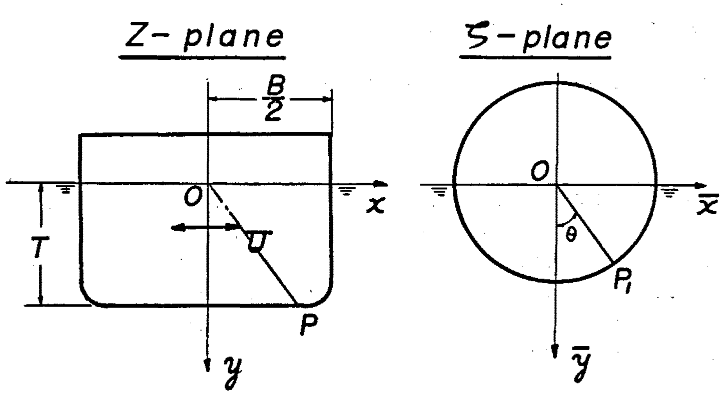


Fig. 1.

on a still water surface and its y-axis vertically downwards and the mean position of the axis of the cylinder at the origin. It is assumed that the depth of water is infinite, the fluid is invicid, incompressible and the fluid motion irrotational.

Suppose now that the cylinder oscillates horizontally with a small displacement $x_S = S\cos(\omega t+\varepsilon)$ and velocity $U = -S\omega\sin(\omega t+\varepsilon)$. As the cylinder is infinitely long the motion of the water will be two-dimensional. The motion of the fluid is not symmetrical about y-axis and then the velocity potential has the following relation;

$$\phi(x, y) = -\phi(-x, y). \tag{1}$$

Linearised free surface condition is expressed as follows:

$$\frac{\omega^2}{g}\phi+\frac{\partial\phi}{\partial y}=0, \text{ at } \left(y=0,\ |x|\geqslant\frac{B}{2}\right). \tag{2}$$

The boundary condition on the surface of the cylinder is given by the next equation

$$\left(\frac{\partial\phi}{\partial\nu}\right)=U\left(\frac{\partial x}{\partial\nu}\right) \tag{3}$$

where ν is the outward normal of the cylinder surface.

As the amplitude of swaying oscillation is infinitesimally small the equation (3) holds, to the first order, at the rest position.

In this paper calculation was performed on the Lewis-form section which can be derived from the unit circle in the ς-plane by the transformation

$$Z=M\left(\varsigma+\frac{a_1}{\zeta}+\frac{a_3}{\varsigma^3}\right) \tag{4}$$

where $z=x+iy$, $\varsigma=ie^{\alpha}e^{-i\theta}$, $M=\dfrac{B}{2(1+a_1+a_3)}$.

Putting $\alpha=0$ in the equation (4) the contour of the Lewis-form is expressed as follows:

$$\left.\begin{aligned} x_0&=M\{(1+a_1)\sin\theta-a_3\sin 3\theta\},\\ y_0&=M\{(1-a_1)\cos\theta+a_3\cos 3\theta\}.\end{aligned}\right\} \tag{5}$$

In consequence of the transformation (4), free surface condition reduces to

$$\xi_B\cdot\phi\left(\frac{e^{\alpha}-a_1e^{-\alpha}-3a_3e^{-3\alpha}}{1+a_1+a_3}\right)\mp\frac{\partial\phi}{\partial\theta}=0 \ \left(\theta=\pm\frac{\pi}{2}\right). \tag{6}$$

Using the stream function ψ the boundary condition on the surface of the cylinder reducces to

$$\left(\frac{-\partial\psi}{\partial\theta}\right)_{\alpha=0}=U\left(\frac{\partial x}{\partial\alpha}\right)_{\alpha=0}=UM\{(1-a_1)\sin\theta+3a_2\sin 3\theta\}. \tag{7}$$

Therefore we obtain

$$\psi(\theta)_{\alpha=0}=UM\{(1-a_1)\cos\theta+a_3\cos 3\theta\}+C(t), \tag{8}$$

where $C(t)$ is a function of the time only.

We take following sets of velocity potentials which satisfy $\dfrac{\partial^2\phi}{\partial x^2}+\dfrac{\partial^2\phi}{\partial y^2}=0$, $\phi(\alpha,\theta)$

$=-\phi(\alpha, -\theta)$ and the free surface condition (6).

$$\phi_{2m}=\Big[e^{-(2m+1)\alpha}\sin(2m+1)\theta+\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{e^{-2m\alpha}}{2m}\sin 2m\theta+\frac{a_1e^{-(2m+2)\alpha}}{2m+2}\sin(2m+2)\theta$$

$$-\frac{3a_3}{2m+4}e^{-(2m+4)\alpha}\sin(2m+4)\theta\Big\}\Big]\begin{matrix}\cos\omega t\\ \sin\omega t\end{matrix}\quad (m=1, 2, 3\cdots\cdots). \tag{9}$$

Then the sets of conjugate stream functions are expressed as follows:

$$\psi_{2m}=\Big[e^{-(2m+1)\alpha}\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{e^{-2m\alpha}}{2m}\cos 2m\theta+\frac{a_1e^{-(2m+2)\alpha}}{2m+2}\cos(2m+2)\theta$$

$$-\frac{3a_3}{2m+4}e^{-(2m+4)\alpha}\cos(2m+4)\theta\Big\}\Big]\begin{matrix}\cos\omega t\\ \sin\omega t\end{matrix}\quad (m=1, 2, 3\cdots\cdots). \tag{10}$$

These sets of functions tend to zero as α tends to infinity. As a stream function representing such a train of waves at infinity we adopt the function of a two-dimensional horizontal doublet at the origin. This stream function is given by the following equations:

$$\Psi=\frac{g\eta}{\pi}[\Psi_C(K, x, y)\cos\omega t+\Psi_S(K, x, y)\sin\omega t] \tag{11}$$

$$\left.\begin{aligned}&\Psi_C=\pi e^{-Ky}\cos Kx\\ &\Psi_S=\pm\pi e^{-Ky}\sin Kx-\int_0^\infty\frac{k\cos ky-K\sin ky}{K^2+k^2}e^{\mp kx}dk-\frac{y}{K(x^2+y^2)}\end{aligned}\right\} \tag{12}$$

where $K=\omega^2/g$ and η is wave amplitude at infinity, and in equ. (12) the upper sign is for $x>0$ and the lower one for $x<0$.

Then we consider ψ which satisfies the basic conditions in addition to the horizontal and vertical velocity being continuous at $x=0$ and $y\geqslant T$ and that it has a simple sine progressive wave at infinity.

Suppose the stream function expressed as follows:

$$\left(\frac{\omega\pi}{g\eta}\right)\psi=\Big[\Psi_C(\xi_B, \alpha, \theta)\cos\omega t+\Psi_S(\xi_B, \alpha, \theta)\sin\omega t\Big]$$

$$+\cos\omega t\sum_{m=1}^{\infty}P_{2m}(\xi_B)\Big[e^{-(2m+1)\alpha}\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{e^{-2m\alpha}}{2m}\cos 2m\theta$$

$$+\frac{a_1e^{-(2m+2)\alpha}}{2m+2}\cos(2m+2)\theta-\frac{3a_3e^{-(2m+4)\alpha}}{2m+4}\cos(2m+4)\theta\Big]$$

$$+\sin\omega t\sum_{m=1}^{\infty}Q_{2m}(\xi_B)\Big[\quad '' \qquad '' \qquad '' \qquad '' \quad\Big] \tag{13}$$

where Ψ_C, Ψ_S have the sign given in the equation (12) according to $x\gtrless 0$.

Putting $\alpha=0$ and using the condition in eq. (8) following equation is obtained.

$$\left(\frac{\omega\pi}{g\eta}\right)\psi(\theta)_{\alpha=0}=\Psi_{C0}(\xi_B, \theta)\cos\omega t+\Psi_{S0}(\xi_B, \theta)\sin\omega t$$

$$+\cos\omega t\sum_{m=1}^{\infty}P_{2m}(\xi_B)\Big[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{\cos 2m\theta}{2m}+\frac{a_1\cos(2m+2)\theta}{2m+2}-$$

$$-\frac{3a_3\cos(2m+4)\theta}{2m+4}\Big\}\Big]$$

$$+\sin\omega t\sum_{m=1}^{\infty}Q_{2m}(\xi_B)\Big[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{\cos 2m\theta}{2m}+\frac{a_1\cos(2m+2)\theta}{2m+2}$$

$$-\frac{3a_3\cos(2m+4)\theta}{2m+4}\Big\}\Big]$$

$$=\left(\frac{\omega\pi}{g\eta}\right)UM\{(1-a_1)\cos\theta+a_3\cos 3\theta\}+C(t), \qquad (14)$$

where Ψ_{C0} and Ψ_{S0} are the values at $\alpha=0$.

Putting $\theta=\frac{\pi}{2}$ we obtain the $C(t)$. Then substituting this equation of $C(t)$ in (14) we have

$$\left[\Psi_{C0}(\xi_B,\theta)-\Psi_{C0}\left(\xi_B,\frac{\pi}{2}\right)\right]\cos\omega t+\left[\Psi_{S0}(\xi_B,\theta)-\Psi_{S0}\left(\xi_B,\frac{\pi}{2}\right)\right]\sin\omega t$$

$$+\cos\omega t\sum_{m=1}^{\infty}P_{2m}(\xi_B)\Big[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{\cos 2m\theta}{2m}+\frac{a_1\cos(2m+2)\theta}{2m+2}$$

$$-\frac{3a_3\cos(m2+4)\theta}{2m+4}\Big\}-\frac{-\xi_B(-1)^{m+1}}{1+a_1+a_3}\left(\frac{1}{2m}-\frac{a_1}{2m+2}-\frac{3a_3}{2m+4}\right)\Big]$$

$$+\sin\omega t\sum_{m=1}^{\infty}Q_{2m}(\xi_B)\Big[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{\cos 2m\theta}{2m}+\frac{a_1\cos(2m+2)\theta}{2m+2}$$

$$-\frac{3a_3\cos(2m+4)\theta}{2m+4}\Big\}-\frac{-\xi_B(-1)^{m+1}}{1+a_1+a_3}\left(\frac{1}{2m}-\frac{a_1}{2m+2}-\frac{3a_3}{2m+4}\right)\Big]$$

$$=\left(\frac{\omega\pi}{g\eta}\right)UM\{(1-a_1)\cos\theta+a_3\cos 3\theta\}. \qquad (15)$$

