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SHINOHARA, Kinji
Research Institute for Applied Mechanics, Kyushu University

TSUBAKI, Toichiro
Research Institute for Applied Mechanics, Kyushu University

SAITO, Takashi
Research Institute for Applied Mechanics, Kyushu University

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LABORATORY INVESTIGATION OF THE EQUILIBRIUM PROFILES OF A SANDY BEACH

By

Kinji SHINOHARA, Tōichirō TSUBAKI
and Takashi SAITŌ

Synopsis In this paper the accuracy and reproducibility of the experiments of the equilibrium profiles of a sandy beach established by the uniform wave action of a constant steepness is discussed and is concluded to be satisfactory. In the next place, the beaches molded from two kinds of bed materials whose median diameters are 0.2 mm and 0.88 mm, and whose initial beach slopes are 1 : 10 and 1 : 20, are subjected to attack by the several kinds of wave action of a constant steepness. In the equilibrium state thus attained the profiles together with their characteristics and the rate of sand transport are measured. From these experiments, the effects of the initial beach slope, wave steepness, the size of bed materials and the original beach profile upon the final equilibrium profile of a sandy beach are made clear, and some new characteristics of the movement of a shore line are found.

1. Introduction

The deformation of the beach profiles will be classified into two kinds, the one occurring in such a short time as a day or a few days and the other taking place over a considerably long period of one to scores of years. Of these two the former is believed to be caused by the transport of bed materials that is directed normally to the shore line. In the present paper, only this short term deformation will be pursued.

When a beach is subjected to attack for long hours by wave action of definite properties, there is established a definite beach profile corresponding to the wave. Such formation will be called the equilibrium profile of a beach. Though it seldom occurs with an actual beach that definite waves attack it continuously for long hours, it is obvious that for a definite wave there is formed a beach profile corresponding to it pretty rapidly and this beach profile successively changes with the change of wave properties. The existence of such an equilibrium profile is, however, denied or doubted by some of the researchers.

Assuming now that there is a definite equilibrium profile in accordance with the wave of definite properties at an arbitrary beach, the beach deformation to which the present beach profile will be submitted in the future may be estimated from the change of wave properties. While observations are being made on such beach deformation at an actual beach, experimental studies have been frequently made on it to grasp the fundamental nature of the problem. In studying the beach deformation through experiments, it is necessary to find out how this is related with an actual beach. According to the studies already published by other research workers, the factors controlling beach profile formation are considered to be as follows,

(1) Properties of deep water wave, i.e. wave length L_0, wave height H_0, especially initial wave steepness $\delta_0 = H_0/L_0$, and

(2) Properties of bed materials of beach, i.e. grain size D_0, specific gravity s.

Bagnold(1) assumes that if R defined by $R = H_0/D_0$ is constant, the similarity of beach profile is to be maintained. J. W. Johnson(2) has attempted to express the beach profile non-dimensionally by dividing the depth from the still water surface and the distance from the shore line by the wave length of deep water. Dr. Yūichi Iwagaki *et al.*(3) state that, when the equilibrium profile obtained at a laboratory is applied to the beach attended by wave refraction or diffraction, a non-dimensional expression due to the characteristics of breakers is better applicable to an actual beach than those of the waves of deep water.

When some of the results of former laboratory studies are compared by means of dimensionless expression method, there seems to exist some differences in the equilibrium profile obtained, even with the experiments made under nearly the similar conditions. In Fig. 1 (a) and (b), the comparison is made on the results of experiments by Dr. Iwagaki *et al.*, Johnson and the authors for the cases of $\delta_0 = 0.018$ and $\delta_0 = 0.06$ respectively.

As seen from the above figures, while the foreshore is nearly in agreement, considerable difference is seen from there toward offshore. According to Dr. Iwa-

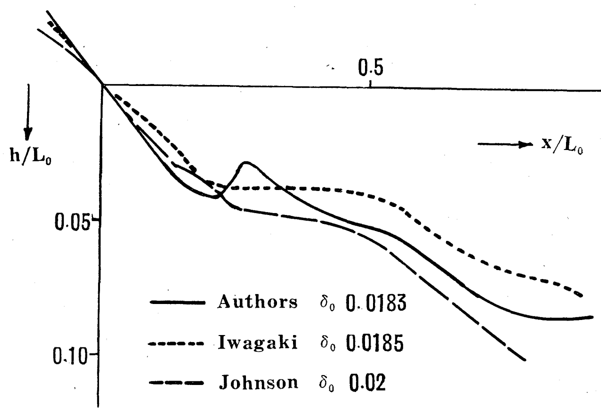

Fig. 1 (a) The difference in the equilibrium profile by researchers ($\delta_0 \doteqdot 0.02$).

(1) M.R.A. Bagnold: Beach formation by waves; some model-experiments in a wave tank, Journal of the Institution of Civil Engineers. Nov., 1940.

(2) J.W. Johnson: Scale effects in hydraulic models involving wave motion, Trans. A.G.U., Vol. 30, No. 4, Aug. 1949.

(3) Y. Iwagaki and T. Sawaragi: Experimental Study on the Equilibrium Slopes of Beaches and Sand Movement by Breakers; Coastal Engineering in Japan, Vol. 1, 1958.

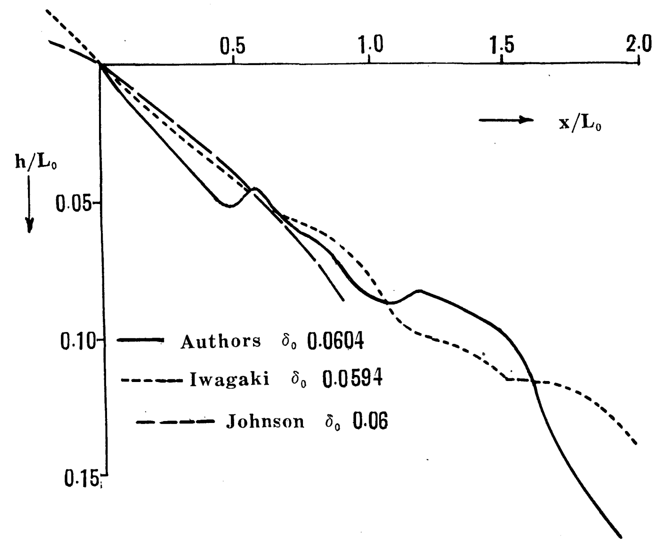

Fig. 1 (b). The difference in the equilibrium profile by researchers ($\delta_0 \doteqdot 0.06$).

gaki's view on Johnson's experiment, either duration of sending waves was too short to induce equilibrium or the grain size of the sand used for experiment was larger than that of Dr. Iwagaki's and the movement of sand on the offshore side was weak. The bed materials used for Dr. Iwagaki's experiment was of homogeneous one of 0.3 mm grain size. In the authors' experiment, the sand used was of 0.2 mm average grain size, and equilibrium condition was attained by sending the wave of a definite wave steepness, similarly as in Dr. Iwagaki's experiment, starting first from a sandy beach that was molded into a beach slope of 1/10, where the experimental conditions were nearly the same except that the bed material was slightly different in grain size. Admitting that there is considerable difference in the equilibrium profile obtained by the experiments thus made under similar condition, it will be difficult to apply the results of experiments to an actual beach, even if dimensionless expressions are used.

The authors considered it necessary to solve the following fundamental problems before studying the problem of the beach deformation experimentally.

(1) Problems of the accuracy of experiment and reproducibility

In carrying out experiments on the beach deformation, it is necessary to send constant waves for long hours. Some degree of variations, however, must be expected since it is difficult to maintain the wave height and period constant throughout the experiment. The sand movement of a sandy beach is sometimes subject to accidental factors and we cannot assert that the equilibrium profile is absolutely independent of the least variation in wave properties. Especially with different experimental apparatuses, there is the fear of such tendency. It is therefore necessary to ascertain the extent of reproducibility of the beach profile obtained by repeating experiments under the same condition, using the same method. Thus the accuracy or the limit of application in adopting the equilibrium profile obtained by the experiment to a practical case is made known.

(2) Effect of the initial beach slope and original profile

In former experiments, an equilibrium profile was established, starting from a sandy beach of uniform beach slope, such as 1/10 or 1/20. With an actual sandy beach, starting from the equilibrium profile established by one wave for a certain steepness, a new equilibrium profile corresponding to the wave of other steepness is formed. In this case, it is considered to be probable that the new equilibrium profile is affected by the former one in some way. It is therefore necessary to clarify how far the equilibrium profile corresponding to one wave steepness is affected by the initial beach slope and original profile.

The present paper is aimed to investigate the above-mentioned problems as well as to find out a clue to quantitative estimation of beach deformation by experiments.

2. Experimental equipment and procedure

The tank used for experiments was the one of organic glass stiffened by steel framed works, 0.5 m wide, 0.5 m high and 20 m long, at the end of which were made models of sandy beach with the slope of 1/10 and 1/20 or in other profiles. Maintaining water depth to 35 cm and sending waves from the other end by means of the flutter type wave generating machine, the equilibrium profile of a sandy beach was established. The sand used for sandy beach was of two kinds, i.e. 0.2 mm and 0.88 mm in average grain size and their specific gravities were 2.66 and 2.65 respectively. The measurement of wave height and wave length was made by an electric point gauge and that of beach profile by a specially manufactured touch gauge. For the equipment to catch the sand that moves near the bottom, the one referred to in the previous report[4] was used. The wave properties used for the experiments were within the limit of wave length of deep water L_0: 72.5 $\sim$375 cm, wave height H_0: 2.1$\sim$6.5 cm and initial wave steepness δ_0: 0.009$\sim$0.079. The time between the first sending of waves and the attainment of nearly equilibrium profile was, in the case of 1/10 initial beach slope, 4$\sim$10 hours, and 8$\sim$14 hours in the case of 1/20, where the larger initial beach slope is, or the smaller initial wave steepness is, the shorter it is. The measurement of beach profile was made every two hours in the case of 1/20 slope and every one hour in the case of 1/10 slope.

3. Accuracy and reproducibility of experiments

Throughout the whole experiments the measurement of beach profile was made on the center section of the beach 50 cm wide. In order to check the difference of profile between the center section and other sections, longitudinal profile at a point 15 cm right or left from center section was examined, and the comparison of this with the profile of center section is shown in Fig. 2. Fig. 2 (a)

(4) K. Shinohara, T. Tsubaki, M. Yoshitaka and C. Agemori: Sand Transport along a Model Sandy Beach by the Wave Action, Reports of Research Institute for Applied Mechanics, Kyushu University, Vol. 6, No. 21, 1958.

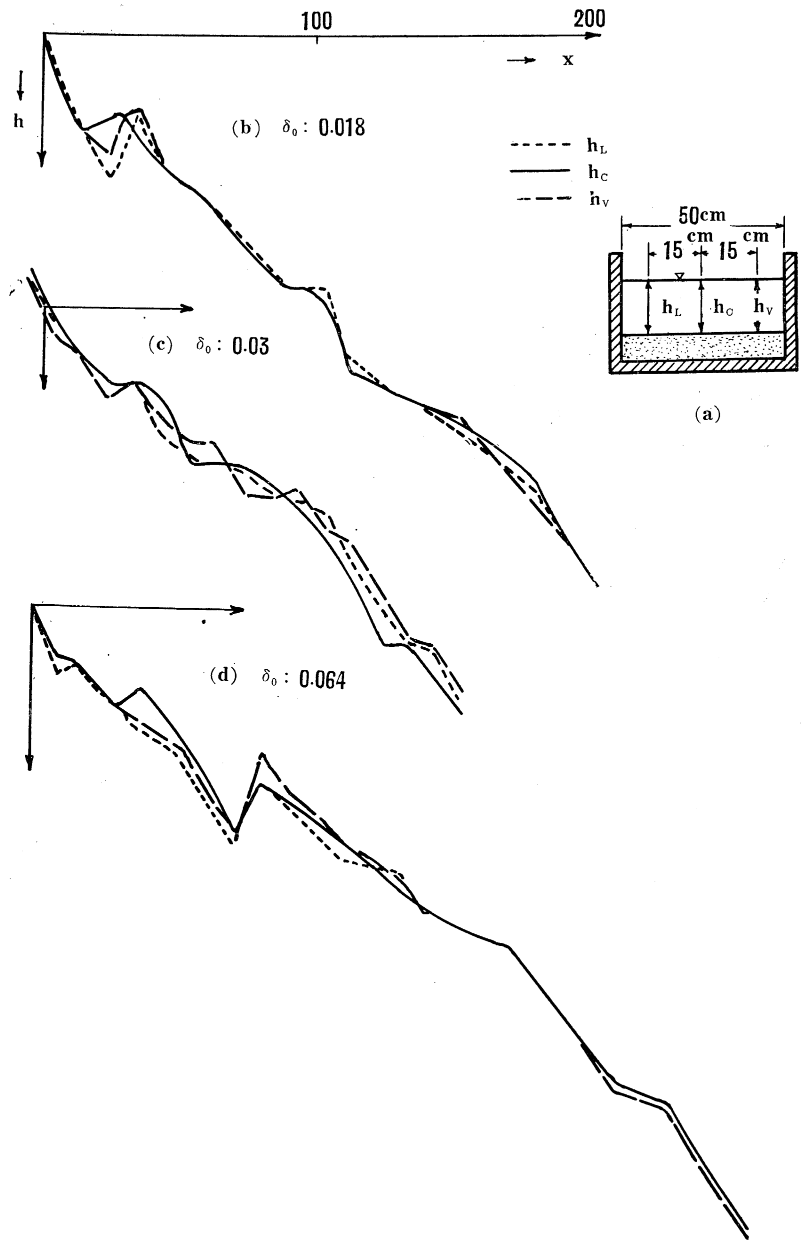

Fig. 2. The accuracy of experiments.

indicates the relative position of the measured section. In the figure, three cases of $\delta_0=0.018$, 0.030 and 0.064 are given. It shows horizontally the distance from the shore line and vertically the depth from the still water surface. According to the figure, the center section does not necessarily give average profile, but seems to be somewhat different from the center section at the section close to the both

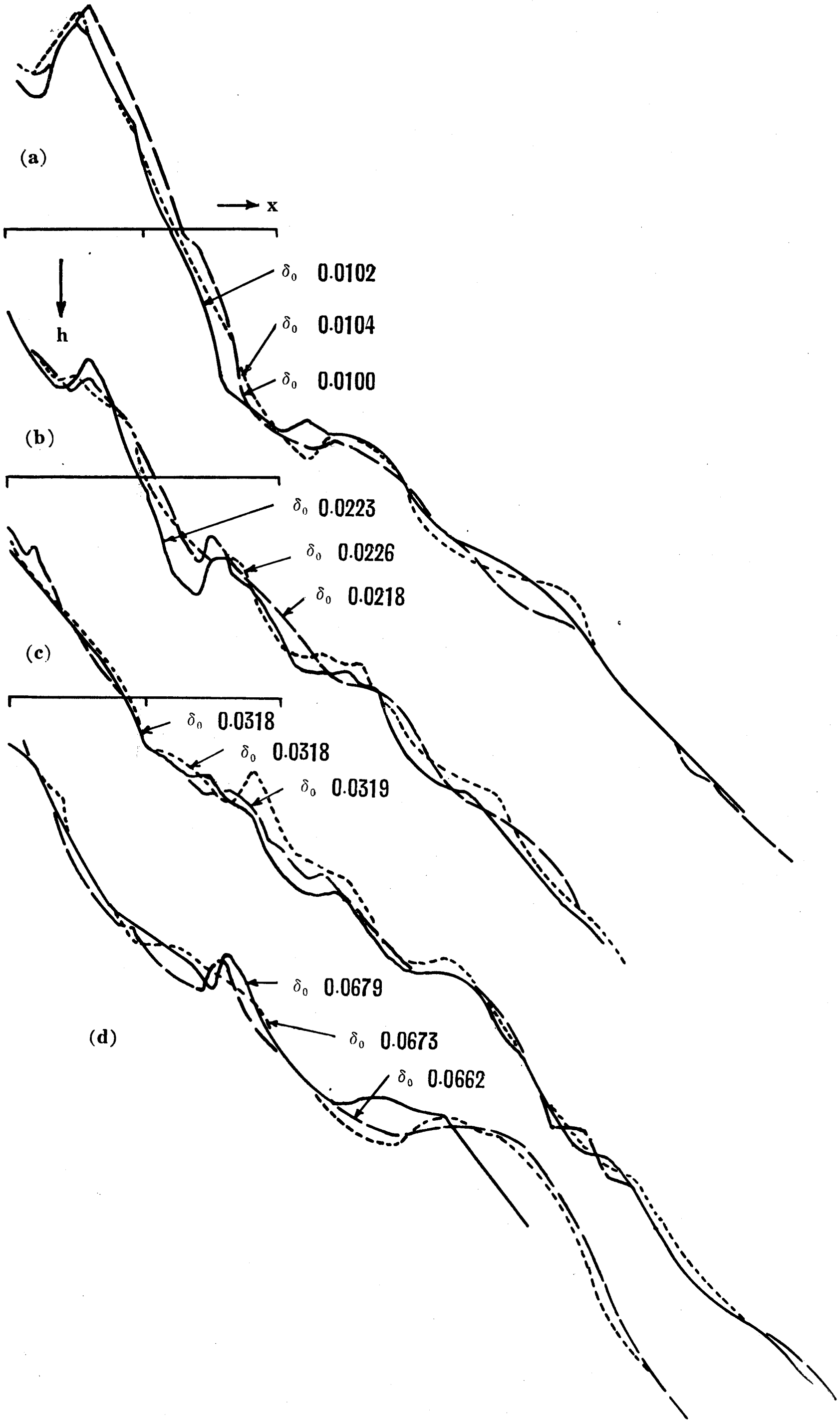

Fig. 3. The reproducibility of experiments.

ends, probably due to the effect of the wall. The discrepancy seems to be large, especially on the onshore part beyond the breaking point. These facts tell us that the profile of sandy beach obtained by the present experiment is not two-dimensional but can give, at some parts, difference about 1~2 cm at most.

While it is difficult to generate waves which are always constant in height and length by our experimental equipment, wave height or wave length seems to change more or less after a duration of its sending over ten hours for some circumstance of the machine. It is therefore necessary in experimenting on the waves of almost equal height and steepness to make sure the degree of similarity of the equilibrium profile obtained or, in short, the reproducibility of experiments.

Fig. 3 shows the reproducibility of experiments, where the comparison is made on the equilibrium profiles obtained when the waves of nearly equal height and steepness are sent in the cases of $\delta_0=0.01$, 0.022, 0.032 and 0.067. It may be said from what is represented in the figure that the difference of the equilibrium profiles due to the discrepancies of wave steepness is more than that resulting from the imcompleteness of reproducibility in the same wave steepness and the reproducibility is almost perfect qualitatively in the case of the same wave steepness. It cannot be said, however, that it is sufficient for the quantitative estimation of the position and the profile of the step or the longshore bar. On the whole, the slope of foreshore is looked upon as having reproducibility.

Though it will hardly be said from the above that the equilibrium profile obtained by experiment is applicable to a real sandy beach due to the waves of the same steepness and H_0/D_0, the profile near the shore line and the characteristics of the beach profile due to the wave steepness are considered to be available.

4. Equilibrium profile in the case when the grain size of bed materials and the initial beach slope are different

What are known as the parameters controlling the profile of a sandy beach by former studies are the properties of deep water wave given by wave length L_0, wave height H_0, and the properties of bed materials of a sandy beach expressed with the representative grain size D_0 (average diameter is generally used) and specific gravity s. Now, taking the x axis offshore with the shore line as the origin and letting the depth from still water surface to bottom surface be h, the equilibrium profile of a sandy beach that is established by the waves of a definite initial steepness ($\delta_0=H_0/L_0$) starting from the uniform initial beach slope I_0 can be expressed by the following functional formula

$$\frac{h}{L_0}=f\left(\frac{x}{L_0},\ \delta_0\ ,\ \frac{H_0}{sD_0},\ I_0\right)\ ,$$

where the specific gravity of bed materials in the water will be used for s. Under the same δ_0, H_0/sD_0 is the parameter related to the ratio of the fluid force caused by the flow due to waves and breakers and the resisting force of bed materials.

Since in former experiments the initial beach slope I_0 was assumed as 1/10 in many cases, the effect upon the equilibrium profile was not clear in the case when I_0 was various. In this experiment, therefore, the comparison has been tentatively made on two cases where I_0 is 1/10 or 1/20.

The effect of grain size of bed materials on the equilibrium profile of a sandy beach has been studied by Dr. Iwagaki *et al.*(5) They carried out the experiment using three kinds of sand of 0.3, 0.52 and 1.0 mm grain size, and made known an interesting result that the position of the longshore bar and the distribution of the rate of sand transport are varied according to the grain size of bed materials. The authors who conducted experiments using sand of two kinds with 0.2 and 0.88 mm average grain size compared the equilibrium profiles whose grain size of bed materials are different.

The range of these experiments are as follows:

No.	D_0(mm)	I_0	δ_0
A 1– 5	0.20	1/10	0.0107–0.0672
A 6–11	0.88	1/10	0.0109–0.0613
A 12–19	0.20	1/20	0.0108–0.0719
A 20–25	0.88	1/20	0.0104–0.0789

The equilibrium profile and sand movement were examined. The equilibrium profile was expressed in dimensionless form where all length were divided by the wave length of deep water L_0. The amount of sand transport was expressed by dimensionless volume of $qs/\sqrt{gH_0^3}$, assuming the amount of sand transport over bottom surface back and forth across the unit width in a unit time as q (gross volume). With the sand collector used for this experiment, some of the sand was observed to pass over the collector, eluding seizure, especially with the 0.2 mm sand, the accuracy of measurement seems to be insufficient.

Fig. 4 (a)~(d) and Fig. 5 (a)~(d) show the equilibrium profile and the amount of sand transport when the waves of $\delta_0 = 0.011$, 0.025, 0.042 and 0.067~0.075 are sent to a sandy beach of $I_0=1/10$ and 1/20 respectively. Again Fig. 6 (a)~(d) are to make the same experimental results clear, especially the effect of the difference of I_0. The results of experiment and the consideration therewith are as follows.

When definite waves of deep water are sent to a sandy beach with the initial beach slope I_0, the sandy beach gradually changes form, approaching to an equilibrium profile corresponding to the characteristics of the waves of given property, the properties of bed materials and the initial beach slope of the sandy beach. The process of the change during the time is somewhat varied according to the grain size and the initial beach slope, as in the following.

(i) In the case of $D_0=0.20$ mm, $I_0=1/10$

When the wave steepness is small, breakers have an action to carry sand on the offshore up to the foreshore from the shore line, in which case, the foreshore part is deposited while the inshore part is eroded. (Fig. 4(a)~(b)). When the wave steepness is large, the foreshore part is eroded by returning flow of breakers, and sand drifts deposit toward offshore from the breaking point into a large-scale longshore bar, assuming an aspect of storm beach. (Fig. 4(c)~(d)). The limit of occurrence of the longshore bar is approximately $\delta_0=0.03$, which perfectly agrees

(5) Y. Iwagaki and T. Sawaragi: On Some Problems about the Equilibrium Slopes of Beaches; Collected Papers Commemorating the 5th Anniversary of the Disaster Prevention Research Institute of Kyoto University, 1956.

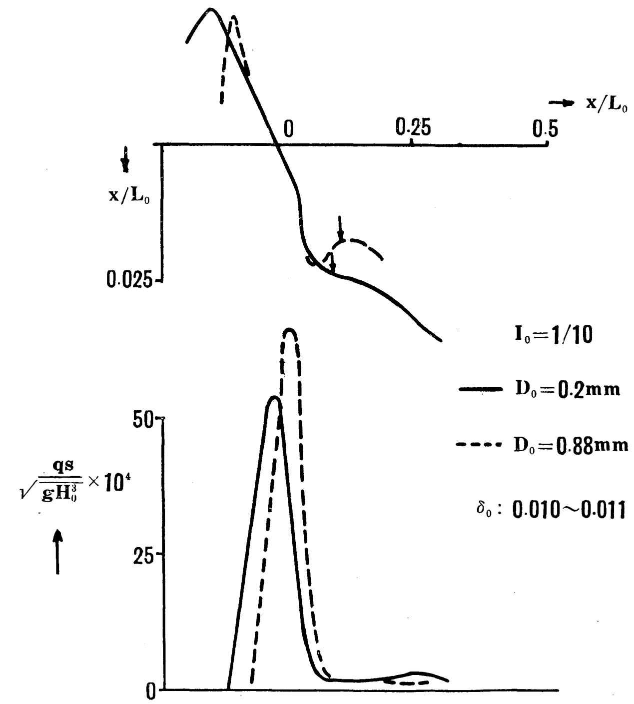

Fig. 4 (a). The equilibrium profile of a sandy beach and the rate of sand transport (a), $I_0=1/10$, $\delta_0=0.01$.