The right side of the equation (15) can be expressed as follows:

$$\left(\frac{\omega\pi}{g\eta}\right)UM\{(1-a_1)\cos\theta+a_3\cos 3\theta\}=h(\theta)\{P_0\cos\omega t+Q_0\sin\omega t\} \qquad (16)$$

where

$$\left.\begin{aligned}h(\theta)&=\frac{\{(1-a_1)\cos\theta+a_3\cos 3\theta\}}{1+a_1+a_3},\\ P_0&=\frac{-\pi S}{\eta}\xi_B\sin\varepsilon,\ Q_0=-\frac{\pi S}{\eta}\xi_B\cos\varepsilon.\end{aligned}\right\} \qquad (17)$$

Substituting (16) into (15) we get the following equation

$$\left.\begin{aligned}\Psi_{C0}(\xi_B,\theta)-\Psi_{C0}\left(\xi_B,\frac{\pi}{2}\right)&=\sum_{m=0}^{\infty}f_{2m}(\theta)\cdot P_{2m}\\ \Psi_{S0}(\xi_B,\theta)-\Psi_{S0}\left(\xi_B,\frac{\pi}{2}\right)&=\sum_{m=0}^{\infty}f_{2m}(\theta)\cdot Q_{2m}\end{aligned}\right\} \qquad (18)$$

where $f_0(\theta)=h(\theta)$

and $$f_{2m}(\theta)_{m\neq 0}=\cos(2m+1)\theta+\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{\cos 2m\theta}{2m}+\frac{a_1\cos(2m+2)\theta}{2m+2}-\frac{3a_3\cos(2m+4)\theta}{2m+4}\Big\}$$

$$+\frac{\xi_B(-1)^{m+1}}{1+a_1+a_3}\left\{\frac{1}{2m}-\frac{a_1}{2m+2}-\frac{3a_3}{2m+4}\right\}. \tag{19}$$

Assuming that the series of (18) converges uniformly in the range of $\alpha \geqslant 0$, $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$ and performing the numerical calculation by the same method as done for heaving [4] we can determine the P_{2m} and Q_{2m}.

Using P_0 and Q_0, the amplitude ratio $\bar{A}$ is given by the following equation:

$$\bar{A}=\frac{\eta}{S}=\frac{\pi\xi_B}{\sqrt{P_0^2+Q_0^2}}. \tag{20}$$

In the case of the circular cylinder, when ξ_B is very small P_0 tends to zero and Q_0 to $\frac{1}{\xi_B}$ respectively. Therefore it is resulted into $\bar{A}\rightarrow\pi\xi_B^2$. (21)

1.2. Added mass

In the next place, the velocity potential corresponding to the equation (13) is given as follows:

$$\left(\frac{\omega\pi}{g\eta}\right)\phi=\Phi_C(\xi_B,\alpha,\theta)\cos\omega t+\Phi_S(\xi_B,\alpha,\theta)\sin\omega t$$

$$+\cos\omega t\sum_{m=1}^{\infty}P_{2m}(\xi_B)\Big[e^{-(2m+1)\alpha}\sin(2m+1)\theta+\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{e^{-2m\alpha}}{2m}\sin 2m\theta$$

$$+\frac{a_1e^{-(2m+2)\alpha}}{2m+2}\sin(2m+2)\theta-\frac{3a_3e^{-(2m+4)\alpha}}{2m+4}\sin(2m+4)\theta\Big\}\Big]$$

$$+\sin\omega t\sum_{m=1}^{\infty}Q_{2m}(\xi_B)\Big[e^{-(2m+1)\alpha}\sin(2m+1)\theta+\frac{\xi_B}{1+a_1+a_3}\Big\{\frac{e^{-2m\alpha}}{2m}\sin 2m\theta$$

$$+\frac{a_1e^{-(2m+2)\alpha}}{2m+2}\sin(2m+2)\theta-\frac{3a_3e^{-(2m+4)\alpha}}{2m+4}\cdot\sin(2m+4)\theta\Big)\Big] \tag{22}$$

where

$$\left.\begin{aligned}\Phi_C&=-\pi e^{-Ky}\sin Kx\\ \Phi_S&=\pm\pi e^{-Ky}\cos Kx\mp\int_0^{\infty}\frac{K\cos ky+k\sin ky}{k^2+K^2}e^{\mp kx}dk+\frac{x}{K(x^2+y^2)}\end{aligned}\right\} \tag{23}$$

in which the upper sign is for $x>0$ and the lower one for $x<0$. Hydrodynamic pressure on the surface of the cylinder can be calculated by the linearised equation

$$p=-\rho\left(\frac{\partial\phi}{\partial t}\right)_{\alpha=0}$$

Then the hydrodynamic force per unit length acting on the cylinder in the direction of x-axis is found by integrating the horizontal component of the hydrodynamic pressure on the surface of the cylinder, and the force becomes

$$F_x=\rho B\left(\frac{g\eta}{\pi}\right)[N_0\sin\omega t-M_0\cos\omega t] \tag{24}$$

provided that

$$N_0=-\int_0^{\pi/2}\Phi_{C0}(\xi_B, a_1, a_3, \theta)\cdot\frac{(1-a_1)\sin\theta+3a_3\sin 3\theta}{(1+a_1+a_3)}d\theta-\frac{3a_3}{1+a_1+a_3}\cdot\frac{\pi}{4}P_2$$

$$-\sum_{m=1}^{\infty}P_{2m}(\xi_B)\frac{\xi_B(-1)^{m-1}}{(1+a_1+a_3)^2}\Big[\Big\{\frac{1}{4m^2-1}-\frac{a_1}{(2m+2)^2-1}-\frac{3a_3}{(2m+4)^2-1}\Big\}(1-a_1)$$

$$+3a_3\Big\{\frac{-1}{4m^2-9}+\frac{a_1}{(2m+2)^2-9}+\frac{3a_3}{(2m+4)^2-9}\Big\}\Big]. \qquad (25)$$

M_0 is obtained by converting Φ_{C0} into Φ_{S0} and P_{2m} into Q_{2m} in eq. (25).

Φ_{C0}, Φ_{S0} are the values of Φ_C, Φ_S at $\alpha=0$, and ρ is the density of water.

The hydrodynamic force F_x is resolved into components in phase with the acceleration and with the horizontal velocity.

The added mass M_S is the ratio of the hydrodynamic force in phase with the acceleration to the acceleration,

$$M_S=2\rho T^2 H_0^2\frac{N_0P_0+M_0Q_0}{P_0^2+Q_0^2} \qquad (26)$$

where T is the draught and $H_0=\frac{B}{2T}$.

The added mass in case of $\omega\to\infty$ is given by Landweber [5]

$$M_S(\omega\to\infty)=\frac{2\rho}{\pi}C_2\cdot T^2,\ C_2=\Big[1+\frac{16}{3}\Big(\frac{a_3}{1-a_1+a_3}\Big)^2\Big]. \qquad (27)$$

Therefore the coefficient of added mass $K_x(\omega\to\infty)$ becomes

$$K_x(\omega\to\infty)=\frac{M_S(\omega\to\infty)}{\frac{1}{2}\rho\pi T^2}=\frac{4}{\pi^2}C_2. \qquad (28)$$

Expressing the coefficient in the ratio of added mass to the sectional area S_0, we obtain

$$K_S(\omega\to\infty)=\frac{M_S(\omega\to\infty)}{\rho S_0}=\frac{C_2}{\pi H_0\sigma}. \qquad (29)$$

According to M_S for a certain ω in eq. (26), we have K_x and K_S:

$$\left.\begin{aligned}K_x&=\frac{4}{\pi}H_0^2\cdot\frac{N_0P_0+M_0Q_0}{P_0^2+Q_0^2},\\ K_S&=\frac{H_0}{\sigma}\cdot\frac{N_0P_0+M_0Q_0}{P_0^2+Q_0^2}.\end{aligned}\right\} \qquad (30)$$

σ is the area coefficient, namely, $\sigma=\frac{S_0}{BT}$.

In the case of $\omega\to 0$, that is, when the period of swaying oscillation is extremely long the free surface reduces to the similar boundary as that of the fixed wall.

Then the added mass is, as is well known, given by the following formula:

$$M_S(\omega\to 0)=\frac{1}{2}\rho\pi T^2 C_3,\ C_3=\frac{(1-a_1)^2+3a_3^2}{(1-a_1+a_3)^2}. \qquad (31)$$

1.3. Rolling moment

In the case of swaying oscillation, the symbols of the hydrodynamic pressure which act upon the right and left sides of the cylinder differ from each other, so that the rolling moment is generally produced outside of F_x.

Letting M_R denote the rolling moment about origin 0, we have

$$M_R = \frac{\rho B^2 g \eta}{\pi}\left[X_R \sin \omega t - Y_R \cos \omega t\right] \tag{32}$$

where

$$X_R = \int_0^{\pi/2} \frac{\Phi_{C0}(\xi_B, a_1, a_3, \theta)}{(1+a_1+a_3)^2}\{a_1(1+a_3)\sin 2\theta - 2a_3 \sin 4\theta\}\,d\theta$$

$$+\frac{\pi\xi_B(a_1P_2 - a_3P_4)}{8(1+a_1+a_3)^2} + \sum_{m=1}^{\infty}\frac{P_{2m}(\xi_B)(-1)^{m+1}}{(1+a_1+a_3)^2}\left\{\frac{2a_1(1+a_3)}{(2m+1)^2-4} + \frac{8a_3}{(2m+1)^2-16}\right\}. \tag{33}$$

Y_R can be obtained by converting Φ_{C0} into Φ_{S0} and P_{2m} into Q_{2m}.

M_R is resolved into two components in phase with the acceleration and velocity of swaying.

That is,

$$M_R = M_{S\varphi}\left(-\frac{d^2x_S}{dt^2}\right) + N_{S\varphi}\left(-\frac{dx_S}{dt}\right).$$

In the above equation, the first term is the inertia moment and the second is the damping moment produced by swaying oscillation.

Both are the coupled terms in the coupled oscillation between swaying and rolling oscillation of a cylinder.

Put

$$M_{S\varphi} = M_S l_{S\varphi} = M_S K_{S\varphi} T. \tag{34}$$

Then the ratio of the lever $l_{S\varphi}$ to the draught leads to

$$K_{S\varphi} = \frac{l_{S\varphi}}{T} = 2H_0\left(\frac{P_0X_R + Q_0Y_R}{N_0P_0 + M_0Q_0}\right). \tag{35}$$

In the next place we have

$$N_{S\varphi} = \frac{\rho B^3\omega}{2}\left(\frac{P_0Y_R - Q_0X_R}{P_0^2 + Q_0^2}\right).$$

On the other hand, the two-dimensional wave-making damping force per unit length of a cylinder and unit swaying velocity, that is, the coefficient of linear wave-making damping of swaying N_S becomes as follows:

$$N_S = \frac{\rho g^2}{\omega^3}\bar{A}^2.$$

Putting now

$$N_{S\varphi} = N_S T \alpha_{S\varphi} \tag{36}$$

we have

$$\alpha_{S\varphi} = \frac{4H_0}{\pi^2}(P_0Y_R - Q_0X_R). \tag{37}$$

As mentioned above, the calculating method for Lewis-form section was given, and it may also be extended for the section of n-parameter family as given in case of heaving [6].