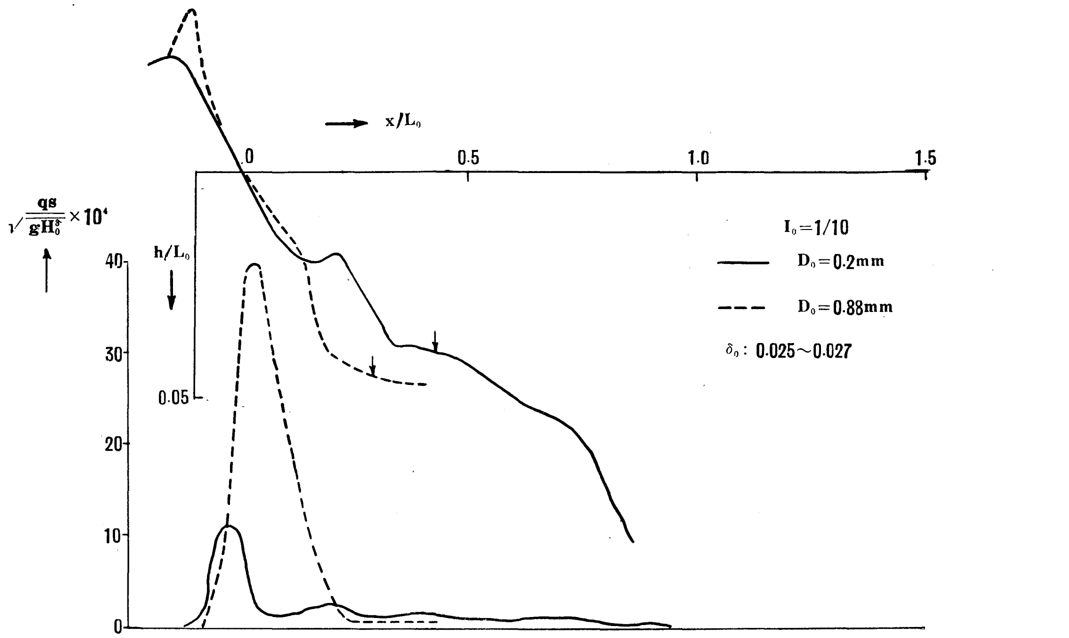

Fig. 4 (b). The equilibrium profile of a sandy beach and the rate of sand transport (b), $I_0=1/10$, $\delta_0=0.026$,

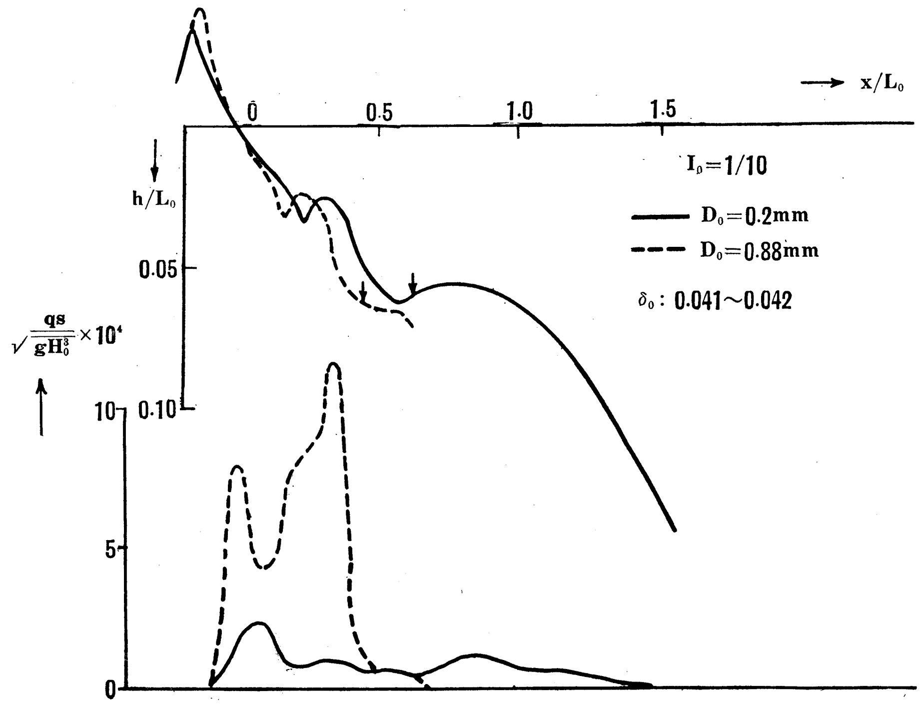

Fig. 4 (c). The equilibrium profile of a sandy beach and the rate of sand transport (c), $I_0=1/10$, $\delta_0=0.04$.

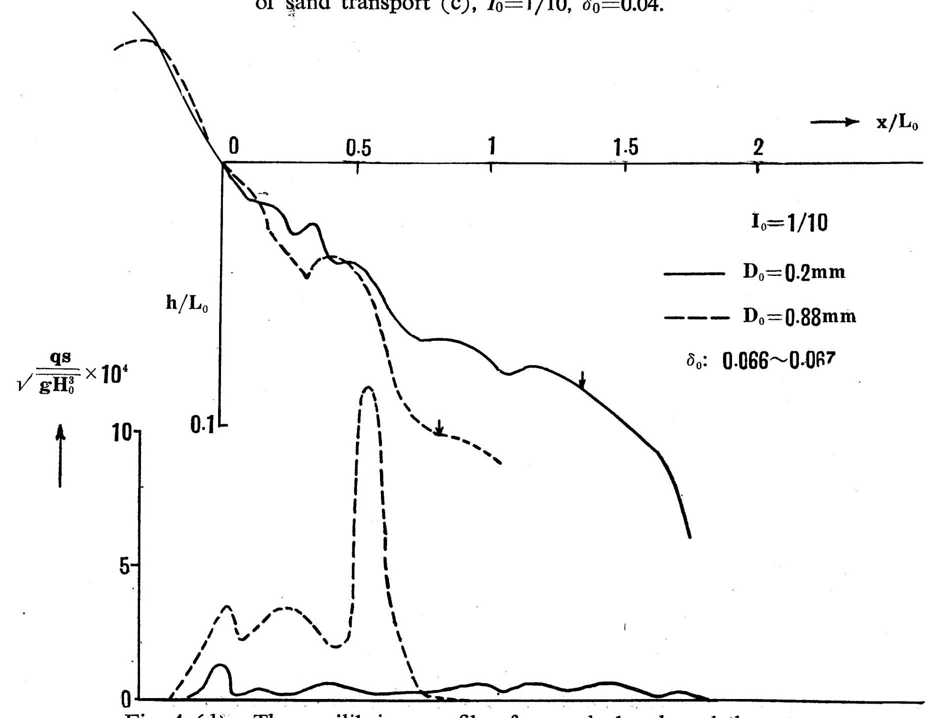

Fig. 4 (d). The equilibrium profile of a sandy beach and the rate of sand transport (d), $I_0=1/10$, $\delta_0=0.066$.

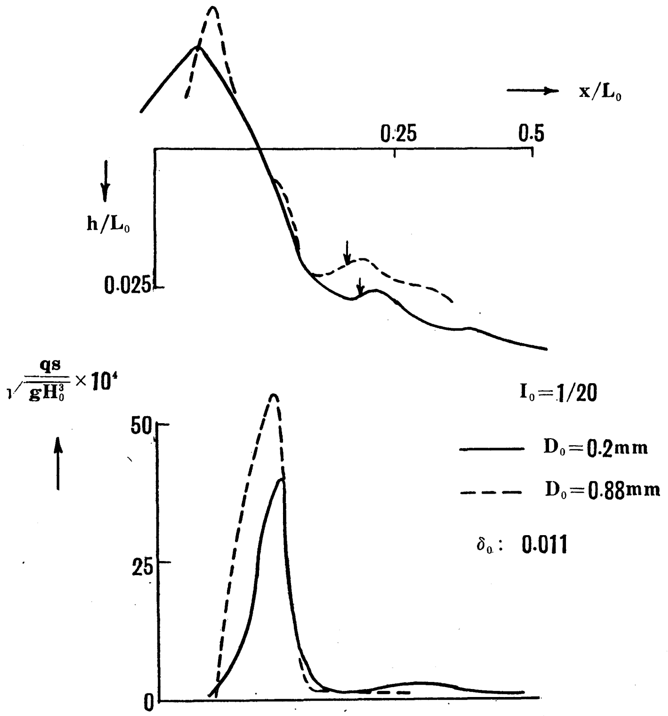

Fig. 5 (a). The equilibrium profile of a sandy beach and the rate of sand transport (a), I_0=1/20, δ_0=0.011.

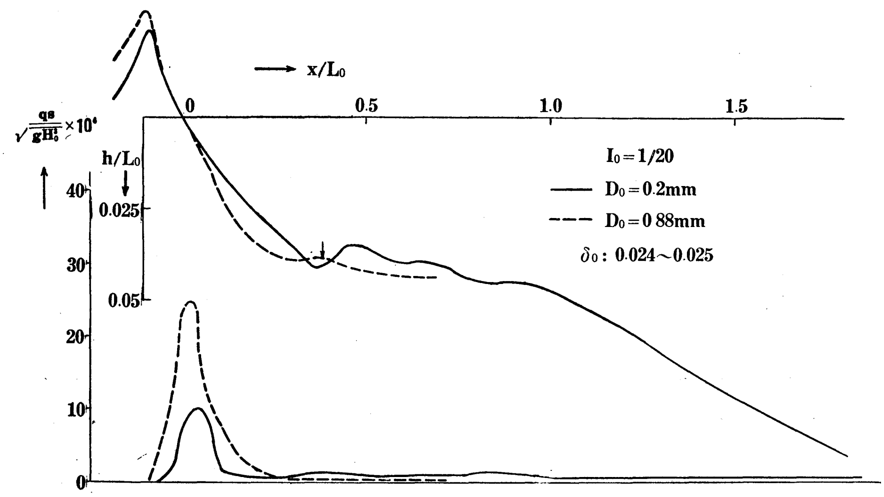

Fig. 5 (b). The equilibrium profile of a sandy beach and the rate of sand transport (b), I_0=1/20, δ_0=0.025.

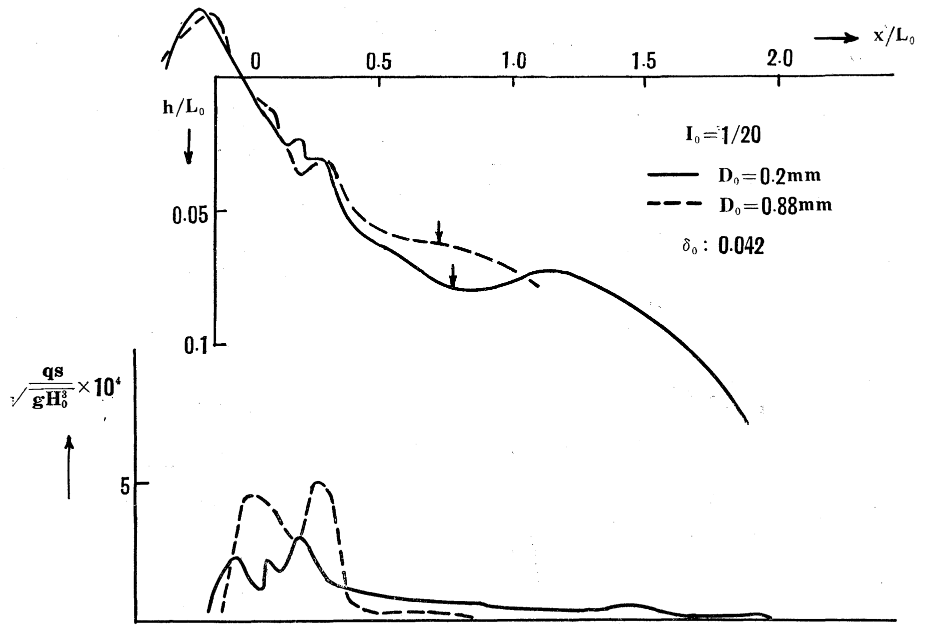

Fig. 5 (c). The equilibrium profile of a sandy beach and the rate of sand transport (c), $I_0=1/20$, $\delta_0=0.042$.

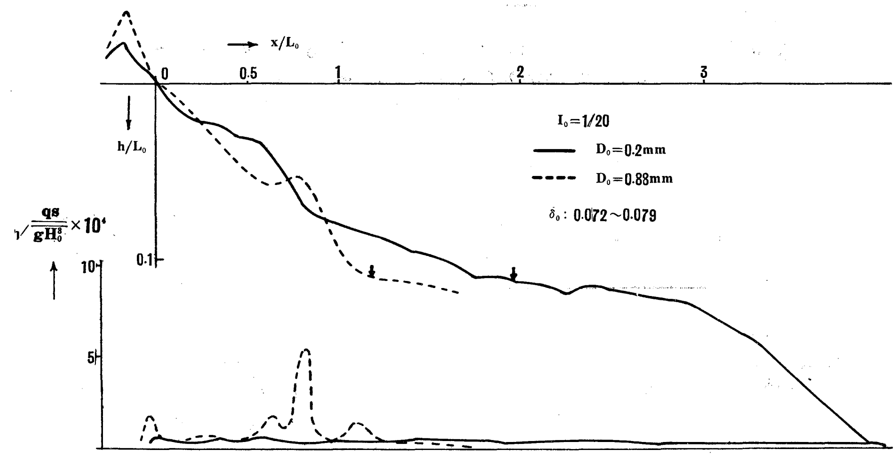

Fig. 5 (d). The equilibrium profile of a sandy beach and the rate of sand transport (d), $I_0=1/20$, $\delta_0=0.07$.

with the results of experiment carried out by Dr. Hamada[6] or Dr.Iwagaki (in the case of 0.3 mm sand). In the case of 0.2 mm sand, sand ripples are formed for all kinds of the initial wave steepness on the bottom surfaces of the inshore and offshore part, and sand grains move periodically in a tractive motion between

(6) T. Hamada: Breakers and Beach Erosion, Report of Transportation Technical Research Institute, Report No. 1, 1951.

sand ripples while considerable suspended sands are observed over the longshore bar.

(ii) In the case of $D_0=0.88$ mm, $I_0=1/10$

When the wave steepness is small, the sand near the breaking point and the inshore part is transported onshore, forming deposits on the foreshore and inshore parts near the shore line. Also in the case of the large wave steepness, this tendency remains the same as a whole, but deposits on the inshore part are reduced and there is formed the longshore bar of small curvature near the foam line between the breaking point and the shore line. (Fig. 4 (c), (d)). The fact that the longshore bar has come nearer to the onshore part than the case of $D_0=0.2$ mm corresponds to the result of experiment made on 0.52 and 1.00 mm sand by Dr. Iwagaki *et al.*

(iii) In the case of $D_0=0.20$ mm, $I_0=1/20$

The movement of sand at the foreshore part is slow in the early period since the initial beach slope is gentle and waves break farther on the offshore side than the case of (i), and the beach deformation is slower than the case of $I_0=1/10$ since the movement range of bed materials is wide. Erosion of a sandy beach and process of deposit are nearly the same with the case of (i) except that deposit formed invariably on the foreshore part for all kinds of initial wave steepness.

(iv) In the case of $D_0=0.88$ mm, $I_0=1/20$

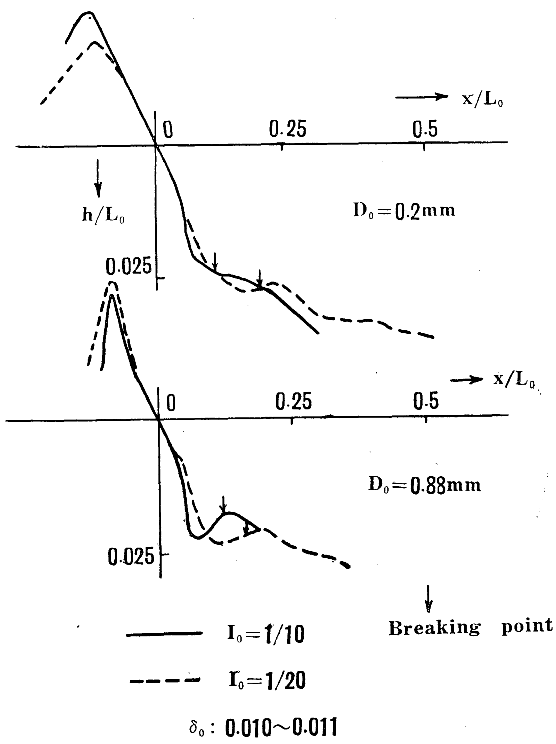

Fig. 6 (a). The effect of initial beach slope (a) ($\delta_0=0.01$).

Though speed of beach deformation is small, the manner of erosion and deposit is almost the same with (ii). Longshore bar does not occur up to $\delta_0=0.032$, as apparent in the experiment of $\delta_0=0.042$. Nor are sand ripples seen to occur while the bottom surface remains smooth.

From the above mentioned results of experiment, the effect of the grain size of bed materials and the initial beach slope upon the equilibrium profile will be put in order as follows.

(A) Effect of grain size of bed materials

As the effect of the difference in the grain size of bed materials upon the equilibrium profile, following matters are observed.

(a) Varying in the position of the breaking point according to the grain size of bed materials, the position of the breaking point occurs on the offshore side in the case of small grain size.

(b) With the bed materials of large grain size, varying in the wave steepness

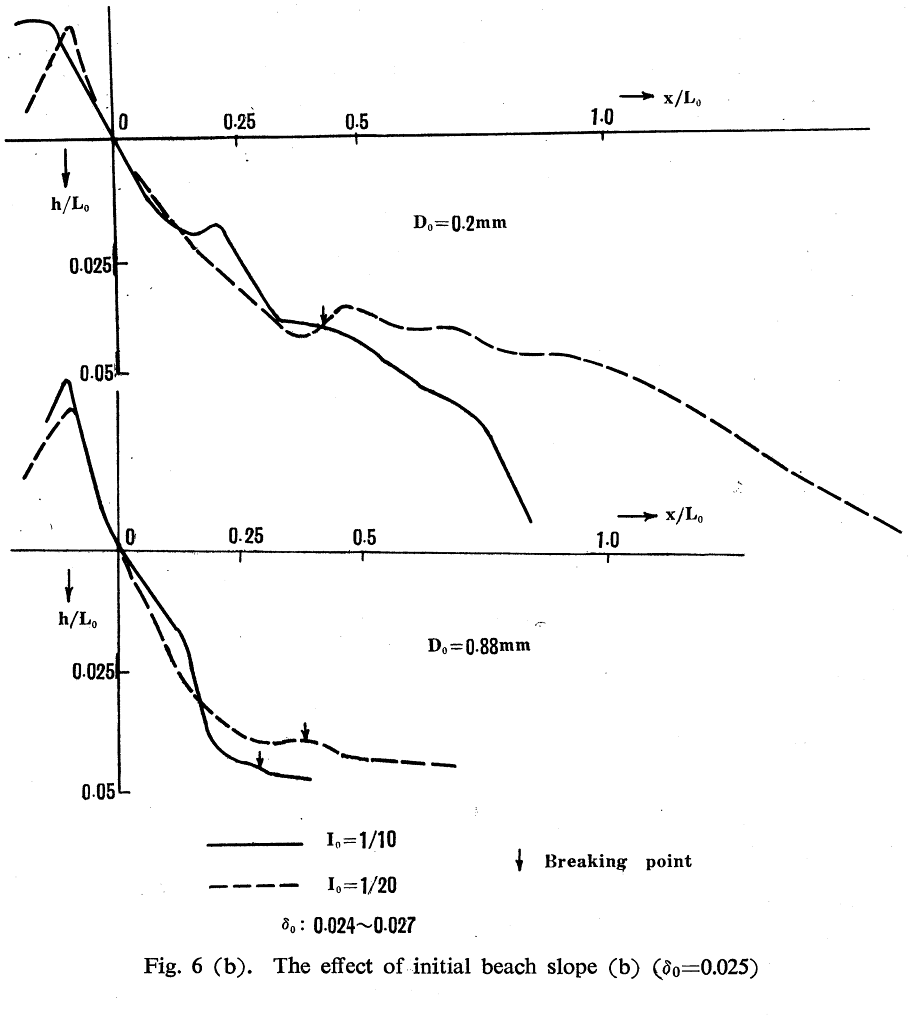

Fig. 6 (b). The effect of initial beach slope (b) ($\delta_0=0.025$)

δ_0 that gives rise to the longshore bar, the effect is not apparent unless δ_0 becomes large.

(c) The range of beach deformation varies by the difference in the grain size of bed materials. Those of large grain size have smaller range of beach deformation.

(d) The profile of the foreshore part is almost definite despite the difference in the grain size.

What is especially noticeable among these is that there is the difference in the position and the scale of longshore bar. Namely, if the wave steepness grows large, the suspended sand is deposited offshore from the breaking point in the case of 0.2 mm sand and local scouring and small bar formation occur near the foam line between the breaking point and shore line. Since these scouring and bar formation are, however, mainly due to the suspension and settling of bed materials by strong turbulence caused by plunging action of breakers, their profiles are extremely unstable. On the other hand, in the case of 0.88 mm sand, no longshore bar is formed on the offshore side from the breaking point since the movement

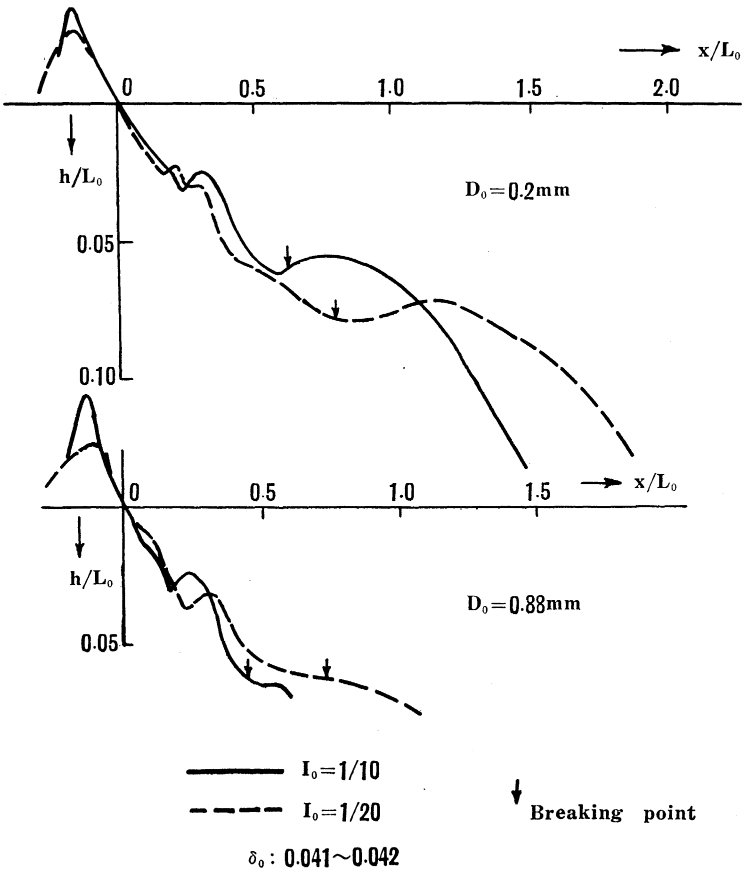

Fig. 6 (c). The effect of initial beach slope (c) (δ_0=0.041).

range of sand is nearly limited to the onshore side from the breaking point and the movement of sand is made by tractive force. But on the inshore part, there is formed a considerably stable bar of small curvature letting strong current due to the plunging action of waves near the foam line cause sand transport of bed load type.