1. 4. Results of calculation

In the equation (24), the component in phase with $\dot{x}_S$ is

$$F_D=\rho B\left(\frac{g\eta}{\pi}\right)\frac{N_0Q_0-M_0P_0}{P_0^2+Q_0^2}(P_0\cos\omega t+Q_0\sin\omega t). \tag{38}$$

The rate of work done by the above force per unit time leads to

$$\frac{\rho g^2\eta^2}{\pi^2\omega}(N_0Q_0-M_0P_0)$$

which equals the energy transmitted from both sides of the cylinder per unit time, that is, $\frac{1}{2}\frac{\rho g^2\eta^2}{\omega}$. Accordingly the following relation is given,

$$N_0Q_0-M_0P_0=\frac{1}{2}\pi^2. \tag{39}$$

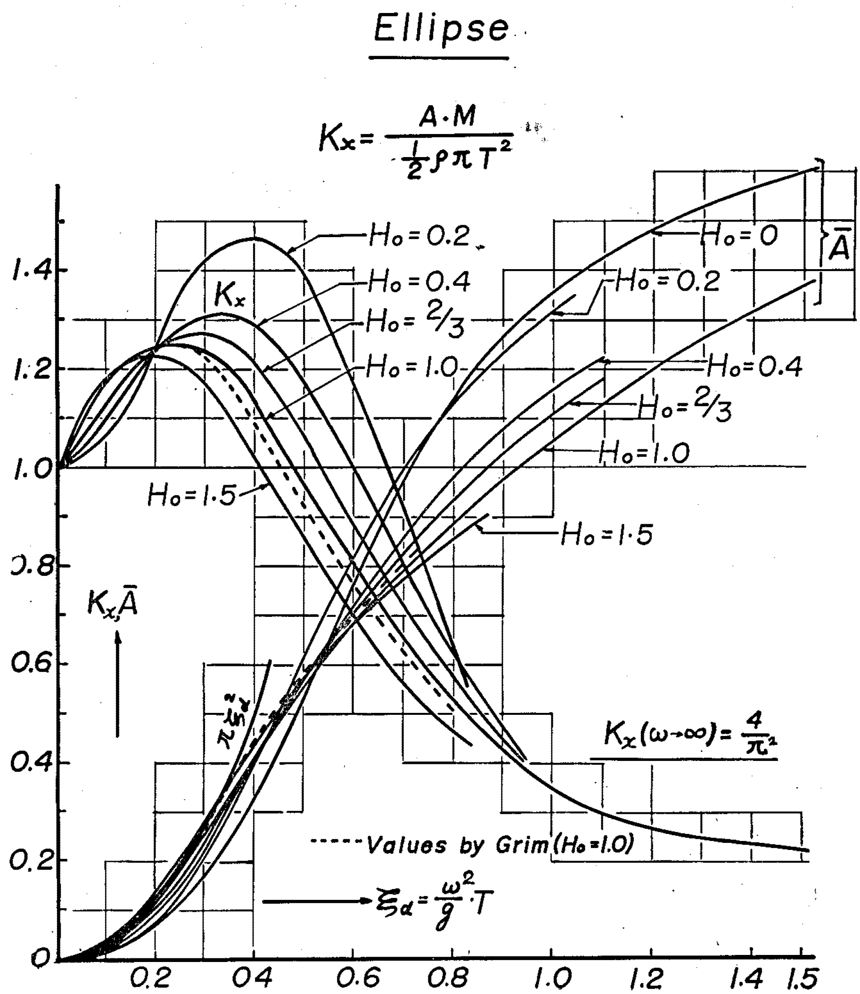


Fig. 2.

The relation was used to check the computation. Making use of the same method as done in the heaving oscillation by F. Ursell [7] we can prove the uniform convergence of the infinite series on the right-hand side of equation (18).

To obtain the coefficients P_{2m}, Q_{2m}, it should be solved using an infinite number of equations in an infinite number of unknowns. As a practical problem, the system of equations was replaced by a system involving only six polynomials $f_{2m}(\theta)$ as done in the case of the heaving [4].

In this paper, calculations were carried out for fourteen sections of Lewis-form. The error was less than 2% in the case of $H_0=0.2$ and 1% in the other sections.

In Fig. 2, K_x and $\bar{A}$ for five elliptic sections, H_0 of which respectively equals 0.2, 0.4, 2/3, 1.0 and 1.5 are given as a function of $\xi_d=\frac{\omega^2}{g}T$, and the dotted line is the result by O. Grim [1]. Though he also calculated for other two elliptic sections, $H_0=2/3$ and $H_0=1.5$, we find a little error in the neighbourhood of the maximum value.

Recently O. Grim and K. Tamura calculated again and obtained these values for Lewis-form sections (unpublished).

As seen in Fig. 2, the maximum of K_x and the ξ_d corresponding to the maximum increase with the decrease of H_0.

In a small ξ_d, $\bar{A}$ is larger in a section with larger H_0, but this tendency reverses with the increase of ξ_d.

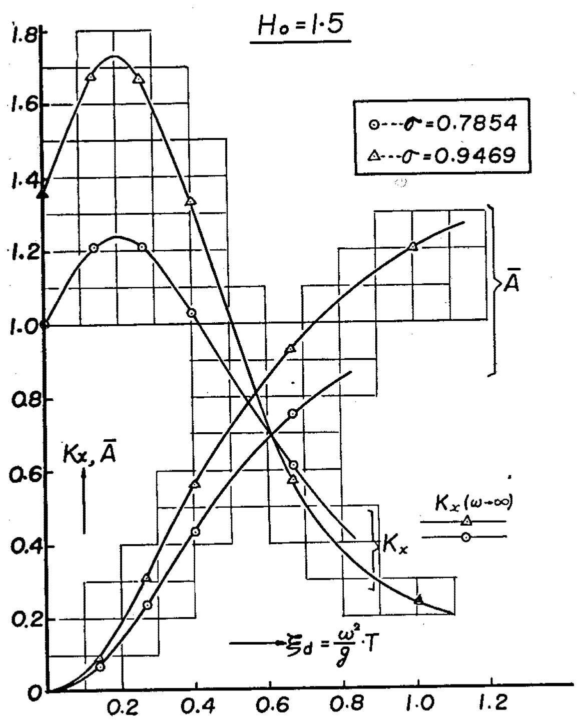
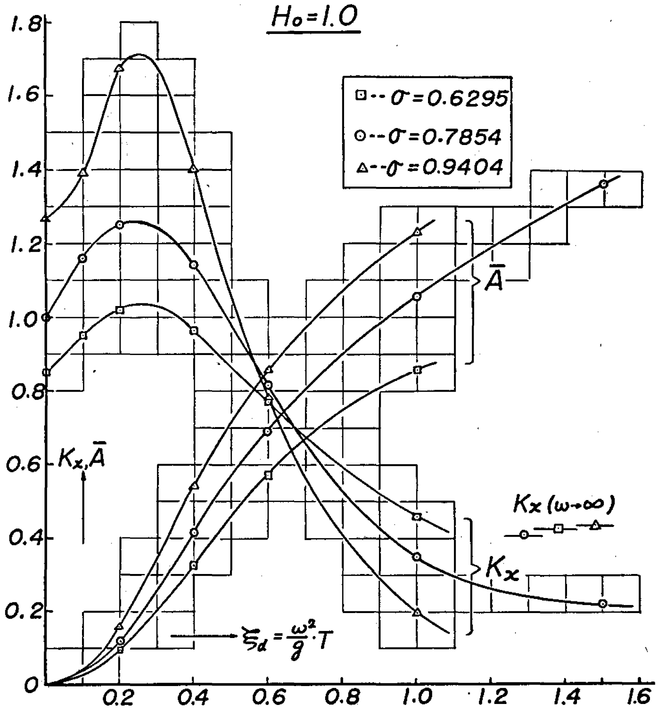
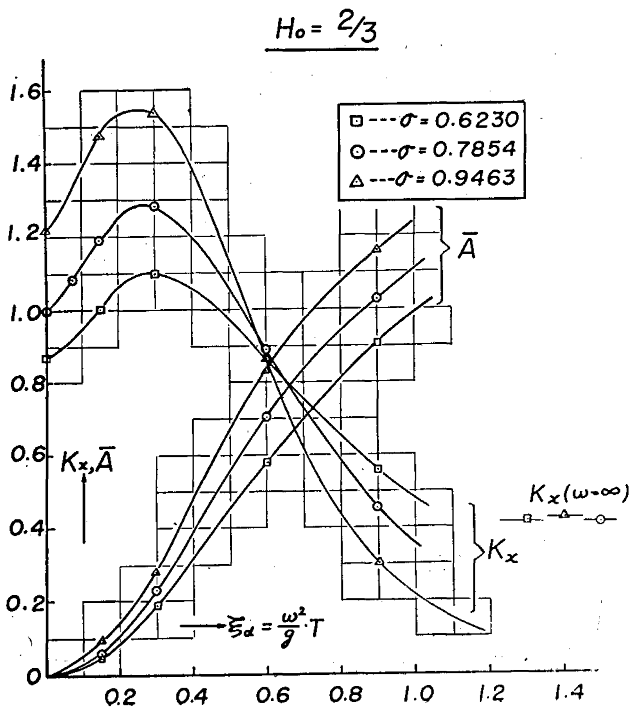


Fig. 3.



H_o=1.0
□-- σ=0.6295
○-- σ=0.7854
△-- σ=0.9404
$\bar{A}$
$K_x, \bar{A}$
$K_x(\omega\to\infty)$
K_x
$\xi_d = \frac{\omega^2}{g} \cdot T$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

Fig. 4.



H_o= 2/3
□--- σ= 0.6230
○--- σ= 0.7854
△--- σ= 0.9463
$\bar{A}$
$K_x, \bar{A}$
$K_x(\omega\to\infty)$
K_x
$\xi_d = \frac{\omega^2}{g} \cdot T$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Fig. 5.