(B) Effect of the initial beach slope (Fig. 6 (a)~(d))

In the case of 0.88 mm sand, the effect of the difference of the initial beach slope on the equilibrium profile is not remarkable, since the movement range of sand is almost limited to onshore from the breaking point. With 0.2 mm sand, however, when the initial wave steepness is large where the movement range of sand is wide, the profile from the shore line to the part near the breaking point is closely resembled, but there is apparent the effect due to the difference of the initial beach slope farther on the offshore side. And it is known that the profile of the foreshore part as well as the inshore part near the shore line hardly suffer

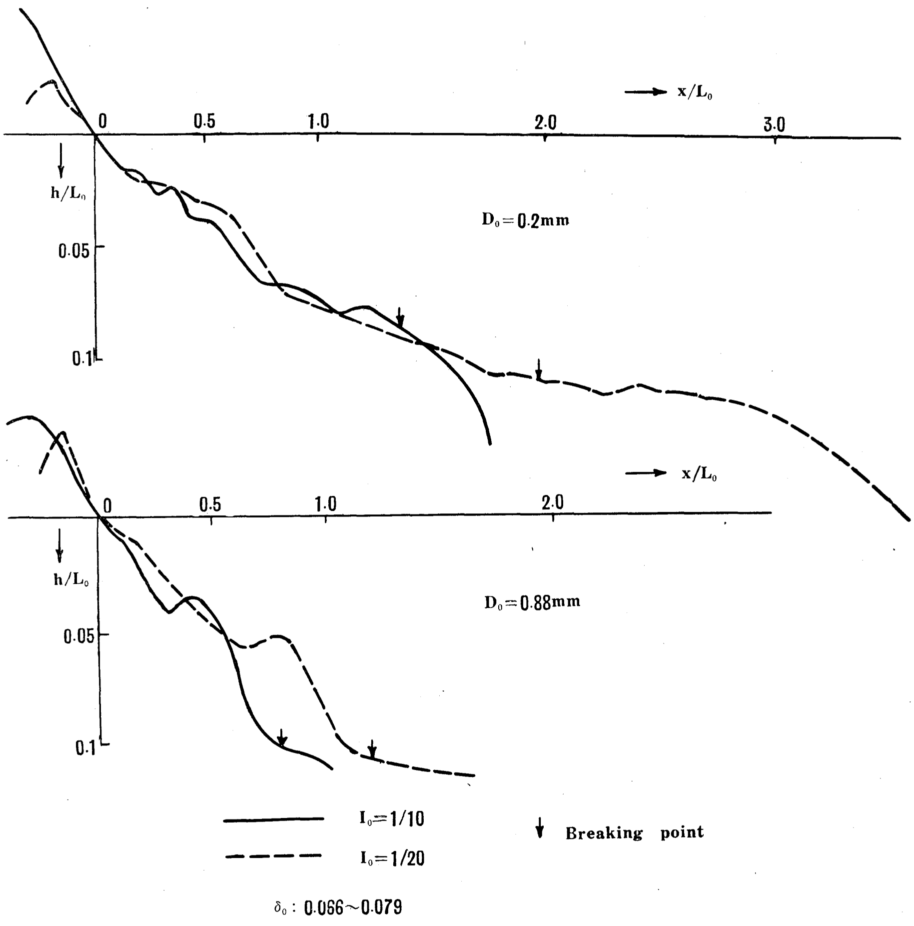

Fig. 6 (d). The effect of initial beach slope (d) (δ_0=0.07).

any effect of the initial beach slope. It is clear from this that when the wave steepness becomes large, the profile of a sandy beach can be divided into two parts, the one independent of the initial beach slope and the other bearing hysteresis effect to the last, and that the effect of the initial beach slope is not to be neglected.

(C) The rate of sand transport (Fig. 4 and 5)

When the wave steepness is small the sand transport on the bottom surface is most abundant near the shore line, in the case of any grain size and initial beach slope, and the movement is also limited to the part near the shore line. If the wave steepness becomes large, the distribution of the rate of sand transport in the case of 0.2 mm sand becomes flat and there is little movement on the bar, though the movement between the shore line and the breaking point, especially that of the inshore, is conspicuous. In the case of 0.88 mm sand, the movement of sand is remarkable on the bar between the breaking point and the shore line as well as near the shore line. This condition remains unchanged even if the initial beach slope is different. Showing now the relation between the rate of the sand transport on the shore line and the wave steepness, the result will be as shown in Fig. 7. It is seen from the figure that $qs/\sqrt{gH_0}^3$ representing the rate of the sand transport rapidly decreases with the increase of δ_0.

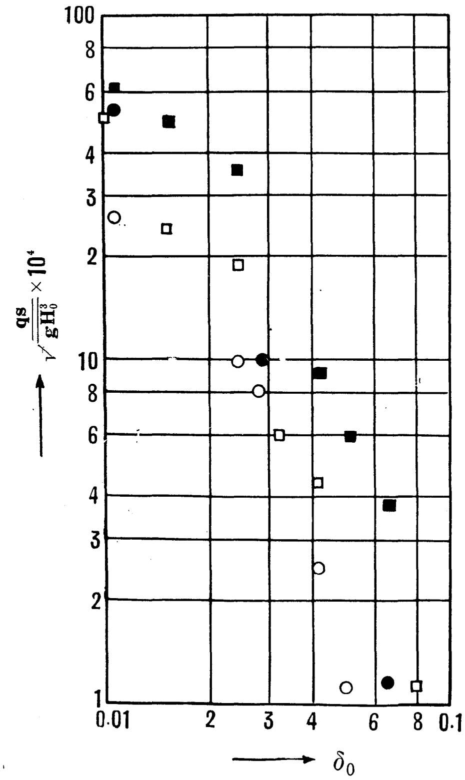

Fig. 7. The relation between the rate of sand transport on the shoreline and the wave steepness.

5. The deformation of a sandy beach and the amount of the movement of shore line

When a sandy beach that has the original profile established by definite waves with wave height H_1, wave length L_1, therefore initial wave steepness $\delta_1 = H_1/L_1$ is newly subjected to attack of the waves with characteristics H_2, L_2 and δ_2, the sandy beach is deformed and the shore line moves generally. Now taking the x axis toward offshore with the shore line of the original beach profile as the

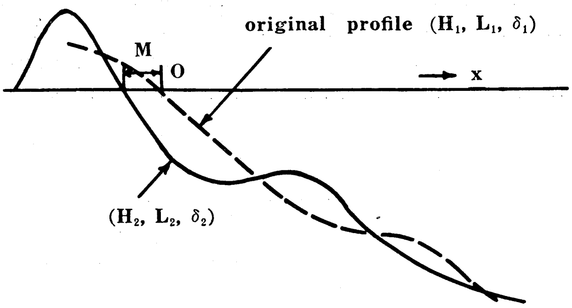

Fig. 8.

origin and expressing the original profile of a sandy beach non-dimensionally and then representing similarly the equilibrium profile to be formed by the waves newly sent in non-dimensional form, the result will be as shown in Fig. 8. M denotes the amount of the movement of shore line of the new sandy beach from the shore line of the original profile.

With a view to investigating how the equilibrium profile or the amount of the movement of shore line made by the subsequent attack of the waves of δ_2 is

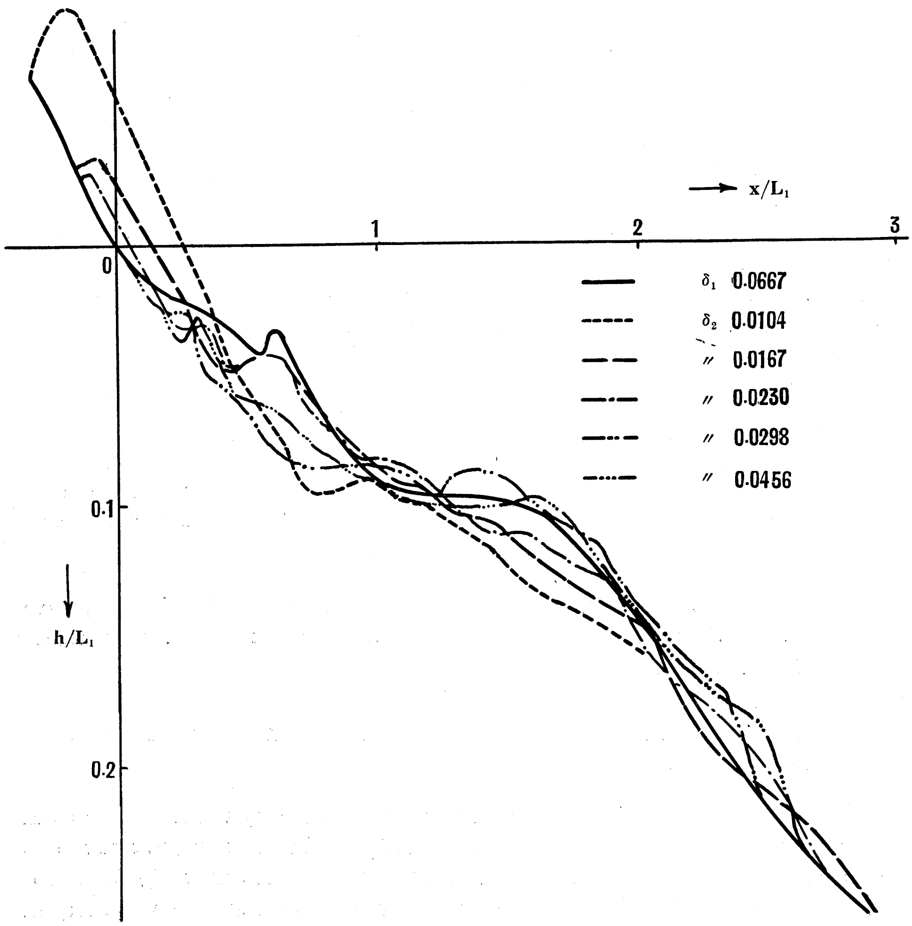

Fig. 9 (a). The equilibrium profile of a sandy beach (case of $\delta_1 > \delta_2$).

affected by the original equilibrium profile that had been established by the waves of δ_1, the experiments were carried out in three cases of (i) $\delta_1 > \delta_2$, (ii) $\delta_1 \lesssim \delta_2$ and (iii) $\delta_1 < \delta_2$.

As for the method of experiment, a sandy beach of initial slope 1/10 was molded with 0.2 mm sand and the equilibrium profile was formed by sending the wave of δ_1 (wave height H_1, wave length L_1). Then the wave of δ_2 (wave height H_2, wave length L_2) was sent and the profile was measured hourly until the beach profile seemingly attained nearly the equilibrium profile. Then the deformation of beach profile and the amount of movement of shore line were examined. The range of experiments was as follows;

No.	δ_1	H_1(cm)	δ_2	H_2(cm)
B 1– 5	0.0649–0.0679	5.07–5.59	0.0104–0.0456	3.52–5.02
B 6–10	0.0100–0.0115	3.29–3.96	0.0167–0.0666	3.95–5.00
B 11–14	0.0148–0.0206	3.71–4.31	0.0101–0.0647	3.75–5.77
B 15–20	0.0183–0.0230	3.52–4.51	0.0086–0.0598	3.23–5.08
B 21–25	0.0304–0.0324	4.90–5.15	0.0090–0.0698	3.42–5.40

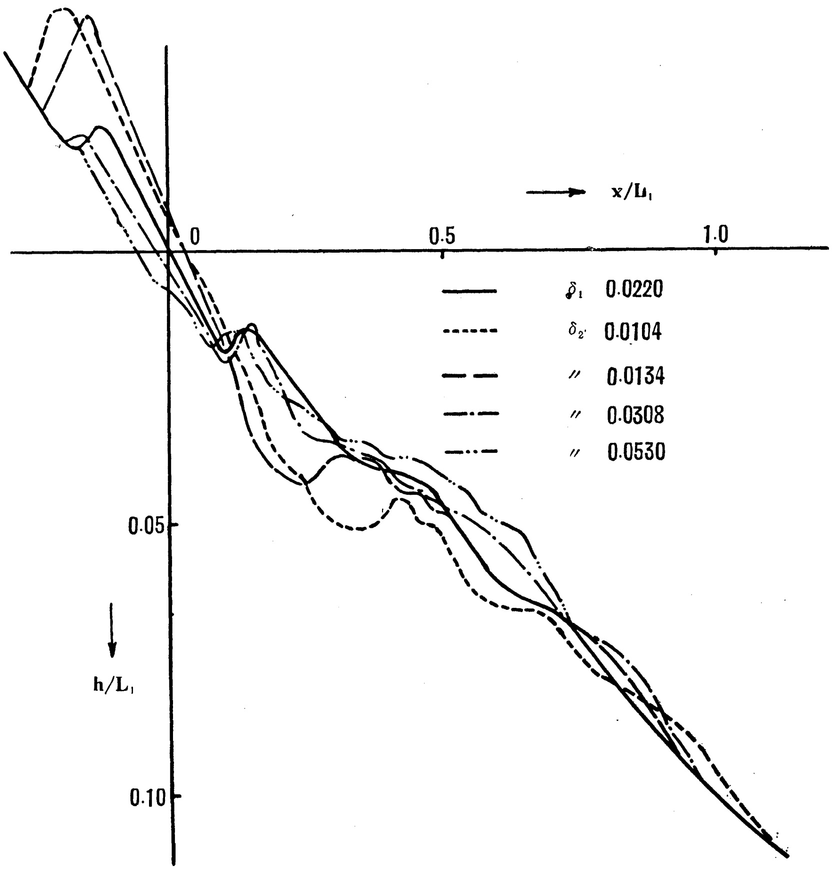

Fig. 9 (b). The equilibrium profile of a sandy beach (case of $\delta_1 \gtrless \delta_2$).

As seen from the above table where experiments have been made on one value of δ_1 by changing the value of δ_2 4~6 times, the value of δ_1 varies more or less, it being difficult to generate waves of the same wave height and wave length every time. As a result, the original profiles for δ_1 which are assumed as equivalent show a little difference at each experiment. As to the reproducibility and accuracy of experiments in such cases, there is no qualitative difference as mentioned already in §3 and the results seem to serve the object of experiments sufficiently.

Among the results of experiments, the one from experiment number B 1~5 is given in Fig. 9(a) as an example of $\delta_1 > \delta_2$ and the other from experiment number B 15~20 is given in Fig. 9(b) as an example of $\delta_1 \lesseqgtr \delta_2$ and then in Fig. 9(c) the result from experiment number B 6~10 is given as an example of $\delta_1 < \delta_2$. Fig. 9(a) represents the case when the storm beach with large δ_1 is subjected to attack of waves of initial steepness smaller than it, where the sand transport is deposited near the shore line, the shore line advanced and the longshore bar toward offshore from the breaking point is eroded. The degree is more conspicuous the

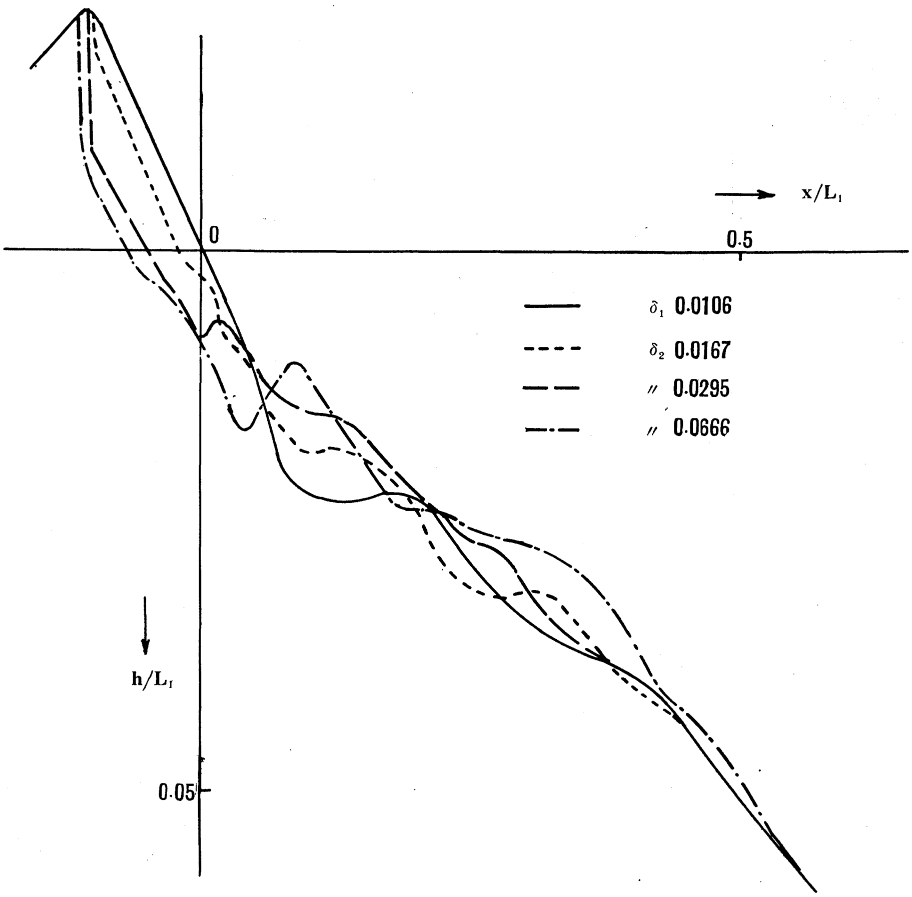

Fig. 9 (c). The equilibrium profile of a sandy beach (case of $\delta_1 < \delta_2$).

smaller initial wave steepness. In the case of δ_2 greater than $\delta_2 \doteqdot 0.03$, no change is observed on the foreshore and the shore line, and the surf zone between the breaking point and shore line is somewhat eroded, forming deposit toward offshore from the breaking point. This is chiefly due to the movement of the small bar (step) that had been formed near the foam line in the original profile. On the offshore side from the breaking point, there is apparently no conspicuous change in the beach profile.

Fig. 9(b) shows the case when a sandy beach which is nearly in the interim between storm beach and normal beach is subjected to attack of new waves, where in the case of $\delta_1 > \delta_2$, deposit is formed on the foreshore part and the shore line advances. Erosion is also observed on the foam line and the inshore part. If δ_2 surpasses 0.03 (in the case of $\delta_1 < \delta_2$), there is observed erosion near the shore line as well as the retreat of shore line and the formation of sand bar on

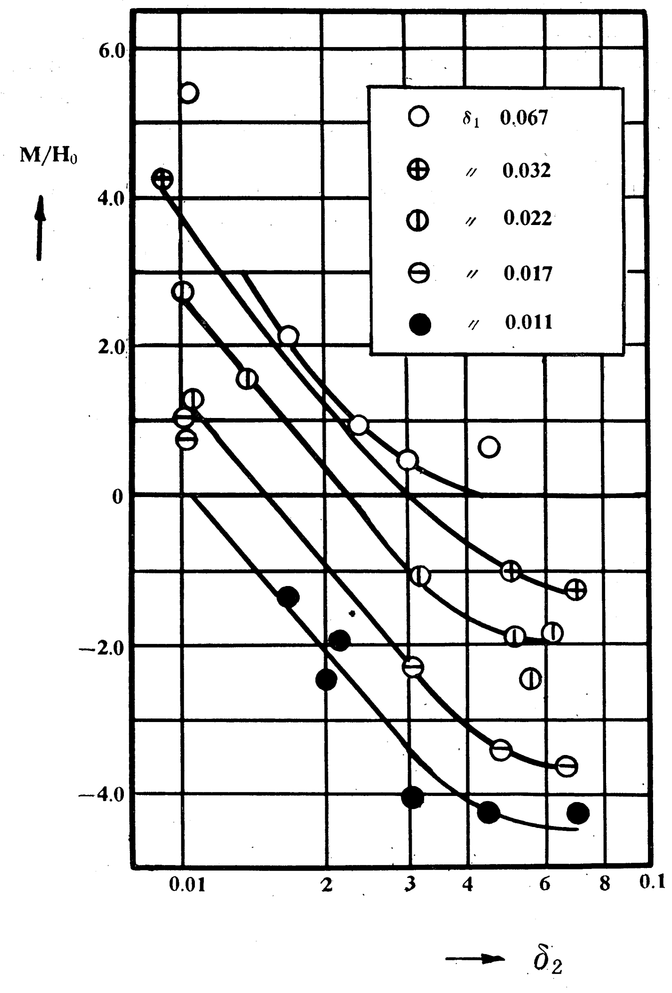

Fig. 10. The relation between the amount of movement of shoreline and the wave steepness.

the inshore part.

Fig. 9(c) is the case when a sandy beach formed by the waves of $\delta_1=0.01$ is subjected to attack of the waves with initial wave steepness larger than this, where there is observed erosion near the shore line and the retreat of shore line. Again, the formation of a sand bar on the foam line and the inshore part can be observed.

Thus, whether erosion or deposit occurs near the shore line or the shore line retreats or advances depends on the relation between the initial wave steepness corresponding to a sandy beach formed in the beginning and that of the waves attacking subsequently and will not be determined only by the initial wave steep-ness of the waves attacking newly. Same thing will be also said about the profiles of surf zone or inshore part.

From these experimental results, expressing the relation between M/H_2 and the wave steepness (δ_2) of the waves attacking subsequently with the wave steepness to the original beach profile (δ_1) as a parameter, Fig. 10 will be obtained. In the figure, $M/H_2>0$ is the case in which the shore line advances while $M/H_2<0$ is the case in which the shore line retreats. It is seen from the above that the ad-vance or retreat of the shore line is not decided by δ_2 but caused by $\delta_2 \lesseqgtr \delta_1$. On the other hand, while M/H_2 changes nearly linearly for log δ_2 within the range of $\delta_2 \leqq 0.03$ when δ_1 is definite, the rate of change slacks if δ_2 surpasses 0.03, and in the case of δ_2 larger than 0.04 the value of M/H_2 will be probably looked upon as nearly independent of δ_2. It is of much interest to note that such difference observed in the change of shore line with $\delta_2=0.03$ as the border line agrees well

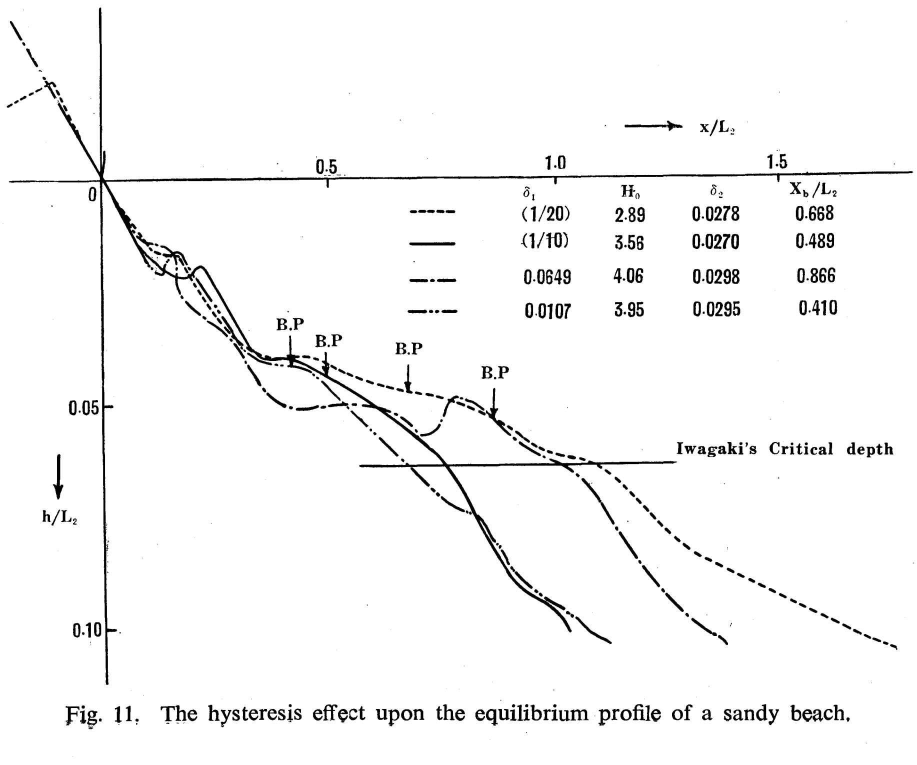

Fig. 11. The hysteresis effect upon the equilibrium profile of a sandy beach.

with what is called by Johnson as "the boundary between the storm, and normal beaches".