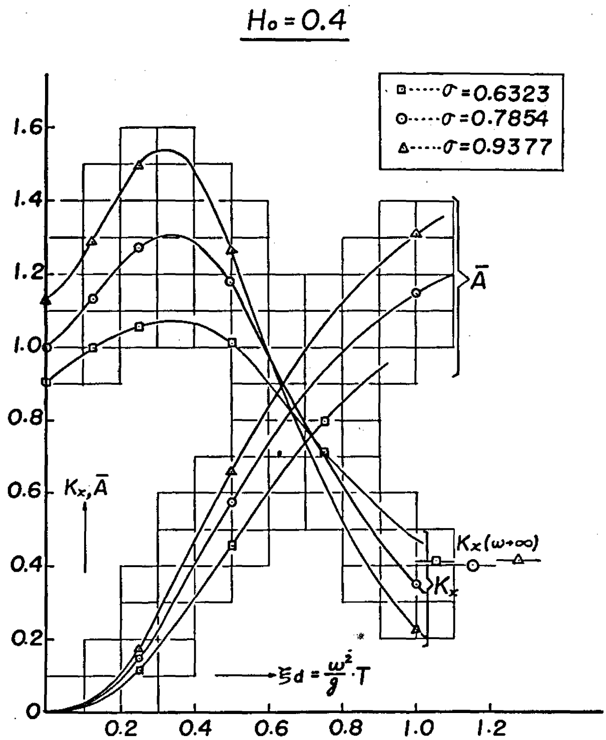

$H_o = 0.4$
□-----σ=0.6323
○-----σ=0.7854
△-----σ=0.9377
$K_x, \bar{A}$
$\bar{A}$
$K_x(\omega\to\infty)$
K_x
$\xi_d = \frac{\omega^2}{g} \cdot T$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2

Fig. 6.

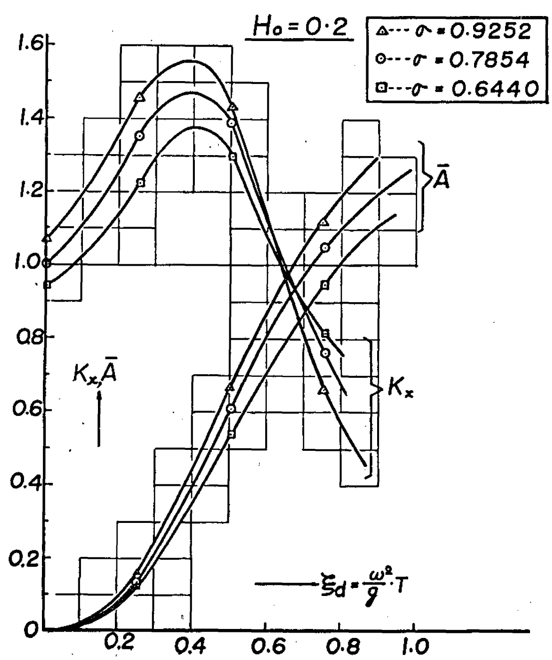

$H_o = 0 \cdot 2$
△--- σ = 0.9252
○--- σ = 0.7854
□--- σ = 0.6440
$K_x, \bar{A}$
$\bar{A}$
K_x
$\xi_d = \frac{\omega^2}{g} \cdot T$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0

Fig. 7.

Generally speaking, with respect to the K_x and $\bar{A}$ of swaying oscillation the variation with H_0 is smaller than that of the heaving for the same magnitude of σ.

In Fig. 2, $\bar{A}$ of $H_0=0$ is the one for the flat plate (See appendix). The effect of the area coefficient σ in each H_0 is shown in the Figures 3 to 7.

As clearly seen in these figures, K_x for fuller sections are larger than the ones for fine sections till $\xi_d \doteqdot 0.6$ and then the matters are adverse with increasing of ξ_d. And also the fuller the section is, the larger $\bar{A}$ becomes.

Results of the calculation are also given in Table 1–1 to 1–5. O. Grim [2] has given the $\bar{A}$ in the case of $\xi_B=\frac{\omega^2}{g}\frac{B}{2}$ tending to 0 by the following equation

$$\bar{A}=\frac{\pi(1-a_1)}{(1+a_1+a_3)^2}\xi_B'^2. \tag{40}$$

Rewriting the above equation as a function of ξ_d

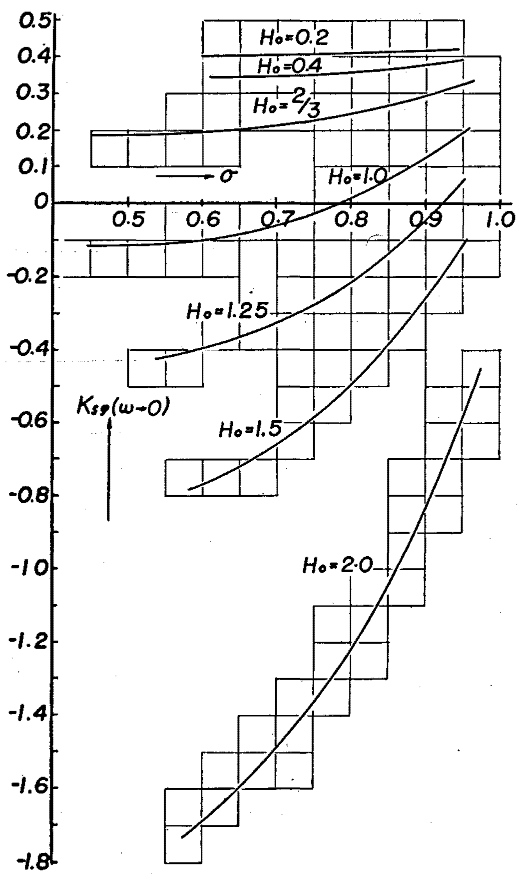


Fig. 8.

we have $$\overline{A}=\frac{\pi(1-a_1)}{(1-a_1+a_3)^2}\xi_d^2. \tag{41}$$

The equation (41) gives a good approximate value till $\xi_d=0.2\sim0.3$. A formula of $K_{S\varphi}$ in the case of $\xi_d\to0$ is also given by O. Grim [2].

$$K_{S\varphi}=\frac{l_{S\varphi}(\omega\to0)}{T}=-\frac{16}{3\pi}\cdot\frac{a_1\left\{(1-a_1)+\frac{4}{5}a_3-a_1a_3+\frac{3}{5}a_3^2\right\}+\frac{4}{5}a_3-\frac{12}{7}a_3^2}{\{(1-a_1)^2+3a_3^2\}(1-a_1+a_3)}. \tag{42}$$

It is shown in Fig. 8. $K_{S\varphi}$ for $\xi_d=0$ shown in Table 1 are that which were obtained from (42).

With regards to the $K_{S\varphi}$, $\alpha_{S\varphi}$ of two elliptic sections with $H_0=2/3$ and 1.5, O. Grim gave his calculated values in Bild. 31 of [1], which are in good agreement with the present ones. $K_{S\varphi}$, $\alpha_{S\varphi}$ for full and fine sections are also shown in Fig. 9 and 10.

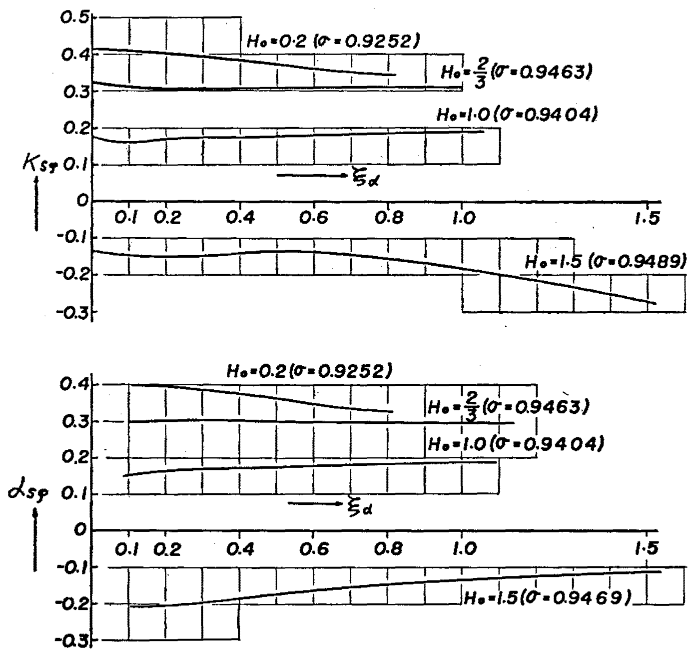


Fig. 9.

In this paper, calculations were carried out in the range of $\xi_d\leqq1.0$ except for the circular section. From these calculations it was found that the variation of $K_{S\varphi}$, $\alpha_{S\varphi}$ with ξ_d is generally small, and that in the elliptic sections and in the ones with samll H_0 the value of $K_{S\varphi}$ nearly equals $\alpha_{S\varphi}$.

Table 1-1

$H_0 = 1.0$

σ	ξ_d	0	0.1	0.2	0.4	0.6	1.0	1.5	2.0	∞
0.6295	$\bar{A}$	0	0.026	0.099	0.332	0.574	0.857			
	K_x	0.851	0.950	1.024	0.964	0.768	0.456			0.423
	$K_{s\varphi}$	-0.094	-0.098	-0.103	-0.101	-0.101	-0.102			
	$\delta_{s\varphi}$		-0.132	-0.146	-0.148	-0.152	-0.163			
0.7854	$\bar{A}$	0	0.032	0.123	0.417	0.686	1.056	1.365	1.605	
	K_x	1.0	1.163	1.248	1.141	0.813	0.352	0.222	0.183	0.405
0.9404	$\bar{A}$	0	0.038	0.159	0.540	0.865	1.231			
	K_x	1.272	1.393	1.677	1.407	0.777	0.197			0.432
	$K_{s\varphi}$	0.178	0.162	0.174	0.174	0.180	0.191			
	$\delta_{s\varphi}$		0.151	0.163	0.170	0.179	0.184			

Table 1-2

$H_0 = 1.5$

σ	ξ_B	0	0.2	0.4	0.6	1.0	1.5	∞
	ξ_d	0	0.133	0.267	0.4	0.667	1.0	∞
0.7854	$\bar{A}$	0	0.067	0.234	0.431	0.746		
	K_x	1.0	1.208	1.210	1.025	0.609		0.405
	$K_{s\varphi}$	-0.530	-0.535	-0.530	-0.524	-0.556		
	$\delta_{s\varphi}$		-0.528	-0.515	-0.501	-0.486		
0.9469	$\bar{A}$	0	0.086	0.308	0.564	0.926	1.201	
	K_x	1.355	1.673	1.668	1.333	0.570	0.239	0.446
	$K_{s\varphi}$	-0.135	-0.153	-0.142	-0.140	-0.186	-0.276	
	$\delta_{s\varphi}$		-0.210	-0.185	-0.161	-0.128	-0.110	

Table 1-3

$H_0 = 2/3$

σ	ξ_B	0	0.05	0.1	0.2	0.4	0.6	∞
	ξ_d	0	0.075	0.15	0.3	0.6	0.9	∞
0.6230	$\bar{A}$	0		0.050	0.194	0.581	0.911	
	K_x	0.871		1.001	1.100	0.857	0.562	0.418
	$K_{s\varphi}$	0.196		0.217	0.208	0.219	0.220	
	$\delta_{s\varphi}$			0.180	0.174	0.163	0.160	
0.7854	$\bar{A}$	0	0.015	0.060	0.236	0.705	1.029	
	K_x	1.0	1.088	1.196	1.287	0.892	0.458	0.405
	$K_{s\varphi}$	0.236	0.234	0.231	0.226	0.217	0.215	
	$\delta_{s\varphi}$		0.233	0.231	0.225	0.210	0.202	
0.9463	$\bar{A}$	0		0.073	0.284	0.835	1.157	
	K_x	1.219		1.478	1.539	0.867	0.304	0.424
	$K_{s\varphi}$	0.323		0.309	0.305	0.321	0.309	
	$\delta_{s\varphi}$			0.300	0.304	0.295	0.298	

Table 1-4

$H_0 = 0.4$

σ	ξ_B	0	0.05	0.1	0.2	0.3	0.4	∞
	ξ_d	0	0.125	0.25	0.5	0.75	1.0	∞
0.6323	$\bar{A}$	0	0.031	0.122	0.459	0.797		
	K_x	0.907	0.998	1.055	1.015	0.714		0.411
	$K_{s\varphi}$	0.343	0.324	0.289	0.275	0.254		
	$\delta_{s\varphi}$		0.321	0.274	0.259	0.237		
0.7854	$\bar{A}$	0	0.036	0.150	0.575		1.149	
	K_x	1.0	1.137	1.272	1.185		0.350	0.405
	$K_{s\varphi}$	0.364	0.377	0.358	0.327		0.290	
	$\delta_{s\varphi}$		0.365	0.344	0.311		0.270	
0.9377	$\bar{A}$	0	0.041	0.175	0.661		1.315	
	K_x	1.137	1.289	1.498	1.271		0.230	0.413
	$K_{s\varphi}$	0.387	0.373	0.362	0.342		0.331	
	$\delta_{s\varphi}$		0.382	0.369	0.342		0.330	

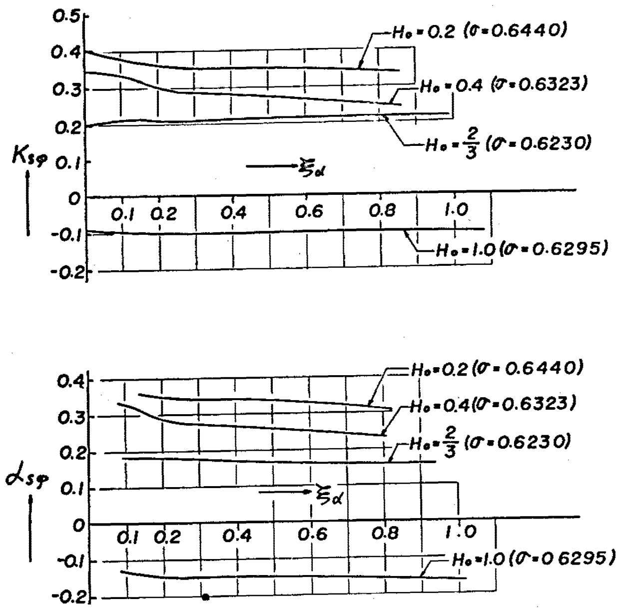


Fig. 10.

Table 1-5

$H_0=0.2$						
σ	ξ_B	0	0.05	0.1	0.15	∞
	ξ_d	0	0.25	0.5	0.75	∞
0.6440	$\bar{A}$	0	0.126	0.537	0.949	
	K_x	0.946	1.223	1.298	0.811	0.407
	$K_{s\varphi}$	0.401	0.351	0.356	0.343	
	$\alpha_{s\varphi}$		0.342	0.337	0.317	
0.7854	$\bar{A}$	0	0.135	0.606	1.050	
	K_x	1.0	1.350	1.390	0.760	0.405
	$K_{s\varphi}$	0.407	0.383	0.356	0.343	
	$\alpha_{s\varphi}$		0.374	0.343	0.325	
0.9252	$\bar{A}$	0	0.155	0.661	1.122	
	K_x	1.067	1.454	1.430	0.660	0.407
	$K_{s\varphi}$	0.415	0.399	0.373	0.347	
	$\alpha_{s\varphi}$		0.390	0.360	0.333	

II. Rolling oscillation

2.1. Progressive wave height

In the case of rolling motion, suppose now that a cylinder performs rolling oscillation about the origin 0 with a small angular displacement $\theta=\theta_0\cos(\omega t+\gamma)$ and velocity $\dot{\theta}=-\theta_0\omega\sin(\omega t+\gamma)$ in the clockwise direction (See Fig. 11).

The fluid motion, in this case, is quite similar to the swaying oscillation, and

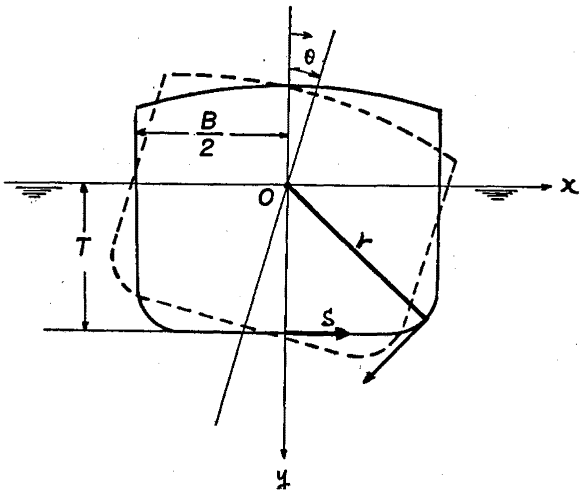


Fig. 11.

equation (1) holds of course on the velocity potential ϕ in the rolling. The boundary condition on the surface of the cylinder is given by the following equation

$$\left(\frac{\partial\phi}{\partial\nu}\right)=r\left(\frac{d\theta}{dt}\right)\frac{dr}{dS}$$

accordingly

$$-\left(\frac{\partial\psi}{\partial S}\right)=\left(\frac{d\theta}{dt}\right)\frac{\partial}{\partial S}\left\{\frac{1}{2}(x_0^2+y_0^2)\right\}, \tag{43}$$

where ϕ is the velocity potential and ψ the stream function of the fluid in rolling motion respectively.

Therefore on the surface of the cylinder we obtain

$$\psi=-\frac{1}{2}\left(\frac{d\theta}{dt}\right)(x_0^2+y_0^2)+C(t). \tag{44}$$

By the similar way as done for swaying oscillation we obtain the following equation which corresponds to (14):

$$\left(\frac{\omega\pi}{g\eta}\right)\psi(\theta)_{\alpha=0}=\Psi_{C0}(\xi_B,\theta)\cos\omega t+\Psi_{S0}(\xi_B,\theta)\sin\omega t$$

$$+\cos\omega t\sum_{m=1}^{\infty}P_{2m}\left[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\left\{\frac{\cos 2m\theta}{2m}\right.\right.$$

$$\left.\left.+\frac{a_1\cos(2m+2)\theta}{2m+2}-\frac{3a_3\cos(2m+4)\theta}{2m+4}\right\}\right]$$

$$+\sin\omega t\sum_{m=1}^{\infty}Q_{2m}\left[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\left\{\frac{\cos 2m\theta}{2m}\right.\right.$$

$$\left.\left.+\frac{a_1\cos(2m+2)\theta}{2m+2}-\frac{3a_3\cos(2m+4)\theta}{2m+4}\right\}\right]$$

$$=-\left(\frac{\pi\omega}{g\eta}\right)\frac{1}{2}\left(\frac{d\theta}{dt}\right)(x_0^2+y_0^2)+C(t) \tag{45}$$

in which Ψ_C and Ψ_S are the same ones as given in (11) and (12), and η is of course the amplitude of progressive wave.

By some reduction, the right-hand side of the equation (45) becomes

$$\frac{\pi\xi_B h_r}{2\eta}\cdot\mu(\theta)\sin(\omega t+\gamma)+C(t)$$

where $h_r=B\theta_0/2$

and $\mu(\theta)=[(1+a_1^2+a_3^2)-2a_1(1+a_3)\cos 2\theta+2a_3\cos 4\theta]/(1+a_1+a_3)^2.$ (46)

Putting $\theta=\frac{\pi}{2}$ in the equation (45) and eliminating the constant $C(t)$ we obtain the following equation:

$$\begin{aligned}&\left(\left\{\Psi_{C0}(\xi_B,\theta)-\Psi_{C0}\left(\xi_B,\frac{\pi}{2}\right)\right\}+\sum_{m=1}^{\infty}P_{2m}\left[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\left\{\frac{\cos 2m\theta}{2m}\right.\right.\right.\\&\left.\left.\left.+\frac{a_1\cos(2m+2)\theta}{2m+2}-\frac{3a_3\cos(2m+4)\theta}{2m+4}\right\}-\frac{\xi_B(-1)^m}{1+a_1+a_3}\left(\frac{1}{2m}-\frac{a_1}{2m+2}-\frac{3a_3}{2m+4}\right)\right]\right)\cos\omega t\\&+\left(\left\{\Psi_{S0}(\xi_B,\theta)-\Psi_{S0}\left(\xi_B,\frac{\pi}{2}\right)\right\}+\sum_{m=1}^{\infty}Q_{2m}\left[-\cos(2m+1)\theta-\frac{\xi_B}{1+a_1+a_3}\left\{\frac{\cos 2m\theta}{2m}\right.\right.\right.\\&\left.\left.\left.+\frac{a_1\cos(2m+2)\theta}{2m+2}-\frac{3a_3\cos(2m+4)\theta}{2m+4}\right\}-\frac{\xi_B(-1)^m}{1+a_1+a_3}\left(\frac{1}{2m}-\frac{a_1}{2m+2}-\frac{3a_3}{2m+4}\right)\right]\right)\sin\omega t\\&=\frac{\pi\xi_B}{2\eta}\cdot h_r\{\mu(\theta)-1\}\sin(\omega t+\gamma).\end{aligned} \tag{47}$$

Put $\mu(\theta)-1\equiv g(\theta)$.