Fig. 11 is an example in which is shown the shore line of new beach profile taking the x axis offshore in order to investigate a hysteresis effect of the original profile upon the beach profile, where the equilibrium profile in the cases of initial beach slope 1/10 and 1/20 is also given for reference. In these cases, the equilibrium profile was made by sending the waves of δ_2 after having established the equilibrium profile by sending the waves of δ_1 to a sandy beach with the initial beach slope 1/10. In Fig. 11, the case of δ_2 being 0.03 is shown. It is seen from the figure that, in case of $\delta_1>\delta_2$, the original profile remains toward offshore from the breaking point. The critical depth for sediment transportation[7] that was found by former researches is in the region subjected to the hysteresis effect of original profile almost invariably. It is therefore difficult to adopt the critical depth for sediment transportation as the basic point of the beach deformation in estimating the equilibrium profile of a sandy beach corresponding to certain waves. From the fact that the deformation of a sandy beach due to waves approaches from the foreshore part to the equilibrium profile, it seems more reasonable to adopt the position of shore line for the basic point of beach profile. That is, it will be possible to estimate roughly the beach profile such that the amount of movement from the shore line of original beach profile M is evaluated from Fig. 9 and the beach profile is drawn up to the breaking point by the method of equilibrium beach slope. Though the part toward offshore from it is not clear being subjected to the hysteresis effect, it will also be possible to estimate the beach profile in reference to the fact that the amount of erosion and deposit is proved to be equivalent as well as the difference in the degree of the development of sand deposit due to the value of initial wave steepness and the critical depth of beach deformation to be mentioned later.

6. The characteristics of the equilibrium profile of a sandy beach

As the parameters expressing the characteristics of the equilibrium profile of a sandy beach, the slope of foreshore, the wave run-up, the depth of neutral point of beach deformation, the critical depth of beach deformation and the critical depth for sediment transportation will be taken up for consideration.

Fig. 12 shows the relation between the slope of foreshore (I_u) and the initial wave steepness (δ_0) where are given average values of the experiment made on 0.3 and 1.0 mm sand grains by Dr. Iwagaki *et al.* Though scattering of points is remarkable, the slope of foreshore decreases with increase of δ_0 and it is slightly larger as the grain size of bed materials becomes larger.

Expressing the wave run-up to a sandy beach R in non-dimensional form with the wave height H_0 and plotting the relation between R/H_0 and δ_0, the result will be as shown in Fig. 13. Since the slope of foreshore is nearly 1/5 and that of surf zone is about 1/5~1/10, there is also given the result of experiments in which R.P. Savage[8] investigated the wave run-up to the slope of smooth surface

(7) The same with (3) and (4).
(8) R.P. Savage: Wave Run Up on Roughened and Permeable Slopes ; Proc. A.S.C.E, Paper No. 1640, 1948.

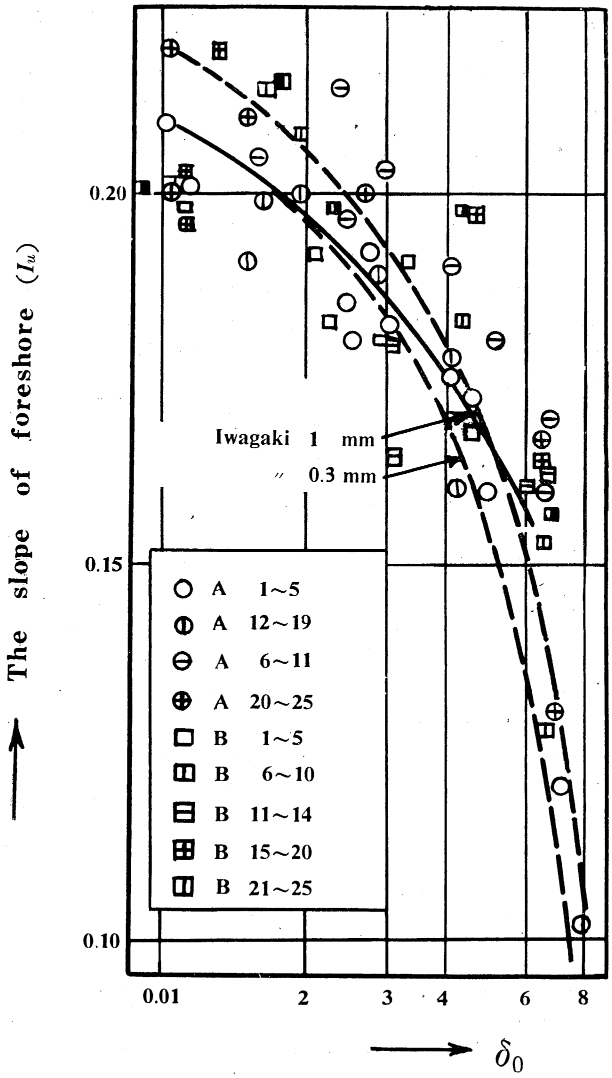

Fig. 12. The relation between the slope of foreshore (I_u) and δ_0.

of 1/5 and 1/10. As clear from the figure, the wave run-up in the case of the equilibrium profile lies midway of the slope of 1/5 and 1/10 of Savage's experiment, but the relation between R/H_0 and δ_0 is considerably different by the effect from the slope near the foreshore as compared with the case of uniform beach slope. The fact that it comes close to the case of 1/10 slope when δ_0 is large and in the case of small δ_0 it is nearer to the case of 1/5 slope is clear from the fact that the slope of foreshore approaches to 0.1 in the case of large δ_0 and in the case of small δ_0, it is nearer to 0.2. It is also known that the relation of $R/H_0 \sim \delta_0$ is almost independent of the difference in the grain size of bed ma-

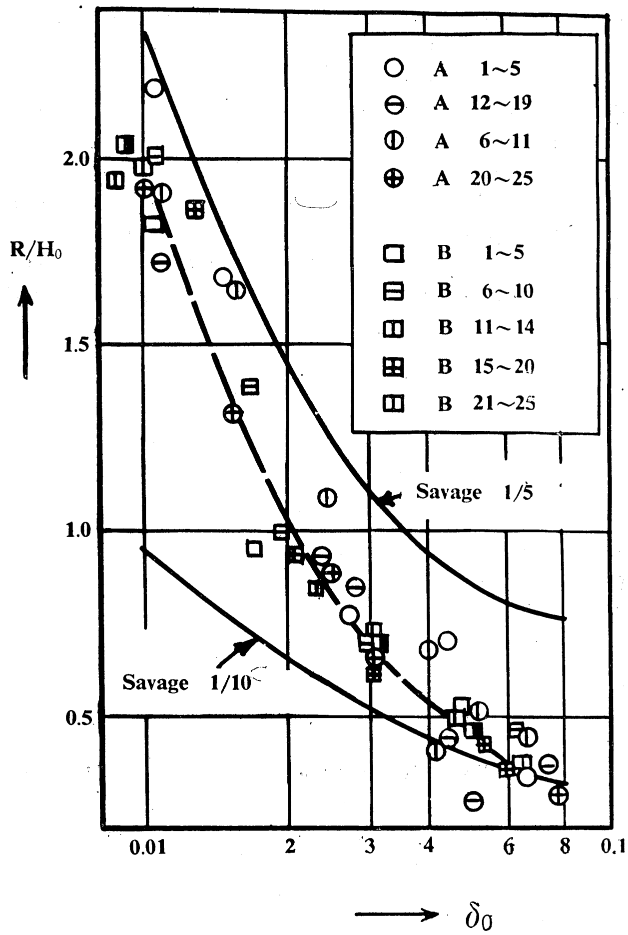

Fig. 13. The relation between R/H_0 and δ_0.

terials. The result of experiment formerly done by one of the authors[9] also tells that R/H_0 is less affected by the degree of roughness of slope than the angle of the slope, which is also proved by the result of this experiment.

Comparing the original profile of a sandy beach with the equilibrium profile formed by the subsequent waves, there is found a point dividing the part of deposit and that of erosion near the breaking point. This will be tentatively called as the neutral point of beach deformation. Also in the equilibrium profile, the point which is nearest to the shore line among the parts where no deformation has taken place since the time of its original profile will be called the critical point of beach deformation. The relation between h_0/L_0, h_c/L_0 and the initial wave steepness δ_0 in which the depth of neutral point of beach deformation h_0 and

(9) K. Shinohara: Studies on the Characteristics of Breakers and Wave Run Up on the Slope; Memoirs of the Research Institute for Applied Mechanics, Kyushu University, No. 11, 1958.

the critical depth of beach deformation h_c are expressed non-dimensionally by dividing them by the wave length will be plotted as shown in Fig. 14. In it are shown Breaker Index and the critical depth for sediment transportation of 0.3 mm sand by Dr. Iwagaki *et al.* for comparison. It is known from the above figure that h_0/L_0 bears a definite relation with δ_0, being a little offshore side than Breaker Index and onshore side more than the critical depth for sediment transportation of Dr. Iwagaki *et al.* On the other hand, there is observed no effect due to the difference in the original profile in the relation of $h_0/L_0 \sim \delta_0$.

The relation between h_c/L_0 and δ_0, as seen from the figure, shows difference in the case when the original profile is the uniform slope beach such as 1/10 or

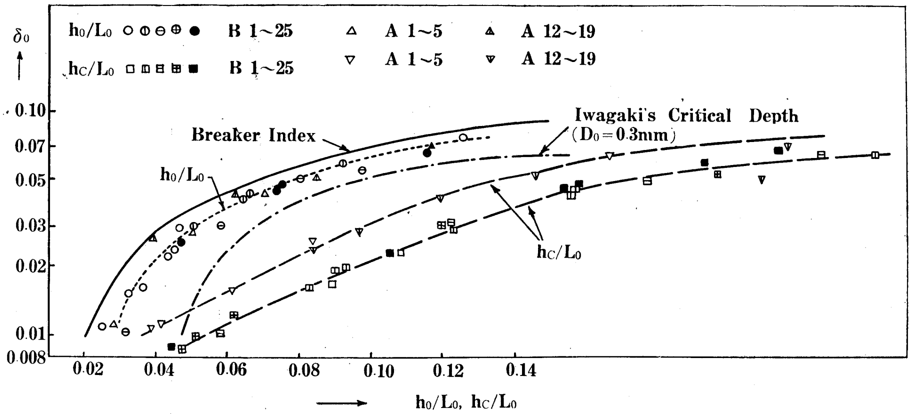

Fig. 14. The relation between h_0/L_0, h_c/L_0 and δ_0 (D_0=0.2 mm).

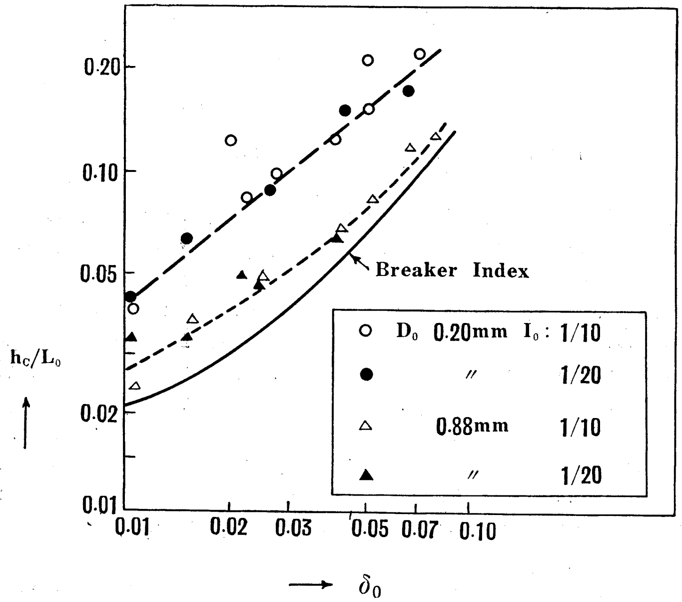

Fig. 15. The relation between h_c/L_0 and δ_0 (effect of grain size of sand).