Then the right-hand side of the equation (47) reduces to

$$\left.\begin{aligned}&g(\theta)\{P_0\cos\omega t+Q_0\sin\omega t\},\\&\text{where}\\&P_0=\frac{\pi\xi_B}{2\eta}h_r\sin\gamma,\quad Q_0=\frac{\pi\xi_B}{2\eta}h_r\cos\gamma\end{aligned}\right\} \tag{48}$$

Finally, we obtain

$$\left.\begin{aligned}\Psi_{C0}(\xi_B,\theta)-\Psi_{C0}\left(\xi_B,\frac{\pi}{2}\right)&=\sum_{m=0}^{\infty}P_{2m}\cdot f_{2m}(\theta)\\\Psi_{S0}(\xi_B,\theta)-\Psi_{S0}\left(\xi_B,\frac{\pi}{2}\right)&=\sum_{m=0}^{\infty}Q_{2m}\cdot f_{2m}(\theta)\end{aligned}\right\} \tag{49}$$

provided that

$$\begin{aligned}&f_0=g(\theta)=\mu(\theta)-1\\&\underset{m\neq0}{f_{2m}}=\cos(2m+1)\theta+\frac{\xi_B}{1+a_1+a_3}\left\{\frac{\cos 2m\theta}{2m}+\frac{a_1\cos(2m+2)\theta}{2m+2}-\frac{3a_3\cos(2m+4)\theta}{2m+4}\right\}\\&\qquad+\frac{\xi_B(-1)^{m+1}}{1+a_1+a_3}\left(\frac{1}{2m}-\frac{a_1}{2m+2}-\frac{3a_3}{2m+4}\right).\end{aligned} \tag{50}$$

From the equation (49)

$$\sqrt{P_0^2+Q_0^2}=\frac{\pi\xi_B}{2\eta}h_r.$$

Therefore we have

$$\bar{A}=\frac{\eta}{h_r}=\frac{\pi\xi_B}{2\sqrt{P_0^2+Q_0^2}}. \tag{51}$$

When the values of P_0 and Q_0 are obtained by solving the equation (49) the progressive wave height created by the rolling motion is given, accordingly the amplitude ratio $\bar{A}$ is found as a function of ξ_B or ξ_d.

2.2. Added moment of inertia and swaying force

Hydrodynamic pressure on the surface of the cylinder is given as follows:

$$p=\frac{\rho g\eta}{\pi}\Bigg(\Bigg[\Phi_{C0}(\xi_B,\theta)+\sum_{m=1}^{\infty}P_{2m}\Bigg\{\sin(2m+1)\theta+\frac{\xi_B}{1+a_1+a_3}\Bigg(\frac{\sin 2m\theta}{2m}+\frac{a_1\sin(2m+2)\theta}{2m+2}-\frac{3a_3\sin(2m+4)\theta}{2m+4}\Bigg)\Bigg\}\Bigg]\sin\omega t$$
$$-\Bigg[\Phi_{S0}(\xi_B,\theta)+\sum_{m=1}^{\infty}Q_{2m}\Bigg\{\sin(2m+1)\theta+\frac{\xi_B}{1+a_1+a_3}\Bigg(\frac{\sin 2m\theta}{2m}+\frac{a_1\sin(2m+2)\theta}{2m+2}-\frac{3a_3\sin(2m+4)\theta}{2m+4}\Bigg)\Bigg\}\Bigg]\cos\omega t\Bigg). \tag{52}$$

Hydrodynamic moment M_ϕ in clockwise direction and hydrodynamic force F_x' in the direction of $-x$ are evaluated using the above pressure (52).

That is,

$$\left.\begin{aligned} M_\phi&=-2\int_0^{\pi/2}p\left(x_0\frac{dx_0}{d\theta}+y_0\frac{dy_0}{d\theta}\right)d\theta\\ F_x'&=-2\int_0^{\pi/2}p\left(\frac{dy_0}{d\theta}\right)d\theta\end{aligned}\right\} \tag{53}$$

These are rewritten:

$$\left.\begin{aligned} M_\phi&=-\frac{\rho gB^2\eta}{\pi}[X_R\sin\omega t-Y_R\cos\omega t]\\ F_x'&=-\frac{\rho gB\eta}{\pi}[N_0\sin\omega t-M_0\cos\omega t]\end{aligned}\right\} \tag{54}$$

where X_R, Y_R, N_0 and M_0 are the same equations as were used in the swaying. Therefore added moment of inertia I_R is

$$I_R=\frac{\rho B^4}{8}\frac{(Q_0Y_R+P_0X_F)}{(P_0^2+Q_0^2)}. \tag{55}$$

The component of moment in phase with $\dot{\theta}$ is

$$\frac{Q_0X_R-P_0Y_R}{P_0^2+Q_0^2}(Q_0\sin\omega t+P_0\cos\omega t)\frac{\rho gB^2\eta}{\pi}. \tag{56}$$

By some reduction as done in the swaying we have the following relation which were used to check the computation,

$$P_0 Y_R - Q_0 X_R = \frac{\pi^2}{8}. \tag{57}$$

Letting $N_R \dot{\theta}$ denote the linear wave-making damping which is proportional to the rolling velocity, N_R leads to

$$N_R = \frac{\rho g^2}{\omega^3} \bar{A}^2 \left(\frac{B}{2}\right)^2. \tag{58}$$

2.3. Coefficient of added moment of inertia

Added moment of inertia in case of $\omega \to \infty$ is given by Professor Kumai [8] for the Lewis-form section, that is,

$$I_{R\infty} = \frac{\rho \pi B^4}{16} \cdot \frac{\{a_1^2(1+a_3)^2 + 2a_3^2\}}{(1+a_1+a_3)^4}. \tag{59}$$

For a ellipse it will be

$$\frac{\rho\pi}{16}\left\{\left(\frac{B}{2}\right)^2 - T^2\right\}^2.$$

On the other hand, added moment of inertia in case of $\omega \to 0$ is obtained by O. Grim [2].

$$I_{R0} = \frac{\rho\pi}{8}\left(\frac{B}{2}\right)^4 \left[\frac{128}{\pi^2} \cdot \frac{a_1^2(1+a_3)^2 + \frac{8}{9}a_1 a_3(1+a_3) + \frac{16}{9}a_3^2}{(1+a_1+a_3)^4}\right]. \tag{60}$$

For a ellipse it will be

$$\frac{\rho}{\pi}\left\{\left(\frac{B}{2}\right)^2 - T^2\right\}$$

and then we have the following relation for the elliptical sections

$I_{R0}/I_{R\infty} = 16/\pi^2 \doteqdot 1.621.$

Generally, for the Lewis form sections $\quad I_{R0} \geqslant I_{R\infty}$. (61)

Writing $K_{\varphi B}$ the ratio of the added moment of inertia to $\frac{\pi}{8}\rho\left(\frac{B}{2}\right)^4$ and $K_{\varphi T}$ the ratio of that to $\frac{\pi}{8}\rho T^4$ we obtain

$$K_{\varphi B}(\omega \to 0) = \frac{128}{\pi^2} \cdot \frac{a_1^2(1+a_3)^2 + \frac{8}{9}a_1 a_3(1+a_3) + \frac{16}{9}a_3^2}{(1+a_1+a_3)^4} \tag{62}$$

and

$$K_{\varphi B}(\omega \to \infty) = \frac{8\{a_1^2(1+a_3)^2 + 2a_3^2\}}{(1+a_1+a_3)^4}. \tag{63}$$

In general for a certain ω

$$\left.\begin{aligned} K_{\varphi B} &= \frac{16}{\pi} \cdot \frac{Q_0 Y_R + P_0 X_R}{P_0^2 + Q_0^2} \\ K_{\varphi T} &= K_{\varphi B} \cdot H_0^4 \end{aligned}\right\} \tag{64}$$

2.4. Coefficient of swaying force

We resolve F_x into two components, the one in phase with $\ddot{\theta}$ and the other in phase with $\dot{\theta}$.

That is,

$$F_x' = F_{RS}\left(-\frac{d^2\theta}{dt^2}\right) + N_{RS}\left(-\frac{d\theta}{dt}\right). \tag{65}$$

For the F_{RS} and N_{RS} we have

$$\left.\begin{aligned} F_{RS} &= \frac{\rho B^3}{8} \cdot \frac{(Q_0 M_0 + P_0 N_0)}{P_0^2 + Q_0^2} \\ N_{RS} &= \frac{\rho\omega B^3}{8} \cdot \frac{(\rho_0 M_0 - Q_0 N_0)}{P_0^2 + Q_0^2} \end{aligned}\right\}. \tag{66}$$

Put $F_{RS} = I_R / l_{RS}$, $l_{RS} = K_{RS} T$. (67)

From eqs. (55) and (66), we get

$$K_{RS} = 2H_0\left(\frac{P_0 X_R + Q_0 Y_R}{P_0 N_0 + Q_0 M_0}\right). \tag{68}$$

In the next place putting

$$N_{RS} \times l'_{RS} = N_R, \quad l'_{RS} = \alpha_{RS} \cdot T \tag{69}$$

we obtain

$$\alpha_{RS} = \left(\frac{\pi}{2}\right)^2 \frac{H_0}{(P_0 M_0 - Q_0 N_0)}. \tag{70}$$

Accordingly, we can evaluate the coefficients of swaying force in rolling oscillation by the following equations:

$$F_{RS} = \frac{I_R}{K_{RS} \cdot T} \text{ and } N_{RS} = \frac{N_R}{\alpha_{RS} \cdot T}. \tag{71}$$

In the case of $\omega \to 0$, K_{RS} is given by O. Grim [2], and it is as follows:

$$K_{RS} = -\frac{6\left[a_1^2(1+a_3)^2 + \frac{8}{9}a_1 a_3(1+a_3) + \frac{16}{9}a_3^2\right]}{\pi\left[a_1(1-a_1)(1+a_3) + a_1 a_3(1+a_3) \times 0.6 + a_3(1-a_1) \times 0.8 - 1.714 a_3^2\right](1-a_1+a_3)} \tag{72}$$

For a ellipse it reduces to

$$K_{RS} = -\frac{6}{\pi}\frac{a_1}{(1-a_1)^2}. \tag{73}$$

The formula (73) is the same one as given by F. Ursell in [13]. $\bar{A}(\omega \to 0)$ given by O. Grim [2] is also

$$\bar{A} = \frac{16}{3}\left|\frac{a_1(1+a_3) + \frac{4}{5}a_3}{(1+a_1+a_3)^3}\right| \xi_B^2. \tag{74}$$

2. 5. Numerical calculations

For the ten Lewis-form sections numerical calculations were carried out.

In Figures 12 to 14, $\bar{A}$ is shown as a function of ξ_d.

The coefficient of added moment of inertia $K_{\varphi T}$ is given in Figures 15 and 16. General tendency of $K_{\varphi T}$ with respect to ξ_d is quite similar to the one of swaying oscillation.

Figures 17 to 19 show K_{RS}, α_{RS}.

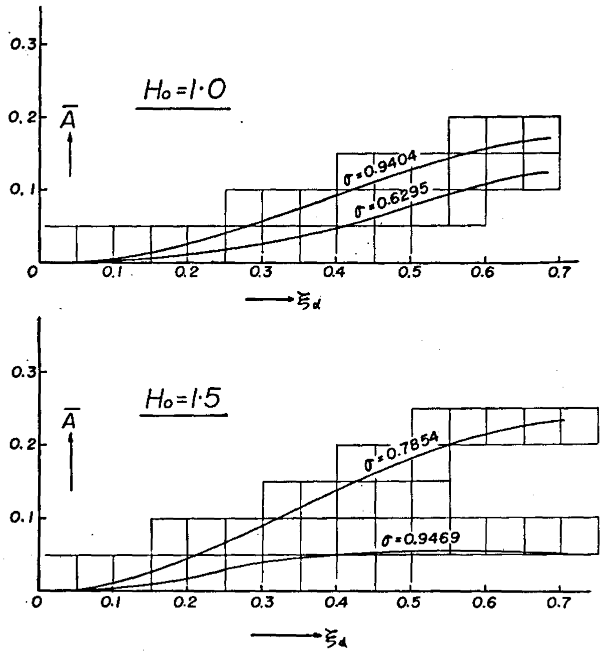


Fig. 12.

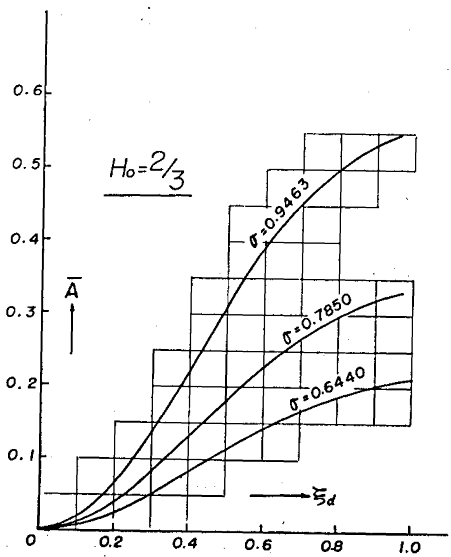


Fig. 13.

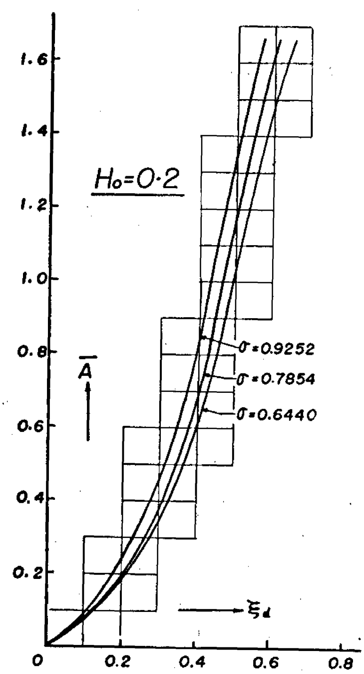
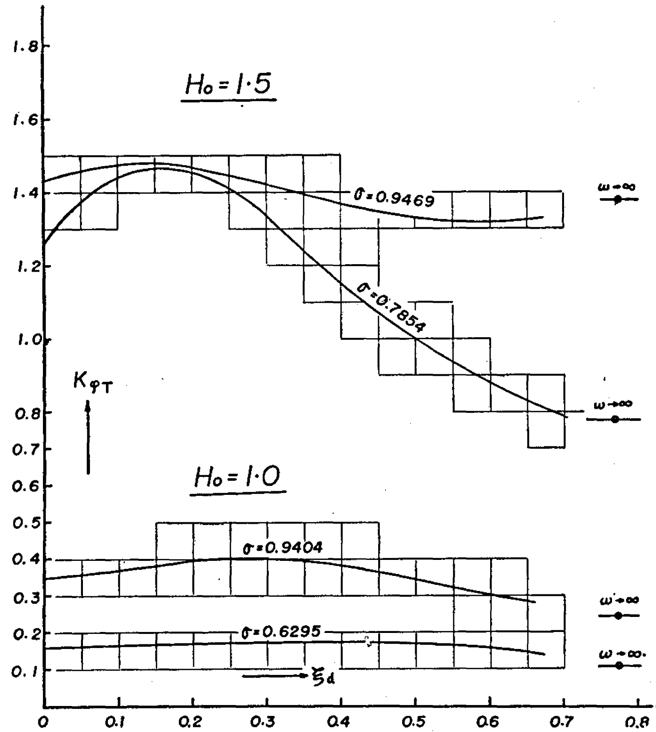
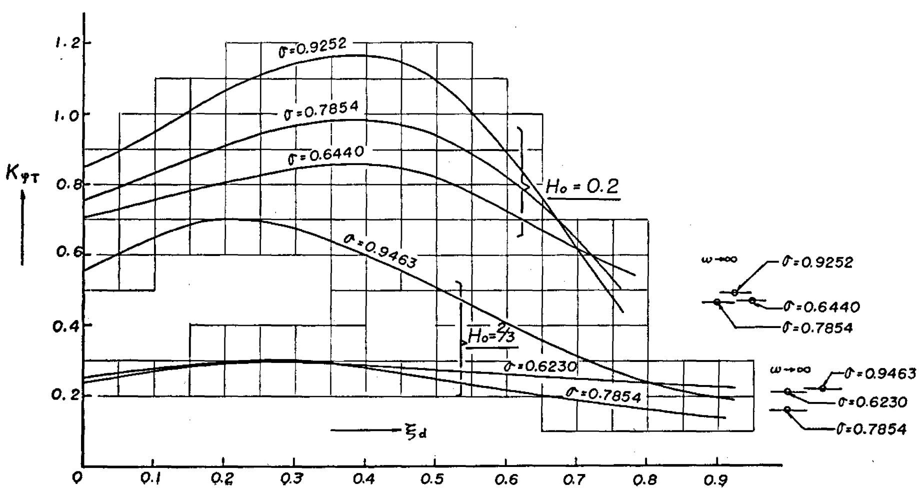


Fig. 14.



Ho = 1·5
σ=0.9469
σ=0.7854
ω→∞
Kφτ
Ho = 1·0
σ=0.9404
σ=0.6295
ξd
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1.8
1.6
1.4
1.2
1.0

Fig. 15.



σ=0.9252
σ=0.7854
σ=0.6440
σ=0.9463
Ho = 0.2
Ho=2/3
σ=0.6230
σ=0.7854
Kφτ
ω→∞
σ=0.9252
σ=0.6440
σ=0.7854
σ=0.9463
σ=0.6230
σ=0.7854
ξd
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.2
1.0

Fig. 16.

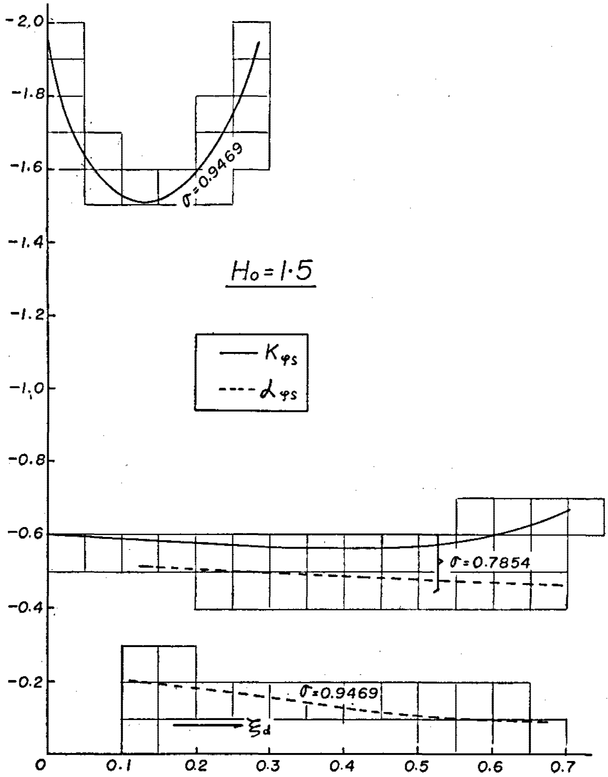

-2.0
-1.8
-1.6
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
σ=0.9469
H_o=1·5
$K_{\varphi s}$
$\alpha_{\varphi s}$
σ=0.7854
σ=0.9469
ξ_d

Fig. 17.

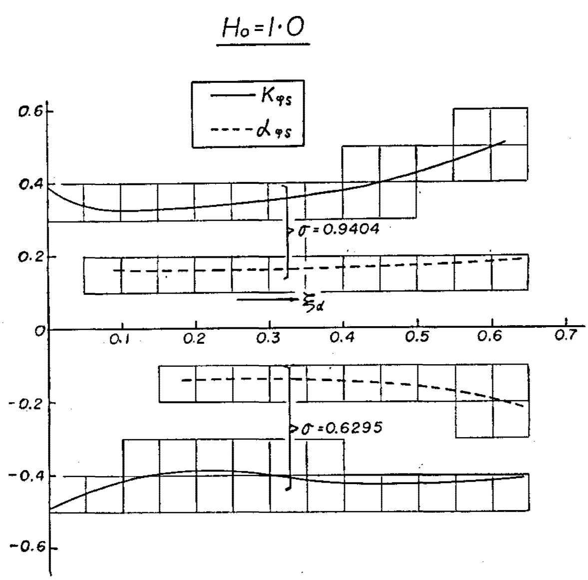

H_o=1·0
$K_{\varphi s}$
$\alpha_{\varphi s}$
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
σ=0.9404
ξ_d
σ=0.6295

Fig. 18.

Generally speaking, the values of K_{RS} differ considerably from the α_{RS}. In the case of $H_0=0.2$, however, the both values are fairly near.

Numerical results are also given in Tables 2–1 to 2–4.