1/20 and that when it is an arbitrary profile. Namely, h_c/L_0 is smaller when start is made from the uniform slope for the same δ_0, or in short, the range of sand movement is smaller. This might be ascribed to the reasons that the resistance of bottom surface is increased by sand ripples formed on the bottom surface by preceding waves and that sand is induced to get in suspension due to the eddies raised on the bottom surface by sand ripples. The critical depth for sediment transportation determined from the distribution of the rate of sand transport along the bottom surface of equilibrium profiled sandy beach by Dr. Iwagaki *et al.* is smaller than the critical depth of beach deformation.

In Fig. 15 the comparison is made on the relation between h_c/L_0 and δ_0 in the case where 0.2 and 0.88 mm sand was used, by which it is known that this relation varies according to grain size and that h_c/L_0 becomes smaller when grain size is large for the same δ_0. On the other hand, there is apparent no effect from the difference in the initial beach profile.

Fig. 16 gives the relation of h_c/L_0 with the non-dimensional parameter $H_0/T\sqrt{sgD_0}$ that controls the limit of movement of bed materials proposed by the authors before.(10) As clear from the above figure, there is a definite relation between h_c/L_0 and $H_0/T\sqrt{sgD_0}$, in which any effect of difference in the initial beach slope and the original profile is hardly apparent. Consequently, it is considered that the non-dimensional parameter $H_0/T\sqrt{sgD_0}$ is more excelled than δ_0 as the parameter that forms connection with h_c/L_0. It seems, however, difficult to eliminate the effect due to the difference in grain size of bed materials even by using this expression, which might be due to the difference in the transport mechanism of bed materials.

Among the characteristics of the equilibrium profile, what is most significant and interesting from practical viewpoint is that the slope of foreshore, wave run-up and neutral point of beach deformation almost depend on the initial wave

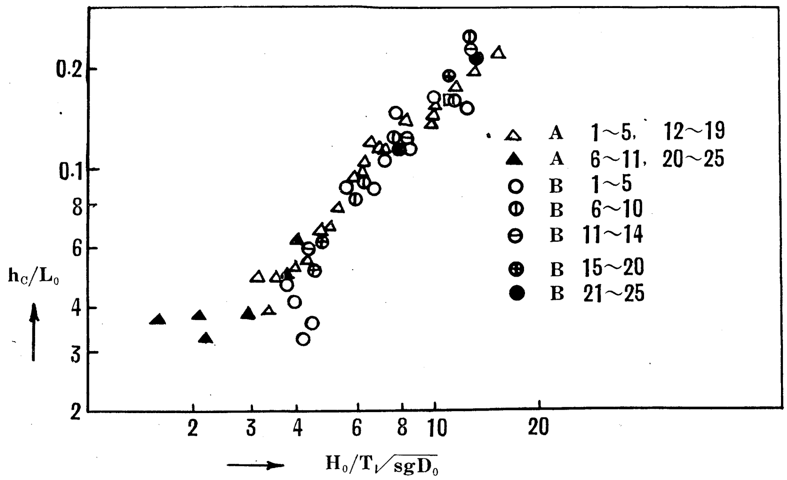

Fig. 16. The relation between h_c/L_{c0} and $H_0/T\sqrt{sgD_0}$.

(10) The same with (4).

steepness, independently of bed materials and hysteresis of beach profile.

7. Conclusion

Models of a sandy beach with the beach slope of 1/10, 1/20 and other profiles were made using two kinds of sand, with average grain size of 0.2 and 0.88 mm and by sending it waves of definite steepness, the equilibrium profile of the beach obtained as well as the rate of bed materials transported and the characteristics of the profile were examined. The result of this experiment will be summed up as follows.

(1) The reproducibility of the experiment on the equilibrium profile of a sandy beach is said to be almost perfect qualitatively, but for any quantitative expressions of the position, profile and others of the offshore side from the surf zone, for instance, step or longshore bar, it cannot be said to be sufficient. The reproducibility is found, even quantitatively, in the foreshore, especially near the shore line.

(2) The profile of the foreshore can be determined by the wave steepness independently of grain size of bed materials, initial beach slope and original beach profile.

(3) The position of the breaking point and the range of beach deformation vary according to the grain size of bed materials, and the one with large grain size has small range of deformation. Also, unless the initial wave steepness becomes large, the longshore bar does not appear.

(4) Since the beach profile toward offshore side from the surf zone is subject to the effect of grain size of bed materials and the hysteresis of the original beach profile, it is not possible to estimate the beach profile by the equilibrium beach slope method with this point as the basic point even though the critical water depth for sediment transportation can be defined by the wave steppness.

(5) Since the amount of movement of the shore line is determined by the relation between the initial wave steepness corresponding to the original profile and that of newly incoming waves, a rough estimation of the beach profile can be done by making the new shore line as the basic point and by evaluating the amount of shore line movement from this relation.

(6) As to the characteristics of the equilibrium profile

(i) The slope of foreshore I_u decreases with the increase of the initial wave steepness. The larger the grain size of bed materials is, the larger I_u becomes, even slightly.

(ii) The rate of R/H_0 obtained by expressing non-dimensionally wave run-up R for a sandy beach by the wave height H_0 has a definite relation with δ_0 and is reduced as δ_0 increases. The difference in the grain size of bed materials gives no effect to this relation.

(iii) Dimensionless parameters h_0/L_0 and h_c/L_0 of the depth of the neutral point of beach deformation h_0 and of the critical depth of beach deformation h_c have a definite relation with δ_0, provided that the relation of h_c/L_0 and δ_0 is various according to the original beach profile and the grain size of bed materials. h_c/L_0 can be connected well with non-dimensional parameter $H_0/T\sqrt{sgD_0}$, and this relation is independent of the difference in the initial beach slope and the original beach profile. The effect of the difference of the grain size of bed materials, however, is apparent.

These points are the conclusions drawn within the range of the present experiment. The deformation of actual seashore is, however, so complicated that the domain of actual seashore cannot be covered by the limited scale of laboratory experiment so far as the parameter H_0/sD_0 controlling the beach profile is concerned. The authors wish to pursue further the connection between experimental result and actual case (the problem of law of similarity) in cooperation with the observation of actual seashore.

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Appendix 1. Experimental Data (1)

No.	D_0 mm	I_0	δ_0	T_1 sec	L_1 cm	H_1 cm	I_u	h_c cm	R cm
A 1	0.2	1/10	0.0107	1.15	206.5	2.22	0.240	8.5	4.85
A 2	〃	〃	0.0152	1.011	160	2.42	0.210	9.9	4.03
A 3	〃	〃	0.0270	0.920	132	3.56	0.200	11.1	2.68
A 4	〃	〃	0.0433	0.847	112	4.85	0.110	16.5	2.0
A 5	〃	〃	0.0672	0.759	90	6.05	0.167	15.0	2.0
A 6	0.88	〃	0.0104	1.14	205	2.13	0.200	6.7	4.07
A 7	〃	〃	0.0151	1.074	180	2.72	0.190	5.9	3.53
A 8	〃	〃	0.0247	0.890	123.5	3.05	0.180	5.8	2.7
A 9	〃	〃	0.0317	0.834	109	3.46	0.210	5.4	2.25
A10	〃	〃	0.0417	0.741	85.5	3.57	0.160	—	1.4
A11	〃	〃	0.0789	0.757	89.5	7.06	0.103	11.0	1.9
A12	0.2	1/20	0.0108	1.105	190	2.06	0.200	7.7	3.5
A13	〃	〃	0.0242	0.949	141	3.41	0.185	11.9	3.9
A14	〃	〃	0.0278	0.836	109.1	2.89	0.192	10.6	2.4
A15	〃	〃	0.0303	0.815	104	3.15	0 220	14.4	—
A16	〃	〃	0.0415	0.722	81.5	3.38	0.178	10.9	2.3
A17	〃	〃	0.0500	0.845	115	5.58	0.145	19.3	1.4
A18	〃	〃	0.0506	0.770	92.5	4.67	0.120	13.5	—
A19	〃	〃	0.0719	0.774	93.5	6.52	0.120	20.1	—
A20	0.88	〃	0.0109	1.129	198.7	2.17	0.260	4.6	4.1
A21	〃	〃	0.0156	1.126	197	3.07	0.220	7.2	5.05
A22	〃	〃	0.0245	0.862	116	2.84	0.215	5.4	3.1
A23	〃	〃	0.0422	0.768	92	3.89	0.190	6.1	2.7
A24	〃	〃	0.0519	0.722	81	4.21	0.180	6.1	2.1
A25	〃	〃	0.0663	0.774	94	6.23	0.160	10.9	2.65

Appendix 2. Experimental Data (2)

No.	δ_1	T_1 sec	L_1 cm	H_1 cm	I_{u1}	R_1 cm	δ_2	T_2 sec	L_2 cm	H_2 cm	I_{u2}	R_2 cm	h_c cm	M cm
B 1	0.0679	0.671	79	5.07	0.118	—	0.0104	1.474	340	3.52	0.201	6.96	11.1	19.0
B 2	0.0662	0.723	81	5.36	0.217	—	0.0167	1.188	221	3.69	0.176	—	19.7	7.9
B 3	0.0673	0.732	83	5.59	0.128	—	0.0230	1.128	199	4.58	0 198	—	21.5	4.2
B 4	0.0649	0.720	81	5.25	—	1.5	0.0298	0.934	136	4.06	0.185	2.86	19.9	1.8
B 5	0.0662	0.725	81	5.36	0.153	—	0.0456	0.841	110	5.02	0.173	2.58	17.3	2.6
B 6	0.0100	1.468	337	3.37	0.193	6.84	0.0167	1.280	255.5	4.26	0.200	5.93	21.2	5.5
B 7	0.0155	1.482	344	3.96	0.215	6.99	0.0201	1.187	220	4.42	0.200	4.11	20.0	8.5
B 8	0.0107	1.477	341	3.65	0.198	6.39	0.0295	0.929	134	3.95	0.190	2.73	16.5	16.2
B 9	0.0104	1.465	335	3.48	0.205	—	0.0429	0.840	110	4.72	0.180	—	17.3	19.9
B10	0.0102	1.442	324	3.29	—	6.58	0.0666	0.694	75	5.00	0.170	2.27	18.3	21.0
B11	0.0206	1.180	183	3.76	0.193	4.01	0.0101	1.541	373	3.75	0.206	7·41	22.0	3.1
B12	0.0148	1.269	251	3.71	—	5.73	0.0304	0.945	139	4.23	0.187	2.93	17.0	9.6
B13	0.0172	1.266	250	4.31	—	4.64	0.0465	0.890	124	5.77	0.198	2.69	22.6	20.1
B14	0.0166	1.280	255	4.16	0.196	6.08	0.0647	0.725	82	5.23	0.160	1.89	18.5	19.2
B15	0.0230	1.116	194	4.47	—	3.50	0.0086	1.546	375	3.23	0.206	6.25	18.0	9.0
B16	0.0223	1.047	170.5	3.80	0.200	3.61	0.0104	1.528	366	3.80	0.203	7.61	18.9	5.0
B17	0.0218	1.045	170	3.70	—	3.06	0.0134	1.266	251	3.35	0.219	6.26	15.5	5.0
B18	0.0226	1.045	170	3.84	0.195	—	0.0308	0.949	140	4.31	0.180	2.56	16.9	4.7
B19	0.0183	1.109	192	3.52	—	3.06	0.0530	0.762	91	4.82	0.160	2.0	17.0	12.0
B20	0.0215	1.16	209.5	4.51	0.192	3.50	0.0598	0.735	85	5.08	0.167	1.07	16.4	8.8
B21	0.0304	1.033	166	5.05	—	2.84	0.0090	1.553	380	3.42	0.205	6.93	16.8	13.5
B22	0.0318	0.997	153	4.90	—	3.24	0.0228	1.160	210	4.80	0.195	4.01	22.0	1.8
B23	0.0318	0.998	155	4.94	0.175	3.09	0.0450	0.866	117	5.26	0.172	2.59	—	9.5
B24	0.0324	1.01	159	5.15	0.196	3.97	0.0477	0.853	113.5	5.40	0.187	2.56	17.8	6.1
B25	0.0319	0.694	74.5	5.04	0.190	2.83	0.0698	0.681	72.5	5.06	0.157	1.83	—	6.7