Wave-making resistance of rolling motion of ships was investigated in detail by Hishida [9]–[10]. First, the problem is dealt with as a two-dimensional wave problem to develop an approximate solution of wave motion produced by

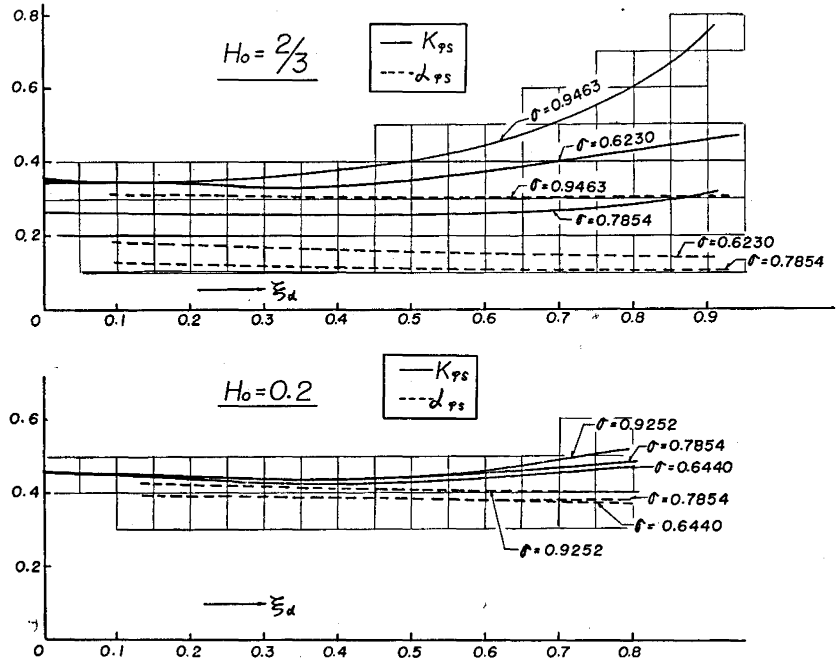


Fig. 19.

Table 2-1 $H_0=1.5$

σ							
	ξ_B	0	0.2	0.4	0.6	1.0	∞
	ξ_d	0	0.1333	0.2667	0.4	0.6667	∞
0.7854	$\bar{A}$	0	0.0229	0.0755	0.1398	0.2321	
	$K_{\varphi T}$	1.2656	1.4671	1.3922	1.1527	0.8120	0.7776
	K_{RS}	-0.5967	-0.5832	-0.5700	-0.5583	-0.6381	
	α_{RS}		-0.5122	-0.4970	-0.4864	-0.4650	
0.9469	$\bar{A}$	0	0.0116	0.0347	0.0511	0.0531	
	$K_{\varphi T}$	1.4291	1.4671	1.4286	1.3745	1.3279	1.3770
	K_{RS}	-1.9441	-1.5056	-1.8390	-3.1599	-23.5290	
	α_{RS}		-0.2000	-0.1660	-0.1278	-0.0873	

rolling of a body floating on a water surface. Using the conformal transformation he obtained the amplitude of two-dimensional progressive waves of various section form representing ship's section.

Comparison between the amplitude ratio obtained by Hishida and the present one will be performed in the near future.

Table 2-2 $H_0 = 1.0$

σ	ξ_B	0	0.1	0.2	0.4	0.6	1.0	∞
	ξ_d	0	0.1	0.2	0.4	0.6	1.0	∞
0.6295	$\bar{A}$	0		0.0136	0.0476	0.1102	0.1221	
	$K_{\varphi T}$	0.1574		0.1711	0.1700	0.1600		0.1088
	K_{RS}	-0.4906		-0.3896	-0.4218	-0.4158		
	α_{RS}			-0.1396	-0.1428	-0.1967		
0.9404	$\bar{A}$	0	0.0063	0.0260	0.0925	0.1564		
	$K_{\varphi T}$	0.3513	0.3700	0.4003	0.3815	0.3035		0.2432
	K_{RS}	0.3885	0.3274	0.3326	0.3800	0.4922		
	α_{RS}		0.1630	01623	0.1704	0.1823		

Table 2-3 $H_0 = 2/3$

σ	ξ_B	0	0.05	0.1	0.2	0.4	0.6	∞
	ξ_d	0	0.075	0.15	0.3	0.6	0.9	∞
0.6230	$\bar{A}$	0		0.0135	0.0519	0.1415	0.1976	
	$K_{\varphi T}$	0.2342		0.2795	0.2910	0.2548	0.2218	0.2070
	K_{RS}	0.3462		0.3461	0.3291	0.3744	0.4585	
	α_{RS}			0.1808	0.1718	0.1568	0.1427	
0.7854	$\bar{A}$	0	0.0053	0.0207	0.0793	0.2260	0.3173	
	$K_{\varphi T}$	0.2502	0.2711	0.2871	0.2988	0.2106	0.1321	0.1542
	K_{RS}	0.2653	0.2613	0.2584	0.2541	0.2623	0.3037	
	α_{RS}		0.2324	0.2282	0.2208	0.2113	0.2044	
0.9463	$\bar{A}$	0		0.0347	0.1328	0.3895	0.5297	
	$K_{\varphi T}$	0.5568		0.6427	0.6745	0.4079	0.1928	0.2127
	K_{RS}	0.3526		0.3436	0.3551	0.4395	0.7409	
	α_{RS}			0.3101	0.3100	0.3059	03034	

Table 2-4 $H_0=0.2$

σ	ξ_B	0	0.05	0.1	0.15	∞
	ξ_d	0	0.25	0.5	0.75	∞
0.6440	$\bar{A}$	0	0.2481	1.0142	1.8705	
	$K_{\varphi T}$	0.7083	0.8202	0.8200	0.5683	0.4656
	K_{RS}	0.4627	0.4348	0.4322	0.4664	
	α_{RS}		0.3940	0.3836	0.3756	
0.7854	$\bar{A}$	0	0.2716	1.1451	2.0000	
	$K_{\varphi T}$	0.7498	0.9400	0.9400	0.5380	0.4624
	K_{RS}	0.4584	0.4430	0.4451	0.4772	
	α_{RS}		0.3923	0.3897	0.3803	
0.9252	$\bar{A}$	0	0.3369	1.3260	2.1645	
	$K_{\varphi T}$	0.8500	1.1098	1.0019	0.4838	0.4890
	K_{RS}	0.4585	0.4399	0.4428	05.015	
	α_{RS}		0.4248	0.4100	0.4022	

III. Conclusions

Two-dimensional hydrodynamic force and moment produced by swaying and/or rolling oscillation were calculated for the Lewis-form cylinders floating on the surface of a fluid. We could gain the following conclusions within the limit $\xi_d \leqslant 1.0$ from the results mentioned above.

First, with respect to the swaying oscillation

(1) As regards K_x and $\bar{A}$ the effect of the variation in H_0 is smaller than that of the coefficient of added mass and amplitude ratio in case of the heaving.

(2) $\bar{A}$, in case of the same H_0, of the fuller sections are larger than the ones of the fine sections. This is opposite to the case of the heaving.

(3) Within the limit $\xi_d \doteqdot 0.6$, the fuller the section is the larger the K_x becomes, however, it has the adverse tendency beyond this limit.

(4) The formula of $\bar{A}$ given by O. Grim [2] gives fairly good approximate values till $\xi_d = 0.2 \sim 0.3$.

(5) As for $K_{\varphi S}$ and $\alpha_{S\varphi}$ the effect of the variation in ξ_d is small.

Second, with respect to the rolling oscillation.

(6) The tendency of $K_{\varphi T}$ with regard to ξ_d is quite similar to the K_x.

(7) The values of K_{RS} differ considerably from the α_{RS}.

Throughout these works, the writer is very grateful to Messrs. Tomioka and Arakawa for their help in numerical calculations as well as in presenting the manuscript.

Appendix

The progressive wave height produced by swaying oscillation of a flat plate with draught T can be obtained by solving the integral equation after the method developed by F. Ursell [11] and [12].

That is, the amplitude ratio $\bar{A}$ is given by the following equation:

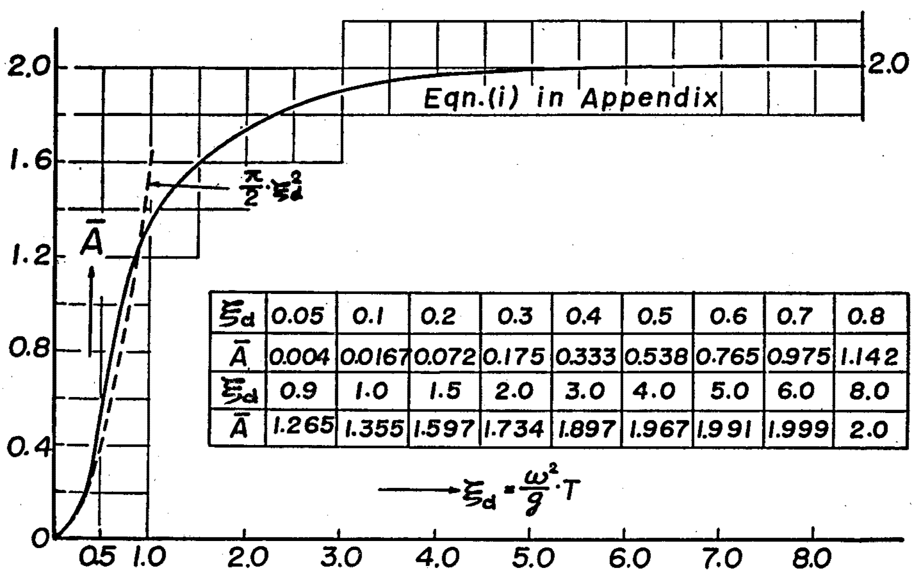
$$\bar{A}=\frac{\pi\,[I_1(\xi_d)+L_1(\xi_d)]}{\sqrt{\pi^2 I_1^2(\xi_d)+K_1^2(\xi_d)}} \tag{i}$$

where $\xi_d=\dfrac{\omega^2}{g}T$ and $I_1(\xi_d)$, $K_1(\xi_d)$ are the modified Bessel functions, and $L_1(\xi_d)$ is

$$L_1(\xi_d)=\sum_{m=0}^{\infty}\frac{\left(\frac{1}{2}\xi_d\right)^{2m+2}}{\Gamma\left(m+\frac{3}{2}\right)\Gamma\left(m+\frac{5}{2}\right)} \tag{ii}$$

(See [14]).

As ξ_d tends to zero in the equation (i), $\bar{A}$ tends to $\frac{\pi}{2}\xi_d^2$. This is the same with the approximate formula which results from putting $a_1=-1.0$, $a_3=0$ in the equation (41). In Fig. 20, $\bar{A}$ is shown as a function of ξ_d.



ξ_d	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$\bar{A}$	0.004	0.0167	0.072	0.175	0.333	0.538	0.765	0.975	1.142
ξ_d	0.9	1.0	1.5	2.0	3.0	4.0	5.0	6.0	8.0
$\bar{A}$	1.265	1.355	1.597	1.734	1.897	1.967	1.991	1.999	2.0

Fig. 20.

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